

2, 9, 16, 17
112/113

Preface

The extensive use of reinforced concrete for a variety of structural members has necessitated a proper understanding of the design in structural concrete members by the structural engineers. The wide spread use of reinforced concrete is the natural outcome of the rapid development in the theory and design procedures with the introduction of the philosophy of limit state design.

The book presents the design of a variety of reinforced concrete structures like Continuous beams, Portal frames, Silos, Bunkers, Chimneys, Shells Overhead Water tanks, Vierendeel girders, Trusses, Deep beams, Box culverts, Folded plates, Hyperbolic cooling towers, Curved girders, Poles, Pipes and Bridge deck systems. The design of these advanced reinforced concrete structures conform to the revised Indian Standard Code IS : 456-1978 and S.I. units have been adopted for all the design examples. The topics covered are intended to meet the requirements of graduate and post graduate curricula of most of the engineering institutions in India. The book is primarily design oriented with more emphasis on types of design with minimum extent of theory, presented wherever required for application in design. The various design steps are identified and provided in a logical sequence.

The book is extensively illustrated with working drawings showing the reinforcement details. The examples for practice provided at the end of each chapter is intended to help the students preparing for university examinations.

The references provided at the end of the book have been extensively used in the preparation of the text and are gratefully acknowledged. The author is grateful to his wife, Pramila and daughters Sarvamangala and Amrutha for extending their fullest co-operation in the preparation of the type script. Finally the author welcomes constructive criticisms and suggestions which will immensely help in updating the contents of the book.

Bangalore
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List of Symbols

A	Cross Sectional area
a	Lever arm
b	Breadth of beam
D	Overall depth of beam
DL	Dead Load
D_f	Thickness of flange
d	Effective depth of beam or slab
d'	Depth of compression reinforcement from the highly compressed face
E_c	Modulus of elasticity of concrete
E_s	Modulus of elasticity of steel
e	Eccentricity
f_{ck}	Characteristic Compressive Strength of Concrete
f_d	Design Strength
f_y	Characteristic Strength of Steel
I	Moment of Inertia of section
K	Stiffness of a Member
k	Constant or Coefficient of Factor
LL	Live Load
L	Span Length
M	Bending Moment
m	Modular ratio
n	Neutral axis depth
P	Axial load on a Compression Member
r	Radius
s	Spacing of Stirrups
T	Torsional Moment
V	Shear force
w	Distributed load per unit area
W	Total load; Concentrated load

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x_u	Neutral axis depth
Z	Modulus of the section
δ	Displacement
γ_f	Partial safety factor for load
γ_m	Partial safety factor for material
σ_{cb}	Permissible Stress in Concrete in bending Compression
σ_{cc}	Permissible Stress in Concrete in direction Compression
σ_{sc}	Permissible stress in steel in Compression
σ_{st}	Permissible stress in steel in Tension
σ_{sv}	Permissible tensile stress in shear reinforcements
τ_c	Permissible shear stress in Concrete
τ_v	Nominal shear stress in Concrete
μ	Coefficient of friction between filling on filling
μ'	Coefficient of friction between wall and Material
ϕ	Diameter of plain mild steel bar
Φ	Diameter of deformed Tor steel bar
α	Angles or Ratio
β	
θ	

**ADVANCED
REINFORCED
CONCRETE
DESIGN**

1

Continuous-Beams

1.1 INTRODUCTION

In case of multistoreyed reinforced concrete framed buildings, the floor slabs are generally cast integral with secondary and main continuous beams supported on columns at regular intervals. The continuous beams framing into main beams and columns are designed for moments and shears developed due to the dead and live loads. It is important to note that the maximum span moments develop, when alternate spans are loaded with live load and maximum negative moments at supports develop when adjacent spans are loaded.

1.2 EFFECTIVE SPAN

According to the Indian Standard Code IS: 456-1978, for a continuous beam having a support width less than $1/12$ clear span, the effective span shall be as per freely supported beams i.e. clear span plus the effective depth of beam or centre to centre of supports whichever is less.

If the supports are wider than $1/12$ the clear span or 600 mm whichever is less, the effective span shall be as given below:

(a) For end span with one end fixed and the other continuous or for intermediate spans, the effective span shall be the clear span between the supports and

(b) For end span with one free and the other continuous, the effective span shall be equal to the clear span plus half the effective depth of beam or the clear span plus half the width of the discontinuous support, whichever is less.

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(c) In the case of spans with roller and rocker bearings, the effective span shall always be the distance between the centres of bearings.

(d) In the case of continuous monolithic frames, the effective span of continuous beams are taken as the centre line distance between the members.

1.3 SPAN/DEPTH RATIO

The span to effective depth ratio has an important influence on the deflection characteristics of the beams. The vertical deflection limits of span/350 or 20 mm, specified in IS: 456-1978, may generally be assumed to be satisfied if the span/effective depth ratio of continuous members are not greater than 26. For spans greater than 10 m, this value is multiplied by the ratio of 10/span in metres.

Generally continuous beams carry very heavy dead and live loads and consequently the span/effective depth ratio recommended in practical designs are normally between 10 to 15. The use of the upper limit of span/effective depth ratio of 26, results in shallow depths requiring high percentages of tensile and compression steel tending towards over-reinforced sections.

The computations of the relative stiffness of members may be based on the moment of inertia calculated on the basis of gross, transformed or cracked sections, with consistency for all the members of the structure throughout the analysis.

1.4 BENDING MOMENTS AND SHEAR FORCES

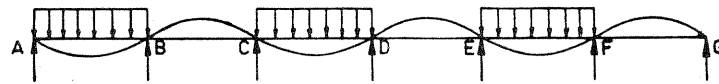
The following arrangement of superimposed loading is generally considered for computation of maximum positive and negative moments at the cross-section of a continuous member.

(a) Design dead load on all spans with full design live load on alternate spans.

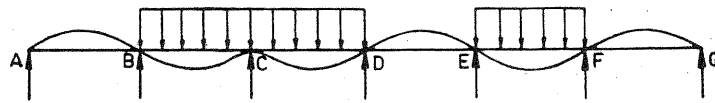
(b) Design dead load on all spans with full design live load on adjacent spans.

The position of incidental or live loads for maximum span and supports moments are shown in Fig. 1.1 The following Tables 1.1 and 1.2 containing the moment and shear coefficients are recommended in IS: 456-1978 for slabs and beams continuous over three or more approximately equal spans. However redistribution is not permitted when using these coefficients.

Appendix-1 shows the bending moment and shear force coefficients for continuous beams of two to five equal spans with uniformly distributed and central point loads. These bending moment coefficients are useful in



a) Maximum Sagging Moment in Span CD
(Alternate Spans Loaded)



b) Maximum Hogging Moment at Support C
(Adjacent Spans Loaded)

Fig. 1.1 Position of live loads for maximum moments in continuous beams.

designing beams allowing for redistribution of moments. According to the IS: 456-78 code, it is permitted to allow 15% redistribution in the working stress method and 30% redistribution in the limit state method of design.

TABLE 1.1 Bending Moment Coefficients

Type of Load	Span Moments		Support Moments	
	Near Middle of End Span	At Middle of Interior Span	At Support Next to the End Support	At Other Interior Supports
Dead load and Imposed Load (fixed)	$+1/12$	$+1/24$	$-1/10$	$-1/12$
Imposed Load (not fixed)	$+1/10$	$+1/12$	$-1/9$	$-1/9$

Note: For obtaining the bending moment, the coefficient shall be multiplied by the total design load and effective span.

TABLE 1.2 Shear Force Coefficients

Type of Load	At End Support	At Support Next to the End Support		At All Other Interior Supports
		Outer side	Inner side	
Dead Load and Imposed Load (fixed)	0.4	0.6	0.55	0.5
Imposed Load (not fixed)	0.45	0.6	0.6	0.7

Note: For obtaining the shear force, the coefficient shall be multiplied by the total design load.

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1.5 DESIGN EXAMPLE

A continuous beam with simple supports has two spans each of 8 m. The characteristic dead load is 10 kN/m and the characteristic live load is 15 kN/m. Design the critical sections of the beam and sketch the details of reinforcements using working stress method. Adopt M-15 concrete and ribbed tor steel.

1. Data

Effective span = $L = 8$ m
Dead load = 15 kN/m
Live load = 25 kN/m
Concrete M-15 grade
Steel:—High strength deformed tor steel

2. Allowable Stresses

$$\begin{array}{ll}\sigma_{cb} = 5 \text{ N/mm}^2 & m = 19 \\ \sigma_{st} = 230 \text{ N/mm}^2 & Q = 0.659 \\ f_y = 415 \text{ N/mm}^2 & j = 0.903\end{array}$$

3. Depth of Beam

As the continuous beam carries heavy loads, a span/effective depth ratio of 10 is provided.

$$\text{Effective depth} = \left(\frac{\text{span}}{10} \right) = \left(\frac{8000}{10} \right) = 800 \text{ mm}$$

Adopt overall depth, $D = 900$ mm and width $b = 300$ mm

Effective depth, $d = 850$ mm

Cover to compression and tension steel = 50 mm.

4. Effective Span

Effective span = $L = 8$ m.

5. Loads

Self weight of beam = $(0.3 \times 0.9 \times 24) = 6.48$ kN/m

Dead load on beam = 10.00 kN/m

Total dead load on beam = $g = \dots = 16.48$ kN/m

Live load on beam = $q = \dots = 15.00$ kN/m

6. Bending Moments and Shear Forces

$$\begin{aligned}\text{Negative B.M at interior support} &= \left(\frac{gL^2}{10} + \frac{qL^2}{9} \right) \\ &= \left(\frac{16.48 \times 8^2}{10} + \frac{15 \times 8^2}{9} \right) = 212 \text{ kN m}\end{aligned}$$

$$\begin{aligned}\text{Positive B.M at centre of span} &= \left(\frac{gL^2}{12} + \frac{qL^2}{10} \right) \\ &= \left(\frac{16.48 \times 8^2}{12} + \frac{15 \times 8^2}{10} \right) = 184 \text{ kN m}\end{aligned}$$

Shear Force at the centre support

$$\begin{aligned} V &= 0.6L(g+q) \\ &= 0.6 \times 8(16.48 + 15.00) \\ &= 151 \text{ kN.} \end{aligned}$$

7. Resisting Moment

Resisting Moment of balanced singly reinforced section

$$\begin{aligned} M_1 &= 0.659b \cdot d^2 \\ &= 0.659 \times 300 \times 850^2 \\ &= 142.8 \times 10^6 \text{ N mm} \\ &= 142.8 \text{ kN m.} \end{aligned}$$

$$\begin{aligned} \text{Balance Moment} = M_2 &= (M - M_1) \\ &= (212 - 142.8) \\ &= 69.2 \text{ kN m.} \end{aligned}$$

8. Tensile Steel

Tensile steel required for balanced section is obtained as

$$A_{s1} = \left(\frac{M_1}{\sigma_{st} \cdot j \cdot d} \right) = \left(\frac{142.8 \times 10^6}{230 \times 0.903 \times 850} \right) = 809 \text{ mm}^2$$

Additional tensile steel for balance Moment M_2 is

$$A_{s2} = \left(\frac{M_2}{\sigma_{st} \cdot j \cdot (d - d_c)} \right) = \left(\frac{69.2 \times 10^6}{230 \times 0.903 \times 800} \right) = 417 \text{ mm}^2$$

Total Tensile Steel = $A_s = (809 + 417) = 1226 \text{ mm}^2$

Use 4 bars of 200 mm diameter ($A_s = 1256 \text{ mm}^2$)

9. Compression Steel

Assuming a cover on compression side $d_c = 50 \text{ mm}$

$$\begin{aligned} A_c &= \left[\frac{m \cdot A_{s2} \cdot (d - n_c)}{(1.5m - 1)(n_c - d_c)} \right] \quad \text{and} \quad n_c = 0.4d \\ &= \left[\frac{19 \times 417(850 - 0.4 \times 850)}{(1.5 \times 19 - 1)(0.4 \times 850 - 50)} \right] = 507 \text{ mm}^2 \end{aligned}$$

Use 2 bars of 20 mm diameter ($A_c = 628 \text{ mm}^2$)

10. Shear Stresses

$$\text{Nominal shear stress, } \tau_v = \left(\frac{V}{bd} \right) = \left(\frac{151 \times 10^3}{300 \times 850} \right) = 0.59 \text{ N/mm}^2$$

$$\left(\frac{100 A_s}{bd} \right) = \left(\frac{100 \times 1256}{300 \times 850} \right) = 0.49$$

From Table 17 of IS: 456, $\tau_c = 0.29 \text{ N/mm}^2$

Since $\tau_v > \tau_c$, shear reinforcements are required. If two bars in the tension side are bent up near supports,

Shear taken by bent up bars is

$$\begin{aligned} V_{sb} &= \sigma_{sv} \cdot A_{sv} \cdot \sin \alpha \\ &= \left(\frac{230 \times 2 \times 314 \times 0.707}{1000} \right) = 102 \text{ kN.} \end{aligned}$$

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$$\begin{aligned}\text{Shear taken by concrete} &= V_c = \tau_c \cdot b \cdot d \\ &= \left(\frac{0.29 \times 300 \times 850}{1000} \right) = 74 \text{ kN}\end{aligned}$$

Total shear resisted = $(102 + 74) = 176 \text{ kN} > 151 \text{ kN}$.

Provide nominal stirrups, using 6 mm diameter 2 legged links,

$$\text{spacing } s_v = \left(\frac{A_{sv} f_y}{0.4b} \right) = \left(\frac{56 \times 415}{0.4 \times 300} \right) = 193 \text{ mm}$$

Use 6 mm diameter, 2 legged stirrups at 180 mm centres throughout the beam as shown in Fig. 1.2.

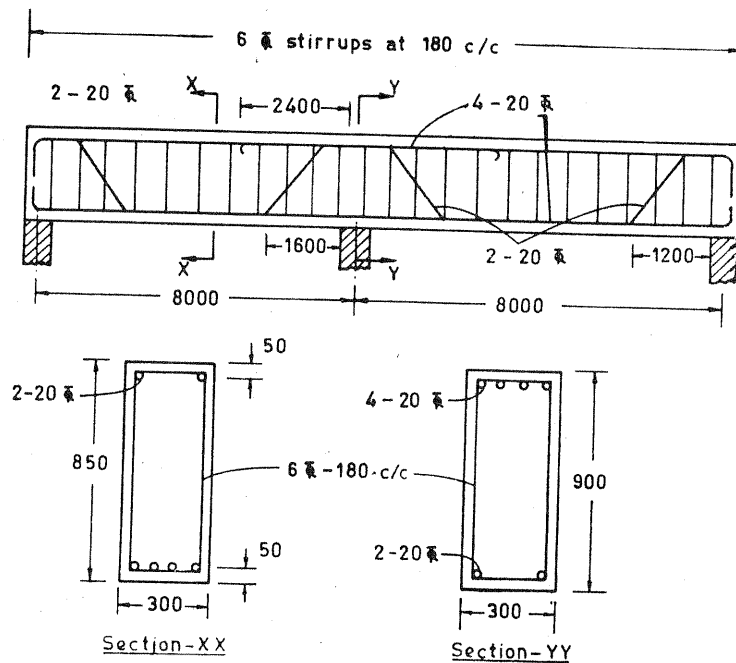


Fig. 1.2 Reinforcements in Continuous beam

1.6 DESIGN EXAMPLE

Design a continuous beam of two spans supported on stone masonry walls using the limit state method and allowing for 15% redistribution of moments. The following data may be used.

1. Data

- Clear span between supports = 6 m
- Width of masonry supports = 300 mm
- Thickness of reinforced concrete slab = 150 mm.
- Spacings of continuous beams = 3 m c/c.
- Self weight of floor finish = 0.4 kN/m^2
- Live load on office floor = 4 kN/m^2

Characteristic cube strength of concrete = $f_{ck} = 15 \text{ N/mm}^2$

Characteristic strength of steel = $f_y = 415 \text{ N/mm}^2$

2. *Characteristic Strength*

$$f_y = 415 \text{ N/mm}^2$$

$$f_{ck} = 15 \text{ N/mm}^2$$

3. *Depth of Beam*

As the continuous beam carries heavy loads, a span/effective depth ratio of 12 is adopted.

$$\text{Effective depth} = \left(\frac{\text{span}}{12} \right) = \left(\frac{6000}{12} \right) = 500 \text{ mm.}$$

$$\text{Overall depth} = 600 \text{ mm}$$

$$\text{Cover} = 50 \text{ mm}$$

$$\text{Effective depth} = 550 \text{ mm}$$

$$\text{Width of beam} = 300 \text{ mm}$$

4. *Effective Span*

$$\text{Width of Support} = 300 \text{ mm.}$$

$$1/12 \text{ clear span} = \left(\frac{1}{12} \times 6000 \right) = 500 \text{ mm}$$

support width is less than 1/12 clear span therefore, effective span is the least of

$$\begin{aligned} 1. \text{ Centre to centre of supports} \\ &= (6000 + 300) = 6300 \text{ mm} = 6.3 \text{ m.} \end{aligned}$$

$$\begin{aligned} 2. \text{ Clear span} + \text{effective depth} \\ &= (6000 + 550) = 6550 \text{ mm} = 6.55 \text{ m.} \\ &\text{Effective span, } L = 6.3 \text{ m.} \end{aligned}$$

5. *Loads*

$$\text{Self weight of R.C slab} = (0.15 \times 24) = 3.6 \text{ kN/m}^2$$

$$\begin{array}{lcl} \text{Floor finish} & \dots\dots\dots & = 0.4 \\ & & \underline{4.0 \text{ kN/m}^2} \end{array}$$

$$\text{Dead load on beam} = (4.0 \times 3) = 12 \text{ kN/m}$$

$$\text{Self weight of beam} = (0.3 \times 0.6 \times 24) = 4.32 \text{ kN/m}$$

$$\text{Total dead load on beam (g)} = 16.32 \text{ kN/m}$$

$$\text{Live load on beam (q)} = (4 \times 3) = 12 \text{ kN/m}$$

The factored ultimate loads on beam are

$$g_u = (1.5 \times 16.32) = 24.48 \text{ kN/m}$$

$$q_u = (1.5 \times 12) = 18.00 \text{ kN/m.}$$

6. *Bending Moments and Shear Forces*

Referring to Appendix 1, the maximum negative and positive moments at support and mid span sections are obtained as:

$$\text{Negative Maximum Moment} = 0.125(g_u + q_u)L^2$$

$$= 0.125(24.48 + 18.0)6.3^2$$

$$= 211 \text{ kN/m}$$

Solving $A_{st} = 1109 \text{ mm}^2$

Provide 4 bars of 20Φ ($A_s = 1256 \text{ mm}^2$)

10. Shear Stresses

Ultimate shear force $V_u = 166 \text{ kN}$.

$$\tau_v = \left(\frac{V_u}{bd} \right) = \left(\frac{166 \times 10^3}{300 \times 550} \right) = 1.00 \text{ N/mm}^2$$

$$\left(\frac{100A_s}{bd} \right) = \left(\frac{100 \times 1256}{300 \times 550} \right) = 0.76$$

From Table 13 of IS : 456, $\tau_c = 0.54 \text{ N/mm}^2$

If two bars are bent up at quarter span points, shear force taken by bent bars is

$$\begin{aligned} V_{us} &= 0.87 f_y \cdot A_{sv} \cdot \sin \alpha \\ &= \left(\frac{0.87 \times 415 \times 2 \times 314 \times 0.707}{1000} \right) = 160 \text{ kN}. \end{aligned}$$

Shear taken by concrete $= \tau_c \cdot b \cdot d$

$$= \left(\frac{0.54 \times 300 \times 550}{1000} \right) = 89 \text{ kN}.$$

Total shear resisted $= (160 + 89) = 249 \text{ kN} > 166 \text{ kN}$

Provide Nominal Shear reinforcements. Using 6 mm two legged stirrups, the spacing is given by

$$s_v = \left(\frac{A_{sv} f_y}{0.46} \right) = \left(\frac{56 \times 415}{0.4 \times 300} \right) = 193 \text{ mm}$$

Adopt 6 mm diameter two legged stirrups at 180 mm centres, throughout the length of the beam.

1.7 EXAMPLES FOR PRACTICE

1. Design a continuous beam of 3 spans supported on stone masonry piers 300 mm by 300 mm. Clear span between the supports = 8 m. Thickness of reinforced concrete slab = 120 mm
 spacings of continuous beams = 2.75 m c/c
 self weight of floor finish = 0.6 kN/m^2
 live load on office floor = 4 kN/m^2
 concrete of M-15 grade and ribbed tor steel are available for use. Design the reinforcements for the critical sections and sketch the details using working stress method.
2. A continuous beam with simple supports has two spans each of 6 m, from centre to centre of supports. The characteristic dead load is 15 kN/m and characteristic live load is 20 kN/m . Design the critical sections of the beam and sketch the details of reinforcements using the limit state method. Characteristic strength of concrete $f_{ck} = 15 \text{ N/mm}^2$, characteristic strength of steel = 415 N/mm^2 . Sketch the details in the continuous beam.

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Positive Maximum Moment

$$\begin{aligned} &= (0.071g_u + 0.096q_u)L^2 \\ &= (0.071 \times 24.48 + 0.096 \times 18.0)6.3^2 \\ &= 138 \text{ kN/m.} \end{aligned}$$

Maximum Support Shear Force

$$\begin{aligned} &= V_u = 0.62(g_u + q_u)L \\ &= 0.62(24.48 + 18.0)6.3 \\ &= 166 \text{ kN.} \end{aligned}$$

7. Redistribution of Moments

The design Maximum moment is obtained by decreasing the negative moment at support 15 percent and increasing the positive span moment by the same magnitude.

$$15 \text{ percent of } M_{\max} = (0.15 \times 211) = 32 \text{ kN/m.}$$

$$\begin{aligned} \text{Design Moment at support section} &= (211 - 32) \\ &= 179 \text{ kN/m} \end{aligned}$$

$$\text{Design Moment at centre of span section} = (138 + 32) = 170 \text{ kN/m}$$

Since the design ultimate moment, positive and negative are almost equal in magnitude, the sections are designed for an ultimate moment $M_u = 179 \text{ kN/m.}$

8. Moment of Resistance

Moment of Resistance of balanced section is

$$M_{ub} = 0.36 f_{ck} \cdot b \cdot x_u (d - 0.42 x_u)$$

$$\text{for } f_y = 415 \text{ N/mm}^2$$

$$x_u (\text{maximum}) = 0.48d$$

$$\begin{aligned} M_{ub} &= 3.36 \times f_{ck} \times b \times 0.48 d (d - 0.42 \times 0.48 d) \\ &= 0.138 f_{ck} \cdot b \cdot d^2 \\ &= 0.138 \times 15 \times 300 \times 550^2 \\ &= 187.8 \times 10^6 \text{ N.mm} \\ &= 187.8 \text{ kN.m} > 179 \text{ kN.m} \end{aligned}$$

Since the Moment of Resistance of the balanced section is greater than the design ultimate moment, the section is under reinforced i.e.

$$\frac{x_u (\max)}{d} < 0.48$$

9. Main Reinforcements in Section

If A_{st} = Area of tensile steel in the section

$$M_u = 0.87 f_y A_{st} \cdot d \left[1 - \frac{A_{st} \cdot f_y}{b \cdot d \cdot f_{ck}} \right]$$

$$179 \times 10^6 = 0.87 \times 415 \times A_{st} \times 550 \left[1 - \frac{A_{st} \times 415}{300 \times 550 \times 5} \right]$$

$$A_{st}^2 - 5966.8 A_{st} + 5.37 \times 10^6 = 0$$

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3. A two span simply supported continuous beam is to be designed to suit the following data :
Distance between centre to centre of supports = 6 m.
The beam supports dead and live loads of 100 kN and 150 kN respectively at the centre of spans. Using M-15 grade concrete and ribbed tor steel, design the continuous beam using working stress method.
4. A continuous beam of two spans is fixed at the end supports and continuous over the central support.
Distance between the centre of supports = 8 m
The beam supports a reinforced concrete slab 150 mm thick with Live Load on floor = 2 kN/m².
spacings of continuous beams = 4 m.
characteristic strength of concrete = 15 N/mm²
characteristic strength of steel = 415 N/mm²
Design the continuous beam using limit state method and allowing for 15 percent redistribution of moments.
5. Design the interior span of a continuous tee beam of effective span 6 m. The beams are placed at 4 m centres and carry a slab of thickness 100 mm. Dead load on slab inclusive of self weight may be taken as 3 kN/m² and live load as 6 kN/m². Use M-20 grade concrete and grade-I mild steel.
6. Design the interior span of a rectangular beam supported on 500 mm square column and beam continuous over 8 m span. The beams support a slab 120 mm thick and are placed 4 m centres and are cast monolithic with the slab. The loading on the slab is due to self weight, floor finish and plastering and live load of magnitude 5 kN/m². Adopt M-20 grade concrete and ribbed tor steel.
7. A Reinforced concrete beam is continuous over two equal spans of length 6 m each. It is simply supported at its ends. It carries a uniformly distributed load of intensity 15 kN/m inclusive of its own weight over its entire length of 12 m.
Design for flexure and shear a suitable rectangular section for the beam. Adopt M-15 grade concrete and Grade-I mild steel.
(Bangalore University—January 1978)
8. A reinforced concrete continuous beam of three equal spans $AB = BC = CD = 7$ m, is simply supported at the ends A and D .
If the beam supports a uniformly distributed live load of 10 kN/m in addition to its own weight, design the beam for flexure and shear and sketch the details of reinforcements in the beam.
(Bangalore University—January 1979)
9. A reinforced concrete beam, continuous over two spans $AB = BC = 5$ m and supported over masonry walls at ends A and C is required to support a uniformly distributed live load of 20 kN/m. Design a suitable rectangular section for the beam and sketch the details of reinforcements. Adopt M-15 grade concrete.
(Bangalore University—July 1979)

2

Bunkers and Silos

2.1 INTRODUCTION

Bunkers and silos may be classified as storage structures generally used for storing coal, cement, food grains and other granular materials. Reinforced concrete bunkers and silos have almost replaced the steel storage structures because of their ease of maintainance and superior architectural qualities. The present day cement factories invariably opt for single or a battery of silos to store the manufactured cement. The development of slip form method of casting of tall cylindrical reinforced concrete structures has resulted in rapid construction of silos.

2.2 DIFFERENCE BETWEEN BUNKER AND SILO

(a) *Bunkers*

Bunkers are shallow structures in which the plane of rupture of the material stored meets the top horizontal surface of the material before meeting the opposite sides of the structure, as shown in Fig. 2.1.

The angle of rupture is at $\left(\frac{90+\phi}{2}\right)$ from the horizontal, where ϕ is the angle of repose of the material.

The side walls resist the lateral pressure and the total load of the material is supported by the floor of the bunker. The intensity of lateral pressure on the sides is determined by Rankine's theory.

(b) *Silos*

In a silo, the vertical walls are considerably taller than the lateral dimensions resulting in a tall structure. Consequently the plane of rupture of the

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material stored meets the opposite sides of the structure before meeting the top horizontal surface of the material as shown in Fig. 2.2.

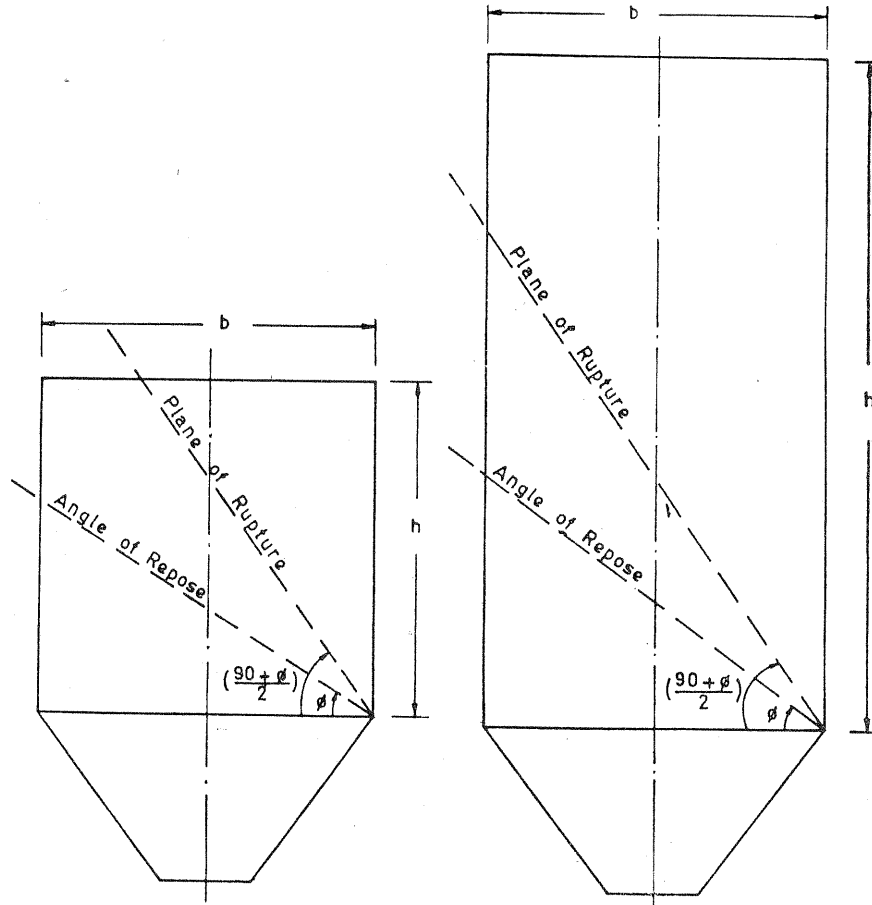


Fig. 2.1 Bunker or shallow bin

Fig. 2.2 Silo or deep bin

Due to the high ratio of height to the lateral dimensions, a significant portion of the load is resisted by friction between the material and the wall. Only a fraction of the total weight of the material acts on the floor of the structure.

If B = Breadth
 H = Height of the structure
 ϕ = Angle of repose

For a structure to be classified as a silo,

$$H > B \tan \left(\frac{90 + \phi}{2} \right)$$

if $\phi = 30^\circ$, $H > 1.732 B$

2.3 DESIGN OF SQUARE OR RECTANGULAR BUNKERS

The structural elements of a bunker are as shown in Fig. 2.3.

The various parts are

1. The vertical side walls
2. Hopper bottom
3. Edge beams
4. Columns.

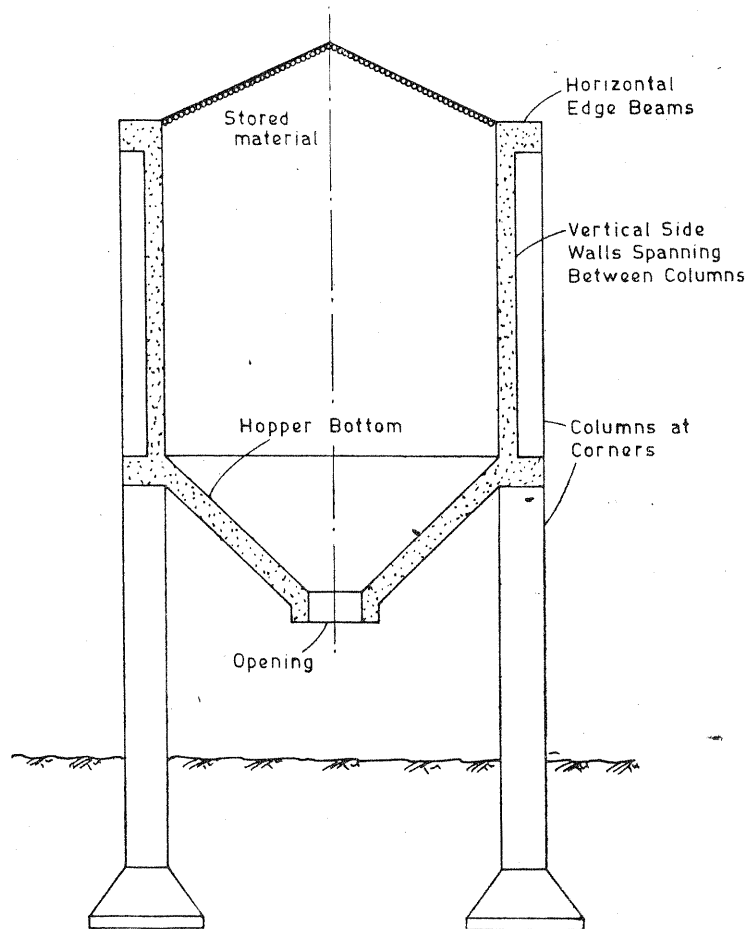


Fig. 2.3 Structural Elements of a bunker

(a) Design of Vertical Side Walls

Figure 2.4 shows the cross sectional plan and elevation of a rectangular bunker of length L breadth B , and height h . If p = Intensity of lateral pressure at a depth h , then according to Rankins theory

$$p_{\alpha} = w h \cos \alpha \left[\frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}} \right]$$

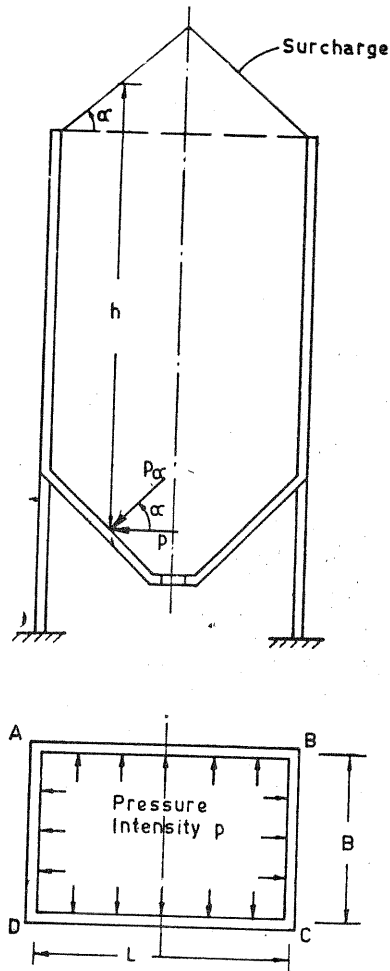


Fig. 2.4 Pressure intensity on walls of a bunker

where α = Angle of surcharge
 ϕ = Angle of repose
 w = density of material stored

The pressure acts in a direction parallel to the surface of the retained material.

$$\text{Horizontal Component} = p = p_{\alpha} \cdot \cos \alpha$$

if $\alpha = \phi$, then $p_{\alpha} = wh \cdot \cos \phi$ and $p = wh \cdot \cos^2 \phi$

The design moments at the supports and centre of long and short walls are given as

$$M_A = M_B = M_C = M_D = -\frac{p}{12} [L^2 + B^2 - BL]$$

Positive Moment at centre of AB or CD

$$= \left[\frac{pL^2}{8} - \frac{p}{12} (L^2 + B^2 - BL) \right]$$

Positive moment at centre of BC or AD

$$= \left[\frac{pB^2}{8} - \frac{p}{12} (L^2 + B^2 - BL) \right]$$

Direct tension in longwalls = $\left(\frac{pB}{2} \right)$

Direct tension in shortwalls = $\left(\frac{pL}{2} \right)$

The thickness of side wall is designed for maximum bending moment. The reinforcement in the walls are designed for bending moment and direct tension.

If M = Bending moment

T = Tension

x = distance between the centre of section and reinforcement position.

σ_{st} = permissible tensile stress in steel

d = effective depth of side walls

Q = Design constant

j = Lever arm factor

b = width of section

A_{st} = Area of tensile steel

then the effective depth is given by

$$d = \sqrt{\frac{(M - T \cdot x)}{Q \cdot b}}$$

and

$$A_{st} = \left(\frac{M - T \cdot x}{\sigma_{st} \cdot j \cdot d} \right) + \left(\frac{T}{\sigma_{st}} \right)$$

These reinforcements are arranged in the horizontal direction. Distribution steel is provided in the vertical direction. At top and bottom of the vertical walls, edge beams of 300 mm by 300 mm section are provided to allow for attachments of conveyer supports.

(b) *Design of Hopper Bottom*

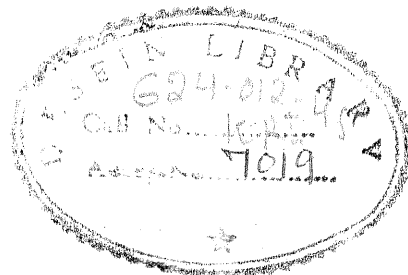
The hopper bottom which is a sloping slab is designed for direct tension developed due to the weight of material and the self weight of the sloping slab.

Referring to Fig. 2.5 (a and b)

If W_t = Weight of the material, sloping bottom etc.

The direct tension = $W_t \operatorname{cosec} \theta$

where θ = Angle between the horizontal and the sloping slab.



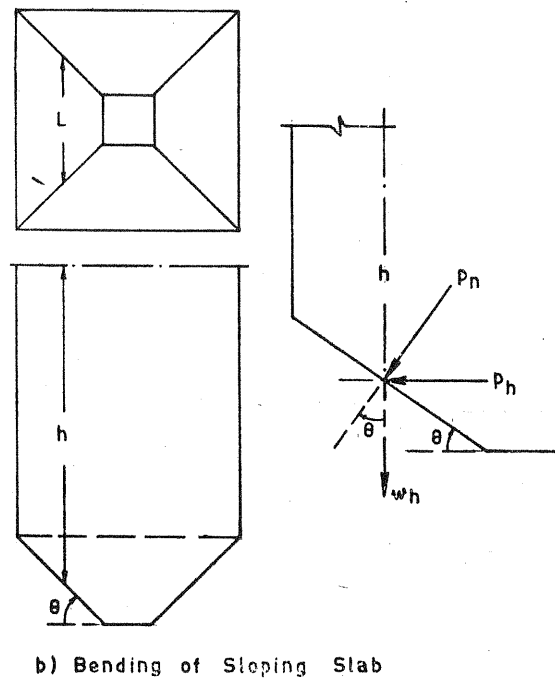
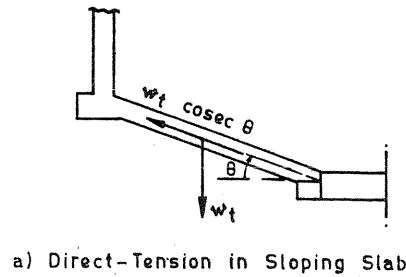


Fig. 2.5 Forces acting on hopper bottom

The sloping slab is considered to span horizontally between the intersections of the adjacent sloping faces. The section of the slab at the centre of the slope is designed.

If W = Density of the material stored
 h = average height at centre of slope
 L = effective span at centre of slope.

Normal pressure intensity at a depth h is given by

$$p_n = wh \cos^2 \theta + p_h \cdot \sin^2 \theta \quad \text{but } p_h = wh \cos^2 \phi$$

$$\therefore p_n = wh [\cos^2 \theta + \cos^2 \phi \cdot \sin^2 \theta]$$

If W_d = self weight of slab per unit length

Normal component = $W_d \cdot \cos \theta$

Total Normal pressure = $P = (p_n + W_d \cdot \cos \theta)$

Maximum Negative Bending Moment at supports = $(PL^2/12)$

Positive bending moment at centre of span = $\left(\frac{PL^2}{24}\right)$

2.4 DESIGN OF CIRCULAR BUNKERS

In the case of bunkers, circular in shape the vertical walls are subjected to hoop tension.

If D = Diameter of the bunker

p_h = horizontal component of pressure at a depth h from the top,

then

$$\text{Hoop tension} = 0.5p_h \cdot D.$$

The reinforcements in the walls are designed to resist the hoop tension. A minimum thickness of 120 mm is recommended from practical considerations for the vertical walls. Distribution reinforcement of 0.15 percent of the gross cross section is provided in the vertical direction.

The hopper bottom is designed for direct tension and hoop tension developed due to the normal pressure on the sloping slab.

The columns of bunkers are designed for compression and bending developed due to the vertical loads such as the stored material and self weight of members and horizontal loads such as wind loads.

2.5 DESIGN EXAMPLE

Design the side walls and hopper bottom of a 3 m by 3 m square bunker to store 30 tonnes of coal. Density of coal = 9 kN/m³.

Angle of repose = 30 degrees

Adopt M-15 grade concrete and ribbed tor steel. Sketch the details of reinforcements in the bunker.

1. Data

Total weight of coal = 300 kN

Density of coal = 9 kN/m³

Size of bunker in plan = 3 m by 3 m

Angle of repose = 30 degrees

Grade of concrete = M-15, $f_{ck} = 15 \text{ N/mm}^2$

Type of steel = Tor steel, $f_y = 415 \text{ N/mm}^2$

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2. Permissible Stresses

$$\sigma_{cb} = 5 \text{ N/mm}^2$$

$$m = 19$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$j = 0.903$$

$$Q = 0.659$$

3. Dimensions of Bunker

$$\text{Volume of bunker} = \left(\frac{300}{9} \right) = 33.33 \text{ m}^3$$

Adopt a bunker of size 3 m by 3 m by 3 m with the depth of the hopper bottom = 1.2 m as shown in Fig. 2.6.

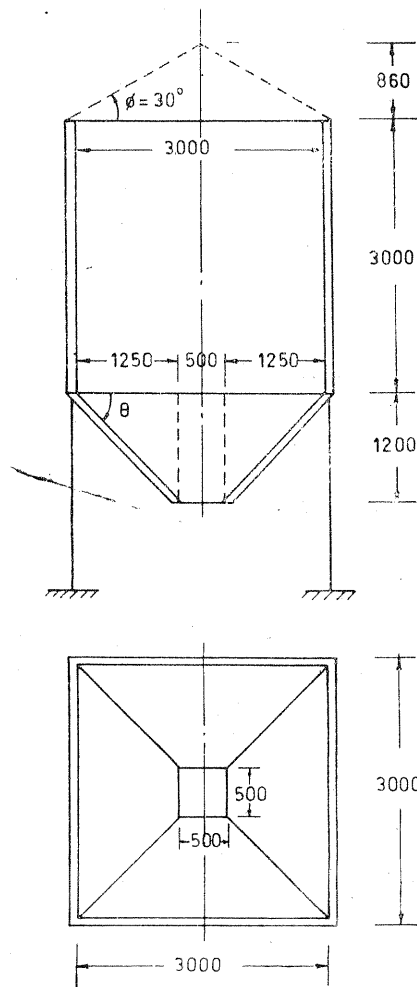


Fig. 2.6 Dimensions of bunker

The volume capacity of bunker is computed given below:

$$\text{Volume of surcharge} = \left(3 \times 3 \times \frac{1}{3} \times 0.86 \right) = 2.58 \text{ m}^3$$

$$\begin{aligned} \text{Volume of cubical portion of bunker} \\ = (3 \times 3 \times 3) \dots = 27.00 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume of hopper bottom} \\ = \frac{1}{3} (3^2 + 0.5^2 + \sqrt{3^2 + 0.5^2}) \dots = 4.29 \text{ m}^3 \\ \text{Total volume} \dots = 33.87 \text{ m}^3 \end{aligned}$$

4. Design of Side Walls

$$\begin{aligned} \text{Horizontal pressure} = p &= wh \cdot \cos^2 \phi \\ &= 9 \times 3 \times 0.866^2 \\ &= 20.25 \text{ kN/m}^2 \end{aligned}$$

Assuming 180 mm thick side walls
Effective span $= (3.00 + 0.18) = 3.18 \text{ m}$

$$\begin{aligned} \text{Maximum B.M at corners} = M &= \left(\frac{pL^2}{12} \right) \\ &= \left(\frac{20.25 \times 3.18^2}{12} \right) = 17.06 \text{ kN} \cdot \text{m} \end{aligned}$$

$$\text{Direct tension in wall} = T = \left(\frac{20.25 \times 3}{2} \right) = 30.375 \text{ kN}$$

Providing a cover of 30 mm, Effective depth $d = 150 \text{ mm}$
Net design B.M $= (M - T \cdot x)$

$$= (17.06 - 30.375 \times 0.06) = 15.24 \text{ kN} \cdot \text{m}.$$

Effective depth required is given by

$$d = \sqrt{\frac{(M - T \cdot x)}{Q \cdot b}} = \sqrt{\frac{15.24 \times 10^6}{0.659 \times 1000}} = 152 \text{ mm}$$

using an effective depth of $d = 150 \text{ mm}$

$$\begin{aligned} A_{st} &= \left[\frac{(M - T \cdot x)}{\sigma_{st} \cdot j \cdot d} \right] + \left[\frac{T}{\sigma_{st}} \right] \\ &= \left(\frac{15.24 \times 10^6}{230 \times 0.903 \times 150} \right) + \left(\frac{30.375 \times 10^3}{230} \right) \\ &= (489 + 132) \\ &= 621 \text{ mm}^2 \end{aligned}$$

Use 12 mm diameter bars at 150 mm centres at corners

($A_{st} = 750 \text{ mm}^2$) in the horizontal direction

Positive B.M at centre

$$= \left(\frac{pL^2}{24} \right) = \left(\frac{20.25 \times 3.18^2}{24} \right) = 8.53 \text{ kN} \cdot \text{m}.$$

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$$(M - T \cdot x) = (8.53 - 30.375 \times 0.06) = 6.71 \text{ kN} \cdot \text{m}.$$

$$\begin{aligned} A_{st} &= \left[\frac{6.71 \times 10^6}{230 \times 0.903 \times 150} \right] + \left[\frac{30.37 \times 10^3}{230} \right] \\ &= 215 + 132 \\ &= 347 \text{ mm}^2 \end{aligned}$$

Use 12 mm diameter bars at 300 mm centres at centre of span ($A_{st} = 375 \text{ mm}^2$).

$$\text{Distribution steel} = (0.0012 \times 1000 \times 180) = 216 \text{ mm}^2$$

Use 6 mm diameter bars at 240 mm centres in the vertical direction on both faces.

5. Design of Hopper Bottom

Weight of coal = 300 kN

Weight of sloping hopper bottom (180 mm thick)

$$= \left(\frac{3 + 0.5}{2} \right) (\sqrt{1.25^2 + 1.2^2}) (4 \times 0.18 \times 24) = 53 \text{ kN}$$

Total load on 4 walls = 353 kN

Load on one wall = $w_t = (0.25 \times 353) = 88.25 \text{ kN}$

$$\tan \theta = \left(\frac{1.20}{1.25} \right) = 1.00 \quad \therefore \theta = 45^\circ$$

$$\text{cosec } \theta = 1.44$$

Direct tension in sloping wall = $w_t \cdot \text{cosec } \theta$

$$= (88.25 \times 1.44) = 127 \text{ kN}$$

$$\text{Tension/metre run} = \left(\frac{127}{3} \right) = 42.33 \text{ kN/m}$$

Steel reinforcement for direct tension

$$= \left(\frac{42.33 \times 10^3}{230} \right) = 184 \text{ mm}^2$$

But Minimum reinforcement = $(0.0012 \times 180 \times 1000) = 216 \text{ mm}^2$

Use 6 mm diameter bars at 240 mm centres in the direction of the sloping faces.

Normal component of coal pressure at centre of sloping slab is given by

$$p_n = wh(\cos^2 \theta + \cos^2 \phi \cdot \sin^2 \theta)$$

where $w = 9 \text{ kN/m}^3$

$$h = [3 + (0.5 \times 1.20) + (0.5 \times 0.86)] = 4.03 \text{ m}$$

$$\theta = 45^\circ$$

$$\phi = 30^\circ$$

$$p_n = 9 \times 4.03 (\cos^2 45 + \cos^2 30 \times \sin^2 45) = 27 \text{ kN/m}^2$$

Normal component due to weight of sloping slab

$$= w_d \cdot \cos \theta$$

$$= (0.180 \times 24 \times \cos 45)$$

$$= 3.00 \text{ kN/m}^2$$

$$\begin{aligned}\text{Total normal pressure} = p &= (p_n + w_d \cdot \cos \theta) \\ &= (27 + 3) \\ &= 30 \text{ kN/m}^2\end{aligned}$$

$$\text{Effective span} = L = \left(\frac{3 + 0.5}{2} \right) + 0.18 = 1.93 \text{ m}$$

$$\begin{aligned}\therefore \text{Maximum Negative B.M} &= \left(\frac{pL^2}{12} \right) \\ &= \left(\frac{30 \times 1.93^2}{12} \right) = 9.31 \text{ kN.m/m}\end{aligned}$$

$$\text{Effective depth available} = (180 - 30) = 150 \text{ mm}$$

$$\therefore A_{st} = \left(\frac{9.31 \times 10^6}{230 \times 0.903 \times 150} \right) = 299 \text{ mm}^2$$

Use 10 mm diameter bars at 260 mm centres at corners.

$$\begin{aligned}\text{Positive maximum B.M at centre} &= \left(\frac{pL^2}{24} \right) \\ &= \left(\frac{30 \times 1.93^2}{24} \right) = 4.66 \text{ kN.m.}\end{aligned}$$

$$A_{st} = \left(\frac{4.66 \times 10^6}{230 \times 0.903 \times 150} \right) = 150 \text{ mm}^2$$

but minimum area of reinforcement

$$= (0.0012 \times 180 \times 1000) = 216 \text{ mm}^2$$

Use 10 mm diameter bars at 260 mm centres at centre of span.

6. Edge Beams

Provide edge beams of 300 mm by 300 mm connecting the corner columns at the top and the junction of vertical walls and sloping slab.

The details of reinforcements in the side walls, hopper bottom and edge beams are shown in Fig. 2.7.

2.6 DESIGN EXAMPLE

Design a circular cylindrical bunker to store 20 tonnes of coal. Density of coal = 9 kN/m³, Angle of repose = 30 degrees. Use limit state method of design and adopt characteristics strength of concrete and steel as 15 and 415 N/mm². Sketch the details of reinforcements in the bunker.

1. Data

Total weight of coal = 200 kN

Density of coal = 9 kN/m³

Angle of repose = 30 degrees

Grade of concrete = M-15

Type of steel = Ribbed tor steel.

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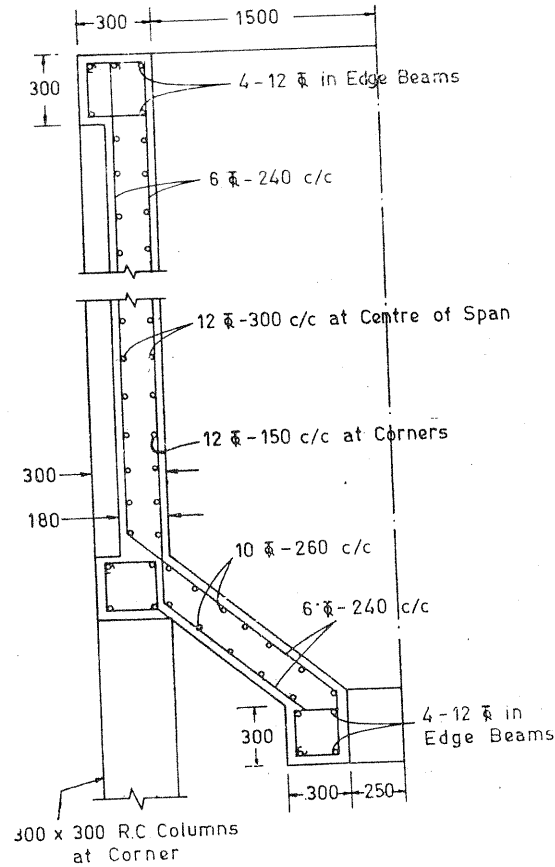


Fig. 2.7 Reinforcement details in bunker

2. Characteristic Strength and Partial Safety Factors

Characteristic strength of concrete = $f_{ck} = 15 \text{ N/mm}^2$

Characteristic strength of steel = $f_y = 415 \text{ N/mm}^2$

Partial safety factor for live and dead loads = 1.5

Partial safety factor for concrete = 1.5

Partial safety factor for steel = 1.15

3. Dimensions of Bunker

$$\text{Volume of bunker} = \left(\frac{200}{9} \right) = 22.2 \text{ m}^3$$

Adopt a bunker of diameter 2.6 m and

height of cylindrical portion = 3 m

Depth of hopper bottom = 1.2 m

The overall dimensions of the bunker is shown in Fig. 2.8.

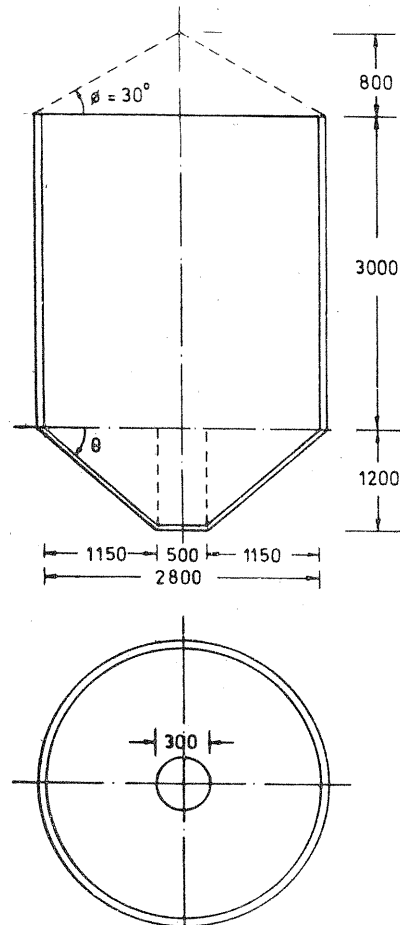


Fig. 2.8 Dimensions of circular bunker

The volume capacity of the bunker is computed as given below:

$$\text{Volume of surcharge} = (1/3 \times \pi \times 1.4^2 \times 1.4 \tan 30) = 1.64 \text{ m}^3$$

$$\text{Volume of cylindrical portion} = \left(\frac{\pi \times 2.8^2}{4} \times 3.00 \right) = 18.46 \text{ m}^3$$

$$\begin{aligned} \text{Volume of frustrum of cone} &= \frac{\pi h}{12} (d_1^2 + d_1 d_2 + d_2^2) \\ &= \frac{\pi \times 1.2}{12} (2.8^2 + 2.8 \times 0.5 + 0.5^2) \\ &= 2.90 \text{ m}^3 \end{aligned}$$

$$\text{Total volume} = 23.00 \text{ m}^3 > 22.22 \text{ m}^3$$

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4. Design of Cylindrical Walls

$$\begin{aligned}\text{Horizontal pressure} &= p = wh \cdot \cos^2 \phi \\ &= 9 \times 3 \times (0.866)^2 \\ &= 20.25 \text{ kN/m}^2\end{aligned}$$

$$\begin{aligned}\text{Hoop tension in the cylindrical wall per metre height} &= 0.5 pD \\ &= (0.5 \times 20.25 \times 2.8) \\ &= 28.35 \text{ kN}\end{aligned}$$

$$\text{Ultimate hoop tension} = (1.5 \times 28.35) = 42.53 \text{ kN}.$$

$$\text{Area of reinforcement } A_{st} = \left(\frac{42.53 \times 10^3}{0.87 \times 415} \right) = 118 \text{ mm}^2$$

Using 150 mm thick walls,

$$\text{Minimum reinforcement} = (0.0012 \times 150 \times 1000) = 180 \text{ mm}^2$$

Use 8 mm diameter hoops at 250 mm centres, provide vertical reinforcement of 8 mm diameter bars at 250 mm centres.

5. Design of Hopper Bottom

Provide a sloping slab 150 mm thick with 30 mm lining.

Total thickness = 180 mm

$$\text{weight of coal} = (23 \times 9) = 207 \text{ kN}$$

weight of sloping bottom

$$= \pi \left[\frac{2.8 + 0.5}{2} + 0.18\sqrt{2} \right] 0.18 \times \sqrt{2} \times 24 = 42 \text{ kN}$$

$$\therefore \text{Total load} = (207 + 42) = 249 \text{ kN}.$$

If T = tension per metre run,

Mean diameter at centre of sloping slab = 1.85 m

$$T = \left(\frac{249}{\pi \times 1.85} \right) \operatorname{cosec} 45 = 60.6 \text{ kN}$$

$$\text{Ultimate tension} = (1.5 \times 60.6) = 90.9 \text{ kN}$$

$$\therefore \text{Steel reinforcement for direct tension} = \left(\frac{90.9 \times 10^3}{0.87 \times 415} \right) = 250 \text{ mm}^2$$

Use 8 mm diameter bars at 200 mm centres in the direction of the sloping slab.

Normal component of coal pressure at centre of sloping slab is given by

$$p_n = wh(\cos^2 \theta + \cos^2 \phi \cdot \sin^2 \theta)$$

where

$$w = 9 \text{ kN/m}^3$$

$$h = 3 + (0.5 \times 1.2) + (0.5 \times 0.80)$$

$$= 4.00 \text{ m}$$

$$\theta = 45^\circ$$

$$\phi = 30^\circ$$

$$p_n = 9 \times 4(\cos^2 45 + \cos^2 30 \times \sin^2 45) = 31.5 \text{ kN/m}^2$$

$$\begin{aligned}\text{Normal component due to weight of the sloping slab} &= w_d \cdot \cos \theta \\ &= (0.18 + 24 \times \cos 45) \\ &= 3 \text{ kN/m}^2\end{aligned}$$

$$\begin{aligned}\text{Total normal pressure } p &= (p_n + w_d \cos \theta) \\ &= (31.5 + 3) = 34.5 \text{ kN/m}^2\end{aligned}$$

$$\begin{aligned}\text{Mean diameter at centre of sloping slab} \\ &= \left(\frac{2.8 + 0.5}{2} \right) + 0.18 \sqrt{2} = 1.90 \text{ m}\end{aligned}$$

$$\text{Hoop tension/metre} = (0.5 \times 34.5 \times 1.90) = 32.8 \text{ kN}$$

$$\text{Ultimate hoop tension} = (1.5 \times 32.8) = 49.2 \text{ kN}$$

$$\text{Area of hoop reinforcement} = \left(\frac{49.2 \times 10^3}{0.87 \times 415} \right) = 137 \text{ mm}^2$$

$$\text{But minimum area of steel} = (0.0012 \times 180 \times 1000) = 216 \text{ mm}^2$$

Use 8 mm diameter hoop reinforcement at 220 mm centres in the sloping slab. The details of reinforcements in the circular bunker is shown in Fig. 2.9.

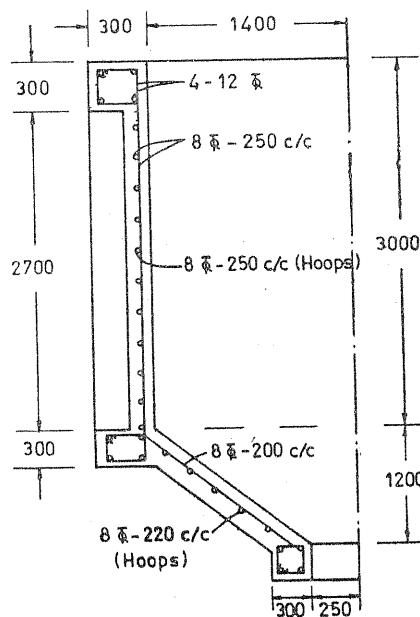


Fig. 2.9 Reinforcement details in circular bunker

6. Edge Beams

At the junction of the cylindrical wall and hopper bottom and at the top of bunker, edge beams of 300 mm by 300 mm are provided to increase the rigidity of the structure.

2.7 DESIGN OF A BATTERY OF BUNKERS

A battery of bunkers consists of more than one compartment interconnected to form a series of cells, generally square or rectangular with different ratios of side length of walls to the height of the bunker. The design moments to be considered are influenced by the ratio of side length to height and the loading pattern of the compartments.

The following two cases are generally considered for obtaining the design positive and negative moments in the side walls of a battery of three bunkers.

Case-a: Battery of Bunkers with High Side Walls

When the sides of the bunker is less than 0.75 times the height, the slabs span horizontally and moments are induced at the junction of side walls and partition walls. The typical direction of moments developed in a battery of bunkers is shown in Fig. 2.10. By analysis the maximum moments developed in each of the walls are determined.

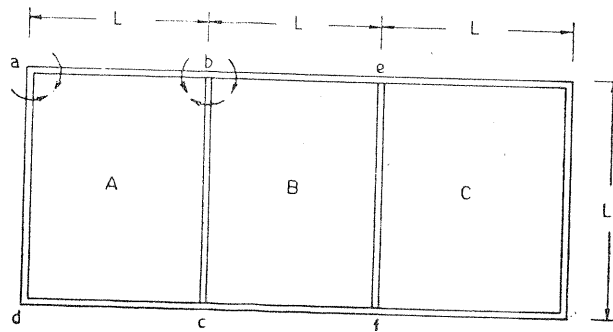


Fig. 2.10 Moments in battery of bunkers

The following moment coefficients compiled in Table 2.1 may be used for a battery of three bunkers as shown in Fig. 2.10.

$$M = \text{Coefficient} \times pL^2$$

p = Maximum pressure at the bottom due to the stored material

$R = \left(\frac{L}{L'}\right)$ = Ratio of shorter side to longer side. Moment is positive when it causes tension on the inside face of walls.

TABLE 2.1 Moment coefficients for battery of bunkers (with high side walls)

Moment	A-Loaded	B-Loaded	C-Loaded
$M_{ab} = M_{ad}$	$0.0362 + 0.470R^2$	$0.0076(R^2 - 1)$	—
M_{ba}	$0.0695 + 0.0138R^2$	$-0.038(R^2 - 1)$	$0.0089(R^2 - 1)$
M_{bc}	$0.0250 + 0.0585R^2$	$-(0.0228 + 0.0605R^2)$	$-0.00535(R^2 - 1)$
M_{be}	$0.445(1 - R^2)$	$0.0606 + 0.0227R^2$	$+0.0143(R^2 - 1)$

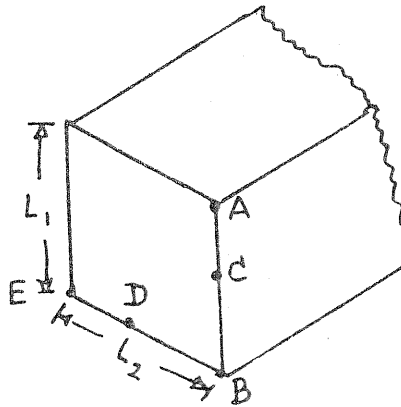
The table gives the support moments developed when each of the bunkers are loaded. The maximum span moments are obtained as the difference between the value of $(pL^2/8)$ and the support moments.

Case-b Battery of Bunkers with Low Side Walls

When the side walls are low, the slabs behave as two way slabs in the vertical as well as horizontal direction. The approximate moment coefficients for practical purposes are those in Table 2.2.

TABLE 2.2 Moment coefficients for battery of bunkers (with low side walls)

Position of moment	Values of coefficients when (L_2/L_1) is		
	1.25	1.20	0.75
<i>Vertical span-L_1</i>			
Top end-A	1/30	1/45	1/90
Bottom-B	1/20	1/30	1/60
Midspan-C	1/40	1/60	1/120
<i>Horizontal span-L_2</i>			
Ends-E	1/80	1/45	1/30
Midspan-D	1/160	1/90	1/60



These coefficients are applicable under the following conditions.

- The bunker is nearly square in plan.
- The side of bunker is more than 0.75 and less than 1.25 times the height.
- The bunker may be an isolated one or any one in the battery.
- In case of battery of bunkers, all walls, top and bottom slabs should be reinforced on both faces for positive and negative moments.

2.8 DESIGN OF SILOS

In deep bins (SILOS), the weight of the material stored is not completely supported by the bottom of the silo. A part of the load is resisted by friction between material and side walls of silo, resulting in the reduction of lateral pressure. Hence Rankine's theory can not be used for determining the pressure intensity. The vertical weight carried by walls causes direct compression in the wall.

The pressure intensity in silos where friction exists between material and wall surface can be determined by H. Janssen's and W. Airy's theories.

(a) *Janssen's Theory*

The following assumptions are made in the design of silos by Janssen's theory;

- (i) The material is uniform in texture
- (ii) The material has a definite angle of repose
- (iii) The coefficient of friction between material and side walls has a constant value.

The following notations are adopted in the analysis by Janssen's theory:

p_h = Horizontal intensity of pressure at a depth h (kN/m²)

p_v = Vertical intensity of pressure at a depth h (kN/m²)

r = radius of the silo (metres)

μ' = coefficient of friction between wall and material

R = Hydraulic mean radius

$$= \left(\frac{\pi r^2}{2\pi r} \right) = \left(\frac{r}{2} \right)$$

n = Ratio of $\left(\frac{p_h}{p_v} \right)$

w = Density of material (kN/m³)

Referring to Fig. 2.11, for the equilibrium of vertical forces in the disc.

$$\pi r^2 \cdot dh \cdot w = \pi r^2 \cdot dp_v + \mu' p_h \cdot 2\pi r \cdot dh$$

Dividing by πr and putting $p_h = np_v$ and rearranging the terms,

$$dh = \left(\frac{r}{wr - 2\mu'np_v} \right) dp_v \quad (1)$$

Integrating, we get

$$h = -\frac{r}{2\mu'n} \cdot \log_e (wr - 2\mu'np_v) + c_1$$

but at

$$h = 0, \quad p_v = 0$$

$\therefore$

$$c_1 = \left(\frac{r}{2\mu'n} \cdot \log_e wr \right)$$

The solution of the differential Eq. 1 is given by

$$h = -\frac{r}{2\mu'n} \log_e (wr - 2\mu'np_v) + \frac{r}{2\mu'n} \log_e wr$$

$$h = -\frac{r}{2\mu'n} \log_e \left(\frac{wr - 2\mu'np_v}{wr} \right)$$

$\therefore$

$$-\frac{2\mu'nh}{r} = \log_e \left(\frac{wr - 2\mu'np_v}{wr} \right)$$

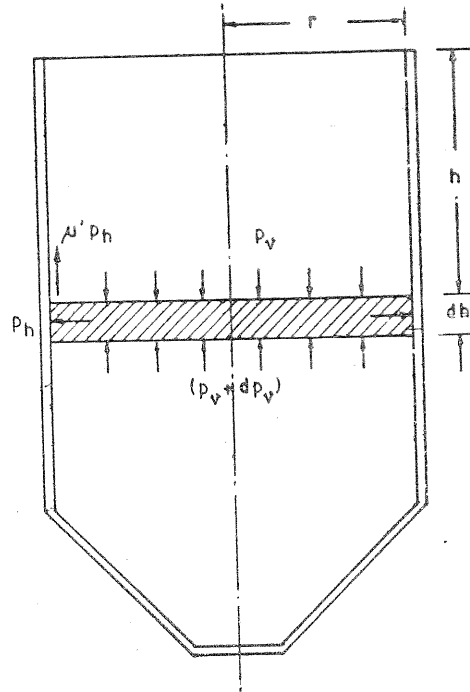


Fig. 2.11 Equilibrium of forces in silos

$$\exp\left(-\left(\frac{2\mu'nh}{r}\right)\right) = \left(1 - \frac{2\mu'np_v}{wr}\right)$$

$$\therefore \left(\frac{2\mu'np_v}{wr}\right) = 1 - \exp\left(-\left(\frac{2\mu'nh}{r}\right)\right)$$

$$\therefore p_v = \frac{wr}{2\mu'n} \left(1 - \exp\left(-\frac{2\mu'nh}{r}\right)\right)$$

$$\text{But Hydraulic mean radius} = R = \left(\frac{r}{2}\right)$$

$$\therefore p_v = \frac{wR}{\mu'n} \left(1 - \exp\left(-\frac{\mu'nh}{R}\right)\right)$$

$$\text{Also } p_h = n \cdot p_v$$

$$\therefore p_h = \frac{wR}{\mu'} \left(1 - \exp\left(-\frac{\mu'nh}{R}\right)\right)$$

For large value of h

$$\exp\left(-\left(\frac{\mu'nh}{R}\right)\right) = \left[\frac{1}{\exp\left(\frac{\mu'nh}{R}\right)}\right] = \text{very small quantity}$$

Hence

$$p_h = \left(\frac{wR}{\mu'}\right)$$

The vertical walls are designed for hoop tension of $p_h \cdot r$ also for the vertical load supported by the wall.

The load taken by wall is due to friction $= \mu' \cdot p_n$

Total vertical load taken by wall for a depth h is given by the expression,

$$\begin{aligned} & \mu' \int_0^h \frac{wR}{\mu'} \left(1 - \exp\left(-\frac{\mu' nh}{R}\right) \right) \cdot 2\pi r \cdot dh \\ & = w \cdot A \int_0^h \left(1 - \exp\left(-\frac{\mu' nh}{R}\right) \right) dh \end{aligned}$$

where

A = cross-sectional area of silo

$$\begin{aligned} & = w \cdot A \left(h + \frac{R}{\mu' n} \cdot \exp\left(-\frac{\mu' nh}{R}\right) \right) \Bigg|_0^h \\ & = w \cdot A \left[h - \frac{R}{\mu' n} \left(1 - \exp\left(-\frac{\mu' nh}{R}\right) \right) \right] \\ & = w \cdot A \cdot h - A \cdot p_v \\ & = A(wh - p_v) \end{aligned}$$

(b) Airy's Theory

Airy's theory of design of silos is based on coulomb's wedge theory of earth pressure. The results obtained from this theory also fairly agree with the experimental results although the basis of the theory is different from that of Janssen's theory. Using Airy's formula, horizontal pressure per unit length of periphery and position of plane of rupture can be determined. Knowing the horizontal pressure, vertical pressure and also vertical load taken by wall can be evaluated. Depending upon the plane of rupture two cases are considered.

Case-1 Plane of rupture cuts the top horizontal surface (shallow-Bin or silo)

Referring to Fig. 2.12, the forces acting on the wedge ACE of grain is given below:

- AE = Plane of Rupture
- W = Weight of wedge
- R_1 = Total Reaction on side AC
- R_2 = Total Reaction on side AE
- P = Reaction from wall
- R = Reaction from material
- b = Diameter of the silo
- h = height of the silo

Resolving the forces in equilibrium along AE and perpendicular to AE

$$\begin{aligned} \mu R + P \cdot \cos \theta &= (W - \mu' P) \sin \theta \\ R - P \cdot \sin \theta &= (W - \mu' P) \cos \theta \end{aligned}$$

solving,
$$P = \frac{W(\tan \theta - \mu)}{\tan \theta(\mu + \mu') + (1 - \mu\mu')}$$

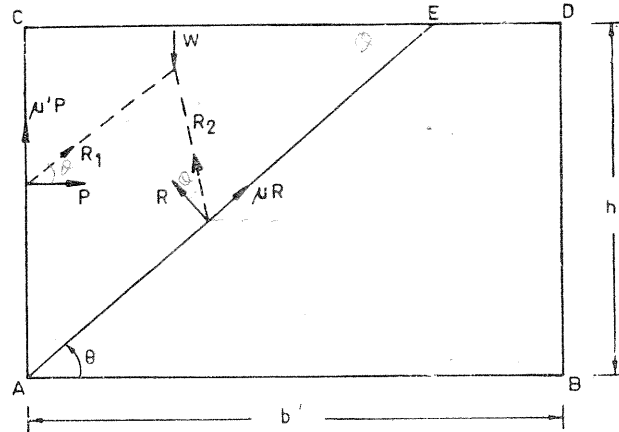


Fig. 2.12 Equilibrium of forces (shallow-bin)

since

$$CE = h \cot \theta$$

$$W = (1/2 \times h \cot \theta \times h \times W) = \left(\frac{wh^2}{2} \right) \cot \theta$$

$$\therefore P = \left[\frac{(\tan \theta - \mu)}{(1 - \mu\mu') + \tan \theta(\mu + \mu')} \right] \frac{wh^2}{2} \cot \theta \quad (1)$$

for Maximum value of P ,

$$\left(\frac{dP}{d\theta} \right) = 0$$

Differentiating and equating to zero it can be shown that

$$\tan \theta = \mu + \sqrt{\frac{\mu(1 + \mu^2)}{(\mu + \mu')}}}$$

Substituting this value in Eq. (1)

$$P = \frac{wh^2}{2} \left[\frac{1}{\{\sqrt{\mu(\mu + \mu')} + \sqrt{(1 + \mu^2)}\}^2} \right]$$

$$P_h = \left(\frac{dP}{dh} \right) = wh \left[\frac{1}{\sqrt{\mu(\mu + \mu')} + \sqrt{1 + \mu^2}} \right]$$

Total lateral pressure = $\pi \cdot b \cdot p$

Vertical load taken by walls = $\pi \cdot b \cdot p \cdot \mu'$

The depth up to which the silo will act as a shallow bin is given by

$$\tan \theta = \left(\frac{h}{b} \right) = \mu + \sqrt{\frac{\mu(1 + \mu^2)}{(\mu + \mu')}}}$$

$$h = b \left[\mu + \sqrt{\frac{\mu(1 + \mu^2)}{(\mu + \mu')}}} \right]$$

Case-2 Plane of rupture cuts opposite side
(Deep bin or silo)

Referring to Fig. 2.13, the forces acting on the wedge $ACDE$ of grain is given below:

W = Weight of wedge $ACDE$

$$DE = (h - b \tan \theta)$$

$$W = wbh - \frac{wb}{2} \cdot b \tan \theta$$

$$W = \frac{wb}{2} (2h - b \tan \theta)$$

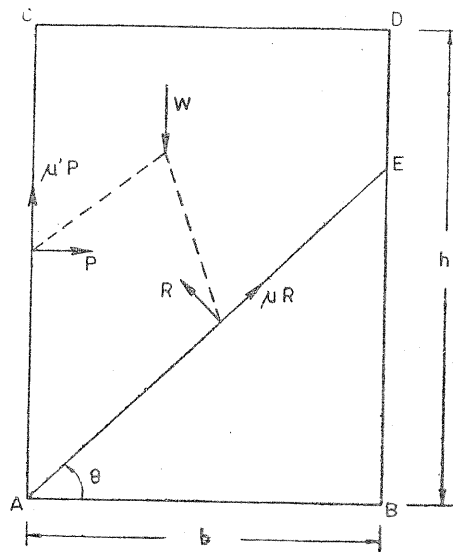


Fig. 2.13 Equilibrium of forces (deep-bin)

As in the case-1, by studying the equilibrium of forces on the wedge

$$P = \frac{W(\tan \theta)}{\tan \theta(\mu + \mu') + (1 - \mu\mu')}$$

Substituting for W and differentiating for maximum value of P , we have

$$\begin{aligned} P &= \frac{wb \left(\frac{2h - b \tan \theta}{2} \right) (\tan \theta - \mu)}{(\mu + \mu') \tan \theta + (1 - \mu\mu')} \\ &= \frac{wb}{2} \left[\frac{(2h - b \tan \theta) (\tan \theta - \mu)}{(\mu + \mu') \tan \theta + (1 - \mu\mu')} \right] \\ &= \frac{wb^2}{2(\mu + \mu')^2} \left[\sqrt{\frac{2h}{b} (\mu + \mu') + (1 - \mu\mu')} - \sqrt{1 + \mu^2} \right]^2 \end{aligned}$$

Differentiating and equating to zero

$$\frac{dp}{dh} = 0, \text{ yields}$$

$$\tan \theta = -\frac{(1 - \mu\mu')}{(\mu + \mu')} + \sqrt{\frac{2h}{b} \cdot \frac{(1 + \mu^2)}{(\mu + \mu')} + \frac{(1 + \mu^2)(1 - \mu\mu')}{(\mu + \mu')^2}}$$

substituting this value for $\tan \theta$,

$$P = \frac{wb}{2} (2h - b \tan \theta) \left[\frac{(\tan \theta - \mu)}{(\mu + \mu') \tan \theta + (1 - \mu\mu')} \right]$$

$$p_h = \frac{dp}{dh} = \frac{wb}{(\mu + \mu')} \left[1 - \frac{\sqrt{(1 + \mu^2)}}{\sqrt{\frac{2h}{b} (\mu + \mu') + (1 - \mu\mu')}} \right]$$

Vertical load taken by walls = $\pi b \cdot P \cdot \mu'$

For the design of conical hopper bottom, the surcharge pressure is given by the expression

$$\begin{aligned} \text{Surcharge pressure} &= \left[\frac{\left(\frac{\pi b^2}{4} \cdot wh \right) - \pi b \cdot p_h \cdot \mu'}{\left(\frac{\pi b^2}{4} \right)} \right] \\ &= \left[\left(wh - \frac{4p_h \cdot \mu'}{b} \right) \right] \end{aligned}$$

The values of the coefficient of friction μ and μ' are compiled in Table 2.3.

TABLE 2.3 Values of Friction Coefficients

Material	Density kN/m ³	Coefficient of Friction		$K = \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)$
		Filling on Filling (μ)	Filling on Concrete (μ')	
Cement	14.40	0.316	0.554	0.5371
Coal	8.00	0.700	0.700	0.2709
Anthracite	8.35	0.510	0.510	0.3753
Coke	4.50	0.839	0.839	0.2174
Sand	16.00	0.674	0.577	0.2830
Wheat	8.50	0.466	0.444	0.4062

2.9 SILOS FOR STORAGE OF CEMENT

The cement manufactured in a factory is generally stored in a battery of silos which are circular in shape with diameters of cylindrical containers up to 10 m and height 25 to 30 m. A typical battery of silos to store 130,000 kN of cement is shown in Fig. 2.14.

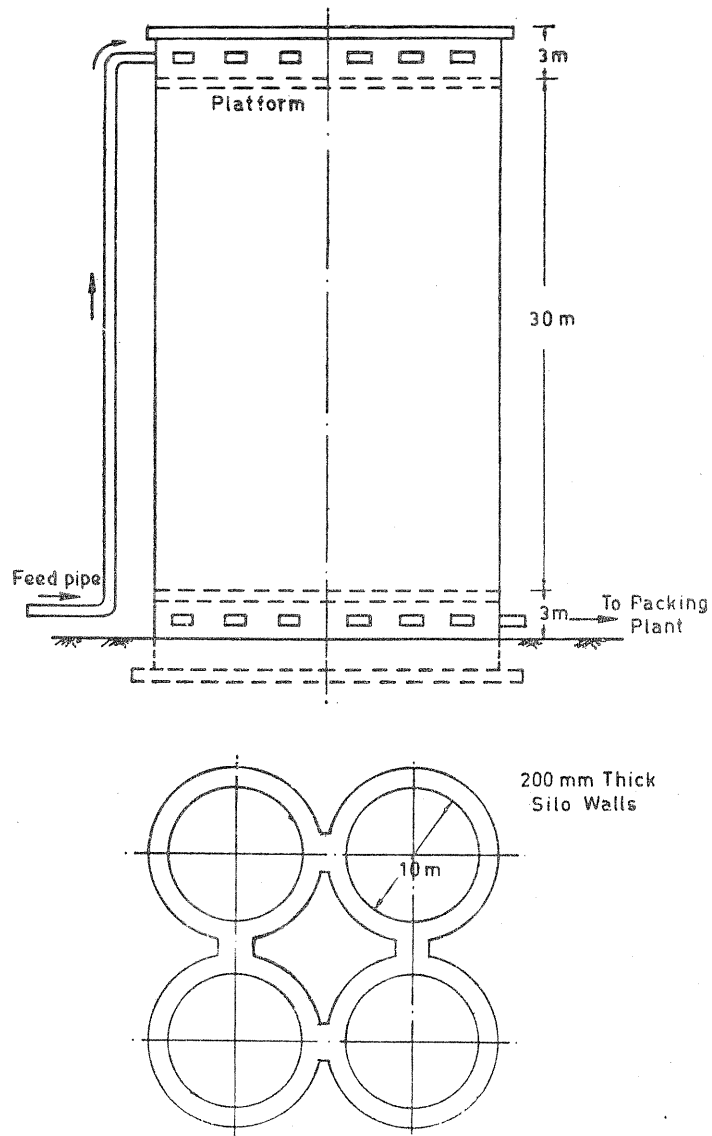


Fig. 2.14 Battery of Cement silos

The horizontal pressure distribution at various depths in cement silos predicted by Janssen's and Airy's theories are compared with the experimental values reported by Faber and Mead in Fig. 2.15. Truly frictional noncohesive materials behave in accordance with Janssen's theory. In materials like cement, due to the erratic cohesive nature of the material, the pressures obtained using Janssen's theory varied widely with the actual experimental observations.

Full scale experiments of cement silos indicate that the lateral pressures are very much underestimated towards the upper half of the silo by the Janssen's theory. In contrast, the lateral pressures are very much over estimated towards the lower half of the silo by the Airy's theory. Towards the bottom of the silo, the lateral pressures reduce due to the arching action.

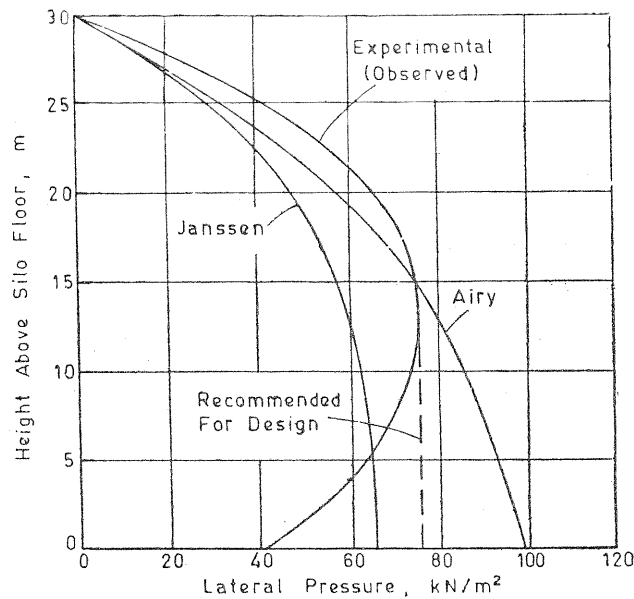


Fig. 2.15 Horizontal pressure distribution in cement silos

The depth of cement occurring below any particular level under consideration influences the lateral pressure. The cement below suffers a cushioning effect packing tighter as the head of cement overburden is increased until it acquires its state of maximum density. At depths where maximum density is reached, the maximum angle of internal friction is also developed. Compacted cement can stand for considerable heights with a vertical face, whereas shallow lightly sprinkled cement exhibits properties not far removed from that of a fluid. Maximum pressures indicated in the figure are useful in design of silos.

2.10 DESIGN EXAMPLE

A cylindrical silo has an internal diameter of 6 m and 20 m deep (cylindrical portion) with a conical hopper bottom. The material stored is wheat with a density of 8 kN/m^3 . The coefficient of friction between wall and material is 0.444. The ratio of horizontal to vertical pressure intensity is 0.40

Angle of repose = 25 degrees. Design the reinforcements in the silo walls. Adopt M-15 grade concrete and ribbed tor steel. Adopt Janssen's theory for pressure calculations.

1. *Data*

Diameter of silo = 6 m
 Depth of cylindrical portion = 20 m
 Density of wheat = 8 kN/m³
 coefficient of friction between wall and material = 0.444
 The ratio of horizontal to vertical pressure intensity = 0.40
 Angle of repose = 25 degrees.

2. *Allowable Stresses*

$\sigma_{cb} = 5 \text{ N/mm}^2$ $m = 19$
 $\sigma_{ct} = 2 \text{ N/mm}^2$
 $\sigma_{st} = 230 \text{ N/mm}^2$

3. *Dimensions of Bunker*

Diameter of bunker = 6 m
 Height of cylindrical portion = 20 m
 Depth of hopper bottom = 2.5 m
 Diameter of opening in hopper bottom = 1 m

4. *Design of Cylindrical Walls*

Using Janssen's theory

$$\text{Horizontal pressure} = p_h = \frac{wR}{\mu'} \left(1 - \exp \left(-\frac{\mu' nh}{R} \right) \right)$$

$$n = \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right) = \left(\frac{1 - \sin 25^\circ}{1 + \sin 25^\circ} \right) = 0.40$$

$$R = \text{Hydraulic mean radius} = \left(\frac{D}{4} \right) = \left(\frac{6}{4} \right) = 1.5$$

$$p_h = \left(\frac{8.00 \times 1.5}{0.444} \right) \left[1 - \exp \left(-\frac{0.444 \times 0.40 \times h}{1.5} \right) \right]$$

$$= 27.03 (1 - e^{-x})$$

For different values of $h = 4, 8, 12, 16, 20$ and 22.5 metres from the top the value of e^{-x} is calculated and the value of p_h is compiled in Table 2.4. Appendix 2 gives the values of e^{-x} for different values of x .

Maximum horizontal pressure in the cylindrical portion (20 m from top) = $p_h = 24.58 \text{ kN/m}^2$

Hoop tension in cylindrical wall per metre height = $0.5 p_h D$
 $= (0.5 \times 24.58 \times 6)$
 $= 73.74 \text{ kN}$

TABLE 2.4 Horizontal Pressures in Silo Walls

Depth from top (h) metres	Horizontal pressure $p_h = \text{kN/m}^2$
4	10.54
8	16.67
12	20.62
16	23.05
20	24.58
22.5	25.13

$$\text{Area of hoop reinforcement} = A_{st} = \left(\frac{73.74 \times 10^3}{230} \right) = 321 \text{ mm}^2$$

Adopt 8 mm diameter hoops at 140 mm centres. (A_{st} provided = 359 mm²)

Using 150 mm thick cylindrical walls,

$$\begin{aligned} \text{Tensile stress in concrete} &= \left(\frac{F_t}{A_c + mA_{st}} \right) \\ &= \left[\frac{73.74 \times 10^3}{(150 \times 10^3) + (19 \times 359)} \right] \\ &= 0.47 \text{ N/mm}^2 < 2 \text{ N/mm}^2 \text{ permissible stress} \end{aligned}$$

Minimum area of steel = 0.12%

$$= (0.0012 \times 150 \times 1000)$$

$$= 180 \text{ mm}^2$$

Adopt 8 mm diameter hoops at 270 mm centres towards the top of silo. The details of reinforcements to be provided at different depths on the cylindrical silo walls is compiled in Table 2.5.

TABLE 2.5 Reinforcement details in silo walls

Depth from top (m)	Spacings of 8 mm diameter hoops	Vertical distribution steel
0-12	180 mm	8 mm diameter at
12-16	160 mm	250 m centres
16-20	140 mm	throughout the whole depth

5. Design of Hooper Bottom

Provide a sloping slab 150 mm thick with 30 mm lining.

Total thickness = 180 mm

Surcharge Load on hooper bottom/metre

$$\begin{aligned} &= \left(wh - \frac{4p_h \mu'}{b} \right) \\ &= \left(8 \times 20 - \frac{4 \times 24.58 \times 0.44}{6} \right) = 153 \text{ kN} \end{aligned}$$

Weight of sloping bottom

$$= \pi \left[\left(\frac{6+1}{2} \right) + 0.18 \sqrt{2} \right] \times 0.18 \times \sqrt{2} \times 24$$

$$= 72 \text{ kN}$$

Total load = (153 + 72) = 225 kN

If T = Tension per metre run

Mean diameter at centre of sloping slab = 3.5 m

$$T = \left(\frac{225}{\pi \times 3.5} \right) \text{ cosec } 45 = 30 \text{ kN}$$

Steel reinforcement for direct tension

$$= \left(\frac{3 \times 10^3}{230} \right) = 130 \text{ mm}^2$$

Use 8 mm diameter bars at 200 mm centres in the direction of the sloping slab.

$$\text{Surcharge pressure on hopper bottom} = \left(\frac{153}{\pi \times 6^2} \right) = 1.35 \text{ kN/m}^2$$

Maximum horizontal pressure in hopper bottom $p_h = 25.13 \text{ kN/m}^2$

If p_n = Normal pressure intensity

$$p_n = (1.35 \cos^2 \theta + p_h \sin^2 \theta)$$

$$= (1.35 \times \cos^2 45 + 25.13 \times \sin^2 45)$$

$$= 13.24 \text{ kN/m}^2$$

Normal component due to self weight of sloping slab

$$= w_d \cdot \cos \theta$$

$$= (0.180 \times 24 \times \cos 45)$$

$$= 3.00 \text{ kN/m}^2$$

Total Normal pressure = $P = (p_n + w_d \cos \theta)$

$$= (13.24 + 3.00)$$

$$= 16.24 \text{ kN/m}^2$$

Mean diameter at centre of sloping slab

$$= \left(\frac{6+1}{2} \right) + 0.18 \sqrt{2} = 3.75 \text{ m}$$

Hoop tension/metre = $(0.5 \times 16.24 \times 3.75)$

$$= 30.45 \text{ kN}$$

$$\text{Area of hoop reinforcement} = \left(\frac{30.45 \times 10^3}{230} \right) = 133 \text{ mm}^2$$

Use 8 mm diameter hoops at 200 mm centre in the hopper bottom.

6. Edge Beams

At the junction of the cylindrical wall and hopper bottom and at the top of the bunker, edge beams of 300 mm by 300 mm with 4 bars of

12 mm diameter are provided to increase the rigidity of the structure. The arrangement of reinforcements are similar to those shown in Fig. 2.9.

2.11 DESIGN EXAMPLE

A cement silo has an internal diameter of 10 m with the height of cylindrical portion being 30 m. The density of cement is 15.2 kN/m³. Coefficient of friction between concrete and material = 0.70. The angle of repose of the material is 17.5 degrees. Adopting M-15 concrete and ribbed tor steel, design the thickness and reinforcements required at the bottom of the cylindrical portion of the silo using Janssen's theory.

1. Data

Diameter of silo = 10 m

Depth of cylindrical portion = 30 m

Density of cement = 15.2 kN/m³

Coefficient of friction between wall and material = $\mu' = 0.70$

Angle of repose of material = $\phi = 17.5^\circ$

Ratio of horizontal to vertical pressure intensity

$$n = \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right) = \left(\frac{1 - 0.3}{1 + 0.3} \right) = 0.54$$

2. Allowable Stresses

$\sigma_{cb} = 5 \text{ N/mm}^2$

$m = 19$

$\sigma_{ct} = 2 \text{ N/mm}^2$

$\sigma_{st} = 230 \text{ N/mm}^2$

3. Design of Cylindrical Walls

Using Janssen's theory

$$\text{Horizontal pressure} = p_h = \frac{wR}{\mu'} \left(1 - \exp \left(- \frac{\mu' nh}{R} \right) \right)$$

$$R = \left(\frac{D}{4} \right) = \left(\frac{10}{4} \right) = 2.5$$

$n = 0.54$

$w = 15.2 \text{ kN/m}^3$

$h = 30 \text{ m}$

$\mu' = 0.70$

At the bottom of the cylindrical portion ($h = 30 \text{ m}$), the horizontal pressure is given by

$$\begin{aligned} p_h &= \left(\frac{15.2 \times 2.5}{0.70} \right) \left[1 - \exp \left(- \frac{0.70 \times 0.54 \times 30}{2.5} \right) \right] \\ &= 54 \text{ kN/m}^2 \end{aligned}$$

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Hoop tension in cylindrical wall per metre height

$$\begin{aligned} &= 0.5p_h \cdot D \\ &= (0.5 \times 54 \times 10) \\ &= 270 \text{ kN} \end{aligned}$$

$$\text{Area of hoop reinforcement} = A_{st} = \left(\frac{270 \times 10^3}{230} \right) = 1174 \text{ mm}^2$$

Adopt 16 mm diameter hoops at 160 mm centres (A_{st} provided = 1256 mm²)

Using 200 mm thick cylindrical walls tensile stress developed in concrete is given by

$$\begin{aligned} \sigma_{ct} &= \left(\frac{F_t}{A_c + m A_{st}} \right) \\ &= \left[\frac{270 \times 10^3}{(200 \times 10^3) + (19 \times 1256)} \right] \\ &= 1.20 \text{ N/mm}^2 < 2 \text{ N/mm}^2 \text{ permissible stress} \end{aligned}$$

Vertical pressure intensity at a depth $h = 30$ m is

$$p_v = \left(\frac{p_h}{0.54} \right) \left(\frac{54}{0.54} \right) = 100 \text{ kN/m}^2$$

$$\text{Equivalent depth of cement} = \left(\frac{100}{15.2} \right) = 6.57 \text{ m}$$

Even though there is 30 m depth of cement, the effective vertical load is only due to 6.57 m of cement.

30 m depth of cement corresponds to a pressure

$$(15.2 \times 30) = 456 \text{ kN/m}^2$$

Vertical pressure intensity at 33 m depth = 100 kN/m²

$$\text{and is} = \left(\frac{100}{456} \times 100 \right) = 21.9\% \text{ of total weight}$$

The remaining 78.1% is transferred to the silo walls by friction.

Provide vertical distribution steel of 0.12% of the cross section equivalent to 8 mm diameter bars at 160 mm centres.

2.12 DESIGN EXAMPLE

Compare the horizontal pressures developed at 5 m intervals in a cement silo of internal diameter 10 m and height 30 m using Janssen and Airy's theories.

Density of cement = 15.2 kN/m³

Coefficient of friction between concrete and cement = $\mu' = 0.554$

Coefficient of friction between filling on filling = $\mu = 0.316$

Angle of repose of cement = $\phi = 17.5$ degrees

$$n = \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right) = 0.54$$

(a) Janssen's Theory

$$p_h = \frac{wR}{\mu'} \left(1 - \exp - \left(\frac{\mu' nh}{R} \right) \right)$$

$$R = \left(\frac{D}{4} \right) = \left(\frac{10}{4} \right) = 2.5$$

$$p_h = \left(\frac{15.2 \times 2.5}{0.554} \right) \left[1 - \exp - \left(\frac{0.554 \times 0.54 \times h}{2.5} \right) \right]$$

$$= 68.59(1 - e^{-x})$$

For different values of $h = 5, 10, 15, 20, 25, 30$ m from top, the value of horizontal pressure p_h is evaluated.

(b) Airy's Theory

1. The depth up to which the silo will act as shallow bin will be given by

$$h = b \left[\mu + \sqrt{\frac{\mu(1+\mu^2)}{(\mu+\mu')}} \right]$$

$$= 10 \left[0.316 + \sqrt{\frac{0.316(1+0.316^2)}{(0.316+0.554)}} \right]$$

$$= 9.48 \text{ m}$$

$$p_h = wh \left[\frac{1}{\{\sqrt{\mu(\mu+\mu')} + \sqrt{1+\mu^2}\}^2} \right]$$

$$= 15.2h \left[\frac{1}{\{\sqrt{0.316(0.316+0.554)} + \sqrt{1+0.316^2}\}^2} \right]$$

$$= 6.148h \text{ kN/m}^2 \text{ valid up to } 9.48 \text{ m}$$

2. The horizontal pressure intensity at depths greater than 9.48 m is given by

$$p_h = \frac{wb}{(\mu+\mu')} \left[1 - \frac{\sqrt{1+\mu^2}}{\sqrt{\frac{2h}{b}(\mu+\mu') + (1-\mu\mu')}}} \right]$$

$$= \frac{15.2 \times 10}{(0.316+0.554)} \left[1 - \frac{\sqrt{1+0.316^2}}{\sqrt{\frac{2h}{10}(0.316+0.554) + (1-0.316 \times 0.554)}}} \right]$$

$$= 174.7 - 183(0.174h + 0.825)^{-1/2}$$

Using this equation the values of p_h at depths of 10, 15, 20, 25 and 30 m is evaluated and the values are compared in Table 2.5.

In deep bins, Airy's theory predicts higher values of horizontal pressure than that resulting from Janssen's theory for increasing depth from top. At greater depths, the pressures computed by Airy's theory is nearly 50% greater than that evaluated by Janssen's theory.

TABLE 2.5 Horizontal pressure in cement silos

Depth from top (m)	Values of p_h (kN/m ²)	
	Janssen's theory	Airy's theory
5	31.00	30.74
10	47.87	60.40
15	57.25	75.88
20	62.41	86.30
25	65.16	94.30
30	66.71	100.00

2.13 EXAMPLES FOR PRACTICE

1. A reinforced concrete circular bunker is 4 m in internal diameter. The circular wall is 4.5 m high. It has a 45 degree conical hopper bottom. The hopper has a concentric circular opening of diameter 0.5 m. The bunker is supported on four columns.

The bunker is used for storing material weighing 8.5 kN/m³. Angle of friction is 30 degrees. The coefficient of friction between the concrete surface and the material is 0.4. Design the circular walls of the bunker and the conical hopper bottom. Adopt M-20 grade concrete and ribbed tor steel Fe-415 grade.

(Bangalore University—January 1978)

2. A cylindrical silo having a ratio of height (cylindrical portion) to the diameter of 4, is required to store 2000 kN of wheat having a density of 8 kN/m³. The coefficient of friction between grain and concrete is 0.444 and the ratio of horizontal to vertical pressure is 0.4. Design the reinforcements in the walls and conical bottom of the silo using Janssen's theory. Adopt M-20 grade concrete and Grade-1 mild steel

(Bangalore University—January 1979)

3. A coal bunker is to be designed to store 300 kN of coal having a unit weight of 8 kN/m³. The bunker should be square with 3 metre sides. The stored coal is to be surcharged at an angle of repose which is 30 degrees for coal. Adopt M-15 grade concrete and Fe-415 steel and design the side walls and hopper bottom and sketch the details of reinforcements.
4. A rectangular shallow bin 3 m by 2.5 m in plan is required to store 240 kN of foamed slag aggregates. The unit weight of the aggregates being 9.5 kN/m³. The bin is supported on four R.C columns at the corners. Using M-15 grade concrete and Grade-1 mild steel, design the side walls, hopper bottom and the column and sketch the structural details. Angle of repose = 30 degrees, Angle of surcharge = 25 degree.
5. A circular grain silo of 3.6 m diameter is reinforced with horizontal hoop reinforcement of 10 mm diameter. Calculate the maximum pitch

for these rods at depth of 24 m below the surface of the grain. The unit weight of grain is 8 kN/m^3 and the coefficient of friction between grain and silo walls is 0.41. The ratio of lateral to vertical pressure at any point is 0.6. Adopt M-15 concrete and Grade-I mild steel and use Janssen's theory.

The vertical reinforcement of the 125 mm thick concrete shell consists of 24 rods of 10 mm diameter equally spaced. Determine the vertical stress in concrete at 24 m depth assuming the silo to be full and neglecting wind forces. Allow for a superimposed load of 11 kN/m^2 at roof level, modular ratio = 19.

6. A cylindrical silo having the ratio of height (cylindrical portion) to the diameter of 4 is required to store 2000 kN of wheat weighing 8 kN/m^3 . The coefficient of friction between grain and concrete is 0.444 and that between grains is 0.466. Compare the lateral pressures developed at 4 m intervals using Janssen's and Airy's theories. Design the reinforcements in the wall and the conical bottom for the worst case. Adopt M-20 grade concrete and ribbed tor steel.

3

Chimneys

3.1 GENERAL

During the last few decades the use of reinforced concrete chimneys in place of brick masonry and steel chimneys have become very popular due to their low cost and durability. Composite material like reinforced concrete is eminently suited for chimney stacks. Brick chimneys are very heavy requiring expensive foundations. In contrast to the steel chimneys, the maintenance costs are minimum in the case of concrete stacks. Also the development of slip form method of constructing cylindrical stacks has resulted in rapid construction in the case of concrete chimneys.

The thickness of the concrete shell generally varying from 120 to 300 mm is considerably smaller than that required in the case of brick chimneys. Concrete stacks with lesser maintenance costs are architecturally superior to masonry and steel chimneys.

3.2 PARTS OF CHIMNEY

A reinforced concrete chimney is generally circular in shape with a rigid concrete shell cast with a rich concrete mix of M-20 to M-25 grade and provided with longitudinal vertical reinforcement and horizontal hoop reinforcement. A fire brick lining 100 to 150 mm thick is provided inside the concrete shell with an air gap of 80 to 150 mm thick to reduce the temperature gradient from the interior surface of fire brick lining to the exterior surface of the concrete shell.

Reinforced concrete brackets with holes are provided at regular intervals to support the fire brick lining. At the bottom of the chimney, provision is

made for a flue opening. The chimney rests on a circular raft foundation. The various parts of the chimney are shown in Fig. 3.1.

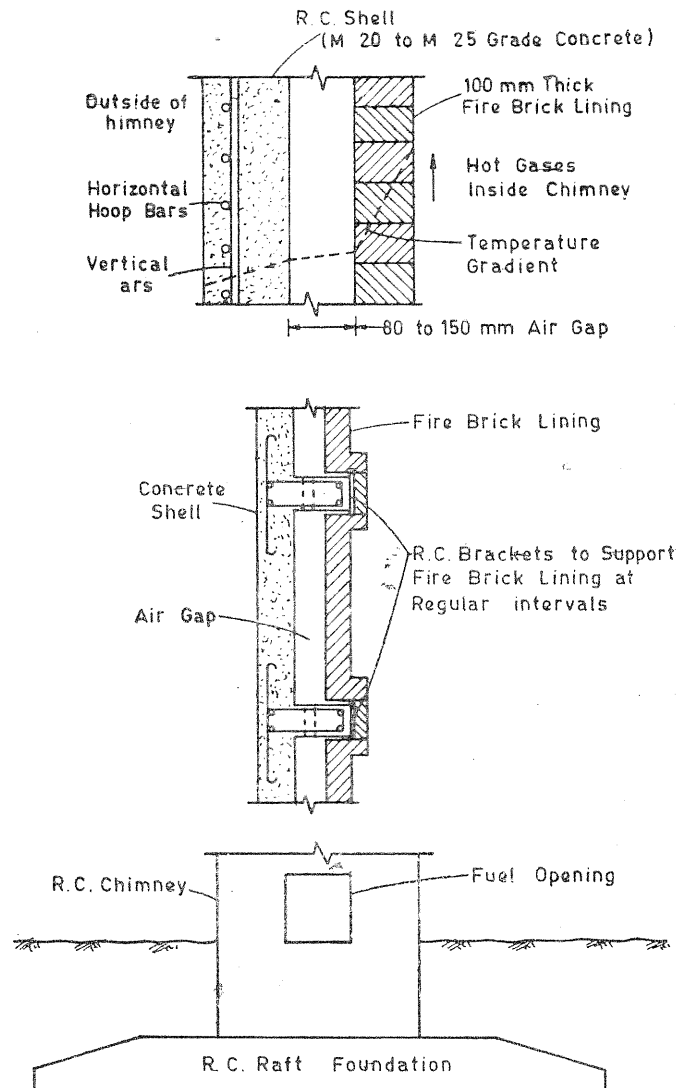


Fig. 3.1 Parts of R.C.C. Chimney

3.3 DESIGN FACTORS

Reinforced concrete chimneys are designed to withstand the stresses developed due to

- (a) Self weight of chimney
- (b) Wind pressure

(c) Temperature stresses

(a) The self weight of the chimney is comprised of the dead weight of the concrete shell and the fire brick lining together with the brackets.

(b) The wind pressure depends upon the velocity of wind at the given place. The velocity of wind increases with the altitude above the ground. The wind pressure in different zones can be obtained from IS: 875-1964 code for purposes of design. The design wind loads on the chimney depends upon the cross sectional shape of the chimney and the values of shape factor for different cross sectional shapes such as, circular octagonal and square are compiled in Table 3.1.

TABLE 3.1 Shape Factors for Chimneys

Ratio of height to base width	0 to 4	4 to 8	8 or over
Cross sectional shape of chimney	Shape Factor		
Circular	0.7	0.7	0.7
Octagonal	0.8	0.9	1.0
Square (wind perpendicular to diagonal)	0.8	0.9	1.0
Square (wind perpendicular to face)	1.0	1.15	1.3

(c) Due to the temperature gradient between the inside and out side faces of the chimney, temperature stresses are induced in the chimney walls both in the vertical and horizontal planes. The inner surface of the shell being at a higher temperature, tends to expand more than the outer surface; which restrains the expansion of the inner fibres to a certain extent. This restrained expansion results in compression of the inner fibres and tension of the outer fibres. In consequence of this effect, the concrete shell is subjected to a bending moment in the vertical plane as shown in Fig. 3.2.

In the vertical cross section, P , Q , R and S represent the temperature gradients in the chimney. Experiments have shown that the rate of fall of temperature is steeper in the fire brick lining than in the concrete shell. The slope of PQ is nearly 6 to 10 times that of RS . The temperature at P is slightly lower than that of flue gases and at S , it is slightly higher than the atmospheric temperature.

In horizontal sections of the chimney, the action due to temperature is similar so that the inner fibres are compressed in a horizontal plane while the outer fibres are stretched.

3.4 STRESSES IN R.C. SHAFTS DUE TO SELF WEIGHT AND WIND LOADS

The following notations are used in the analysis of stresses in R.C. shafts.

W = Total weight of the shaft above the considered section

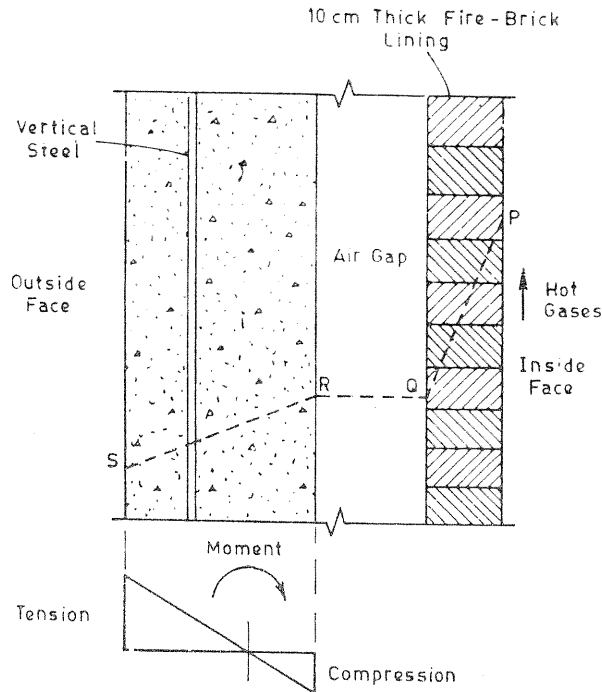


Fig. 3.2 Temperature Stresses in R.C. Chimney

P = Resultant wind force acting at a distance h from the section

A_s = Area of reinforcement assumed to be in the form of a ring at the centre of thickness of shell

t_s = Thickness of steel ring

R = Radius of the centre of thickness

$$t_s = \left(\frac{A_s}{2\pi R} \right)$$

d = Outside diameter of the shell

t_c = Thickness of concrete shell

n = Coefficient of neutral axis depth

σ_c = Compressive stress in concrete at the centre of thickness of shell

σ_s = Tensile stress in steel

α = Angle subtended by the neutral axis at the centre

m = modular ratio

M = Bending moment at the section

Referring to Fig. 3.3, consider a strip $R \cdot d\theta$ at an angle θ from xx . Stress in concrete at the level of elementary strip,

$$\sigma'_c = \sigma_c \left[\frac{R \cos \theta + R \cos \alpha}{R + R \cos \alpha} \right] = \sigma_c \left[\frac{\cos \theta + \cos \alpha}{1 + \cos \alpha} \right]$$

$$\text{Area of strip} = (R \cdot d\theta \cdot t_c)$$

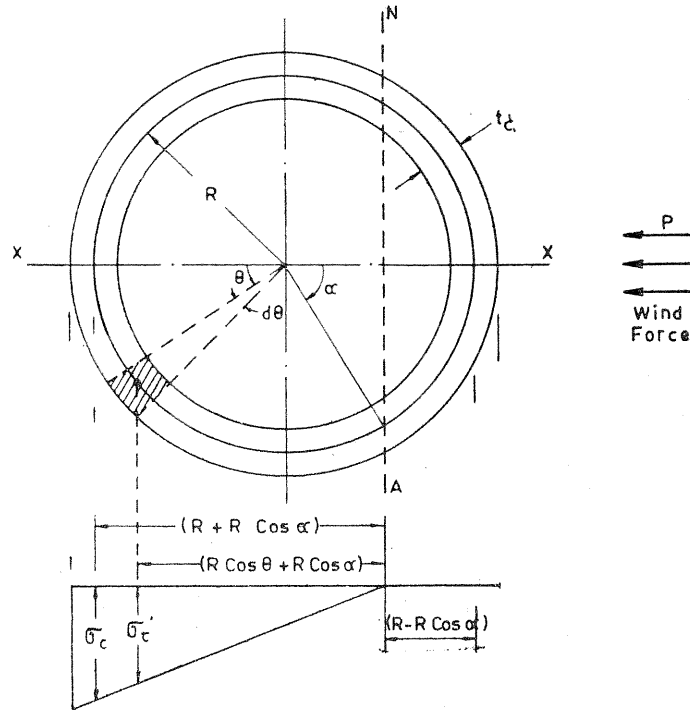


Fig. 3.3 Stresses in Chimney Shafts

Area of steel in strip = $(R \cdot d\theta \cdot t_s)$

$\therefore$ Total compressive force in concrete and steel

$$C = 2 \int_0^{(\pi-\alpha)} (R \cdot d\theta) t_c \cdot \sigma_c \left[\frac{\cos \theta + \cos \alpha}{1 + \cos \alpha} \right] \\ + 2 \int_0^{(\pi-\alpha)} (R \cdot d\theta) t_s \cdot (m-1) \cdot \sigma_c \left[\frac{\cos \theta + \cos \alpha}{1 + \cos \alpha} \right] \\ C = \left[\frac{2R\sigma_c t_c}{1 + \cos \alpha} \right] \int_0^{(\pi-\alpha)} (\cos \theta + \cos \alpha) d\theta \\ + \left[\frac{2R\sigma_c t_s (m-1)}{1 + \cos \alpha} \right] \int_0^{(\pi-\alpha)} (\cos \theta + \cos \alpha) d\theta$$

Integrating and simplifying

$$C = \left[\frac{2R\sigma_c}{1 + \cos \alpha} \right] [t_c + (m-1)t_s] [\sin \alpha + (\pi - \alpha) \cos \alpha]$$

Similarly total tension in steel is given by

$$T = 2 \int_0^{\alpha} (R \cdot d\theta) t_s \cdot m \sigma_c \left(\frac{R \cos \theta - R \cos \alpha}{R + R \cos \alpha} \right) \\ = \left[\frac{2R\sigma_c \cdot m t_s}{1 + \cos \alpha} \right] [\sin \alpha - \alpha \cos \alpha]$$

Equating the sum of the internal forces to external load W

$$W = (C - T)$$

$$W = \left[\frac{2R\sigma_c}{1 + \cos \alpha} \right] [(t_c - t_s) \{\sin \alpha + (\pi - \alpha) \cos \alpha\} + \pi m t_s \cdot \cos \alpha] \quad (1)$$

Equating the element of external forces to the sum of the moments of the internal forces we have

$$\begin{aligned} M &= \int_0^{(\pi-\alpha)} C \cdot R \cdot \cos \theta + \int_0^\pi T \cdot R \cdot \cos \theta \\ M &= \left[\frac{2R^2\sigma_c}{1 + \cos \alpha} \right] [(t_c + m - 1)t_s] \int_0^{(\pi-\alpha)} (\cos^2 \theta + \cos \theta \cos \alpha) d\theta \\ &\quad + \left[\frac{2R^2\sigma_c \cdot m \cdot t_s}{1 + \cos \alpha} \right] \int_0^\pi (\cos^2 \theta - \cos \theta \cos \alpha) d\theta \end{aligned}$$

Integrating and simplifying we have the final equation for the moment M , as

$$\begin{aligned} M &= \left[\frac{2R^2 \cdot \sigma_c}{1 + \cos \alpha} \right] [(t_c + (m - 1)t_s)] \left[\frac{\sin 2\alpha}{4} + \frac{(\pi - \alpha)}{2} \right] \\ &\quad + \left[\frac{2R^2\sigma_c \cdot m t_s}{1 + \cos \alpha} \right] \left[\frac{\alpha}{2} - \frac{\sin 2\alpha}{4} \right] \\ \therefore M &= \left[\frac{2R^2\sigma_c}{1 + \cos \alpha} \right] \left[(t_c - t_s) \left(\frac{\sin 2\alpha}{4} + \frac{\pi - \alpha}{2} \right) + \frac{m t_s \cdot \pi}{2} \right] \quad (2) \end{aligned}$$

Eccentricity $e = (M/W)$

$$e = R \left\{ \frac{[(t_c - t_s) \left(\frac{\sin 2\alpha}{4} + \frac{\pi - \alpha}{2} \right) + \frac{m \cdot t_s \cdot \pi}{2}]}{(t_c - t_s)[\sin \alpha + (\pi - \alpha) \cos \alpha] + \pi m t_s \cdot \cos \alpha} \right\} \quad (3)$$

The value of α which satisfies the Eq. 3 is determined by Trial and Error. Knowing α the stresses in concrete and steel can be evaluated using Eq. 1.

3.5 STRESSES IN HORIZONTAL REINFORCEMENT DUE TO SHEAR FORCE

If H = Horizontal shear force at the section

d = diameter of the chimney

S = pitch of hoop bars

A_t = Area of hoop bars in one pitch length

Area of steel resisting shear in one metre height = $\left(\frac{2A_t}{S} \cdot 1000 \right)$ If σ_c = stress in steel

$$\text{Shear force resisted} = \left(\frac{2 \cdot A_t \cdot 1000 \times \sigma_s}{S} \right) \quad (a)$$

If horizontal distance between reinforcement on both sides is assumed as $0.8d$,

$$\text{shear/metre} = \left(\frac{H \times 1000}{\text{Lever arm}} \right) = \left(\frac{1000H}{0.8d} \right) \quad (b)$$

Equating (a) and (b)

$$\left(\frac{2A_t \times 1000 \times \sigma_s}{s} \right) = \left(\frac{1000H}{0.8d} \right)$$

$$\sigma_s = \left(\frac{H \cdot s}{1.6A_t \cdot d} \right) \quad (4)$$

Where d and s are expressed in mm units and A_t is in mm² units.

3.6 STRESSES DUE TO TEMPERATURE DIFFERENCE

In the walls of a reinforced concrete chimney, stresses are developed due to the temperature gradient between the inner and outer surface of the walls. This temperature drop from inside to outside surface tends to expand the inner surface relative to the outer one. Due to the monolithic action of the entire wall, differential expansion is not possible and hence equal expansion takes place so that the shell is compressed on its inside surface and pulled on the outside surface. As a whole there is an average increase in the length of the shell due to the temperature gradient.

3.6.1 Effect of Temperature only on Stresses in Chimney Walls

Let T° = Temperature difference between inside and outside with a linear temperature gradient.

α = Coefficient of expansion of steel and concrete

e = strain due to temperature difference

m = modular ratio

t_s = Area of reinforcement per unit width

t_c = Area of concrete per unit width

σ_{ct} = stress in concrete due to temperature

σ_{st} = stress in steel due to temperature

$p = (t_s/t_c)$

k = Neutral axis depth constant

Referring to Fig. 3.4 and considering the force equilibrium we have the following relations.

$$(1/2)\sigma_{ct} \cdot k \cdot t_c = t_s \cdot \sigma_{st} = p \cdot t_c \cdot \sigma_{st}$$

$$\therefore \sigma_{st} = \left(\frac{\sigma_{ct} \cdot k}{2p} \right) = m \cdot \sigma_{ct} \left(\frac{at_c - kt_c}{kt_c} \right) = m\sigma_{ct} \frac{(a-k)}{k}$$

$$\therefore k^2 = 2pm(a-k) \text{ and}$$

$$k = -mp + \sqrt{2mpa + p^2m^2} \quad (5)$$

Rise in temperature in reinforcement = $(1-a)T$

Free expansion of steel = $(1-a)\alpha T$

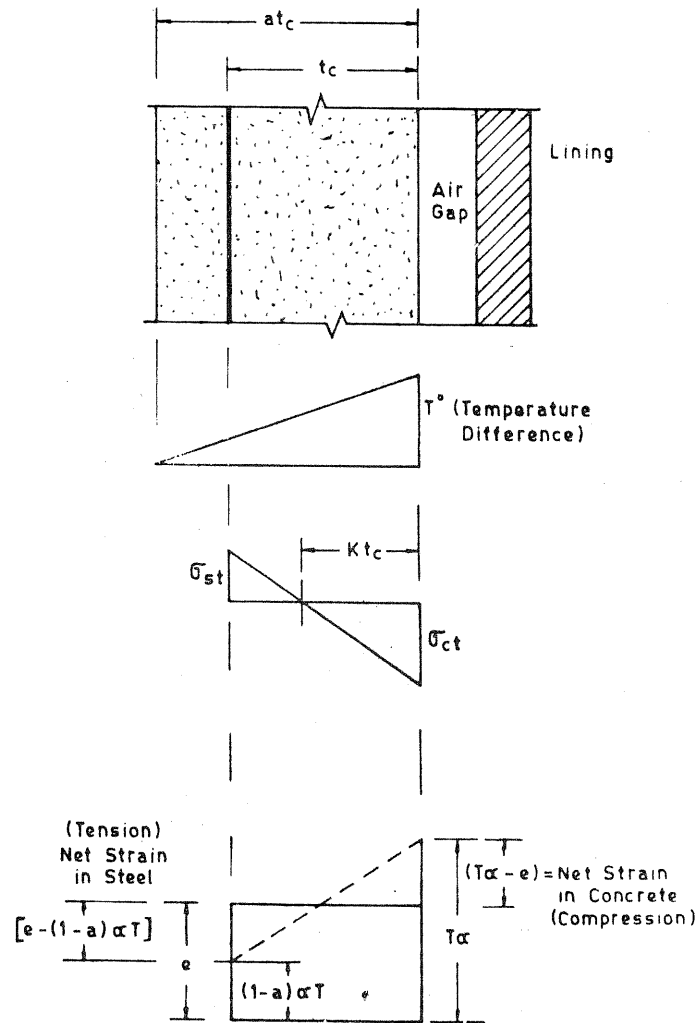


Fig. 3.4 Temperature-Stresses in Chimney Walls

The tensile stress in steel is due to the difference between that due to strain e and due to temperature rise $(1-a)T$

$$\therefore \text{Stress in steel } \sigma_{st} = E_s[e - (1-a)\alpha T]$$

At the neutral axis, there is free expansion due to strain e

$$e = (1-k)\alpha T$$

$$\therefore \text{Stress in steel } \sigma_{st} = E_s[(1-k)\alpha T - (1-a)\alpha T]$$

$$\therefore \sigma_{st} = E_s \cdot \alpha \cdot T(a-k) \quad (6)$$

$$\text{Stress in concrete } \sigma_{ct} = E_c(T\alpha - e) = E_c[T\alpha - (1-k)\alpha T]$$

$$\therefore \sigma_{ct} = E_c \cdot \alpha k \cdot T \quad (7)$$

3.6.2 Combined Effect of Wind Loads, Self Weight and Temperature on Stresses in Chimney Walls

The stresses developed at the neutral axis, compression zone (Leeward side) and tension zone (Wind ward side) of the chimney due to the combined effect of wind loads, self weight and temperature will be examined separately.

Case 1. Stresses at Neutral axis

There are no stresses at the neutral axis due to the external loads. But there are stresses developed due to temperature difference

$$\text{Stress in steel} = E_s \alpha T(a - k) \quad (6)$$

$$\text{Stress in concrete} = E_c \alpha k T \quad (7)$$

Case 2. Stresses in Compression Zone (Leeward side)

The Leeward side of the chimney is under compression due to the effect of wind loads and self weight. If the temperature stresses are superposed, the final effect will be to increase the compressive stress in concrete and decrease the stress in steel.

Referring to Fig. 3.5 and using the following notations:

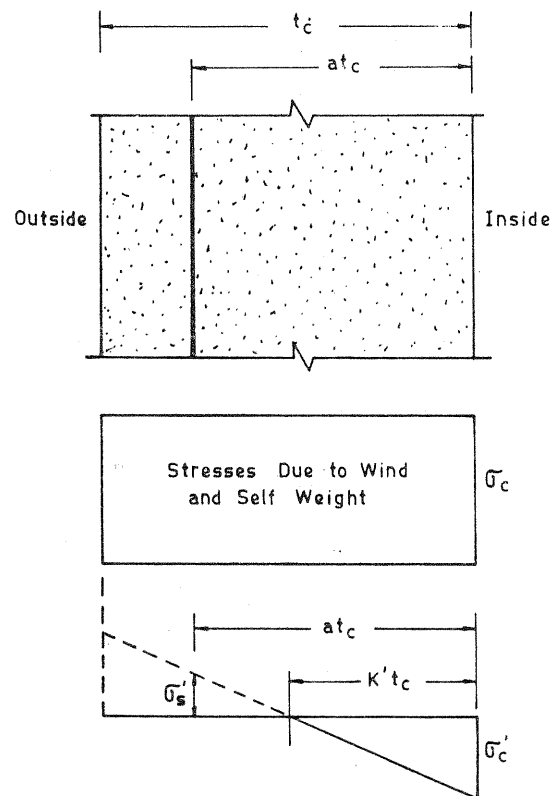


Fig. 3.5 Temperature Stresses, Compression Zone (Leeward Side)

σ_c = Compressive stress in concrete assumed uniform due to the effect of self weight and wind loads

$k't_c$ = Position of neutral axis

σ'_c = Compressive stress in concrete due to the combined effect

σ'_s = Stress in steel due to the combined effect

e = Strain due to temperature difference of T^0

Total compression will remain unchanged in the section.

Considering the force equilibrium, we have

$$\sigma_c \cdot t_c + (m-1)t_s \cdot \sigma_c = \left(\frac{1}{2}\sigma'_c \cdot k't_c\right) - t_s \cdot \sigma'_s$$

If

$$t_s = p \cdot t_c$$

$$\sigma_c \cdot t_c + (m-1)p \cdot t_c \sigma_c = \left(\frac{1}{2}\sigma'_c \cdot k' \cdot t_c\right) - p \cdot t_c m \sigma'_c \left(\frac{a-k'}{k'}\right)$$

$$\therefore \sigma_c \cdot t_c [1 + (m-1)p] = \sigma'_c \cdot t_c \left[k'/2 - mp \left(\frac{a-k'}{k'}\right) \right]$$

$$\therefore \sigma_c = \left\{ \frac{\sigma_c [1 + (m-1)p]}{k'/2 - mp \left(\frac{a-k'}{k'}\right)} \right\} \quad (8)$$

$$\text{Change in stress in concrete at inside face} = (\sigma'_c - \sigma_c) \quad (a)$$

$$\therefore (\sigma'_c - \sigma_c) = E_c(T\alpha - e)$$

$$\text{Change of stress in steel} = (\sigma'_s + m\sigma_c)$$

$$\therefore (\sigma'_s + m\sigma_c) = E_s[e - (1-a)\alpha T] \quad (b)$$

$$\text{But } \sigma'_s = \sigma'_c m \left(\frac{a-k'}{k'}\right)$$

$$\therefore \left[\frac{\sigma'_c m \left(\frac{a-k'}{k'}\right) + m \cdot \sigma_c}{E_s} \right] + (1-a)\alpha T = e \quad (9)$$

Also from Eq. (a), we have

$$T\alpha - \frac{(\sigma'_c - \sigma_c)}{E_c} = e \quad (d)$$

Equating Eqs. (c) and (d), we get

$$\sigma'_c = \left[\frac{E_c \cdot a \cdot T\alpha}{1 + \frac{(a-k')}{k'}} \right] \quad (9)$$

Equating Eqs. (8) and (9)

$$\frac{\sigma_c [1 + (m-1)p]}{k'/2 - mp \left(\frac{a-k'}{k'}\right)} = \left[\frac{E_c \cdot a \cdot T \cdot \alpha}{1 + \frac{(a-k')}{k'}} \right] \quad (10)$$

For given values of the various variables, we can evaluate k' from Eq. (10) and then compute σ_c from Eq. (9) and σ'_s , the stress in steel.

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If k' is more than unity, the whole thickness of concrete t_c will be in compression and the stresses can be analysed using the same procedure.

Case 3. Stresses in Tension zone (Wind ward side)

The chimney section in the wind ward zone is in tension due to the effect of self weight and wind loads. Concrete is assumed to take negligible tension and hence the whole tension is resisted by steel.

Referring to Fig. 3.6 and using the following notations:

σ_s = Tensile stress in steel due to the wind loads and self weight

$k't_c$ = position of neutral axis

σ_c = Compressive stress in concrete

σ_s = Stress in steel due to the combined effect

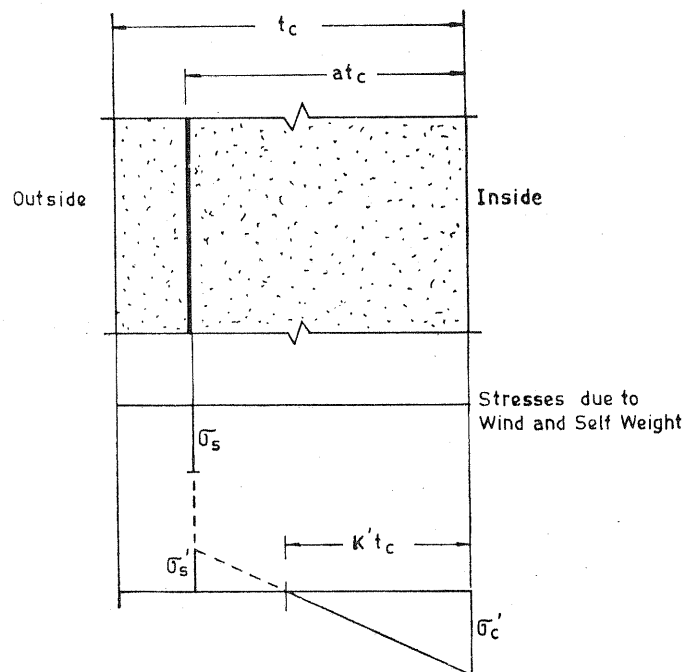


Fig. 3.6 Temperature Stresses, Tension Zone (Wind Ward Side)

The effect of temperature is to develop compressive stress in concrete and increase the tensile stress in steel.

The total force in the section remains unchanged, considering the equilibrium of forces

$$t_s \cdot \sigma_s = (t_s \cdot \sigma_s' - \frac{1}{2} \sigma_c' k' t_c)$$

But
$$\sigma_s' = m \sigma_c' \left(\frac{a - k'}{k'} \right) \quad \text{and} \quad t_s = p \cdot t_c$$

$$\begin{aligned}
 \therefore p \cdot t_c \sigma_s &= p \cdot t_c m \cdot \sigma'_c \left(\frac{a - k'}{k'} \right) - \frac{1}{2} \cdot \sigma'_c k' t_c \\
 \therefore \sigma'_c &= \left\{ \frac{p \cdot \sigma_s}{pm \left(\frac{a - k'}{k'} \right) - \frac{k'}{2}} \right\} \quad (11)
 \end{aligned}$$

Change in strain in concrete at inside face is

$$\begin{aligned}
 &= \left[\frac{\sigma_s}{E_s} + \frac{\sigma'_c}{E_c} \right] \\
 \therefore \left[\frac{\sigma_s}{E_s} + \frac{\sigma'_c}{E_c} \right] &= (T \cdot \alpha - e) \\
 \therefore e &= \left[T \alpha - \frac{\sigma_s}{E_s} - \frac{\sigma'_c}{E_c} \right] \quad (a)
 \end{aligned}$$

Change of strain in steel is given by,

$$\begin{aligned}
 \left(\frac{\sigma'_s - \sigma_s}{E_s} \right) &= [e - (1 - a) \alpha T] \\
 \therefore e &= \left(\frac{\sigma'_s - \sigma_s}{E_s} \right) + (1 - a) \alpha T \quad (b)
 \end{aligned}$$

Equating the Eqs. (a) and (b) we have the relation,

$$\sigma'_c = [a \cdot \alpha T E_c - \sigma'_s / m]$$

substituting for σ'_s and simplifying

$$\sigma'_c = \alpha T E_c k' \quad (12)$$

Equating the Eqs. (11) and (12), we have

$$\left[\frac{p \cdot \sigma_s}{pm \left(\frac{a - k'}{k'} \right) - k'/2} \right] = \alpha T \cdot E_c k' \quad (13)$$

The value of k' can be evaluated using the known values of the other variables and then the values of the stress in concrete σ'_c and the stress in steel σ'_s can be estimated using relevant equations.

3.6.3 Stresses in Horizontal Reinforcement due to Temperature Difference

At high temperatures, the inner surface of the chimney is prevented from expansion and therefore gets compressed. The outer surface will expand more than the natural expansion and will be in tension. Due to temperature stresses, generally the hoops tries to expand and consequently tensile stresses develop in the hoop reinforcement.

Taking unit height of wall and referring to Fig. 3.7 and using the following notations.

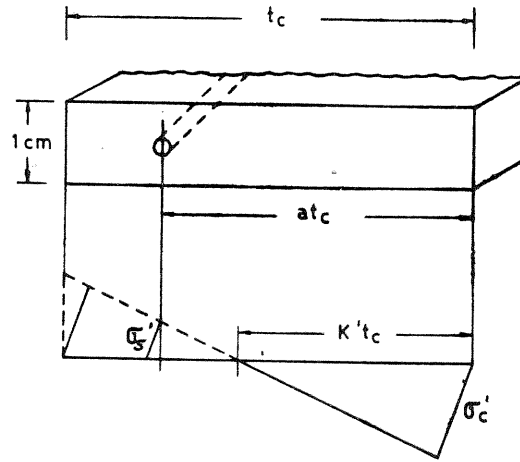


Fig. 3.7 Stresses in Horizontal Reinforcement due to Temperature Difference

$k't_c$ = position of neutral axis

σ'_c = Compressive stress in concrete

σ'_s = Tensile stress in steel

A'_c = Area of hoop reinforcement/unit height

A_s = Cross sectional area of horizontal steel

s = Spacing

$A'_s = (A_s/s) = p \cdot t_c$

$\sigma'_s = m \cdot \sigma'_c (a - k')/k'$ (14)

Consider the force equilibrium of the section, compressive force in concrete on the inner side = tensile force in horizontal reinforcement

$$\therefore \frac{1}{2} \sigma'_c k' t_c = A'_s \sigma'_s = p \cdot t_c m \sigma'_c \left(\frac{a - k'}{k'} \right)$$

$$\therefore k'/2 = mp \left(\frac{a - k'}{k'} \right)$$

$$k' = \sqrt{2pma + p^2 m^2} - pm \quad (15)$$

Using this equation the position of the neutral axis is determined.

Let e = actual strain

stress in concrete $= \sigma'_c = (\alpha T - e) E_c$

stress in steel $= \sigma'_s = [e - (1 - a)\alpha T] E_s$

From these two relations,

$$e = [\alpha T - \sigma'_c/E_c] \text{ and } e = [\sigma'_s/E_s + \alpha T(1 - a)]$$

$$\therefore [\alpha T - \sigma'_c/E_c] = [\sigma'_s/E_s + \alpha T(1 - a)]$$

$$\therefore [\sigma'_s + m\sigma'_c] = E_s \alpha T \cdot a \quad (16)$$

Knowing the value of k' , the stresses in steel and concrete σ'_c and σ'_s can be obtained by solving Eqs. (14) and (16).

3.7 EXAMPLE

A reinforced concrete chimney 50 m high above ground has an outside diameter of 4 m. The thickness of the shell is 20 cm at the top and it is increased to 25 cm and 30 cm at 18 m and 30 m from the top. Vertical steel bars = 1 percent of the cross sectional area throughout. The total wind load above the section at 18 m from top may be taken as 93 kN. Find the stresses developed due to wind and dead loads at the Section 18 m from the top of the top of the chimney. Assume modular ratio $m = 13$.

Weight of concrete shell $= (\pi \times 3.8 \times 0.2 \times 18 \times 24)$

$$W = 1031 \text{ kN}$$

$$A_s = 0.01 \times \pi(2^2 - 1.8^2)10^4 = 240 \text{ cm}^2$$

$$t_s = \left(\frac{A_s}{2\pi R} \right) = \left(\frac{240}{2 \times \pi \times 190} \right) = 0.20 \text{ cm}$$

Equivalent second moment of area

$$\begin{aligned} I_c &= [\pi/64(D^4 - d^4) + (m-1)\pi r^3 t_s] \\ &= [\pi/64(400^4 - 360^4) + (13-1)\pi \times 190^3 \times 0.2] \\ &= 480 \times 10^6 \text{ cm}^4 \end{aligned}$$

Equivalent area $= A_e = \pi/4(D^2 - d^2) + (m-1)A_s$

$$A_e = \pi/4(400^2 - 360^2) + (13-1)240 = 26,300 \text{ cm}^2$$

Horizontal wind load $= P = 93 \text{ kN}$

Moment due to wind load at a Section 18 m from top

$$= M = (93 \times 9) = 837 \text{ kN.m.}$$

$\therefore$ eccentricity $e = (M/W) = (837/1031) = 0.81 \text{ m}$

For no tension to develop

$$e_{\max} = \left(\frac{2I}{AD} \right) = \left(\frac{2 \times 480 \times 10^6}{26,300 \times 400} \right) = 91 \text{ cm}$$

Hence the entire section is in compression

Maximum compressive stress in concrete

$$\sigma_c = \left[\frac{W}{A} + \frac{My}{I} \right] = \left[\frac{1031 \times 10^3}{26,300 \times 10^2} + \frac{837 \times 10^6 \times 2000}{480 \times 10^{10}} \right] = 0.74 \text{ N/mm}^2$$

Compressive stress in steel $= m \cdot \sigma_c = (13 \times 0.74) = 9.62 \text{ N/mm}^2$

3.8 DESIGN EXAMPLE

A concrete chimney of height 80 m with the external diameter of the shaft being 4 m at top and 5 m at bottom is required in a place where the wind

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intensity is 1.5 kN/m^2 . Thickness of fire brick lining = 10 cm. Temperature difference between the inside and outside of shift = 75°C . Permissible bearing pressure on soil at site = 150 kN/m^2 .

Adopt M-25 grade concrete mix and tor steel Fe = 415 grade and design the following,

- (a) Base section of the chimney
- (b) Foundation for the chimney.
- (1) *Permissible Stesses*

$$\begin{aligned} \text{M-25 grade concrete} & \quad \therefore \sigma_{cb} = 8.5 \text{ N/mm}^2 \\ \text{Fe = 415 grade steel} & \quad m = 11 \\ & \quad \sigma_{st} = 230 \text{ N/mm}^2 \end{aligned}$$

- (2) *Loads* Referring to Fig. 3.8

$$\text{Weight of chimney} = (\pi \times 4.2 \times 0.3)80 \times 24 = 7600 \text{ kN}$$

$$\begin{aligned} \text{Weight of fire brick lining (10 cm thick)} \\ = (\pi \times 3.8 \times 0.1)80 \times 20 = 1920 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{Total wind load above base} &= [(0.7 \times 1.5 \times (4 + 5)/2 \times 80)] = 375 \text{ kN} \\ \text{acting at a height of 40 m above base} \end{aligned}$$

$$\text{Total dead load above base (W)} = (7600 + 1920) = 9520 \text{ kN}$$

Bending moment at base due to wind loads

$$M = (375 \times 40) = 15,000 \text{ kN}\cdot\text{m}$$

$$\text{Eccentricity } e = \left(\frac{M}{W} \right) = \left(\frac{15,000}{9520} \right) = 1.575 \text{ m}$$

- (3) *Reinforcements*

Providing reinforcements of 1% of the cross sectional area,

$$A_{st} = 0.01(\pi \times 4600 \times 400) = 57,805 \text{ mm}^2$$

Using 25 mm Φ bars

$$\text{Number of bars} = \left(\frac{57,805}{491} \right) = 117$$

provide 120 bars of 25 mm diameter bars

Equivalent thickness of steel ring is given by

$$t_s = \left(\frac{A_{st}}{\pi d_m} \right) = \left(\frac{57,805}{\pi \times 4600} \right) = 4 \text{ mm}$$

- (4) *Analysis of Stresses at Base Section*

Referring to Fig. 3.8, If α = angle subtended by the neutral axis at the centre, the eccentricity is written as,

$$e = R \left\{ \frac{\left[(t_c - t_s) \left(\frac{\sin 2\alpha}{4} + \frac{\pi - \alpha}{2} \right) + \frac{m \cdot t_s \pi}{2} \right]}{(t_c - t_s) \{ \sin \alpha + (\pi - \alpha) \cos \alpha \} + m \cdot t_s \cdot \cos \alpha} \right\}$$

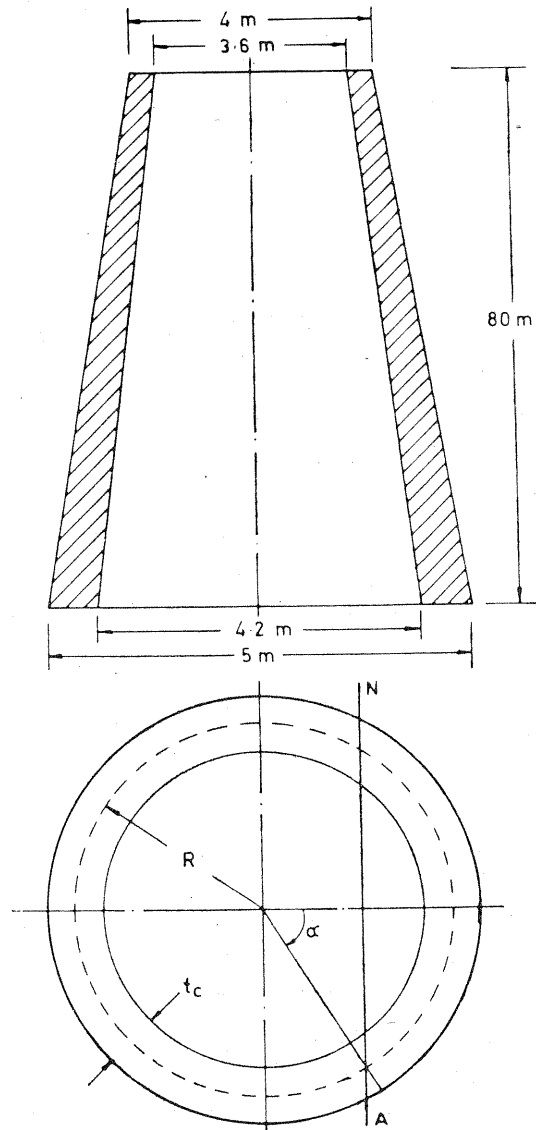


Fig. 3.8 Details of R.C. Chimney

In this equation

$$e = 1575 \text{ mm}$$

$$t_s = 4 \text{ mm}$$

$$R = 2300 \text{ mm}$$

$$t_c = 400 \text{ mm}$$

Trial values of α (varying from 70° to 80°C)

Assumed until the value of calculated eccentricity coincides with the actual value of $e = 1575 \text{ mm}$.

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Trial 1: Assume $\alpha = 70^\circ$

The right hand side of equation = $e = 1742$ mm

which is greater than 1575 mm

Hence reduce the value of α for the second trial.

Trial 2: Assume $\alpha = 60^\circ$

$$(\pi m t_s \cdot \cos \alpha) = (\pi \times 11 \times 4 \times 0.5) = 69.11$$

$$\left(\frac{\sin 2\alpha}{4} \right) = 0.216$$

$$\left(\frac{\pi - \alpha}{2} \right) = \left(\frac{\pi - 1.047}{2} \right) = 1.047$$

$$\sin \alpha = 0.866$$

$$(\pi - \alpha) \cos \alpha = 1.045$$

$$\text{R.H.S} = 2300$$

$$\left\{ \frac{[(400 - 4) \left(\frac{\sin 120}{4} + \frac{\pi - 1.047}{2} \right) + \frac{11 \times 4 \times \pi}{2}]}{(400 - 4) \{ \sin 60 + (\pi - 1.047) \cos 60 \} + \pi \times 11 \times 4 \times \cos 60} \right\}$$

$$= 1586 \text{ mm} \approx 1575 \text{ mm}$$

Hence the value of angle $\alpha = 60^\circ$

Using this value of α in Eq. (1) given by,

$$W = \left[\frac{2R\sigma_c}{1 + \cos \alpha} \right] [t_c - t_s] \{ \sin \alpha + (\pi - \alpha) \cos \alpha \} + \pi m \cdot t_s \cdot \cos \alpha$$

$$9520 \times 10^3 = \left[\frac{2 \times 2300 \sigma_c}{1 + \cos 60^\circ} \right] [(400 - 4) \{ \sin 60 + (\pi - 1.047) \times \cos 60^\circ \} + \pi \times 11 \times 4 \times \cos 60]$$

$$\text{solving stress in concrete } \sigma_c = 3.75 \text{ N/mm}^2$$

$$\text{Stress in steel } \sigma_s = m \cdot \sigma_c \left[\frac{R(1 - \cos \alpha)}{R(1 + \cos \alpha)} \right]$$

$$= 11 \times 3.75 \left[\frac{(1 - \cos 60)}{1 + \cos 60} \right] = 13.73 \text{ N/mm}^2$$

The stresses in concrete and steel are within safe permissible limits.

5. Design of Hoop Reinforcement

Shear at the base of chimney = 375 kN

Mean diameter at base = 4600 mm

using 10 mm diameter hoops at 200 mm centres, using Eq. 4

$$\text{Stress in steel } = \sigma_s = \left(\frac{\text{H.S}}{1.6 A_t \cdot d} \right) = \left(\frac{375 \times 10^3 \times 200}{1.6 \times 79 \times 4600} \right)$$

$$= 129 \text{ N/mm}^2 < 230 \text{ N/mm}^2$$

Hence stresses are within permissible limits.

6. *Temperature Stresses (Combined effect of wind loads, self weight and temperature)*

(a) *Compression Zone (Leeward side)*

Providing an effective cover of 50 mm to steel

$$t_c = 400 \text{ mm} \quad t_s = 4 \text{ mm}$$

$$at_c = 350 \text{ mm} \quad \therefore a = 0.875$$

$$p = (t_s/t_c) = (4/400) = 0.01$$

$$T = 75^\circ\text{C}$$

$$\alpha = 11 \times 10^{-6}/^\circ\text{C} \quad m = 11$$

$$E_c = (E_s/m) = \left(\frac{210 \times 10^3}{11} \right)$$

$$\sigma_c = 3.75 \text{ N/mm}^2$$

Using Eq. 10

$$\frac{\sigma_c [1 + (m-1)p]}{\left[\frac{k'}{2} - mp \left(\frac{a-k'}{k'} \right) \right]} = \left[\frac{E_c \cdot a T \alpha}{1 + \left(\frac{a-k'}{k'} \right)} \right]$$

$$\frac{3.75 [1 + 10 \times 0.01]}{0.5k' - 11 \times 0.01 \left(\frac{0.875 - k'}{k'} \right)} = \frac{\left(\frac{210 \times 10^3}{11} \times 0.875 \times 75 \times 11 \times 10^{-6} \right)}{\left[1 + \left(\frac{0.875 - k'}{k'} \right) \right]}$$

solving $k' = 0.70$

From Eq. 9, the stress in concrete σ'_c is

$$\sigma'_c = \left[\frac{E_c \cdot a \cdot T \pi}{1 + \frac{a-k'}{k'}} \right] = \frac{\left(\frac{210 \times 10^3}{11} \times 0.875 \times 75 \times 11 \times 10^{-6} \right)}{\left(1 + \frac{0.875 - 0.70}{0.70} \right)} = 11.0 \text{ N/mm}^2$$

The permissible compressive stress for M-25 grade concrete when wind loads are also considered is $(1.33 \times 8.5) = 11.33 \text{ N/mm}^2$.

$$\text{Stress in steel} = \sigma'_s = m \sigma'_c \left(\frac{a-k'}{k'} \right)$$

$$= 11 \times 11.0 \left(\frac{0.875 - 0.70}{0.70} \right) = 30.25 \text{ N/mm}^2 < 230 \text{ N/mm}^2$$

permissible stress

(b) *Tension Zone (Wind ward side)*

Using the Eq. 13

$$\left[pm \left(\frac{a-k'}{k'} \right) - \frac{k'}{2} \right] = \alpha T \cdot E_c k'$$

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$$\frac{0.01 \times 13.73}{\left[0.01 \times 11 \left(\frac{0.875 - k'}{k'}\right) - \frac{k'}{2}\right]} = \left[11 \times 10^{-6} \times 75 \times \frac{210 \times 10^3}{11} \times k'\right]$$

Solving $k' = 0.55$

Stress in concrete $\sigma_c = \alpha T E_c k'$

$$\begin{aligned} \text{(Equation 12)} &= \left(11 \times 10^{-6} \times 75 \times \frac{210 \times 10^3}{11} \times 0.55\right) \\ &= 8.66 \text{ N/mm}^2 < 11.33 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Stress in steel } \sigma_s &= m \cdot \sigma'_c \left(\frac{a - k'}{k'}\right) = 11 \times 8.66 \left(\frac{0.875 - 0.55}{0.55}\right) \\ &= 56.28 \text{ N/mm}^2 < 230 \text{ N/mm}^2 \end{aligned}$$

(c) *Stresses at Neutral Axis*

Using Eq. 5

$$k = -mp + \sqrt{2mpa + p^2m^2}$$

$$p = 0.01 \quad m = 11 \quad a = 0.875$$

$$k = -11 \times 0.01 + \sqrt{2 \times 11 \times 0.01 \times 0.875 + (0.01)^2(11)^2} = 0.342$$

Stress in concrete $= \sigma_{ct} = E_c \alpha k T$

(Equation 7)

$$= \left[\frac{210 \times 10^3}{11} \times 11 \times 10^{-6} \times 0.342 \times 75\right] = 5.386 \text{ N/mm}^2$$

Stress in steel $= \sigma_{st} = E_s \cdot \alpha T(a - k)$

(Equation 6)

$$\begin{aligned} &= 210 \times 10^3 \times 11 \times 10^{-6} \times 75(0.875 - 0.342) \\ &= 92.34 \text{ N/mm}^2 \end{aligned}$$

The stresses are within safe permissible limits

7. *Stresses in Hoop Steel due to Temperature*

Hoop steel of 10 mm diameter at 200 mm centres provided at base section.

$$p = \left(\frac{As}{s \cdot t_c}\right) = \left(\frac{79}{200 \times 400}\right) = 0.00098$$

$$a = 0.875 \quad m = 11$$

Using Eq. 15

$$\begin{aligned} k' &= \sqrt{2pma + p^2m^2} - pm \\ &= \sqrt{2 \times 0.00098 \times 11 \times 0.875 + (0.00098)^2(11)^2} - (0.00098 \times 11) \\ &= 0.127 \end{aligned}$$

using Eqs. 14 and 16 we have

$$\sigma'_s = m \cdot \sigma'_c \left(\frac{a - k'}{k'}\right) = 11 \times \sigma'_c \left(\frac{0.875 - 0.127}{0.127}\right) = 60.45 \sigma'_c$$

Also $(\sigma'_s + m\sigma'_c) = E_s \alpha T \cdot a$

$\therefore (60.45\sigma'_c + 11\sigma'_c) = (210 \times 10^3 \times 11 \times 10^{-6} \times 75 \times 0.875)$

solving $\sigma'_c = 2.12 \text{ N/mm}^2$

and $\sigma'_s = 128.25 \text{ N/mm}^2$

Total stress in hoop steel = (stress due to shear)
 + (stress due to temperature difference)
 $= (129 + 128.15)$
 $= 257.25 \text{ N/mm}^2 > 230 \text{ N/mm}^2$

Hence the spacing of the hoop reinforcement can be reduced to 150 mm instead of 200 mm centres.

8. Design of Foundations

A circular R.C. slab foundation is designed for the chimney.

Total vertical load on base = 9520 kN

Bending moment = 15,000 kN·m

Allowable bearing pressure = 150 kN/m²

Self weight of footing (assumed at 10%) = 950 kN

Total load on soil = (9520 + 950) = 10,470 kN

If D = diameter of the circular footing for no tension to develop,

$$\left(\frac{W}{A}\right) = \left(\frac{M}{Z}\right) \therefore \left(\frac{10,470}{\frac{\pi D^2}{4}}\right) = \left(\frac{15,000}{\frac{\pi D^3}{32}}\right) \text{ solving } D = 11.4 \text{ m}$$

The loading on the base is taken as annular loading on the mean diameter. The bending moments in the base obtained by superposing the two types of loading shown in Fig. 3.9.

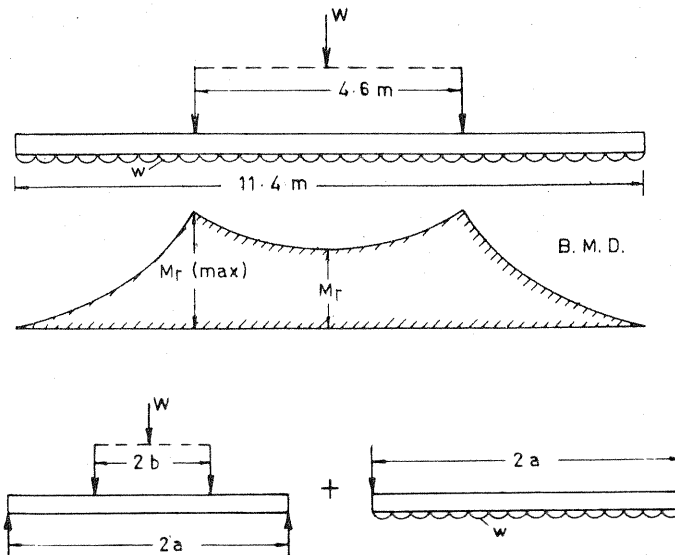


Fig. 3.9 Bending Moments in Circular Foundation Footing

$$\text{Intensity of soil pressure } (w) = \left[\frac{10,470}{\pi \times 11.4^2} \right] = 103 \text{ kN/m}^2$$

$$2a = 11.4 \text{ m} \quad 2b = 4.6 \text{ m}$$

Maximum bending moment in the section is governed by the radial moment.

M_r = bending moment at centre of footing

$$\begin{aligned} &= \frac{W}{8\pi} \left[2 \log_e (a/b) + 1 - (b/a)^2 \right] \frac{3}{16} w \cdot a^2 \\ &= \frac{10,470}{8\pi} \left[2 \log_e \left(\frac{5.7}{2.3} \right) + 1 - \left(\frac{2.3}{5.7} \right)^2 \right] - \frac{3}{16} \times 103 \times 5.7^2 \\ &= 476.28 \text{ kN} \cdot \text{m/m} \end{aligned}$$

$M_{r(\max)}$ = Moment at junction of footing and chimney walls at a radius of 2.3 m.

$$\begin{aligned} &= \frac{W}{8\pi} \left[2 \log_e (a/b) + 1 - (b/a)^2 \right] - \frac{3}{16} w (a^2 - b^2) \\ &= \frac{10,470}{8\pi} \left[2 \log_e \left(\frac{5.7}{2.3} \right) + 1 - \left(\frac{2.3}{5.7} \right)^2 \right] - \frac{3}{16} \times 103 (5.7^2 - 2.3^2) \\ &= 578.38 \text{ kN} \cdot \text{m/m} \end{aligned}$$

Using M-20 grade concrete Fe-415 Tor steel

$$\text{Effective depth } d = \sqrt{\frac{578.38 \times 10^6}{0.897 \times 10^3}} = 802 \text{ mm}$$

Adopt an effective depth $d = 800 \text{ mm}$ and
over all depth $h = 850 \text{ mm}$

$$A_{st} = \left(\frac{578.38 \times 10^6}{230 \times 0.9 \times 800} \right) = 3492 \text{ mm}^2$$

$$\text{spacing of 25 mm diameter bars} = \left(\frac{1000 \times 491}{3492} \right) = 140 \text{ mm}$$

Provide 25 mm bars at 140 mm centres in perpendicular directions both ways as shown in Fig. 3.10. Also provide 12 mm diameter bars both ways at top footing.

3.9 DESIGN EXAMPLE

A circular R.C. chimney has a constant shell thickness of 300 mm and an external diameter of 4 m. The section is reinforced with one percent steel located at 50 mm from the outer face. The temperature different between the inside and outside face of concrete is 70°C.

Modulus of elasticity of steel = 210 kN/mm²

Modulus of elasticity of concrete = 19 kN/mm²

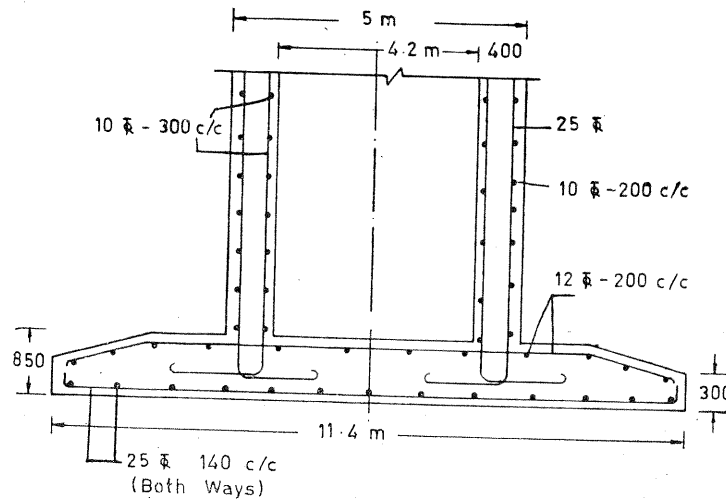


Fig. 3.10 Reinforcement Details in Chimney and Foundations

Coefficient of expansion of concrete and steel $= 11 \times 10^{-6}/^{\circ}\text{C}$
 Compute the stresses developed in concrete and steel due to the temperature gradient.

1. Shell Properties

$$t_c = 300 \text{ mm} \quad A_{st} = 0.01(\pi \times 3700 \times 300) = 34,872 \text{ mm}^2$$

$$t_s = \left(\frac{A_{st}}{\pi d_m} \right) = \left(\frac{34,872}{\pi \times 3700} \right) = 3 \text{ mm}$$

$$m = (E_s/E_c) = (210/19) = 11$$

$$p = (t_s/t_c) = (3/300) = 0.01$$

$$a t_c = 250 \text{ mm}$$

$$\therefore a = \left(\frac{250}{300} \right) = 0.833$$

If k = neutral axis depth constant

$$k = -mp + \sqrt{2mpa + p^2 m^2}$$

$$\therefore k = (-11 \times 0.01) + \sqrt{(1 \times 11 \times 0.01 \times 0.833) + (0.01)^2 \times (0.833)^2}$$

$$= 0.338$$

$$\begin{aligned} \text{stress in steel } \sigma_{st} &= E_s \alpha T(a - k) \\ &= 210 \times 10^3 \times 11 \times 10^{-6} \times 70(0.833 - 0.338) \\ &= 80.04 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{stress in concrete } \sigma_{ct} &= E_c \cdot \alpha \cdot k T \\ &= (19 \times 10^3 \times 11 \times 10^{-6} \times 0.338 \times 70) = 4.944 \text{ N/mm}^2 \end{aligned}$$

3.10 DESIGN OF REINFORCEMENTS IN CHIMNEYS USING CHARTS

Design the vertical reinforcements at the base section of a chimney using the following data and the design chart shown in Fig. 3.11.

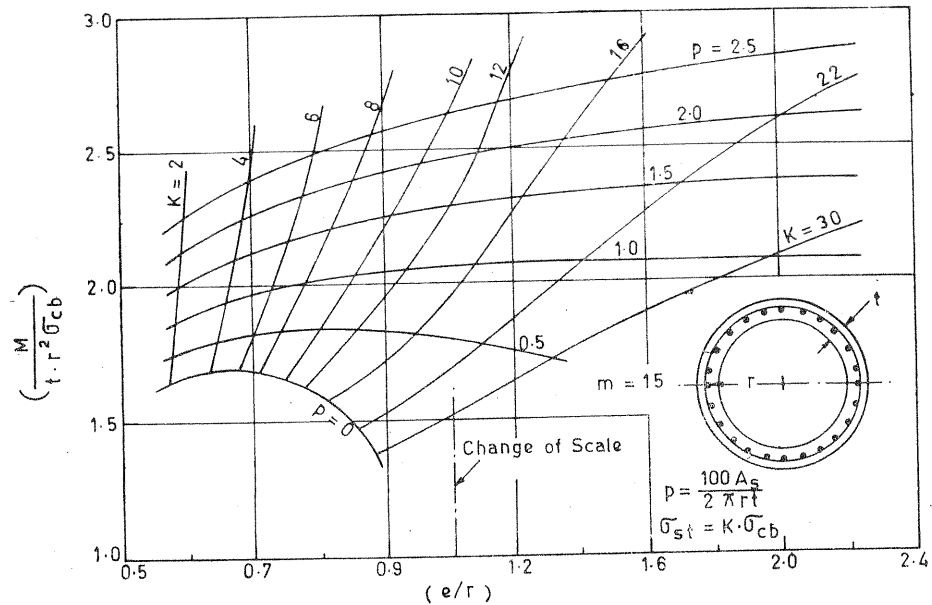


Fig. 3.11 Interaction Diagram for Circular Chimney Shell in Bending

Total vertical load at base section = $W = 10,000$ kN

Bending moment at base section = $M = 17,000$ kN·m

Thickness of chimney shell = $t = 300$ mm

Mean radius of the shell = $r = 2$ m

Concrete M-20 grade and Grade-I—mild steel

The following parameters are computed for using the chart.

$$\text{eccentricity} = e = \left(\frac{M}{W} \right) = \left(\frac{17,000}{10,000} \right) = 1.7$$

$$(e/r) = \left(\frac{1.7}{2.0} \right) = 0.85 \quad \sigma_{cb} = 7 \text{ N/mm}^2 = 7000 \text{ kN/m}^2$$

$$\left(\frac{M}{t \cdot r^2 \cdot \sigma_{cb}} \right) = \left(\frac{17,000}{0.3 \times 2^2 \times 7000} \right) = 2.02$$

From the chart the value of percentage of reinforcement p is obtained as.

$p = 1.0\%$ and $K = 10$

stress in tensile steel = $K \cdot \sigma_{cb} = (10 \times 7) = 70 \text{ N/mm}^2$

3.11 ANALYSIS AND DESIGN OF R.C. CHIMNEYS SUBJECTED TO DYNAMIC LOADS

Tall and slender structures like chimneys are subjected to severe wind loads and earth quake forces. Hence a dynamic load analysis of the structure is essential to study the performance of the structure under dynamic loads. The dynamic analysis for wind loads involves the computation of the natural frequencies and fundamental periods of the structure using the several empirical formulas suggested in the Indian Standard Codes IS: 1893-1975, IS: 4998 (Part-1 1975), ACI-505 and the methods proposed by Housner, Keightly and Mitchell.

Due to air pollution control, tall chimneys in the range of 300 to 400 m height have been constructed. The advent of modern high speed digital computers has made it possible to analyse the tall chimneys subjected to dynamic loads and to compute stresses for various loading conditions in a rigorous and at the same time accurate and economical methods. Typical investigations of the dynamic analysis of a chimney stack of height 180 m and outer diameter varying from 6 m at top to 15 m at bottom and the thickness of concrete shell being 22 cm at top and 40 cm at bottom, when subjected to wind loads varying from 2 kN/m² at bottom to 2.88 kN/m² at top indicates that the design bending moments under dynamic loads are considerably higher than the static load computations. Under dynamic analysis, the permissible stresses in concrete are restricted to $0.4 f_{ck}$ and the permissible stresses in steel are limited to $0.6 f_{sy}$. A separate analysis is made to calculate the temperature stresses.

3.12 EXAMPLES FOR PRACTICE

1. A reinforced concrete chimney having a wall thickness of 15 cm has a mean diameter of 2.5 m. The section is reinforced with sixty bars of 16 mm diameter. If the effective wind pressure is 1.4 kN/m² on the projected area, evaluate the maximum stresses in concrete and steel at a section 25 m from the top of the chimney. Assume modular ratio = 15.
2. A reinforced concrete chimney of 60 m height above ground level has an outside diameter of 4 m. Thickness of fire brick lining provided up to 40 m above ground level is 10 cm. Density of fire brick lining = 20 kN/m³. The lining is supported at every 6 m intervals. The thickness of concrete shell is 200 mm at the top and is increased to 250 mm and 300 mm at 24 m and 48 m from the top respectively. Vertical steel bars of 1% of cross section is provided throughout the chimney. Wind load is 1.65 kN/m² for the top 24 m and 1.5 kN/m² over the rest of the chimney height. Temperature difference between inside and outside of concrete shell = 70° C. Coefficient of expansion of concrete and steel = $11 \times 10^{-6}/^{\circ}\text{C}$. Modular ratio = 11.

Using M-25 grade concrete and Grade-I mild steel design the chimney and check for stresses at the base section. Also design suitable foundations for the chimney assuming the safe bearing capacity of the soil as 150 kN/m^2 .

3. A circular R.C. chimney 50 m high above ground has a constant shell thickness of 300 mm and an external diameter of 4 m. The effective wind pressure on the projected surface is 2 kN/m^2 . The section is reinforced with 1 percent steel in the vertical direction. If modular ratio = 11, find the position of the section from the top where the resultant stress distribution due to dead and wind loads is such that there is maximum compressive stress in concrete at the Leeward side and zero stress at the wind ward face. Evaluate the maximum stresses in concrete and steel at this section.
4. A reinforced concrete chimney 100 m high above ground has an external diameter 4 m at the top and 5 m at the ground level. The thickness of concrete shell varies from 200 mm at the top to 400 mm at the bottom. The wind pressure at site may be taken as 2 kN/m^2 . Assuming a modular ratio of 15, design suitable reinforcements in the shell walls.

Adopt M-15 grade concrete and Grade-I steel.

4

Curved Beams

4.1 GENERAL

Beams curved in plan are often used to support the circular walls of reinforced concrete water tanks, curved balconies and circular domes. The curved beams are generally supported on columns spaced at regular intervals. Since the loads and reactions do not lie along the axis at any point of the beam, torsional moments develop at certain cross sections of a curved beam. However in the case of circular beams supported by symmetrically placed columns, the vertical reactions are provided by the columns and due to the symmetry, the torsional moments at the centre of the curved beam between any two consecutive supports will be zero. Also the maximum negative bending moment develops at the support sections and positive maximum bending moment at sections in between the supports.

Maximum torsional moments will develop at sections nearer the supports and where the bending moment is zero. In other words, the maximum torque occurs at points of contraflexure. Also the shear forces will be maximum at the support sections.

Hence the support sections have to be designed for maximum negative bending moment and shear and sections where the torque is maximum has to be designed for maximum torsional moment and the corresponding shear force at the section.

4.2 ANALYSIS OF BENDING AND TORSIONAL MOMENTS IN CIRCULAR BEAMS

The bending and torsional moments developed in curved girders can be analyzed by strain energy methods. The magnitude of moments and their

location is influenced by the number of supports and the radius of the curved beam. Figure 4.1, shows a typical curved beam circular in plan supported on eight columns. Referring to Fig. 4.2, the maximum positive and negative bending moments and the torsional moments can be expressed in the following form:

$$\text{Negative maximum bending moment} = K_1 \cdot W \cdot R$$

$$\text{Positive maximum bending moment} = K_2 \cdot W \cdot R$$

$$\text{Maximum torsional moment} = K_3 \cdot W \cdot R$$

where W = total load on the curved beam $= 2\pi R \cdot w$

where w = uniformly distributed load per unit length of beam

θ = Angle subtended at the centre by the ends of the beam

R = Radius of the circular girder

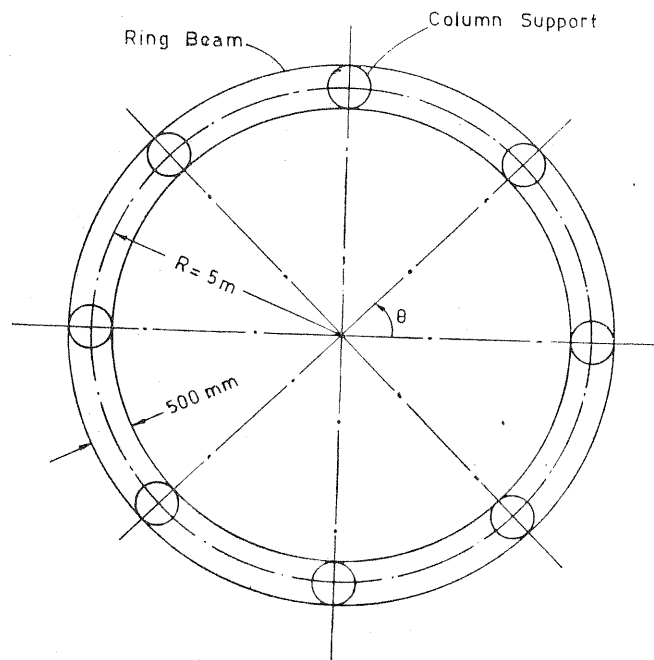


Fig. 4.1 Ring-Girder Supported on Eight Columns

K_1 , K_2 and K_3 are moment coefficients, the values of which are compiled in Table 4.1.

The critical sections to be designed are the support sections subjected to maximum negative and positive bending moments and the section subjected to maximum twisting moment associated with some shear force. At this section, the bending moment is zero. Hence this section has to be designed for combined torsion and shear.

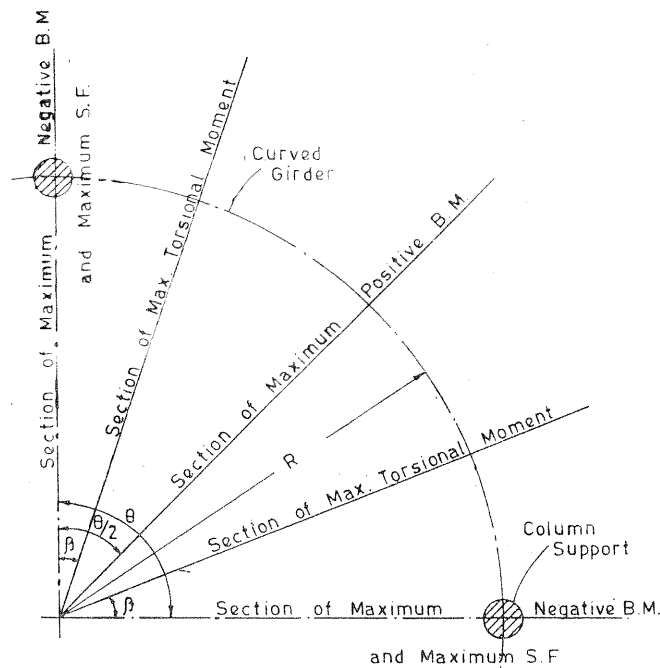


Fig. 4.2 Position of Maximum Moments in Curved Girders

TABLE 4.1 Moment Coefficients in Circular Girders Supported on Columns
Moment Coefficients

Number of columns n		Negative Bending moment at support K_1	Positive Bending moment at centre of spans K_2	Maximum Twisting moment or Torque K_3	Angular distance for maximum torsion
4	90°	0.0342	0.0176	0.0053	19°-12'
5	60°	0.0148	0.0075	0.0015	12°-44'
8	45°	0.0083	0.0041	0.0006	9°-33'
10	36°	0.0054	0.0023	0.0003	7°-30'
12	30°	0.0037	0.0014	0.0017	7°-15'

4.3 MOMENTS IN SEMI CIRCULAR BEAMS SUPPORTED ON THREE COLUMNS

The magnitude and position of maximum positive and negative bending moments and the twisting moments in a semicircular beam supported on three equally spaced supports are given by the following relations:

Maximum positive B.M. = $0.1520 wR^2$ at sections 29°-44' from the end columns.

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Maximum negative B.M. over the central support = $-0.4290 wR^2$

Maximum twisting moment = $0.103wR^2$ at sections $59^\circ-29'$ from the end columns.

4.4 DESIGN EXAMPLE

A circular R.C. girder for the foundation raft of a water tower has a mean diameter of 10 m. The uniformly distributed load transmitted by eight symmetrically placed columns on the girder being 300 kN/m. The width of the beam is 500 mm and the overall depth is 1000 mm. Using M-20 grade concrete and Fe-415 grade for steel, design suitable reinforcements in the circular girder and sketch the details of reinforcements at critical sections.

1. Data

Radius of girder = $R = 5$ m
Width of girder = $b = 500$ mm
Depth of girder = $D = 1000$ mm
Load on girder = $q = 300$ kN/m
Angle = $\theta = 45^\circ$

2. Permissible Stresses

M-20 grade concrete and Fe-415 grade steel

$\sigma_{cb} = 7$ N/mm² $Q = 0.897$
 $\sigma_{st} = 230$ N/mm² $j = 0.906$
 $m = 13$

3. Loads

Self weight of beam = $(0.5 \times 1 \times 24) = 12$ kN/m
uniformly distributed load = 300 kN/m
 $\therefore$ Total design load $w = 312$ kN/m
 $\therefore$ Total load on circular girder = $W = (2\pi R \cdot w) = (2 \times \pi \times 5 \times 312)$
 $= 9802$ kN

4. Bending Moments and Shear Forces

Maximum negative B.M. at support sections

$$M_n = 0.0083 W \cdot R = (0.0083 \times 9802 \times 5) = 407 \text{ kN} \cdot \text{m}$$

Maximum positive B.M. at centre of supports

$$M_p = 0.0041 W \cdot R = (0.0041 \times 9802 \times 5) = 201 \text{ kN} \cdot \text{m}$$

Maximum torsional moment at an angle of 9.5 degrees from support is given by

$$T = 0.0006 W \cdot R = (0.0006 \times 9802 \times 5) = 30 \text{ kN} \cdot \text{m}$$

Shear force at support section is

$$V = \left[\frac{312 \times 5 \times (\pi/4)}{2} \right] = 612 \text{ kN}$$

Shear force at section of maximum torsion is

$$V = \left[612 - \frac{312 \times 5 \times \pi \times 9.5}{180} \right] = 354 \text{ kN}$$

5. Design of Support Section

$$M = 407 \text{ kN} \cdot \text{m}$$

$$V = 612 \text{ kN}$$

$$d = \sqrt{\frac{M}{Q \cdot b}} = \sqrt{\frac{407 \times 10^6}{0.897 \times 500}} = 952 \text{ mm}$$

Adopt effective depth $d = 950 \text{ mm}$

over all depth = 1000 mm

$$A_{st} = \left(\frac{M}{\sigma_{st} \cdot j \cdot d} \right) = \left(\frac{407 \times 10^6}{230 \times 0.9 \times 950} \right) = 2056 \text{ mm}^2$$

Provide 5 bars of 25 Φ ($A_{st} = 2455 \text{ mm}^2$)

$$\tau_v = \left(\frac{V}{bd} \right) = \left(\frac{612 \times 10^3}{500 \times 950} \right) = 1.28 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd} \right) = \left(\frac{100 \times 2455}{500 \times 950} \right) = 0.516$$

From Table 17 of IS: 456 code, permissible shear stress in concrete

$$\tau_c = 0.3 \text{ N/mm}^2 < \tau_v$$

Hence shear reinforcements are required. Shear taken by concrete

$$V_c = (\tau_c \cdot b \cdot d) = \left(\frac{0.3 \times 500 \times 950}{1000} \right) = 142 \text{ kN}$$

Balance shear $= V_s = (612 - 142) = 470 \text{ kN}$

Using 10 mm Φ 2 legged stirrups, the spacing is

$$s_v = \left(\frac{\sigma_{sv} A_{sv} \cdot d}{V_s} \right) = \left(\frac{230 \times 2 \times 79 \times 950}{470 \times 10^3} \right) = 73 \text{ mm}$$

Adopt 10 mm Φ 2 legged stirrups at 70 mm centres.

6. Design of Mid Span Section

Maximum positive moment $M = 201 \text{ kN} \cdot \text{m}$

$$A_s = \left(\frac{201 \times 10^6}{230 \times 0.9 \times 950} \right) = 1022 \text{ mm}^2$$

But minimum area of steel is given by

$$A_s = \left(\frac{0.85bd}{f_y} \right) = \left(\frac{0.85 \times 500 \times 950}{415} \right) = 984 \text{ mm}^2$$

$\therefore$ provide 3 bars of 22 - Φ ($A_{st} = 1140 \text{ mm}^2$)

7. Design of Section Subjected to Maximum Torsion and Shear

$$\begin{aligned} T &= 30 \text{ kN}\cdot\text{m} & D &= 1000 \text{ mm} \\ V &= 354 \text{ kN} & d &= 500 \text{ mm} \\ M &= 0 \end{aligned}$$

$$M_t = T \left[\frac{1 + (D/b)}{1.7} \right] = 30 \left[\frac{1 + (1000/500)}{1.7} \right] = 54 \text{ kN}\cdot\text{m}$$

$$M_{e1} = (M + M_t) = 54 \text{ kN}\cdot\text{m}$$

$$A_{st} = \left(\frac{54 \times 10^6}{230 \times 0.9 \times 950} \right) = 275 \text{ mm}^2$$

But minimum area of tension reinforcement should be not less than that given by,

$$A_s = \left(\frac{0.85bd}{f_y} \right) = \left(\frac{0.85 \times 500 \times 950}{415} \right) = 984 \text{ mm}^2$$

Provide 3 bars of 22 Φ as main tension reinforcement ($A_{st} = 1140 \text{ mm}^2$)

Equivalent shear $= V_e = (V + 1.6 T/b)$

$$= \left(354 + 1.6 \times \frac{30}{0.5} \right) = 453 \text{ kN}$$

$$\tau_{ve} = \left(\frac{V_e}{bd} \right) = \left(\frac{453 \times 10^3}{500 \times 950} \right) = 0.953 \text{ N/mm}^2$$

$$\left(\frac{100A_{st}}{bd} \right) = \left(\frac{100 \times 982}{500 \times 950} \right) = 0.206$$

From Table 17 (IS: 456), $\tau_c = 0.20 \text{ N/mm}^2 < 0.953 \text{ N/mm}^2$

$\therefore$ Shear reinforcements are required.

Using 10 mm two legged stirrups with side covers of 25 mm and bottom covers of 50 mm,

$$b_1 = 450 \text{ mm} \quad d_1 = 900 \text{ mm} \quad A_{sv} = (2 \times 79) \text{ mm}^2$$

$$\begin{aligned} s_v &= A_{sv} \cdot \sigma_{sv} \left[\frac{b_1 d_1}{T} + \frac{2.5 d_1}{V} \right] \\ &= 2 \times 79 \times 230 \left[\frac{450 \times 950}{30 \times 10^5} + \frac{2.5 \times 900}{354 \times 10^3} \right] = 746 \text{ mm} \end{aligned}$$

$$\text{But } A_{sv} < \left(\frac{\tau_{ve} - \tau_c}{s_v} \right) \cdot b \cdot s_v$$

$$\therefore s_v = \frac{A_{sv} \cdot \sigma_{sv}}{(\tau_{ve} - \tau_c) b} = \frac{2 \times 79 \times 230}{(0.953 - 0.20) 500} = 96.5 \text{ mm}$$

Adopt 10 mm Φ - 2 legged stirrups at 90 mm centres. The typical cross sections are shown in Fig. 4.3.

4.5 DESIGN EXAMPLE

Design a semicircular beam supported on 3 columns equally spaced. The centre of the columns are on a curve of diameter 8 m. The superimposed

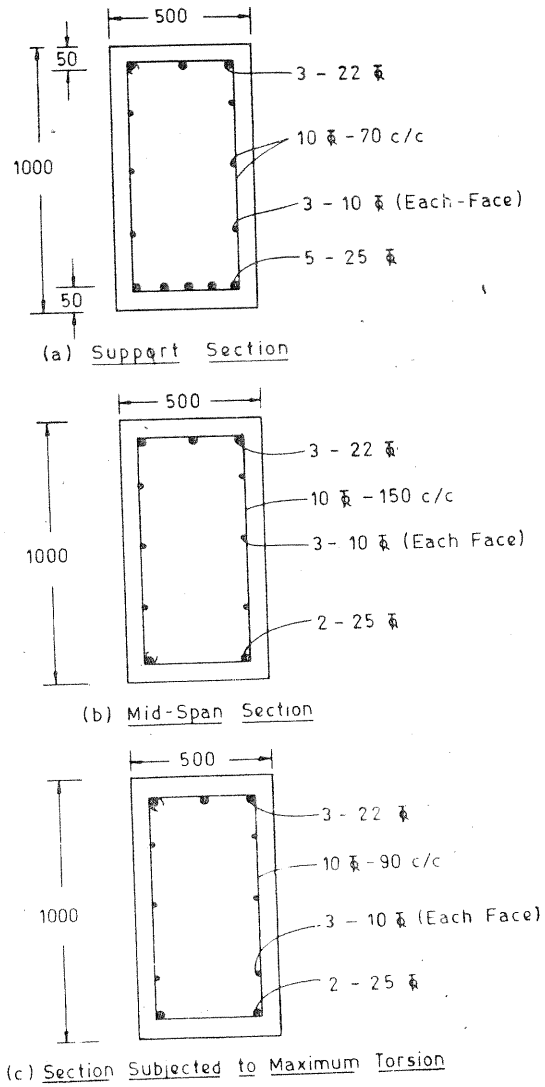


Fig. 4.3 Typical Cross Sections of Circular R C Beam

load on the beam per metre length is 20 kN. Adopt M-20 grade concrete and Fe-415 grade tor steel.

1. *Data*

Radius of girder = $R = 4$ m

Assume width of girder = $b = 400$ mm

Depth of girder = $D = 800$ mm

Live load on girder = $q = 20$ kN/m

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2. Permissible Stresses

M-20 grade concrete and

Fe-415 grade steel

$$\sigma_{cb} = 7 \text{ N/mm}^2$$

$$Q = 0.897$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$j = 0.906$$

$$m = 13$$

3. Loads

$$\text{Self weight of beam} = (0.4 \times 0.8 \times 24) = 7.68 \text{ kN/m}$$

$$\text{Live load on beam} = 20.00 \text{ kN/m}$$

$$\text{Finishes etc.} = 2.32 \text{ kN/m}$$

$$\therefore \text{Total load} = w = 30.00 \text{ kN/m}$$

4. Bending Moments and Shear Forces

$$\text{Maximum negative B.M. at middle support} = -0.429w \cdot R^2$$

$$= (-0.429 \times 30 \times 4^2) \text{ kN} \cdot \text{m} = -206 \text{ kN} \cdot \text{m}$$

$$\text{Maximum positive moment} = 0.152w \cdot R^2 = (0.152 \times 30 \times 4^2) = 73 \text{ kN} \cdot \text{m}$$

$$\text{Maximum torsional moment} = 0.103w \cdot R^2 = (0.103 \times 30 \times 4^2) = 50 \text{ kN} \cdot \text{m}$$

$$\text{Reaction at end supports} = R_1$$

$$\text{Reaction at mid support} = R_2$$

$$\text{Then } R_1 = \frac{wR}{2}(\pi - 2) \quad R_2 = 2w \cdot R$$

$$\text{Shear at end support } R_1 = \frac{wR}{2}(\pi - 2) = \frac{30 \times 4}{2}(\pi - 2) = 69 \text{ kN}$$

$$\text{Shear at mid support section} = \left(\frac{2wR}{2}\right) = wR = (30 \times 4) = 120 \text{ kN}$$

$$\text{Shear at the section of maximum twisting moment} = (R_1 - wR\theta)$$

$$\text{where } \theta = 59^\circ - 39^\circ = (69 - 30 \times 4 \times 1.039) = -55 \text{ kN}$$

5. Design of Mid Support Section

$$\text{Moment } M = 206 \text{ kN} \cdot \text{m} \quad V = 120 \text{ kN}$$

$$d = \sqrt{\frac{M}{Q \cdot b}} = \sqrt{\frac{206 \times 10^6}{0.879 \times 400}} = 757.7 \text{ mm}$$

$$\text{Adopt effective depth} = 760 \text{ mm}$$

$$\text{Over all depth} = 800 \text{ mm}$$

$$A_{st} = \left(\frac{M}{\sigma_{st} \cdot j \cdot d}\right) = \left(\frac{206 \times 10^6}{230 \times 0.9 \times 760}\right) = 1310 \text{ mm}^2$$

$$\text{Provide 4 bars of 22 mm diameter at the top } (A_{st} = 1520 \text{ mm}^2)$$

$$\tau_v = \left(\frac{V}{bd}\right) = \left(\frac{120 \times 10^3}{400 \times 760}\right) = 0.394 \text{ N/mm}^2$$

$$\left(\frac{100A_{st}}{b d^2}\right) = \left(\frac{100 \times 1520}{400 \times 760}\right) = 0.5$$

From Table 17 of IS : 456 code permissible shear stress in concrete $\tau_c = 0.3 \text{ N/mm}^2$, which is less than $\tau_v = 0.394 \text{ N/mm}^2$. Hence shear reinforcements are required. Shear taken by concrete

$$= V_c = \tau_c \cdot b \cdot d = \left(\frac{0.3 \times 400 \times 760}{1000} \right) = 91 \text{ kN}$$

Balance shear $= V_s = (V - V_c) = (120 - 91) = 29 \text{ kN}$

Using 10 mm Φ - 2 legged stirrups, the spacing is given by,

$$s_v = \left(\frac{\sigma_{sv} \cdot A_{sv} \cdot d}{V_s} \right) = \left(\frac{230 \times 2 \times 79 \times 760}{29 \times 10^3} \right) = 952 \text{ mm}$$

$$\text{Also } s_v = \left(\frac{A_{sv} f_y}{0.4b} \right) = \left(\frac{2 \times 79 \times 415}{0.4 \times 400} \right) = 409 \text{ mm}$$

Adopt 10 mm - 2 legged stirrups at 400 mm centres.

6. Design of Maximum Positive Moment Section

Maximum positive moment $M = 73 \text{ kN}\cdot\text{m}$

$$\therefore A_{st} = \left(\frac{73 \times 10^6}{230 \times 0.9 \times 760} \right) = 464 \text{ mm}^2$$

But minimum area of tension reinforcement should be not less than that given by

$$A_{st} = \left(\frac{0.85bd}{f_y} \right) = \left(\frac{0.85 \times 400 \times 760}{415} \right) = 623 \text{ mm}^2$$

Provide 2 bars of 22 mm Φ as main tension reinforcement

$$(A_{st} = 760 \text{ mm}^2)$$

7. Design of Section Subjected to Maximum Torsion and Shear

$T = 50 \text{ kN}\cdot\text{m}$

$D = 800 \text{ mm}$

$V = 55 \text{ kN}$

$b = 400 \text{ mm}$

$M = 0$

$$M_t = T \left[\frac{1 + (D/b)}{1.7} \right] = 50 \left[\frac{1 + (800/400)}{1.7} \right] = 89 \text{ kN}\cdot\text{m}$$

$$M_{e1} = (M + M_t) = 89 \text{ kN}\cdot\text{m}$$

$$A_{st} = \left(\frac{89 \times 10^6}{230 \times 0.9 \times 760} \right) = 566 \text{ mm}^2$$

But minimum area of tension reinforcement as calculated earlier is

$$A_s = \left(\frac{0.85bd}{f_y} \right) = 623 \text{ mm}^2$$

Provide 2 bars of 22 mm Φ at top and bottom ($A_{st} = 760 \text{ mm}^2$)

$$\text{Equivalent shear} = V_e = (V + 1.6T/b) = \left(55 + 1.6 \times \frac{50}{0.4} \right) = 255 \text{ kN}$$

$$\tau_{ve} = \left(\frac{V_e}{bd} \right) = \left(\frac{255 \times 10^3}{400 \times 760} \right) = 0.838 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd} \right) = \left(\frac{100 \times 760}{400 \times 760} \right) = 0.25$$

From Table 17 (IS : 456), $\tau_c = 0.22 \text{ N/mm}^2 < 0.838 \text{ N/mm}^2$

$\therefore$ shear reinforcements are required. Using 10 mm 2 legged stirrups with side curves of 25 mm and top and bottom covers of 40 mm,

$$b_1 = 350 \text{ mm} \quad d_1 = 720 \text{ mm} \quad A_{sv} = (2 \times 79) = 158 \text{ mm}^2$$

$$s_v = A_{sv} \sigma_{sv} \left[\frac{b_1 d_1}{T} + \frac{2.5 d_1}{V} \right]$$

$$= 158 \times 230 \left[\frac{350 \times 760}{50 \times 10^6} + \frac{2.5 \times 720}{55 \times 10^3} \right] = 1380 \text{ mm}$$

But $A_{sv} < \left(\frac{\tau_{ve} - \tau_c}{\sigma_{sv}} \right) b \cdot s_v$

$$\therefore s_v = \frac{A_{sv} \cdot \sigma_{sv}}{(\tau_{ve} - \tau_c)} = \frac{2 \times 79 \times 230}{(0.838 - 0.22)440} = 147 \text{ mm}$$

Adopt 10 mm Φ - 2 legged stirrups at 140 mm centres.

4.6 EXAMPLES FOR PRACTICE

1. A circular girder of a water tank has a mean diameter of 10 m, and it is supported on six symmetrically placed columns. The uniformly distributed load on the girder is 200 kN/m. Design the critical sections of the girder using M-20 grade concrete and Fe-415 grade Tor steel, and sketch the details of reinforcements.

2. The circular girder of an intz type water tank of mean diameter 10 m and supported on eight symmetrically placed columns, supports an uniformly distributed load of 500 kN/m exclusives of its own self weight. Design the girder using M-20 grade concrete and Fe-415 grade Tor steel.

3. Design a semicircular R.C. beam supported on 3 columns equally spaced and supporting an uniformly distributed load of 30 kN/m. The radius of the centre line of the beam is 5 m. Adopt M-20 grade concrete and Fe-415 grade Tor steel.

5

Towers

5.1 GENERAL

Reinforced concrete tower frame works comprising of the columns and braces are generally used to support over head water tanks. The columns having the same cross section are symmetrically placed to resist the dead loads and wind loads. The effective length of the columns are reduced by providing a horizontal frame work of braces at suitable intervals. The braces reduce the bending moments developed in the columns.

The columns and braces are designed for direct loads and bending moment developed at critical cross sections based on an approximate analysis. An accurate procedure for determination of forces and moments in the columns and braces involves a sway analysis of wind loads using any of the well established methods.

A typical tower frame work for supporting a water tank is shown in Fig. 5.1. Generally the braces are spaced at intervals of 2 to 3 m.

5.2 DESIGN PRINCIPLES

The approximate method of analysis is based on the following assumptions:

- (a) The braces are stiff and integral with columns so that they are held in position and direction.
- (b) Contraflexure points are assumed to develop at mid points of columns and braces.
- (c) The inner columns are assumed to resist, twice the shear taken by the outer columns since the inner columns are stiffened by double bracing.

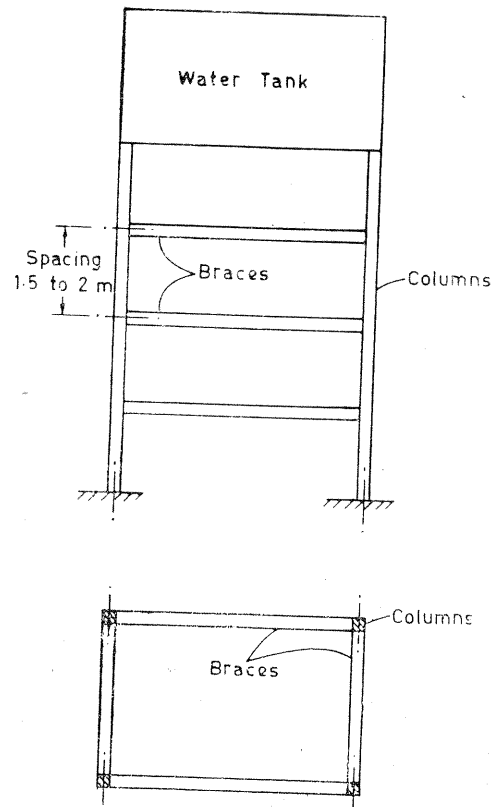


Fig. 5.1 R.C. Supporting Tower

- (d) The end conditions of the columns at footing are assumed to be hinged unless the footing is made rigid by means of piles or rafts or by horizontal braces.

The moment at each end of a brace is the sum of the moments in the column above and below the brace. The shearing force in a brace is equal to the change of bending moment from one end of the brace to the other, divided by the length of the brace.

The brace is designed for bending moment and shear developed due to external loads and self weight. The brace is generally reinforced with equal areas of steel at top and bottom so that it is safe to resist wind from one side or the other.

5.3 COMPUTATION OF MOMENTS DUE TO WIND LOADS

Case a: Two columns hinged at base

Referring to Fig. 5.2, let

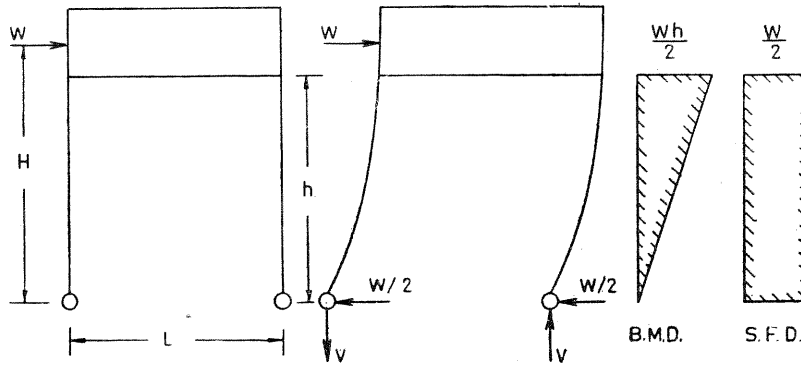


Fig. 5.2 Two Columns Hinged at Base

W = Horizontal wind force resisted by one row of columns.

V = Vertical reaction

taking moments about B , we have the relation

$$V \times L = W \times H$$

$$\therefore V = \left(\frac{WH}{L} \right)$$

$$\text{B.M. in each column} = \left(\frac{wh}{2} \right)$$

$$\text{S.F. in each column} = W/2$$

$$\text{Vertical thrust in each column} = \left(\frac{wH}{L} \right)$$

Case b: Three columns hinged at base

Referring to Fig. 5.3.

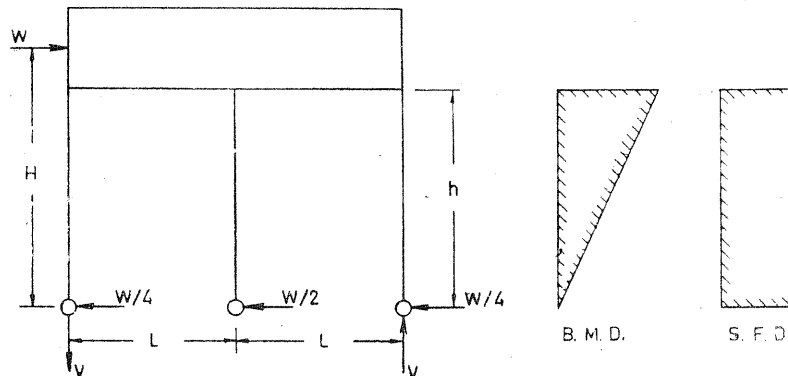


Fig. 5.3 Three Columns Hinged at Base

The central column is assumed to take double the shear resisted by the exterior columns. Hence we have

$$V \cdot 2L = W \cdot H \quad \therefore V = \left(\frac{WH}{2L} \right)$$

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B.M. in exterior column = $(wh/4)$

S.F. in exterior column = $(w/4)$

Vertical thrust in exterior column = $(WH/2L)$

B.M. in interior column = $(WH/2)$

S.F. in interior column = $(w/2)$

Vertical thrust in interior column = 0

Case c: Two columns fixed at base

Referring to Fig. 5.4, let

$M_A = M_B$ = Moment developed in columns at A and B

V = Vertical thrust in the columns by taking moments about B.

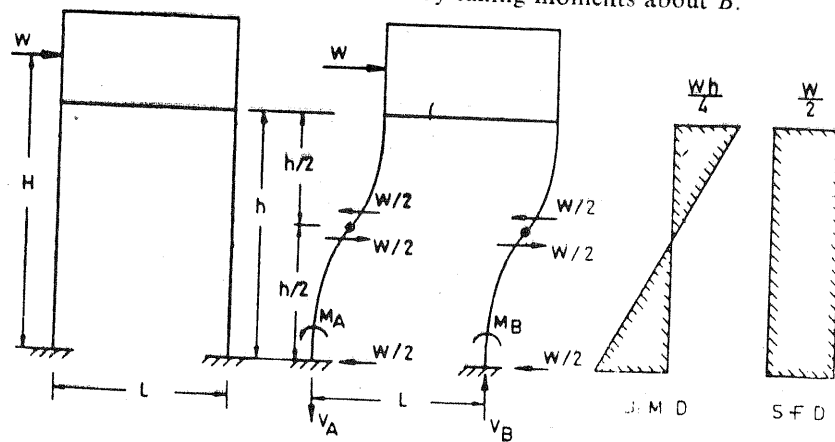


Fig. 5.4 Two Columns Fixed at Base

we have

$$W \cdot H = V \cdot L + M_A + M_B$$

$$V \cdot L = W \cdot H - (M_A + M_B) = W \cdot H - (Wh/4 + Wh/4)$$

$$V = W/L(H - h/2)$$

$$\text{B.M. in each column} = \frac{Wh}{4}$$

$$\text{S.F. in each column} = W/2$$

$$\text{Vertical thrust in each column} = W/L(H - h/2)$$

Case d: Columns braced at intervals with fixed base

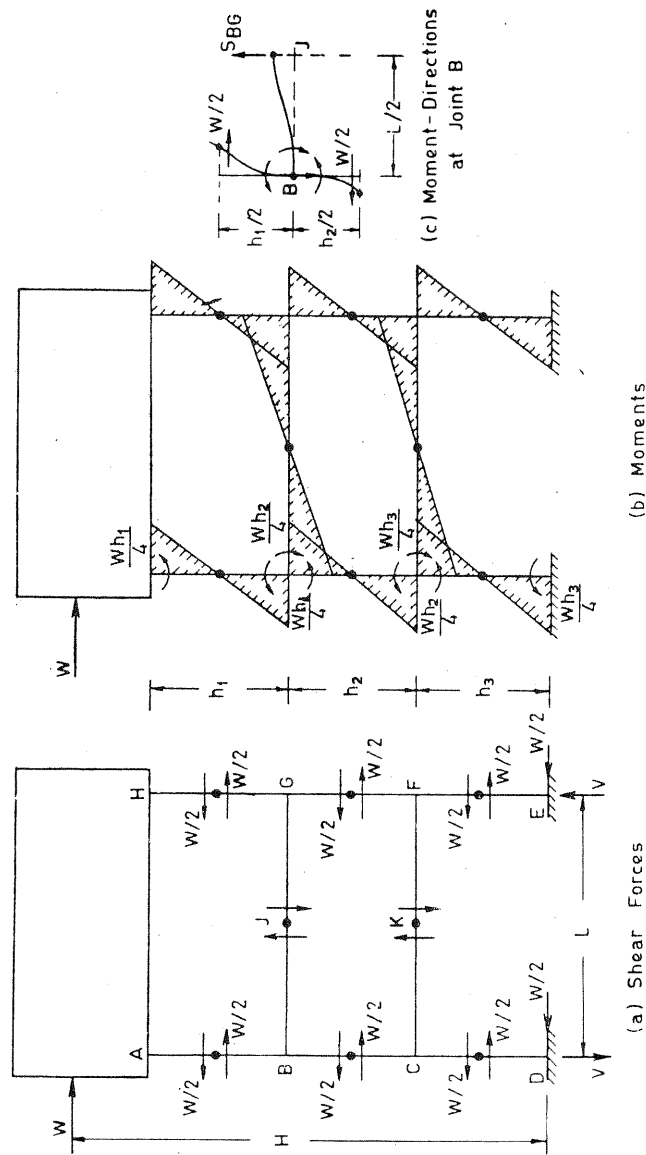
Referring to Fig. 5.5, let

W = load resisted by one row of columns assuming contraflexure points at centre of panels,

Shear force in each column = $W/2$

Moments

$$M_{AB} = \left(\frac{W}{2}\right)\left(\frac{h_1}{2}\right) = \left(\frac{Wh_1}{4}\right)$$



(a) Shear Forces
(b) Moments
(c) Moment-Directions at Joint B

Fig. 5.5 Columns Braced at Intervals with Fixed Base

$$M_{BA} = \left(\frac{Wh_1}{4} \right)$$

$$M_{BC} = M_{CB} = \left(\frac{Wh_2}{4} \right)$$

$$M_{CD} = M_{DC} = \left(\frac{Wh_3}{4} \right)$$

Moments in brace at any junction = sum of the moments in the column above and below the brace.

$$M_{BG} = M_{GB} = (M_{BA} + M_{BC}) = \frac{W}{4}(h_1 + h_2)$$

$$M_{CF} = M_{FC} = (M_{CB} + M_{CD}) = \frac{W}{4}(h_2 + h_3)$$

Shear force in brace is obtained as

$$S_{BG} = \frac{W}{2L}(h_1 + h_2)$$

$$S_{CF} = \frac{W}{2L}(h_2 + h_3)$$

Case e: Columns braced at intervals with hinged base

Referring to Fig. 5.6, let

W = load resisted by one row of columns. Assuming contraflexure points at centre of panels, shear force in each column = $W/2$

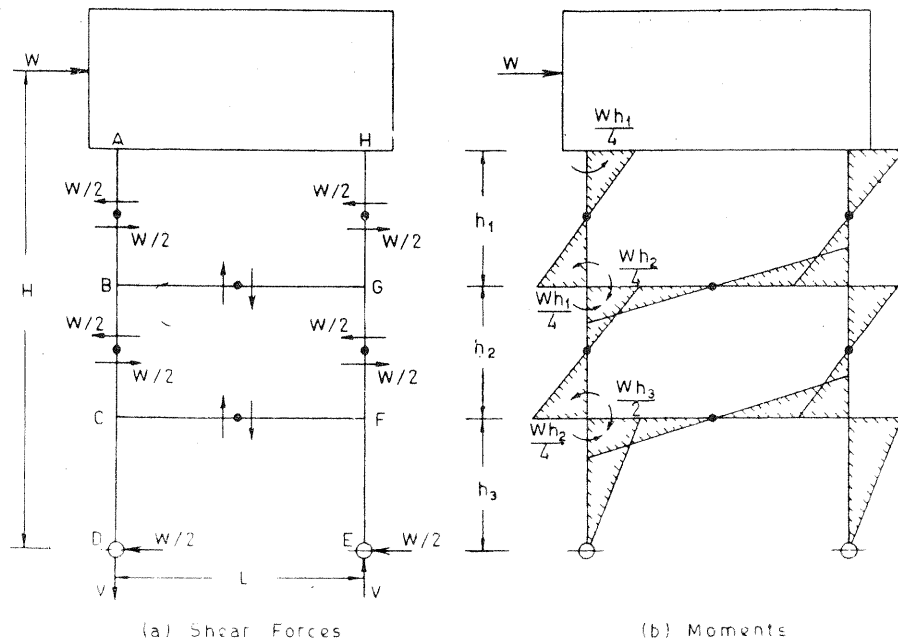


Fig. 5.6 Column Braced at Intervals with Hinged Base

Moments

$$M_{AB} = M_{BA} = \left(\frac{W}{2}\right)\left(\frac{h_1}{2}\right) = \left(\frac{Wh_1}{4}\right)$$

$$M_{BC} = M_{CB} = (Wh_2/4)$$

$$M_{CD} = \left(\frac{Wh_3}{2}\right)$$

$$M_{DC} = 0$$

$$M_{BG} = M_{GB} = (M_{BA} + M_{BC}) = W/4(h_1 + h_2)$$

$$M_{CF} = M_{FC} = (M_{CB} + M_{CD}) = W/4(h_2 + h_3)$$

$$S_{BG} = W/2L(h_1 + h_2)$$

$$S_{CF} = W/2L(h_2 + h_3)$$

5.4 WIND LOAD ANALYSIS OF A TOWER WITH CIRCULAR GROUP OF COLUMNS

In the case of over head circular tanks, the tower consists of a number of columns braced together on the periphery of a circle. The moments and shears in the columns and braces due to wind loads are analysed by assuming contraflexure points at the mid heights of columns between the braces. Referring to Fig. 5.7, let

W, W_1, W_2, W_3 and W_4 be the wind loads acting on the tank and columns at different nodal points maximum. Moments and shears occur in the bottom panel.

Shear in the bottom panel = $(W + W_1 + W_2 + W_3)$

Moment about the base due to wind is calculated as,

$$M_1 = W \cdot H + W_1(h_1 + h_2 + h_3) + W_2(h_2 + h_3) + W_3 \cdot h_3$$

Taking moments of all the forces about the axis yy at the base, we have

$$M_1 = \sum M + 2Vr_1 + 4\left(V \cdot \frac{r_2}{r_1}\right)r_2$$

where $\sum M = (W + W_1 + W_2 + W_3) \frac{h_3}{2}$

and V = the vertical reaction developed in the exterior column B . r_1 and r_2 are the distances of the column B and C respectively measured from the central axis yy .

$$\therefore M_1 = \sum M + \frac{V}{r_1} [\sum r^2]$$

The maximum moment in brace - BC , occurs when wind is blowing normal to brace BL .

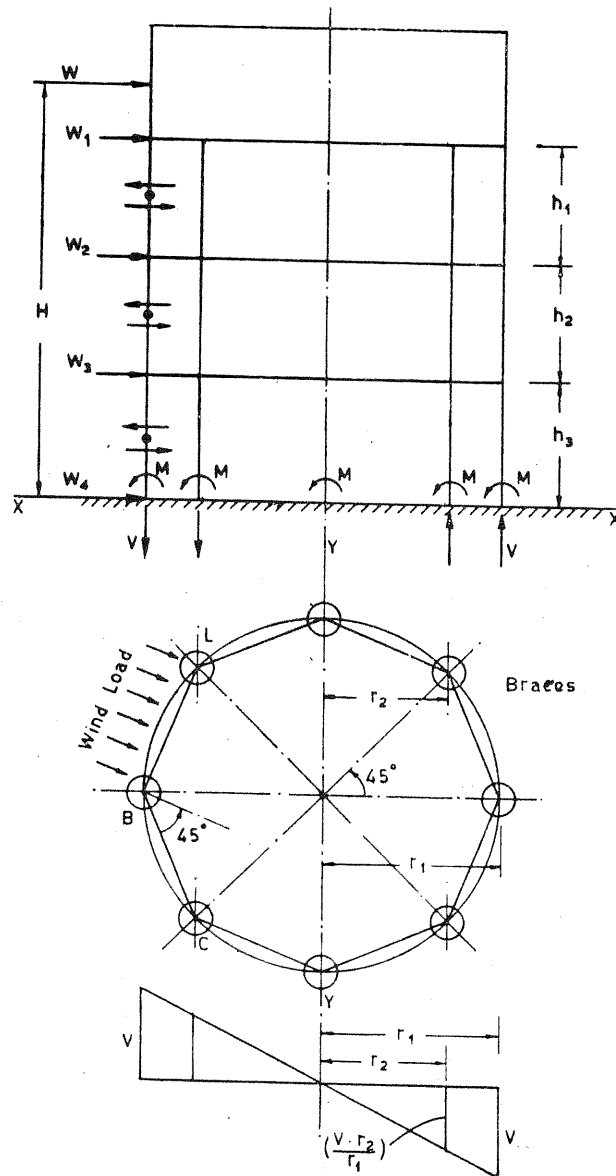


Fig. 5.7 Wind Load Analysis of Circular Tower

Moment at the junction of column in the direction $BK = (M_{BA} + M_{BD})$
 Moment in the direction of brace BC is given by

$$M_{BC} = (M_{BA} + M_{BD}) \sec. 45^\circ = (M_{BA} + M_{BD}) \sqrt{2}$$

Shear force in the brace is given by

$$S = \left[\frac{\text{Moment in the brace}}{\frac{1}{2} (\text{length of brace})} \right]$$

5.5 DESIGN EXAMPLE

A rectangular water tank 3 m by 3 m in plan and of depth 3 m is supported on a tower 6 m height. Number of columns = 4. The columns are braced at mid height. The wind pressure on the tank may be taken as 1 kN/m². Assume dead weight of tank = 160 kN. Weight of water in tank = 280 kN. Adopt M-15 grade concrete and Fe-415 grade Tor steel and design the columns and brace of the supporting tower. The columns are provided with a rigid foundation so that fixity conditions may be assumed at the column base.

1. Data

Height of columns = 3 m
 Length of brace = 3 m
 Dead weight of tank = 160 kN
 Weight of water = 280 kN
 Wind pressure = 1 kN/m²
 M-15 grade concrete and Fe-415 grade tor steel

2. Permissible Stresses

$\sigma_{cc} = 4 \text{ N/mm}^2$ $Q = 0.659$
 $\sigma_{cb} = 5 \text{ N/mm}^2$ $j = 0.903$
 $\sigma_{sc} = 190 \text{ N/mm}^2$ $m = 19$
 $\sigma_{st} = 230 \text{ N/mm}^2$

3. Loads and Moments

Referring to Fig. 5.8
 Wind load on tank = $(3 \times 3 \times 1) = 9 \text{ kN}$

Load on columns:

Dead weight of tank	160 kN
Weight of water	280 kN
Self weight of columns = $(4 \times 0.3 \times 0.3 \times 6 \times 24)$	52 kN
Self weight of braces = $(4 \times 0.3 \times 0.3 \times 3 \times 24)$	26 kN

—
 Total dead load = 518 kN

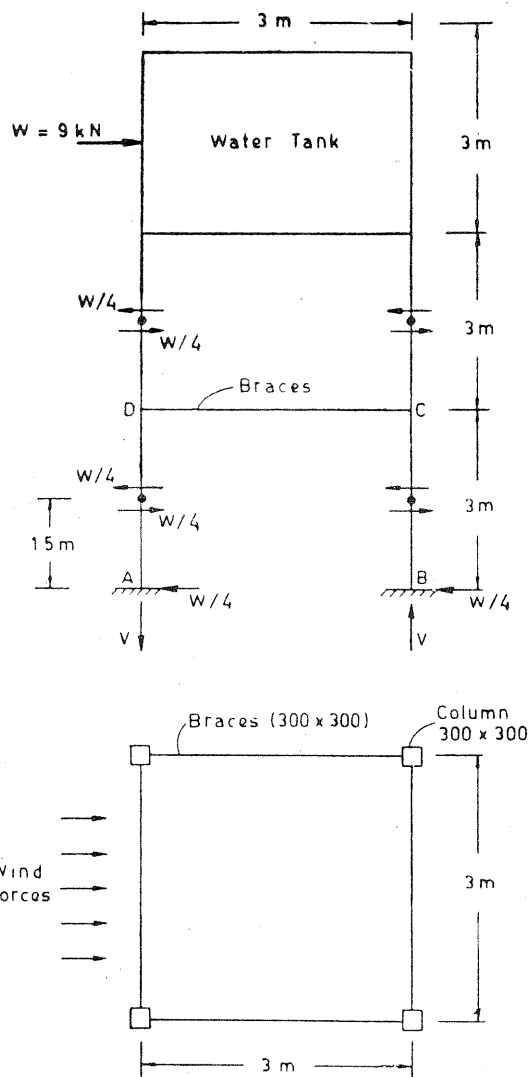


Fig. 5.8 Wind Load Analysis of Water-Tower

$$\text{Dead load per column} = \left(\frac{518}{4} \right) = 130 \text{ kN}$$

shear force in each column due to wind = $9/4 = 2.25 \text{ kN}$

bending moment in column = $(2.25 \times 1.5) = 3.375 \text{ kN} \cdot \text{m}$

If V = direct load due to wind, taking moments about B , we have

$$(2V \times 3) + (3.375 \times 4) = (9 \times 7.5) \quad \therefore V = 9 \text{ kN}$$

4. Design of column section

Size of column = $300 \text{ mm} \times 300 \text{ mm}$

Axial load = $P = (130 + 9) = 139 \text{ kN}$

Bending moment = $M = 3.375 \text{ kN} \cdot \text{m}$

$$\text{eccentricity } e = M/P = \left(\frac{3.375 \times 10^6}{139 \times 10^3} \right) = 24 \text{ mm}$$

The load and eccentricity being small, try minimum steel of 0.8% of concrete section.

$$A_{sc} = \left(\frac{0.8}{100} \times 300 \times 300 \right) = 720 \text{ mm}^2$$

Provide 4 bars of 16 mm diameter ($A_{sc} = 804 \text{ mm}^2$)

$$A_e = (300 \times 300) + (1.5 \times 19 \times 804) = 112914 \text{ mm}^2$$

$$I_e = \left[\frac{bd^3}{12} + 1.5mA_{sc} \cdot h^2 \right] = \left[\frac{300 \times 300^3}{12} + 1.2 \times 19 \times 804 \times 100^2 \right]$$

$$= 9.04 \times 10^8 \text{ mm}^4$$

using a cover = 50 mm $h = 100 \text{ mm}$

5. Stresses in concrete

$$\text{Compressive stress} = \left(\frac{139 \times 10^3}{112914} \right) = 1.23$$

$$\text{Bending stress} = \left(\frac{3.375 \times 10^6 \times 150}{9.04 \times 10^8} \right) = 0.56$$

$$\therefore \left(\frac{\sigma'_{cc}}{\sigma_{cc}} + \frac{\sigma'_{cb}}{\sigma_{cb}} \right) < 1 \quad \left(\frac{1.23}{4} + \frac{0.56}{5} \right) = 0.412 < 1$$

$\therefore$ Use $300 \times 300 \text{ mm}$ column with 4-20 mm diameter bars.

6. Lateral Reinforcement

Diameter of ties = $(20/4) = 5 \text{ mm}$

Pitch shall be the least of

(a) least lateral dimension of column = 300 mm

(b) 16 times the longitudinal bar = $(16 \times 20) = 320 \text{ mm}$

(c) 48 times the lateral reinforcement = $(48 \times 5) = 240 \text{ mm}$

use 5 mm ϕ ties at 240 mm centres.

7. Design of brace

Moment in the brace = $(2 \times 2.25 \times 1.5) = 6.75 \text{ kN} \cdot \text{m}$

$$\text{Shear force in the brace} = \left(\frac{\text{Moment in brace}}{\frac{1}{2} \text{ length of brace}} \right) = \left(\frac{6.75}{1.5} \right) = 4.5 \text{ kN}$$

The cross section assumed for the brace is 300 mm by 300 mm

$$A_{st} = \left(\frac{M}{\sigma_{st} \cdot j \cdot d} \right) = \left(\frac{6.75 \times 10^6}{230 \times 0.9 \times 270} \right) = 121 \text{ mm}^2$$

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But minimum area of steel is given by

$$A_{st} = \left(\frac{0.85bd}{f_y} \right) = \left(\frac{0.85 \times 300 \times 270}{415} \right) = 166 \text{ mm}^2$$

Provide 2 bars of 12 mm diameter ($V_{st} = 226 \text{ mm}^2$), both at top and bottom with a cover of 30 mm.

$$\text{Nominal shear stress } \tau_v = \left(\frac{V}{bd} \right) = \left(\frac{4.5 \times 10^3}{300 \times 270} \right) = 0.055 \text{ N/mm}^2$$

From Table 17 of IS: 456-78, The permissible shear stress in concrete $\tau_c = 0.29 \text{ N/mm}^2$ since $\tau_v < \tau_c$, Nominal shear reinforcement are provided.

Using 6 mm diameter 2 legged stirrups the spacing is given by,

$$s_v = \left(\frac{A_{sv}f_y}{0.4b} \right) = \left(\frac{2 \times 28 \times 415}{0.4 \times 300} \right) = 193 \text{ mm} > (0.75 \times 270) > 202 \text{ mm}$$

Provide a spacing of 190 mm for the stirrups.

5.6 DESIGN EXAMPLE

A circular over head water tank has a diameter of 10 m and a height of 5 m. The tank is supported on a tower 13 m in height which is braced at intervals of 5 m. The R.C. columns have a cross section 300 mm by 500 mm and the braces are 400 mm by 450 mm. Number of columns in the tower = 8. The columns are arranged at intervals of 45° . The total wind loads acting on the tank and tower are as given below:

Wind load (kN)	Distance from base (m)
$W = 63$	18
$W_1 = 23$	15
$W_2 = 36$	10
$W_3 = 36$	5

The total dead load due to the self weight of concrete and weight of water acting on each column at the base is equal to 700 kN. If $f_{ck} = 20 \text{ N/mm}^2$ and $f_y = 250 \text{ N/mm}^2$. Design the reinforcements in the column section at base and in the braces assuming raft foundations.

1. Data

Column section = 300 mm by 500 mm

Brace section = 400 mm by 450 mm

Wind loads: $W = 63 \text{ kN}$, $W_1 = 23 \text{ kN}$, $W_2 = W_3 = 36 \text{ kN}$

$h_1 = h_2 = h_3 = 5 \text{ m}$ $H = 18 \text{ m}$

Dead load on each column at base = 700 kN

M-20 grade concrete and Fe-250 grade mild steel.

2. Stresses

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 250 \text{ N/mm}^2$$

3. Loads and moments

Referring to Fig. 5.9, we have

M = Moment at the base of the columns

$$= (W + W_1 + W_2 + W_3) \frac{h_3}{2}$$

$$= (63 + 23 + 36 + 36) 5/2 = 395 \text{ kN} \cdot \text{m}$$

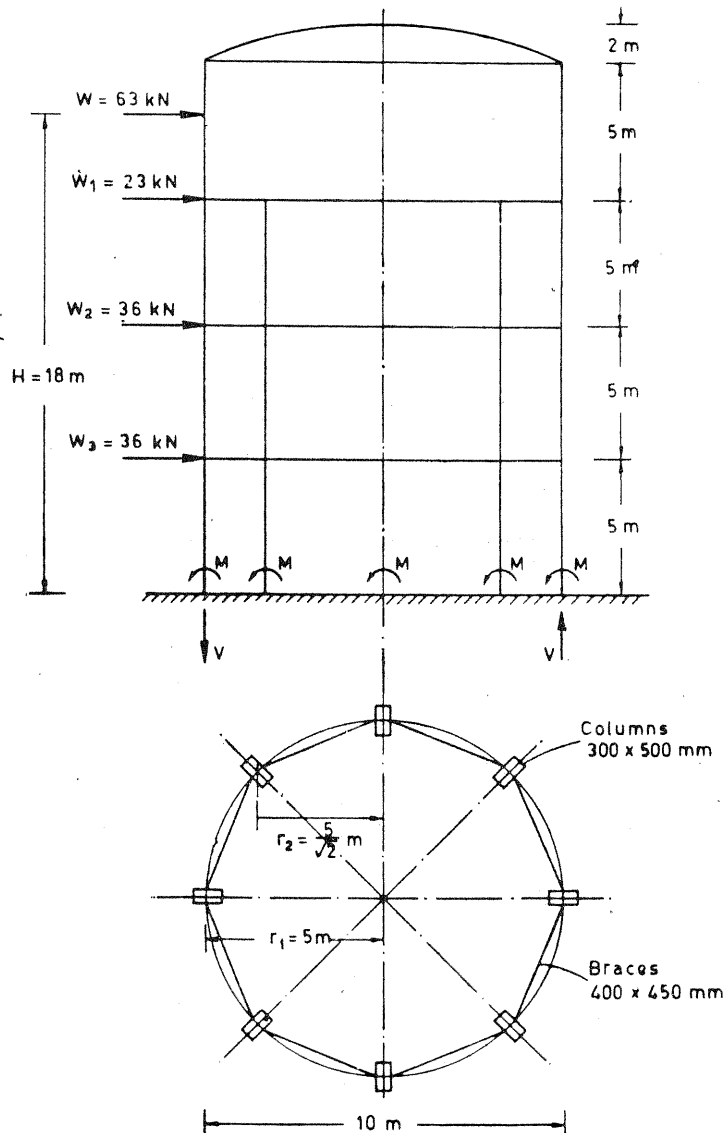


Fig. 5.9 Wind Loads on Water Tank Tower

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If M_1 = Moment due to wind loads about base

$$M_1 = (63 \times 18) + (23 \times 15) + (36 \times 10) + (36 \times 5) = 2019 \text{ kN} \cdot \text{m}$$

If V = reaction developed at the base of exterior columns

$$M_1 = \sum M + \frac{V}{r_1} [\sum r^2]$$

$$2019 = 395 + V/5[(2 \times 5^2) + 4(5/\sqrt{2})^2] \quad \therefore V = 89 \text{ kN}$$

$$\therefore \text{Total load on Leeward Column at base} = (700 + 82) = 782 \text{ kN}$$

$$\text{Moment in column at base} = \left(\frac{395}{8}\right) = 50 \text{ kN} \cdot \text{m}$$

Moments in brace BC

$$= \left[\frac{(63 + 23 + 36)}{8} \times \frac{5}{2} + \frac{(63 + 23 + 36 + 36)}{8} \times \frac{5}{2} \right] \sqrt{2} = 124 \text{ kN} \cdot \text{m}$$

$$\text{shear force in brace} = \left(\frac{124}{\frac{1}{2} \times 3.8}\right) = 65.3 \text{ kN} \cdot \text{m}$$

4. Design of Column section

$$\text{Ultimate load } P_u = (1.5 \times 782) = 1173 \text{ kN}$$

$$\text{Ultimate moment } M_u = (1.5 \times 50) = 74 \text{ kN} \cdot \text{m}$$

$$b = 300 \text{ mm} \quad D = 500 \quad d' = 50 \text{ mm}$$

$$\text{Ratio } (d'/D) = 0.10$$

$$\left(\frac{P_u}{f_{ck} \cdot b \cdot D}\right) = \left(\frac{1173 \times 10^3}{20 \times 300 \times 500}\right) = 0.391$$

$$\left(\frac{M_u}{f_{ck} \cdot b \cdot d^2}\right) = \left(\frac{75 \times 10^6}{20 \times 300 \times 500^2}\right) = 0.05$$

Referring to the interaction diagram, corresponding to

$$f_y = 250 \text{ N/mm}^2 \quad \text{and} \quad (d'/D) = 0.1$$

(Refer page 181 of book Design of R.C. Structures by Author)

$$(p/f_{ck}) = 0.05 \quad \therefore p = (0.05 \times 20) = 1\%$$

$$\therefore A_s = \left(\frac{pbD}{100}\right) = \left(\frac{1 \times 300 \times 500}{100}\right) = 1500 \text{ mm}^2$$

Provide 6 bars of 20 mm ϕ distributing 3 bars on either face.

Transverse Reinforcement

Using 6 mm diameter ties, spacing of the ties is the least of

(a) Least lateral dimension = 300 mm

(b) $(16 \times 20) = 320 \text{ mm}$

(c) $(48 \times 6) = 288 \text{ mm}$

Adopt 6 mm diameter ties at 280 mm centres.

5. Design of Braces

$$M_u = (1.5 \times 124) = 186 \text{ kN} \cdot \text{m}$$

$$V_u = (1.5 \times 65.3) = 98 \text{ kN}$$

Section of brace is $b = 400 \text{ mm}$

$$D = 450 \text{ mm}$$

$$d = 400 \text{ mm}$$

$$\therefore M_{u, \text{lim}} = (0.148 f_{ck} \cdot b \cdot d^2) = (0.148 \times 20 \times 400 \times 400^2) \\ = 189.4 \times 10^6 \text{ N} \cdot \text{mm} = 189.4 \text{ kN} \cdot \text{m}$$

$$\therefore A_{st} = \left[\frac{0.36 f_{ck} b (0.53 d)}{0.87 f_y} \right] = \left[\frac{0.36 \times 20 \times 400 \times 0.53 \times 400}{0.87 \times 250} \right] \\ = 2807 \text{ mm}^2$$

Use 6 bars of 25ϕ on each side ($A_{st} = 2946 \text{ mm}^2$)

$$\tau_v = \left(\frac{V_u}{bd} \right) = \left(\frac{98 \times 10^3}{400 \times 400} \right) = 0.61 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd} \right) = \left(\frac{100 \times 2946}{400 \times 400} \right) = 1.84$$

From Table 13 of IS: 456-1978, permissible shear stress

$$\tau_c = 0.76 \text{ N/mm}^2 > \tau_v$$

$\therefore$ using $10 \text{ mm } \phi$ - 2 legged stirrups as nominal shear reinforcements, the spacing is given by,

$$s_v = \left(\frac{A_{sv} \cdot f_y}{0.4 b} \right) = \left(\frac{2 \times 79 \times 250}{0.4 \times 400} \right) = 250 \text{ mm, centres.}$$

5.7 EXAMPLES FOR PRACTICE

1. A square water tank 4 m by 4 m by 4 m is to be supported by a 4 column tower of height 4 m . The columns have independent footings and their base may be considered as hinged. If the dead weight of the tank is 400 kN and weight of water in the tank is 640 kN , design the supporting tower allowing for a wind load of 1.5 kN/m^2 . Adopt M-20 grade concrete and Fe-415 grade tor steel.

2. A rectangular water tank 3 m by 4 m with a tank depth of 3 m is supported on a four column tower 6 m in height, braced at the mid height. Assuming the dead weight of tank to be 300 kN and the weight of water as 400 kN , design the columns and braces of the supporting tower. Assume the columns as fixed at the base. Intensity of wind pressure is 1.5 kN/m^2 . Adopt M-200 concrete mix and Fe-415 grade tor steel.

3. A circular R.C. water tank 6 m diameter and 5 m height is supported by a tower consisting of 6 R.C columns on a circle of diameter 5 m . The tower height is 8 m with bracing at a height of 4 m from the ground. The

tank is designed to hold water up to a depth of 5.5 m. The self weight of the tank is estimated to be 1750 kN. Intensity of wind on tank is 1.5 kN/m^2 . Using M-20 grade concrete and Fe-415 grade tor steel design the supporting tower of the tank.

4. The staging for a water tank of 12 m diameter and 6 m height comprises of 12 R.C columns arranged concentrically on a circle of 12 m diameter. The height of the staging is 12 m with bracing at 3 m intervals. The columns are 450 mm square while the braces are 400 mm square. If the dead weight of the concrete tank is 2250 kN and the weight of water is three times the self weight of tank, design the reinforcements in the column and brace assuming a wind intensity of 1.5 kN/m^2 . The columns may be assumed to be fixed at the base.

5. A reinforced concrete rectangular water tank is 3 m by 6 m with a depth of 3 m. The tank is supported on six columns provided with rigid foundation. The dead weight of the tank is 400 kN and the weight of water may be taken as 500 kN. The height of the staging is 6 m with braces at 3 m height above the ground. Assuming a wind pressure of 1.5 kN/m^2 and M-20 grade concrete and Fe-415 grade tor steel, design the columns and the brace.

6

Elevated Water Tanks

6.1 TYPES OF OVER HEAD WATER TANKS

The most common types of over head water tanks are,

- (a) The circular tank
- (b) The intz type tank
- (c) The conical or funnel shaped tank.

Circular tanks with a horizontal or flat floor slab is economical for smaller storage capacity of up to 200,000 litres and with diameters in the range of 5 to 8 m. The depth of storage is generally between 3 to 4 m. The side walls are designed for circumferential hoop tension and bending moment since the walls are fixed to the floor slab at the junction. The design forces are determined using coefficients recommended in IS: 3370 (Part IV). The coefficients are expressed as a function of the nondimensional parameter (H^2/Dt) ,

where H = Height of the water tank

D = Diameter of the tank

t = Thickness of the tank wall.

The circular floor slab of the tank is designed for circumferential and radial moments developed in the slab.

Design-examples of circular slabs fixed at the edges and circular tank walls fixed at the base are presented in Chapter 5 and Chapter 14 of the text book 'Design of R.C. Structures' by the author.

6.2 INTZ TYPE TANK

In the case of large diameter elevated circular tanks, thicker floor slabs are

required resulting in uneconomical designs. In such cases, intz type tank with conical and bottom spherical domes provides an economical solution. The proportions of the conical and the spherical bottom domes are selected so that the outward thrust from the bottom dome balances the inward thrust due to the conical domed part of the tank floor.

(a) *Structural elements of Intz tank*

Referring to Fig. 6.1, the various structural elements of an intz type tank comprises of the following:

1. Top spherical dome
2. Top ring beam
3. Circular side walls

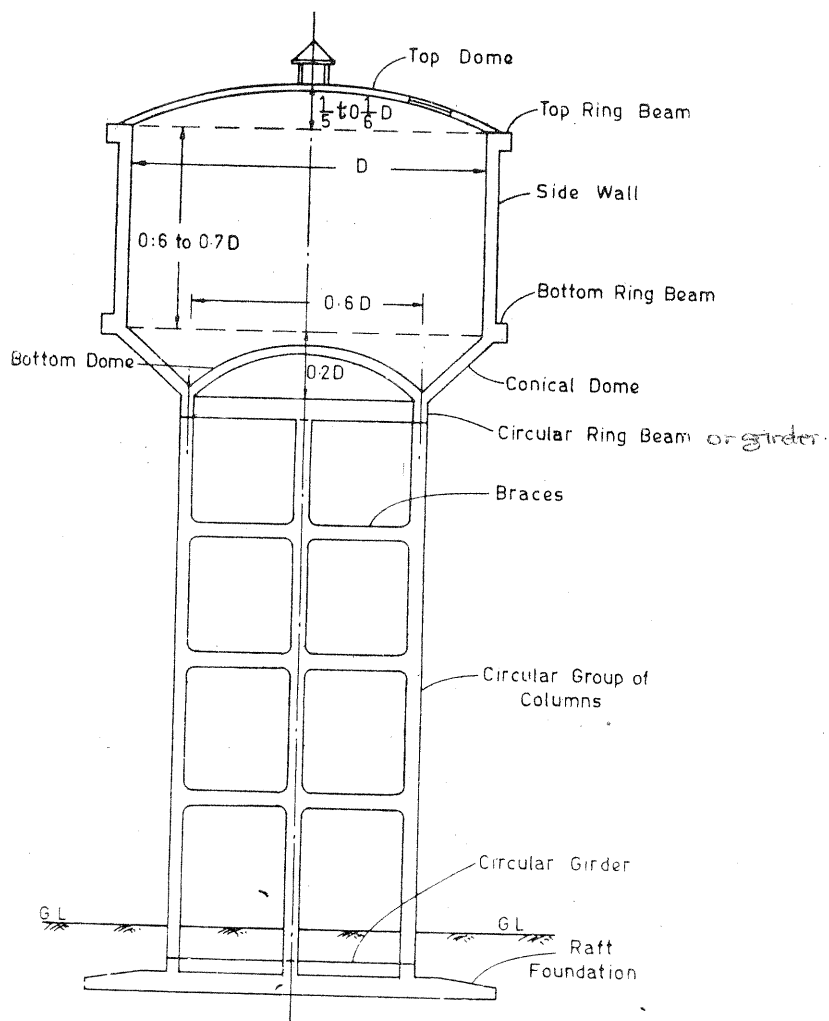


Fig. 6.1 Structural element of intz type tank.

4. Bottom ring beam
5. Conical dome
6. Bottom spherical dome
7. Bottom circular girder
8. Tower with columns and braces
9. Foundations.

If D = diameter of the tank, the proportions of the various other structural elements in terms of the diameter are shown in Fig. 6.1.

(b) Design principles of intz tank

1. *Top spherical dome*

Referring to Fig. 6.2

t = thickness of dome generally varies in the range of 75 to 100 mm.

h = rise of dome = $1/5$ to $1/6$ diameter of tank (D)

R = radius of dome

w = uniformly distributed load per unit area of surface.

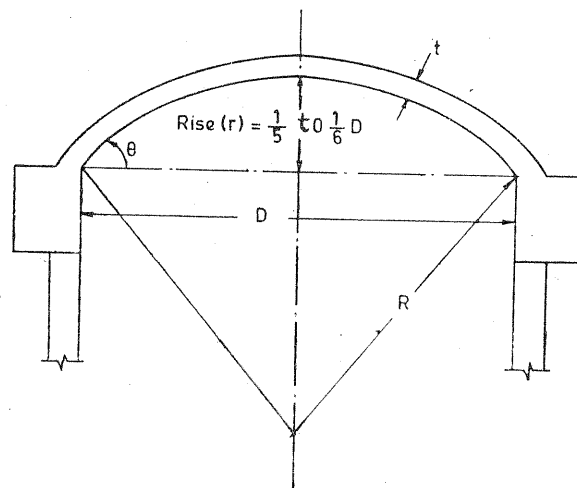


Fig. 6.2 Dimensions of top dome

The reinforcements in the dome are designed for maximum meridional thrust and circumferential force given by,

$$T_1 = \text{Meridional thrust} = \left(\frac{wR}{1 + \cos \theta} \right)$$

$$T_2 = \text{Circumferential force} = wR \left(\cos \theta - \frac{1}{1 + \cos \theta} \right)$$

2. *Top ring beam*

$$\text{Hoop tension in ring beam} = \left(\frac{T_1 \cos \theta \cdot D}{2} \right)$$

The cross sectional area of ring beam is determined by limiting the tensile stress in the ring beam to values specified in IS: 456-1978,

depending upon the grade of concrete. The tensile stress is calculated by the equation, given as

$$\left(\frac{F_t}{A_c + mA_{st}} \right)$$

where F_t = direct or hoop tension
 A_c = cross sectional area of concrete
 m = modular ratio
 A_{st} = cross sectional area of steel.

3. Side Walls of Tank

The side walls of the tank are designed for hoop tension developed due to the water pressure in the tank.

$$\text{Maximum hoop tension} = \left(\frac{w \cdot H \cdot D}{2} \right)$$

where w = Density of water (10 kN/m³)
 H = Height of vertical walls
 D = Diameter of tank

A minimum thickness of 150 to 200 is provided at the top of the tank and the thickness at the base of the vertical wall is designed by limiting the tensile stress. The spacing of the hoop reinforcement is gradually increased towards the top of the tank. Distribution and temperature reinforcement of 0.3% of the gross section is provided in the vertical direction.

4. Bottom Ring Beam

Referring to Fig. 6.3

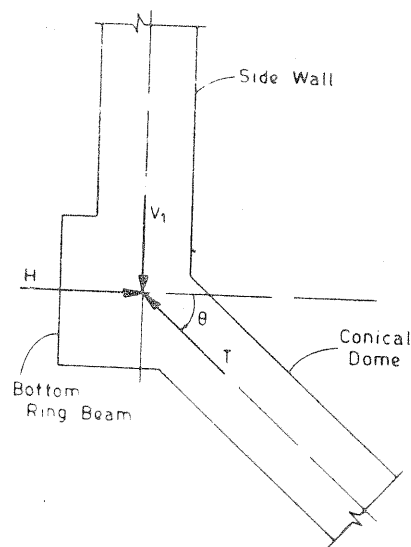


Fig. 6.3 Forces in bottom ring beam

If V_1 = weight of roof, side wall and top ring beam per metre run of the ring beam

h = height of water above the ring beam

T = Thrust in the conical dome

D = Diameter of the ring beam

H = Horizontal force developed at the junction

d = depth of ring beam

Then for equilibrium of the forces, we have

$$T \cdot \sin \theta = V_1 \quad \therefore (H/V_1) = \cot \theta$$

$$T \cdot \cos \theta = H \quad \therefore H = V_1 \cot \theta$$

$\therefore$ Hoop tension in the beam

$$= \left(\frac{H \cdot D}{2} + \frac{wh \cdot d \cdot D}{2} \right)$$

The reinforcements in the ring beam are designed to resist the hoop tension and the section is designed limiting the tensile stress in concrete.

4. Conical dome

Referring to Fig. 6.4

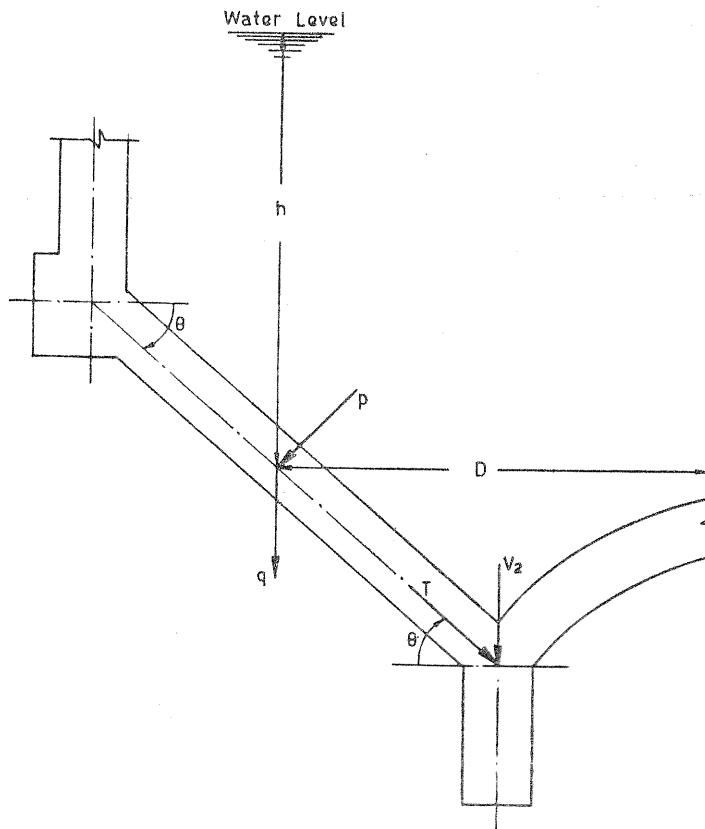


Fig. 6.4 Forces acting on conical dome

- If V_2 = Total load/metre run at the base of the conical dome
 T = Meridional thrust in the slab due to V_2
 H = Hoop tension due to water pressure and self weight of conical dome slab
 p = Intensity of water pressure at a depth h below the water level
 q = Weight of the conical slab per square metre of the surface area
 θ = Angle made by the conical slab with the horizontal
 D = Diameter at a depth h from top.

The meridional thrust and hoop tension in the conical dome is computed using the following equations:

$$\text{Meridional thrust } T = V_2 \operatorname{cosec} \theta$$

$$\text{Hoop tension} = H = (p \cdot \operatorname{cosec} \theta + q \cdot \cot \theta) D/2$$

The reinforcements in the conical dome is designed for hoop tension and meridional thrust.

5. Bottom spherical dome

The design of the floor dome is similar to that of the top dome. The design load for the dome includes the self weight of the dome and the weight of water column above the dome. The dome reinforcements are designed for meridional thrust and circumferential forces.

6. Bottom circular girder

Referring to Fig. 6.5, let

T_1 = Thrust in the direction of the conical dome acting at an angle α to the horizontal

T_2 = Thrust from the bottom spherical dome, acting at an angle β to the horizontal

P = Net horizontal force on the ring beam

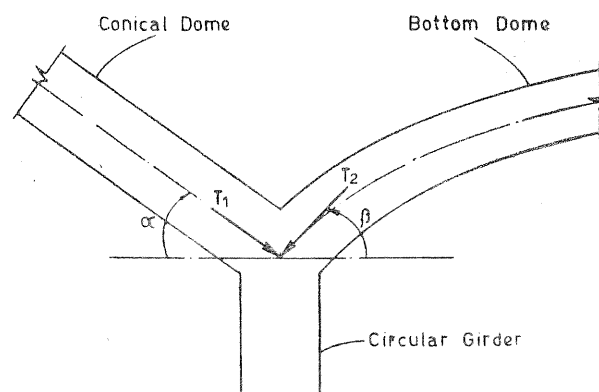


Fig. 6.5 Forces acting on circular girder

If $T_1 \cos \alpha > T_2 \cos \beta$, the ring beam is subjected to a compressive force. The magnitude of this compressive force is negligibly small in well proportioned tanks.

The vertical load on the ring beam is obtained by the relation, $(T_1 \sin \alpha + T_2 \sin \alpha)$ or alternatively by dividing the total vertical loads by the perimeter of the bottom ring beam.

The ring beam is supported by a number of columns equally spaced along the periphery of the circle. Depending upon the number of columns, moment coefficients compiled in Table 4.1, (refer Ch. 4, curved girders) are useful in computing the maximum bending and torsional moments in the circular girder. The procedure outlined in Ch. 4 can be used to design the reinforcements in the ring beam.

8. Tower with columns and braces

The procedure explained in Sec. 5.4 is useful in the design of a tower with circular group of columns braced together at regular intervals. The columns and braces are designed for the maximum forces and moments developed due to the dead loads and wind loads acting the tower.

9. Foundations

The foundations for a circular group of columns generally comprises of a ring beam with raft slab. The ring beam is designed for maximum bending and torsional moments while the annular raft slab is designed for maximum soil pressure from the bottom.

The design procedure is illustrated by the following example.

6.3 DESIGN EXAMPLE OF INTZ TYPE WATER TANK

Design an intz type water tank of 1 million litres capacity, supported on an elevated lower comprising of 8 columns. The base of the tank is 16 m above ground level. Depth of foundations 1 m below ground level. Adopt M-20 grade concrete and Fe-415 grade tor steel. The design of the tank should conform to the stresses specified in IS: 3370 and IS: 456.

1. Data

Capacity of tank = 1 million litres = 1000 m^3

Height of supporting tower = 16 m

Number of columns = 8

Depth of foundations = 1 m below ground level.

2. Permissible Stresses

M-20 grade concrete and Fe-415 grade tor steel, for calculations relating to resistance to cracking (IS: 3310)

$$\sigma_{ct} = 1.2 \text{ N/mm}^2 \quad \sigma_{cb} = 1.7 \text{ N/mm}^2 \quad \sigma_{st} = 150 \text{ N/mm}^2$$

For strength calculations the stresses in concrete and steel are same as that recommended in IS: 456.

$$\begin{aligned}\sigma_{cc} &= 5 \text{ N/mm}^2 & m &= 13 \\ \sigma_{cb} &= 7 \text{ N/mm}^2 & Q &= 0.897 \\ & & j &= 0.906\end{aligned}$$

3. Dimensions of Tank

Referring to Fig. 6.1 let

D = Inside diameter of the tank. Assuming the average depth = $0.75 D$, we have

$$\left(\frac{\pi D^2}{4} \times 0.75 D \right) = 1000 \text{ m}^3$$

$$\therefore D = 12 \text{ m}$$

Height of cylindrical portion of tank = 8 m

Depth of conical dome = 2 m

Diameter of the supporting tower = 8 m

spacings of bracing = 4 m.

The salient dimensions of the tank and the staging is shown in Fig. 6.6.

4. Design of Top Dome

Thickness of dome slab = $t = 100 \text{ mm}$

Live load on dome = 1.5 kN/m^2

Self weight of dome = $(0.1 \times 24) = 2.4 \text{ kN/m}^2$

Live load = 1.5

Finishes = 0.1

Total load = $w = 4.0 \text{ kN/m}^2$

If R = Radius of the dome

D = Diameter at base = 12 m

r = central rise = $(1/6 \times 12) = 2 \text{ m}$

$$R = \left[\frac{(D/2)^2 + r^2}{2r} \right] = \left[\frac{6^2 + 2^2}{2 \times 2} \right] = 10 \text{ m}$$

$$\cos \theta = (8/10) = 0.8$$

$$= 36^\circ 50'$$

$$\text{Meridional thrust} = T_1 = \left(\frac{wR}{1 + \cos \theta} \right) = \left(\frac{4 \times 10}{1 + 0.8} \right) = 22.22 \text{ kN/m}$$

$$\begin{aligned}\text{Circumferential force} &= wR \left[\cos \theta - \frac{1}{1 + \cos \theta} \right] = \left[4 \times 10 \times 0.8 - \frac{1}{1.8} \right] \\ &= 10 \text{ kN/m}\end{aligned}$$

$$\text{Meridional stress} = \left(\frac{22.22 \times 10^3}{1000 \times 100} \right) = 0.22 \text{ N/mm}^2 < 5 \text{ N/mm}^2$$

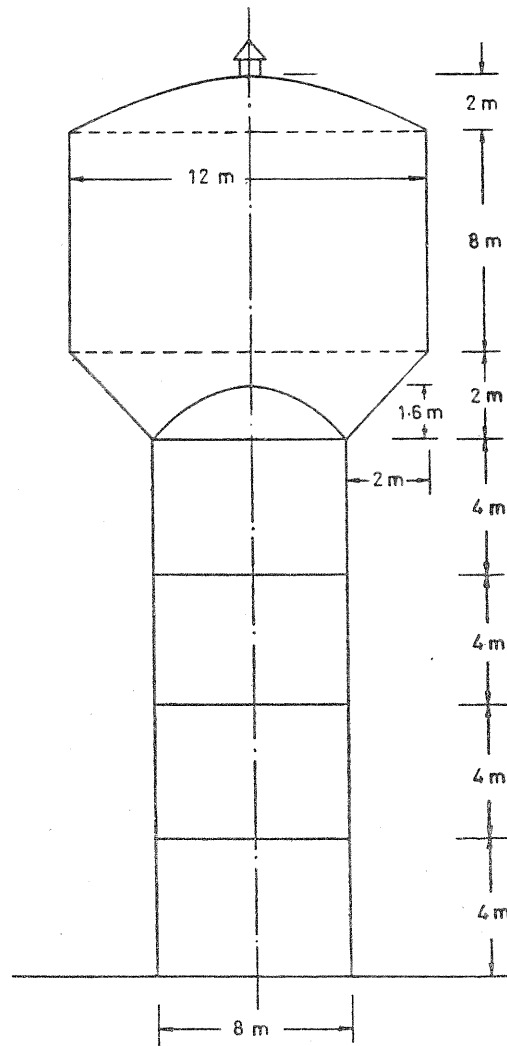


Fig. 6.6 Salient dimensions of intz tank

$$\text{Hoop stress} = \left(\frac{10 \times 10^3}{1000 \times 100} \right) = 0.10 \text{ N/mm}^2 < 5 \text{ N/mm}^2$$

The stresses are within safe limits.

Providing nominal reinforcements of 0.3%

$$A_{st} = \left(\frac{0.3 \times 100 \times 1000}{100} \right) = 300 \text{ mm}^2$$

Provide 8 mm Φ at 160 mm centres both circumferentially and meridionally.

5. Design of Top Ring Beam

$$\text{Hoop tension} = F_t = \left(\frac{T_1 \cos \theta \cdot D}{2} \right) = \left(\frac{22.22 \times 0.8 \times 12}{2} \right) = 106.6 \text{ kN}$$

$$A_{st} = \left(\frac{106.6 \times 10^3}{150} \right) = 710 \text{ mm}^2$$

Provide 8 bars of 12 mm Φ ($A_{st} = 904 \text{ mm}^2$)

If A_c = cross sectional area of ring beam

$$\left(\frac{106.6 \times 10^3}{A_c + 13 \times 904} \right) = 1.2 \quad \therefore A_c = 77,082 \text{ mm}^2$$

Provide 300 mm by 300 mm size top ring beam, with 8 bars of 12 mm Φ as main reinforcement and 6 mm Φ stirrups at 200 mm centres.

6. Design of Cylindrical Tank Wall

Maximum hoop tension at base of wall

$$= F_t = \left(\frac{wh \cdot D}{2} \right)$$

where w = density of water and

$$h = \text{depth of water} \quad \therefore F_t = \left(\frac{19 \times 8 \times 12}{2} \right) = 480 \text{ kN/m}$$

$$A_{st} = \left(\frac{480 \times 10^3}{150} \right) = 3200 \text{ mm}^2/\text{m height}$$

Provide 20 mm ϕ at 180 mm centres on each face ($A_{st} = 3492 \text{ mm}^2$)

steel area required at 2 m below top is $A_{st} = (2/8 \times 3200) = 800 \text{ mm}^2$

Provide 10 mm Φ at 180 mm centres on each face

If t = thickness of side wall at bottom

$$\left[\frac{480 \times 10^3}{1000t + (13 \times 3492)} \right] = 1.2 \quad \therefore t = 358 \text{ mm}$$

Adopt 400 mm thick walls at bottom gradually reducing to 200 mm at top.

Distribution Steel

$$\text{At bottom, } A_{st} = \left(\frac{0.2 \times 400 \times 1000}{100} \right) = 800 \text{ mm}^2$$

Provide 10 mm Φ at 100 mm centres on both face

$$\text{At top } A_{st} = \left(\frac{0.3 \times 200 \times 1000}{100} \right) = 600 \text{ mm}^2$$

Provide 10 mm Φ at 250 mm centres on both faces.

The details of reinforcements provided in the cylindrical tank walls at different heights are as follows:

Distance from top	Main hoop steel (each face)	Vertical distribution steel (each face)
0-2 m	10 mm-180 mm c/c	10 mm-250 mm c/c
2 m-4 m	16 mm-200 mm c/c	10 mm-250 mm c/c
4 m-8 m	20 mm-180 mm c/c	10 mm-180 mm c/c

7. Design of Bottom Ring Beam

Loads on ring beam

(a) Load due to top dome = (Meridional thrust $\times \sin \theta$)

$$= (22.22 \times \sin 36^\circ-50') = 13.3 \text{ kN/m}$$

(b) Load due to top ring beam = $(0.3 \times 0.3 \times 24) = 2.16 \text{ kN/m}$

(c) Load due to cylindrical wall

$$= \left(\frac{0.4 + 0.2}{2} \right) \times 8 \times 24 = 57.6 \text{ kN/m}$$

(d) Self weight of ring beam (Assuming a section 1.2 m by 0.6 m)

$$= 1.2 \times 0.6 \times 24 = 17.28 \text{ kN/m}$$

$\therefore$ Total vertical load = $V_1 = 91 \text{ kN/m}$

Horizontal force = $H = V_1 \cot \theta = (91 \times \cot 45^\circ) = 91 \text{ kN}$

$\therefore$ Hoop tension due to vertical loads is given by

$$H_g = \left(\frac{H \cdot D}{2} \right) = \left(\frac{91 \times 12}{9} \right) = 546 \text{ kN}$$

Hoop tension due to water pressure is

$$H_w = \left(\frac{wh \cdot d \cdot D}{2} \right) = \left(\frac{10 \times 8 \times 0.6 \times 12}{2} \right) = 288 \text{ kN}$$

$\therefore$ Total hoop tension = $(H_g + H_w) = (546 + 288) = 834 \text{ kN}$

$\therefore$ $A_{st} = \left(\frac{834 \times 10^3}{150} \right) = 5560 \text{ mm}^2$

Provide 18 bars of 20 mm ($A_{st} = 5562 \text{ mm}^2$)

Max. tensile stress

$$= \left[\frac{834 \times 10^3}{(1200 \times 600) + (18 \times 5652)} \right] = 1.05 \text{ N/mm}^2 < 1.2 \text{ N/mm}^2$$

Provide a ring beam of 1200 mm wide by 600 mm deep with 18 bars of 20 mm Φ and distribution bars of 10 mm Φ from cylindrical wall taken round the main bars as stirrups at 180 mm centres.

$\frac{A_{st}}{D \cdot B} > \frac{c}{f_y}$

8. Design of Conical Dome

Average diameter of conical dome = $\left(\frac{12 + 8}{2} \right) = 10 \text{ m}$

Average depth of water = $(8 + 2/2) = 9 \text{ m}$

Weight of water above conical dome $= (\pi \times 10 \times 9 \times 2 \times 10) = 5655 \text{ kN}$

Assuming 600 mm thick slab,

Self weight of slab $= (\pi \times 10 \times 2.83 \times 0.6 \times 24) = 1280 \text{ kN}$

Load from top dome, top ring beam, cylindrical wall and bottom ring beam $= (\pi \times 12 \times 91) = 3430 \text{ kN}$

$\therefore$ Total load at base of conical slab

$$= (5655 + 1280 + 3430) = 10365 \text{ kN}$$

$$\text{Load/unit length} = V_2 = \left(\frac{10365}{\pi \times 8} \right) = 413 \text{ kN/m}$$

Meridional thrust

$$= T = V_2 \cdot \operatorname{cosec} \theta = (413 \times \operatorname{cosec} 45^\circ) = 584 \text{ kN}$$

Meridional stress

$$= \left(\frac{584 \times 10^3}{600 \times 1000} \right) = 0.973 \text{ N/mm}^2 < 5 \text{ N/mm}^2 \quad (\text{safe})$$

Hoop tension in conical dome will be maximum at the top of the conical dome slab since diameter D is maximum at this section,

Hoop tension $= H = (p \cdot \operatorname{cosec} \theta + q \cot \theta) D/2$

Water pressure $= p = (10 \times 8) = 80 \text{ kN/m}^2$

Weight of conical dome slab per m^2 is computed as,

$$q = (0.6 \times 24) = 14.4 \text{ kN/m}^2$$

$$\theta = 45^\circ$$

$$D = 12 \text{ m}$$

$$\therefore H = (80 \times \operatorname{cosec} 45^\circ + 14.4 \times \cot 45^\circ) 12/2 = 765 \text{ kN}$$

$$\therefore A_{st} = \left(\frac{765 \times 10^3}{150} \right) = 5100 \text{ mm}^2$$

Provide 25 mm Φ at 180 mm centres ($A_{st} = 5470 \text{ mm}^2$) on both faces of the slab

$$\text{Distribution steel} = \left(\frac{0.2 \times 600 \times 1000}{100} \right) = 1200 \text{ mm}^2$$

Provide 10 mm Φ at 130 mm centres on both faces along the meridians

Max. tensile stress =

$$= \left[\frac{765 \times 10^3}{(600 \times 1000) + (13 \times 5470)} \right] = 1.13 \text{ N/mm}^2 < 1.2 \text{ N/mm}^2$$

(safe)

9. Design of Bottom Spherical Dome

Thickness of dome slab assumed as 300 mm

Diameter at base $= D = 8 \text{ m}$

Central rise $= r = (1/5 \times 8) = 1.6 \text{ m}$

If R = radius of the dome

$$(2R - r)r = (D/2)^2$$

$$(2R - 1.6)1.6 = 4^2 \quad \therefore R = 5.8 \text{ m}$$

Self weight of dome slab = $(2 \times \pi \times 5.8 \times 1.6 \times 0.3 \times 24) = 420 \text{ kN}$

Volume of water above the dome is = $\pi \times 4^2(8 + 2) -$

$$\left[\frac{2\pi \times 5.8^2 \times 1.6}{3} - \frac{\pi \times 4^2}{3} (5.8 - 1.6) \right] = 440 \text{ m}^3$$

$\therefore$ Weight of water = $(440 \times 10) = 4400 \text{ kN}$

$\therefore$ Total load on dome = $(420 + 4400) = 5320 \text{ kN}$

$$\text{Load/unit area} = w = \left(\frac{5320}{\pi \times 4^2} \right) = 106 \text{ kN/m}^2$$

$$\text{Meridional thrust} = T_1 = \left(\frac{wR}{1 + \cos \theta} \right)$$

$$\cos \theta = \left(\frac{4.2}{5.8} \right) = 0.724 \quad \therefore \theta = 44.5^\circ$$

$$\therefore T_1 = \left(\frac{106 \times 5.8}{1 + 0.724} \right) = 357 \text{ kN/m} \quad \checkmark_{2.5}$$

$$\text{Meridional stress} = \left(\frac{357 \times 10^3}{300 \times 1000} \right) = 1.19 \text{ N/mm}^2 \text{ (safe)}$$

$$\text{Circumferential force} = wR \left(\cos \theta - \frac{1}{1 + \cos \theta} \right) = 106 \times 5.8$$

$$\left[0.724 - \frac{1}{1.724} \right] = 88.5 \text{ kN/m}$$

$$\therefore \text{Hoop stress} = \left(\frac{88.5 \times 10^3}{300 \times 1000} \right) = 0.3 \text{ N/mm}^2 \text{ (safe)}$$

Provide nominal reinforcement of 0.3%

$$A_{st} = \left(\frac{0.3 \times 300 \times 1000}{100} \right) = 900 \text{ mm}^2$$

Provide 12 mm at 120 mm centres circumferentially and along the meridians.

10. Design of Bottom Circular Girder

Thrust from conical dome = $T_1 = 413 \text{ kN/m}$ $\checkmark_{2.0}$
acting at an angle of $\alpha = 45^\circ$ to the horizontal.

Thrust from spherical dome = $T_2 = 357 \text{ kN/m}$ $\checkmark_{2.5}$
acting at an angle of $\beta = 44.5^\circ$ to the horizontal.

Net horizontal force on ring beam = $(T_1 \cos \alpha - T_2 \cos \beta)$

$$[(413 \times 0.707) - (357 \times 0.713)] = 38 \text{ kN/m}$$

$$\text{Hoop compression in the beam} = \left(\frac{38 \times 8}{2} \right) = 152 \text{ kN} \quad \boxed{F_{x0.2}}$$

Assuming the size of the ring girder as 600 mm wide by 1200 mm deep,

$$\text{Hoop stress} = \left(\frac{152 \times 10^3}{600 \times 1200} \right) = 0.21 \text{ N/mm}^2 \text{ (safe)}$$

Vertical load on ring beam = $T_1 \sin \alpha + T_2 \sin \beta$

$$= [(413 \times 0.707) + (357 \times 0.70)] = 542 \text{ kN/m}$$

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Self weight of beam = $(0.6 \times 1.2 \times 24) = 17.28 \text{ kN/m}$

$\therefore$ Total load = $(542 + 17.28) = 560 \text{ kN/m}$

$$\rightarrow \text{Total design load on the ring girder} = W = (\pi \cdot D \cdot w) = (\pi \times 8 \times 560) = 14074 \text{ kN}$$

The circular girder is supported on 8 column. Using the moment coefficients given in Table 4.1,

Maximum negative B.M. at support section = 0.0083 W.R.

$$= (0.0083 \times 14074 \times 4) = 467 \text{ kN}\cdot\text{m}$$

Maximum positive B.M. at mid span section = 0.0041 W.R.

$$= (0.0041 \times 14074 \times 4) = 231 \text{ kN}\cdot\text{m}$$

Torsional moment = $0.0006 \text{ WR} = (0.0006 \times 14074 \times 4) = 34 \text{ kN}\cdot\text{m}$

Shear force at support section is

$$V = \left[\frac{560 \times 4 \times (\pi/4)}{2} \right] = 880 \text{ kN}$$

Shear force at section of maximum torsion is (At an angle of 9.5° from higher support)

$$V = \left[880 - \frac{560 \times \pi \times 4 \times 9.5}{180} \right] = 521 \text{ kN}$$

Design of support section

$$M = 467 \text{ kN}\cdot\text{m}$$

$$V = 880 \text{ kN}$$

$$\therefore d = \sqrt{\frac{467 \times 10^6}{0.897 \times 600}} = 931 \text{ mm}$$

Adopt effective depth $d = 1150 \text{ mm}$ cover = 50 mm

$$A_{st} = \left(\frac{467 \times 10^6}{150 \times 0.9 \times 1150} \right) = 3008 \text{ mm}^2$$

Provide 6 bars of $25 \text{ mm } \Phi$ ($A_{st} = 2946 \text{ mm}^2$) ✓

$$\tau_v = \left(\frac{880 \times 10^3}{600 \times 1150} \right) = 1.27 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd} \right) = \left(\frac{100 \times 2946}{600 \times 1150} \right) = 0.426 \quad \therefore \tau_c = 0.27 \text{ N/mm}^2$$

since $\tau_c < \tau_v$, shear reinforcements are required.

$$\text{shear taken by concrete} = \left(\frac{0.27 \times 600 \times 1150}{1000} \right) = 186 \text{ kN}$$

Balance shear = $(880 - 186) = 694 \text{ kN}$

using $12 \text{ mm } \Phi$ - 4 legged stirrups, spacing is,

$$s_v = \left(\frac{150 \times 4 \times 113 \times 1150}{694 \times 10^3} \right) = 112 \text{ mm}$$

Adopt 12 mm Φ - 4 legged stirrups at 110 mm centres near supports.

Design of mid span section

Maximum positive moment $M = 231 \text{ kN}\cdot\text{m}$

$$A_{st} = \left(\frac{231 \times 10^6}{150 \times 0.9 \times 1150} \right) = 1488 \text{ mm}^2$$

But minimum area of steel in the section is

$$= \left(\frac{0.3 \times 600 \times 1200}{100} \right) = 2160 \text{ mm}^2$$

Provide 5 bars of 25 mm at mid span section, stirrups 10 mm 4 legged at 300 mm centres.

Design of section subjected to maximum torsion

$$T = 34 \text{ kN}\cdot\text{m}$$

$$D = 1200 \text{ mm}$$

$$V = 521 \text{ kN}$$

$$b = 600 \text{ mm}$$

$$M = 0$$

$$d = 1150 \text{ mm}$$

$$M_t = T \left[\frac{1 + (D/d)}{1.7} \right] = 34 \left[\frac{1 + (1200/6000)}{1.7} \right] = 60 \text{ kN}\cdot\text{m}$$

$$\therefore M_{e1} = (M + M_t) = (0 + 60) = 60 \text{ kN}\cdot\text{m}$$

$$A_{st} = \left(\frac{60 \times 10^6}{150 \times 0.9 \times 1150} \right) = 387 \text{ mm}^2$$

But minimum area of tension steel is,

$$A_{st} = \left(\frac{0.3 \times 600 \times 1200}{100} \right) = 2160 \text{ mm}^2$$

Provide 5 bars of 25 mm Φ ($A_{st} = 2455 \text{ mm}^2$)

$$\text{Equivalent shear } V_e = (V + 1.6 T/d) = \left(521 + 1.6 \times \frac{34}{0.6} \right) = 612 \text{ kN}$$

$$\tau_{ve} = \left(\frac{V_e}{bd} \right) = \left(\frac{612 \times 10^3}{600 \times 1150} \right) = 0.88 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{b \cdot d} \right) = \left(\frac{100 \times 2455}{600 \times 1150} \right) = 0.32 \quad \therefore \tau_c = 0.24 \text{ N/mm}^2$$

since $\tau_v > \tau_c$, shear reinforcements are required.

Using 12 mm Φ - 4 legged stirrups with side covers of 25 mm and top and bottom covers of 50 mm, spacing is,

$$s_v = \left[\frac{A_{sv} \cdot \sigma_{sv}}{(\tau_v - \tau_c)b} \right] = \left[\frac{4 \times 113 \times 150}{(0.88 - 0.24)600} \right] = 188 \text{ mm}$$

Adopt 12 mm Φ - 4 legged stirrups at 180 mm centres

11. Design of Columns of Supporting Tower

The supporting tower comprises of 8 columns equally spaced on a circle of 8 m diameter.

Loads on Columns

- (a) Vertical load on each column = $\left(\frac{14074}{8}\right) = 1760 \text{ kN}$
 (b) Self weight of column of height 16 m and diameter 650 mm is
 $= (\pi/4 \times 0.65^2 \times 16 \times 24) = 127 \text{ kN}$
 (c) Self weight of bracings (3 numbers of 4 m intervals, size of
 bracing is 500 mm $\times$ 500 mm)
 $= (3 \times 0.5 \times 0.5 \times \pi/8 \times 24) = 57 \text{ kN}$

Total vertical load on each column

$$= (1760 + 127 + 57) = 1944 \text{ kN}$$

Wind Forces on Column

Intensity of wind pressure = 1.5 kN/m²

Reduction coefficient for circular shapes = 0.7

- (a) Wind force on top dome and cylindrical wall
 $= (8 + 2/2) \times 0.7 \times 1.5 \times 12 = 114 \text{ kN}$
 (b) Wind force on conical dome = $(2 \times 10 \times 0.7 \times 1.5) = 21 \text{ kN}$
 (c) Wind force on bottom ring beam = $(1.2 \times 8 \times 0.7 \times 1.5) = 11 \text{ kN}$
 (d) Wind force on five columns = $(5 \times 0.65 \times 16 \times 0.7 \times 1.5) = 55 \text{ kN}$
 (e) Wind force on bracings = $(0.5 \times 8 \times 3 \times 1.5) = 18 \text{ kN}$
 $\therefore$ Total horizontal wind force = $(114 + 21 + 11 + 55 + 18) = 219 \text{ kN}$

Assuming contraflexure points at mid height of columns and fixity at the base due to raft foundations, the moment at the base of columns is computed as

$$M = \left(\frac{219 \times 4}{2}\right) = 438 \text{ kN} \cdot \text{m}$$

If M_1 = Moment at the base of the columns due to wind loads
 $= (114 \times 23) + (21 \times 17) + (11 \times 16) + (6 \times 12) + (6 \times 8) + (6 \times 4)$
 $= 3299 \text{ kN} \cdot \text{m}$

If V = Reaction developed at the base of exterior columns

$$M_1 = \sum M + \frac{V}{r_1} [\sum r^2]$$

$$3299 = 438 + V/4[(2 \times 4^2) + 4(4/\sqrt{2})^2] \quad \therefore V = 179 \text{ kN}$$

$\therefore$ Total load on Leeward column at base = $(1944 + 179) = 2123 \text{ kN}$

Moment in each column at base = $\left(\frac{438}{8}\right) = 55 \text{ kN} \cdot \text{m}$

Reinforcement in column

Axial load = $P = 2123 \text{ kN}$ Bending moment = $M = 55 \text{ kN} \cdot \text{m}$

$$\text{eccentricity} = e = (M/P) = \left(\frac{55 \times 10^6}{2123 \times 10^3}\right) = 25.9 \text{ mm}$$

Since eccentricity is small, direct stresses are predominant.

Using 8 bars of 32 mm Φ and lateral ties of 10 Φ at 300 mm centres

$$A_{sc} = (8 \times 804) = 6432 \text{ mm}^2$$

$$A_e = \left[\frac{\pi(650)^2}{4} + (1.5 \times 13 \times 6432) \right] = 0.45 \times 10^6 \text{ mm}^2$$

$$I_e = \left(\frac{\pi \times 325^4}{4} \right) + (1.5 \times 13) \left[(2 \times 804 \times 275^2) + 4 \times 804 \left(\frac{275}{\sqrt{2}} \right)^2 \right]$$

$$= 13.48 \times 10^9 \text{ mm}^4$$

$$\text{Direct compressive stress} = \sigma'_{cc} = \left(\frac{2123 \times 10^3}{0.45 \times 10^6} \right) = 4.71 \text{ N/mm}^2$$

$$\text{Bending stress} = \sigma'_{cb} = \left(\frac{55 \times 10^6 \times 325}{13.48 \times 10^9} \right) = 1.32 \text{ N/mm}^2$$

Permissible stresses in concrete are increased by 33.33% while considering the wind effect.

$$\left(\frac{\sigma'_{cc}}{\sigma_{cc}} + \frac{\sigma'_{cb}}{\sigma_{cb}} \right) < 1$$

$$\therefore \left(\frac{4.71}{5 \times 1.33} + \frac{1.32}{7 \times 1.33} \right) = 0.84 < 1.0 \text{ (safe)}$$

12. Design of Bracings

$$\text{Moment in brace} = (2 \times \text{moment in column} \times \sqrt{2}) = (2 \times 55 \times \sqrt{2})$$

$$M = 156 \text{ kN.m}$$

Section of brace = 500 mm by 500 mm

$$\therefore b = 500 \text{ mm} \quad d = 450 \text{ mm}$$

Moment of resistance of section is

$$M_1 = (0.897 \times 500 \times 450^2) = 91 \times 10^6 \text{ N.mm} = 91 \text{ kN.m}$$

$$\text{Balance moment} = M_2 = (M - M_1) = (156 - 91) = 65 \text{ kN.m}$$

$$A_{st1} = \left(\frac{91 \times 10^6}{230 \times 0.9 \times 450} \right) = 976 \text{ mm}^2$$

$$A_{st2} = \left(\frac{65 \times 10^6}{230 \times 0.9 \times 400} \right) = 785 \text{ mm}^2$$

$$\therefore A_{st} = (A_{st1} + A_{st2}) = 1761 \text{ mm}^2$$

Provide 4 bars of 25 mm Φ ($A_{st} = 1964 \text{ mm}^2$) at top and bottom since wind direction is reversible.

$$\text{Length of brace} = L = (2 \times 4 \times \sin 22.5^\circ) = 3.06 \text{ m}$$

Maximum shear force in brace

$$= \left(\frac{\text{Moment in brace}}{\frac{1}{2} \text{ length of brace}} \right) = \left(\frac{156}{0.5 \times 3.06} \right) = 102 \text{ kN}$$

$$\tau_v = \left(\frac{102 \times 10^3}{500 \times 450} \right) = 0.45 \text{ N/mm}^2$$

$$\left(\frac{100A_{st}}{bd}\right) = \left(\frac{100 \times 1964}{500 \times 450}\right) = 0.87 \quad \therefore \tau_c = 0.37 \text{ N/mm}^2$$

since $\tau_v > \tau_c$, shear reinforcements are required.

$$\text{shear carried by concrete} = \left(\frac{0.37 \times 500 \times 450}{1000}\right) = 83 \text{ kN}$$

$$\text{Balance shear} = (102 - 83) = 19 \text{ kN}$$

using 10 mm Φ - 2 legged stirrups, spacing is given by

$$s_v = \left(\frac{230 \times 2 \times 79 \times 450}{19 \times 10^3}\right) = 860 \text{ mm}$$

But $s_v > 0.75d$ or 450 mm whichever is less $= (0.75 \times 450) = 337 \text{ mm}$

Adopt 10 mm Φ - 2 legged stirrups at 330 mm centres.

13. Design of Foundations

A circular girder with a raft slab is provided for the tower foundations.

$$\text{Total load on foundations} = (1944 \times 8) = 15552 \text{ kN}$$

$$\text{Self weight of foundations at 10\%} = 1555$$

$$\text{Total load} \dots \dots \dots = 17107 \text{ kN}$$

$$\text{Safe bearing capacity of soil at site} = 250 \text{ kN/m}^2$$

$$\therefore \text{Area of foundation} = \left(\frac{17107}{250}\right) = 68.4 \text{ m}^2$$

Providing a raft slab with equal projections on either side of a circular ring beam and if b = width of raft slab, then

$$(\pi \times 8 \times b) = 68.4$$

$$b = 2.72 \text{ m}$$

Adopt a raft slab having 5 m, inner diameter and 11 m outer diameter ($b = 3 \text{ m}$)

Design of Circular girder of Raft slab

$$\text{Total load on circular girder} = W = 15552 \text{ kN}$$

$$\text{Load per metre run on girder} = \left(\frac{15552}{\pi \times 8}\right) = 618.8 \text{ kN/m}$$

Referring to moment coefficients given in Table 4.1, the maximum moments in the circular girder is computed.

Maximum negative moment at support

$$= 0.0083 W.R = (0.0083 \times 15552 \times 4) = 516 \text{ kN} \cdot \text{m}$$

Maximum positive moment at mid span $= 0.0041 W.R$

$$= (0.0041 \times 15552 \times 4) = 255 \text{ kN} \cdot \text{m}$$

Maximum torsional moment (At 9.5° from either support)

$$= 0.0006 W.R = (0.0006 \times 15552 \times 4) = 37.3 \text{ kN} \cdot \text{m}$$

shear force at support section is,

$$V = \frac{618.8 \times 4 \times (\pi/4)}{2} = 972 \text{ kN}$$

shear force at section of maximum torsion is

$$V = \left[972 - \frac{618 \times \pi \times 4 \times 9.5}{180} \right] = 562 \text{ kN}$$

The support section is designed for a maximum negative moment of $M = 516 \text{ kN}\cdot\text{m}$ shear force $V = 972 \text{ kN}$

Assuming the width of section $b = 750 \text{ mm}$

$$\text{Effective depth} = d = \sqrt{\frac{516 \times 10^6}{0.897 \times 750}} = 875 \text{ mm}$$

Adopt $d = 930 \text{ mm}$ and over all depth $D = 1000 \text{ mm}$ cover = 70 mm

$$A_{st} = \left(\frac{516 \times 10^6}{230 \times 0.9 \times 930} \right) = 2680 \text{ mm}^2$$

Provide 6 bars of $25 \text{ mm } \Phi$ ($A_{st} = 2946 \text{ mm}^2$)

$$\tau_v = \left(\frac{972 \times 10^3}{750 \times 930} \right) = 1.39 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd} \right) = \left(\frac{100 \times 2946}{750 \times 930} \right) = 0.42 \quad \tau_c = 0.26 \text{ N/mm}^2$$

since $\tau_v > \tau_c$, shear reinforcements are required.

$$\text{Shear taken by concrete} = \left(\frac{0.26 \times 750 \times 930}{1000} \right) = 181 \text{ kN}$$

$\therefore$ Balance shear = $(972 - 181) = 791 \text{ kN}$

Using $12 \text{ mm } \Phi$ - 4 legged stirrups, spacing is,

$$s_v = \left(\frac{230 \times 4 \times 113 \times 930}{791 \times 10^3} \right) = 122 \text{ mm}$$

Adopt 120 mm spacing for stirrups. Steel required for mid span section is

$$A_{st} = \left(\frac{255 \times 10^6}{230 \times 0.9 \times 930} \right) = 1325 \text{ mm}^2$$

$$\text{But minimum steel } A_{st} = \left(\frac{0.85 b d}{f_y} \right) = \left(\frac{0.85 \times 750 \times 930}{415} \right) = 1429 \text{ mm}^2$$

Provide 3 bars of 25 mm at mid span section.

The section subjected to maximum torsional moment and shear should be designed for the following forces:

$$T = 37.3 \text{ kN}\cdot\text{m}$$

$$D = 1000 \text{ mm}$$

$$V = 562 \text{ kN}$$

$$b = 750 \text{ mm}$$

$$M = 0$$

$$d = 930 \text{ mm}$$

$$M_t = T \left[\frac{1 + (D/b)}{1.7} \right] = 37.3 \left[\frac{1 + (1000/750)}{1.7} \right] = 51 \text{ kN}\cdot\text{m}$$

$$\therefore M_{e1} = (M + M_i) = (0 + 51) = 51 \text{ kN} \cdot \text{m}$$

$$A_{st} = \left(\frac{51 \times 10^6}{230 \times 0.9 \times 930} \right) = 265 \text{ mm}^2$$

Minimum area of steel $A_{st} = 1429 \text{ mm}^2$

Provide 3 bars of 25 mm Φ ($A_{st} = 1473 \text{ mm}^2$)

Equivalent shear

$$V_e = (V + 1.6T/b) = [562 + 1.6 (37.3/0.75)] = 641 \text{ kN}$$

$$\tau_v = \left(\frac{641 \times 10^3}{750 \times 930} \right) = 0.92 \text{ N/mm}^2$$

$$\left(\frac{100A_{st}}{bd} \right) = \left(\frac{100 \times 1473}{750 \times 930} \right) = 0.1$$

From Tables $\tau_c = 0.21 \text{ N/mm}^2$, $\tau_v > \tau_c$, shear reinforcements are required,

$$\text{Balance shear} = \left[641 - \frac{0.21 \times 750 \times 930}{1000} \right] = 495 \text{ kN}$$

using 12 mm Φ - 4 legged stirrups, spacing is,

$$s_v = \left[\frac{A_{sv} \cdot \sigma_{sv}}{(\tau_v - \tau_c)b} \right] = \left[\frac{4 \times 113 \times 230}{(0.92 - 0.21) 750} \right] = 195 \text{ mm}$$

Adopt 12 mm Φ - 4 legged stirrups at 190 mm centres.

Design of Raft slab

Maximum projection of raft slab from face of column

$$= \left(\frac{3 - 0.75}{2} \right) = 1.125 \text{ m}$$

$$\text{Soil pressure} = \frac{155.52}{(5.5^2 - 2.5^2)} = 206 \text{ kN/m}^2$$

Considering 1 m width of raft slab along the circular arc,

$$\text{Maximum B.M.} = \left(\frac{206 \times 1.1^2}{2} \right) = 124.63 \text{ kN} \cdot \text{m}$$

$$d = \sqrt{\frac{124.63 \times 10^6}{0.897 \times 1000}} = 373 \text{ mm}$$

Provide 500 mm over all depth with effective depth $d = 450 \text{ mm}$ to contain the shear stresses within permissible limits.

$$A_{st} = \left(\frac{124.63 \times 10^6}{230 \times 0.9 \times 450} \right) = 1205 \text{ mm}^2$$

Provide 25 mm bars at 200 mm centres to reduce shear stresses ($A_{st} = 2454 \text{ mm}^2$)

$$\text{Distribution steel} = \left(\frac{0.12 \times 500 \times 1000}{100} \right)$$

$$= 600 \text{ mm}^2 - \text{provide } 12\Phi - 180 \text{ c/c}$$

shear force at a section 450 mm from face of column is

$$V = (206 \times 0.65 \times 1.00) = 134 \text{ kN}$$

$$\tau_v = \left(\frac{134 \times 10^3}{1000 \times 450} \right) = 0.29 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd} \right) = \left(\frac{100 \times 2454}{1000 \times 450} \right) = 0.545$$

From Tables, $\tau_c = 0.31 \text{ N/mm}^2$

since $\tau_v < \tau_c$, no shear reinforcements are required.

The thickness of the footing is retained at 500 mm up to a distance of 500 mm from the column face and there after gradually decreased to 250 mm at the edges. The details of reinforcements in the various structural elements of the intz type tank are detailed in Figs. 6.7, 6.8 and 6.9.

6.4 CONICAL OR FUNNEL SHAPED TANK

Conical or funnel shaped over head water tanks are often preferred to other shapes mainly due to their aesthetic and superior architectural features in comparison with other types of over head tanks. Basically a conical over head tank comprises of the following structural components as shown in Fig. 6.10.

- (a) Conical dome covering the tank
- (b) Top ring beam
- (c) Conical shell
- (d) Bottom spherical dome and internal shaft
- (e) Bottom ring beam
- (f) Supporting cylindrical shaft
- (g) Raft foundation

For supporting towers higher than 25 m, reinforced concrete cylindrical shells are economical and can be rapidly constructed using the slip form process of casting. They can also be built using precast concrete elements. The conical shell walls are sloping at 45° to the horizontal and the thickness of the walls gradually increased towards the bottom of the tank and designed for hoop tension and meridional thrust. The top and bottom ring beams are designed for hoop tension.

The supporting cylindrical shell tower is designed for combined thrust and bending due to wind and seismic forces. A rigid raft slab foundation is provided to support the shaft. The design of a conical or funnel shaped elevated water tank is illustrated by the following example.

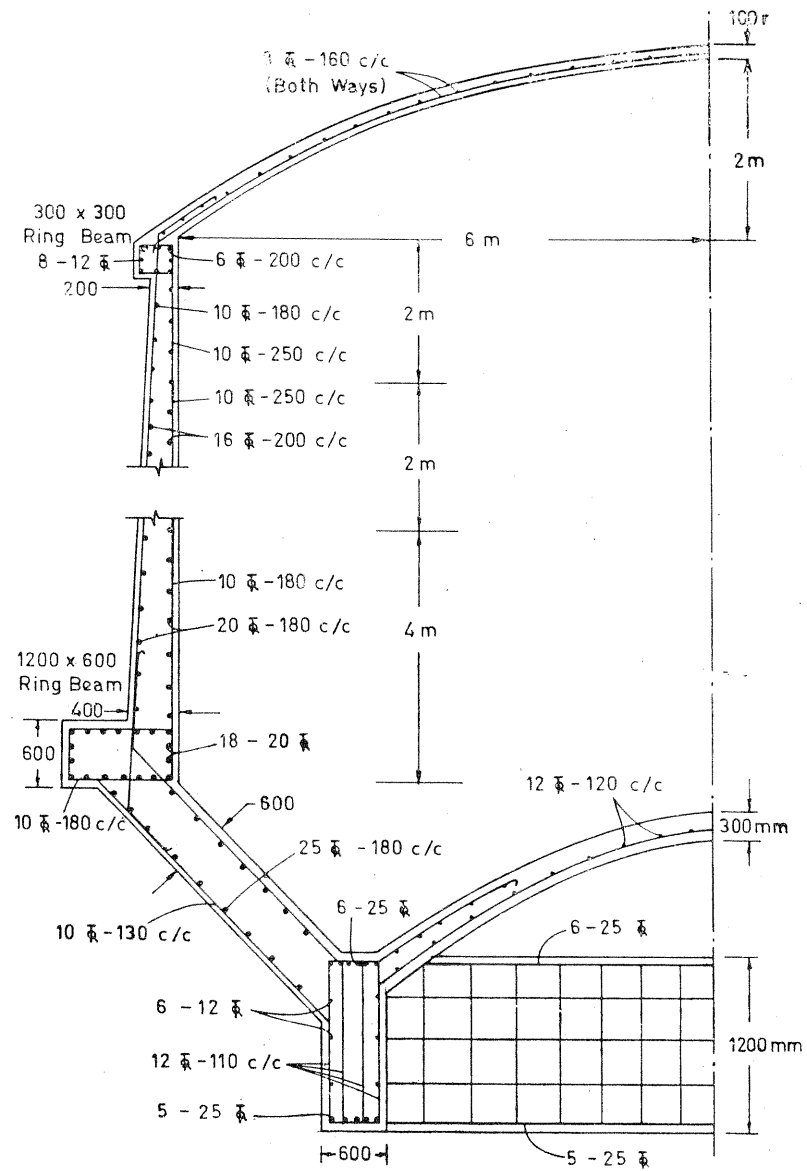


Fig. 6.7 Reinforcement details in intz tank

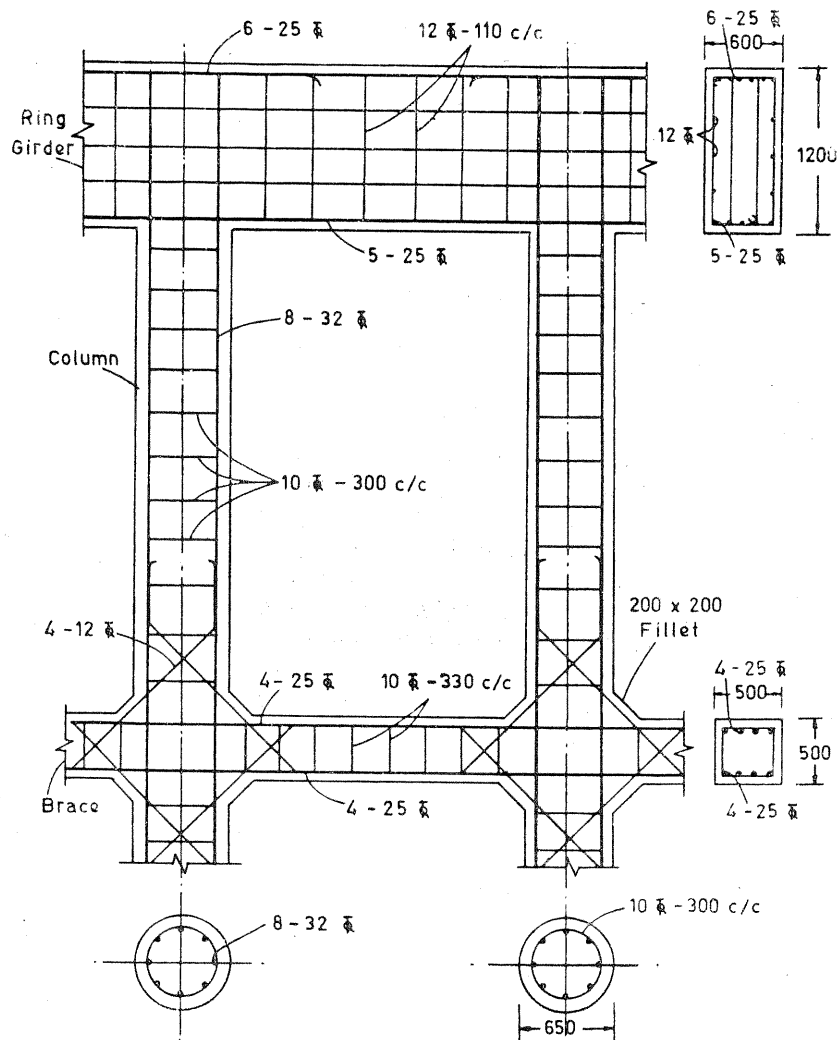


Fig. 6.8 Reinforcement details in ring girder columns and braces.

6.5 DESIGN EXAMPLE OF FUNNEL SHAPED OVER HEAD TANK

Design a funnel shaped over head water tank to suit the following data:

Capacity of the tank = 350,000 litres

Height of tower shaft = 20 m above G.L.

Basic wind pressure = 1.5 kN/m²

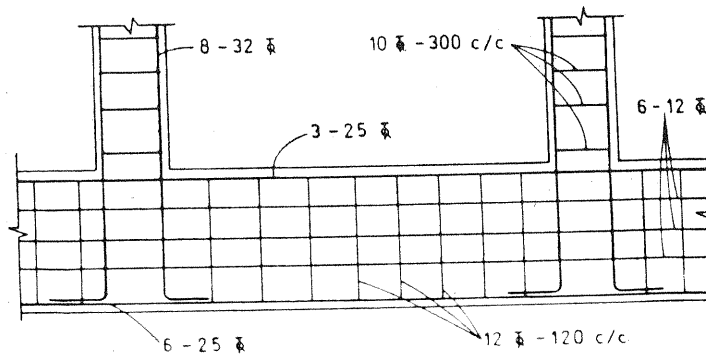
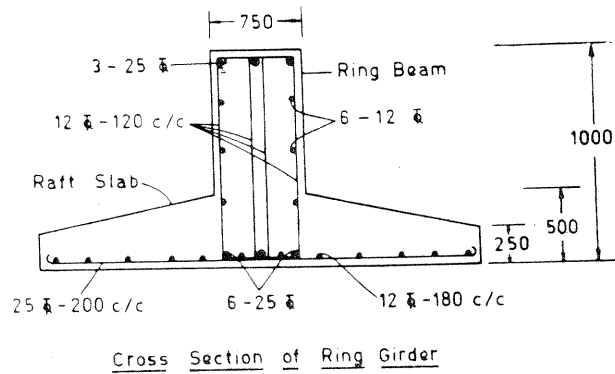


Fig. 6.9 Reinforcement details in ring beam and raft slab.

Adopt M-20 grade concrete and Fe-415 grade tor steel for all R.C.C. work.

Depth of foundations = 1.5 m

Safe bearing capacity of soil = 200 kN/m²

Design the conical shaped tank, supporting cylindrical shaft and foundations for the tank and sketch the typical reinforcement details in the various structural components.

1. Data

Capacity $V = 350,000$ litres

Height of supporting shaft = 20 m

Wind pressure = 1.5 kN/m²

S.B.C. of soil = 200 kN/m²

Depth of foundations = 1.5 m

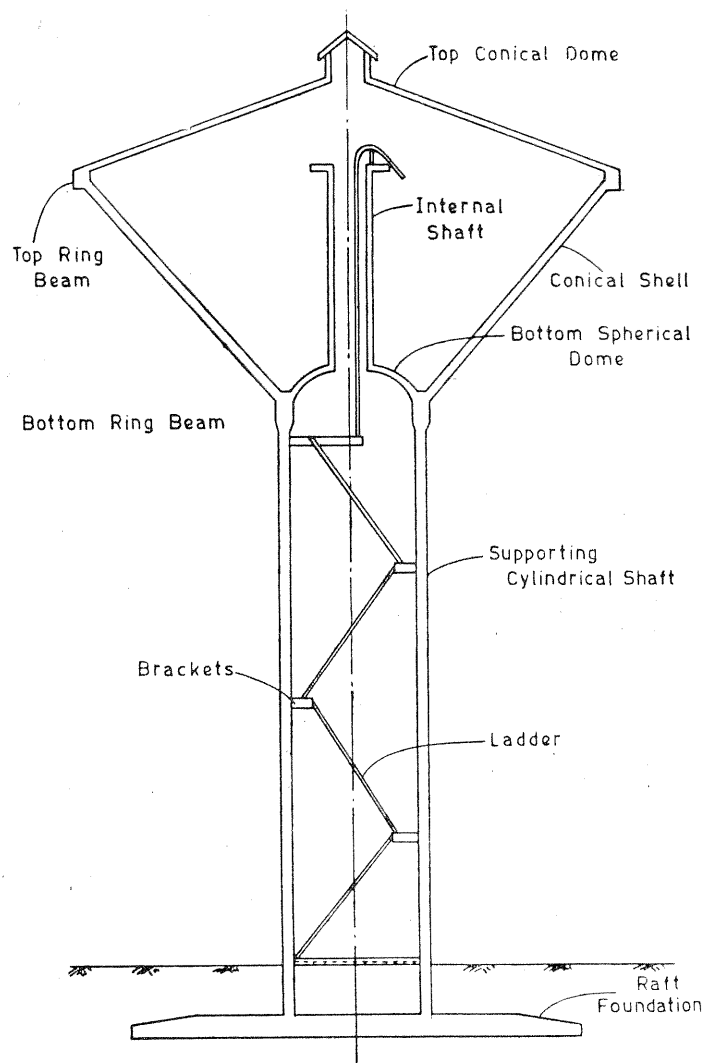


Fig. 6.10 Structural components of funnel shaped tank.

2. Permissible Stresses

For M-20 grade concrete and Fe-415 grade steel

$$\sigma_{ct} = 1.2 \text{ N/mm}^2$$

$$m = 13$$

$$\sigma_{cc} = 5 \text{ N/mm}^2$$

$$Q = 0.897$$

$$\sigma_{cb} = 7 \text{ N/mm}^2$$

$$j = 0.906$$

$$\sigma_{st} = 150 \text{ N/mm}^2$$

3. Dimensions of Tank

Diameter of the supporting shaft is assumed as 5 m.

Let R = radius of the conical shell. The dimensions of the tank can be expressed in terms of R as shown in Fig. 6.11.

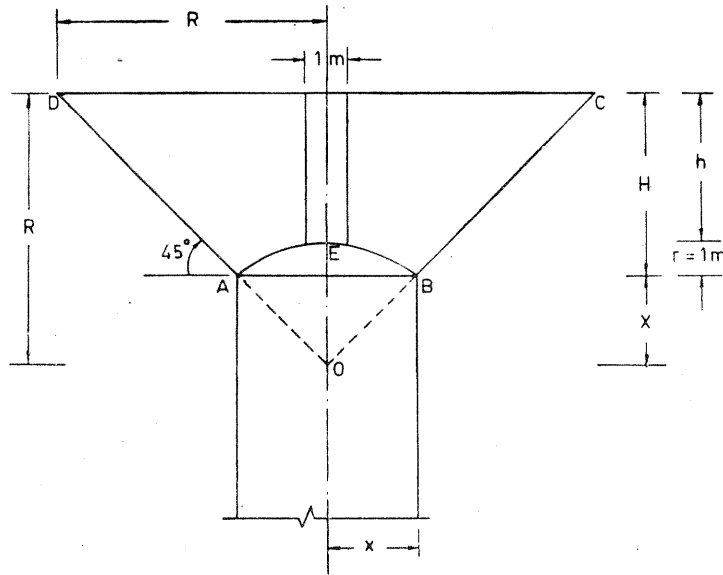


Fig. 6.11 Dimensions of conical shell

x = Radius of shaft = 2.5 m

$H = (R - x) = (R - 2.5)$

$h = (H - r) = (R - 2.5 - 1) = (R - 3.5)$

If

V = volume of the tank

Then,

$V = (\text{Volume of cone } ODC) - (\text{Volume of cone } OAB) - (\text{Volume of dome } AEB) - (\text{Volume of inner shaft})$

V_1 = Volume of cone ODC

$$= \frac{1}{3} \pi R \cdot R^2 = \frac{1}{3} \pi R^3$$

V_2 = Volume of cone OAB

$$= \frac{1}{3} \pi \cdot x^2 = \frac{1}{3} \pi \times 2.5^3$$

V_3 = Volume of dome AEB

$$= \frac{\pi r^2}{3} (3R - r) = \frac{\pi \times 1^2}{3} (3R - 1)$$

V_4 = Volume of inner shaft = $\pi(0.5)^2 (R - 3.5)$

$$\therefore V = (V_1 - V_2 - V_3 - V_4)$$

$$= \frac{1}{3} \pi R^3 - \frac{1}{3} \pi (2.5)^3 - \frac{\pi}{3} (3R - 1) - \pi(0.5)^2 (R - 3.5)$$

But

$$V = \left(\frac{350,000}{1000} \right) = 350 \text{ m}^3$$

$$\begin{aligned} \therefore 350 &= 1.047 R^3 - 16.36 - 3.14 R \\ &+ 1.047 - 0.785 R + 2.7489 \\ 1.047 R^3 - 3.925 R - 362.57 &= 0 \end{aligned}$$

solving $R = 7.15$ m

Adopt $R = 7.5$ m to allow for free board. The capacity of the tank with a radius $R = 7.5$ m is computed as $V = 388 \text{ m}^3$. Which is marginally greater than the required volume capacity of 350 m^3 .

4. Design of Top Conical Dome

The geometry of the conical dome is shown in Fig. 6.12.

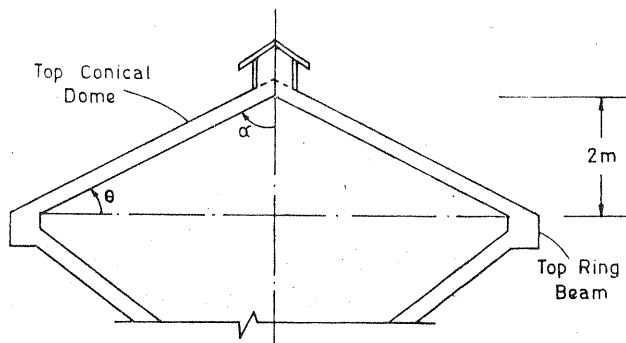


Fig. 6.12 Top conical dome.

Assuming a central rise = 2 m

If α = semi-vertical angle at apex, $\tan \alpha = \left(\frac{7.5}{2} \right) = 3.75$

$\therefore \alpha = 75^\circ$ $\theta = 15^\circ$

t = thickness of slab = 100 mm

Self weight of slab = $(0.1 \times 24) = 2.4 \text{ kN/m}^2$

Live load = 0.6 kN/m^2

Total load $w = 3.0 \text{ kN/m}^2$

Maximum meridional thrust at base

$$T_1 = \left(\frac{wh}{2 \cos^2 \alpha} \right) = \left(\frac{3 \times 2}{2 \cos^2 75^\circ} \right) = 45 \text{ kN/m}$$

Maximum circumferential force at base,

$$N_\theta = wh \cdot \tan^2 \alpha = (3 \times 2 \times \tan^2 75^\circ) = 84.3 \text{ kN/m}$$

$$\therefore \text{Meridional stress} = \left(\frac{45 \times 10^3}{1000 \times 100} \right)$$

$$= 0.45 \text{ kN/mm}^2 < 5 \text{ N/mm}^2 \text{ (safe)}$$

$$\text{Hoop stress} = \left(\frac{84.3 \times 10^3}{1000 \times 100} \right) = 0.843 \text{ N/mm}^2 < 5 \text{ N/mm}^2 \text{ (safe)}$$

Since stresses are within permissible limits, nominal reinforcements are provided in the conical dome.

$$A_{st} = \left(\frac{0.3 \times 1000 \times 100}{100} \right) = 300 \text{ mm}^2$$

Provide 8 mm Φ at 160 mm centres both meridionally and circumferentially.

5. Design of Top Ring Beam

$$\text{Hoop tension} = F_t = \left(\frac{T_1 \cos \theta \cdot D}{2} \right) = \left(\frac{45 \times \cos 15^\circ \times 14.30}{2} \right) = 311 \text{ kN}$$

$$\therefore A_{st} = \left(\frac{311 \times 10^3}{150} \right) = 2074 \text{ mm}^2$$

Provide 8 bars of 20 Φ ($A_{st} = 2512 \text{ mm}^2$)

If A_c = cross sectional area of ring beam

$$\left(\frac{311 \times 10^3}{A_c + 13 \times 2512} \right) = 1.2 \quad \therefore A_c = 225,610 \text{ mm}^2$$

Provide a ring beam of size 600 mm by 400 mm.

Provide 8 Φ stirrups at 200 mm centres.

6. Design of Conical Shell

The geometry of the conical shell is shown in Fig. 6.13.

Thickness of shell at top = 200 mm, at bottom = 300 mm

Average thickness = 250 mm $r_2 = 7.5 \text{ m}$

Slope of wall = 45° $r_3 = 2.5 \text{ m}$

Height of cone = $h_3 = 5 \text{ m}$,

Self weight of slab = $(0.25 \times 24) = 6.0 \text{ kN/m}^2$

Length $L_c = \sqrt{(r_2 - r_3)^2 + h_3^2} = \sqrt{(7.5 - 2.5)^2 + 5^2} = 7.07 \text{ m}$

Weight of conical wall

$$= W_1 = 2 \times \pi \left(\frac{7.5 + 2.5}{2} \right) \times 0.25 \times 7.07 \times 24 = 1333 \text{ kN}$$

Weight of water over the conical wall

$$= W_2 = \pi(5^2 - 2.5^2) \times 5 \times 10 = 2945 \text{ kN}$$

Self weight of top dome = $(\pi \times 7.5 \times 7.76 \times 3) = 549 \text{ kN}$

Self weight of top ring beam = $(0.4 \times 0.6 \times \pi \times 15.4 \times 24) = 279 \text{ kN}$

$\therefore$ Total vertical load at base of conical shell

$$= W = (549 + 279 + 1333 + 2945) \text{ kN} = 5106 \text{ kN}$$

$$\therefore \text{Meridional thrust } T = \left(\frac{W \cdot \operatorname{cosec} \theta}{2\pi r_3} \right) = \left(\frac{5106 \times \operatorname{cosec} 45^\circ}{2\pi \times 2.5} \right) = 460 \text{ kN/m}$$

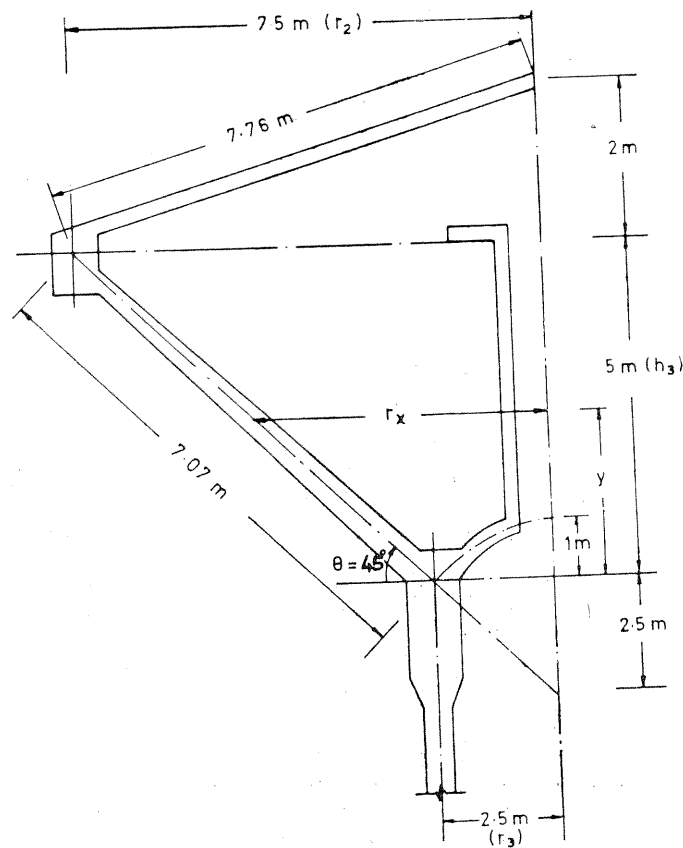


Fig. 6.13 Geometry of conical shell.

Assuming thickness of conical shell at base = $t = 300$ mm

$$\text{Meridional stress} = \left(\frac{460 \times 10^3}{1000 \times 300} \right) = 1.53 \text{ N/mm}^2 < 5 \text{ N/mm}^2 \text{ (safe)}$$

The hoop tension in the conical shell varies with the depth of water and is given by

$$\text{Hoop tension } F_t = (p \cdot \operatorname{cosec} \theta + q \cdot \cot \theta) D/2$$

where, D = diameter at any horizontal section

$$p = \text{Intensity of water pressure normal to the inclined walls} \\ = (10 h) \text{ kN/m}^2$$

h = height of water above the section

θ = angle made by the conical shell with the horizontal

q = weight of conical slab per square metre of surface area.

Hoop Tension at bottom of Conical shell

$$h = 5 \text{ m} \quad D = 5 \text{ m} \quad \theta = 45^\circ$$

$$p = (10 \times 5) = 50 \text{ kN/m}^2$$

$$q = (0.3 \times 24) = 7.2 \text{ kN/m}^2$$

$$F_t = (50 \times \operatorname{cosec} 45^\circ + 7.2 \cot 45^\circ) 5/2 = 195 \text{ kN}$$

Hoop tension at 1 m above base

$$h = 4 \text{ m} \quad D = 7 \text{ m} \quad \theta = 45^\circ$$

$$p = (10 \times 4) = 40 \text{ kN/m}^2$$

$$q = (0.28 \times 24) = 6.72 \text{ kN/m}^2$$

$$\therefore F_t = (40 \times \operatorname{cosec} 45^\circ + 6.72 \cot 45^\circ) 7/2 = 222 \text{ kN}$$

Hoop tension at 2 m above base

$$h = 3 \text{ m} \quad D = 9 \text{ m} \quad \theta = 45^\circ$$

$$p = (10 \times 3) = 30 \text{ kN/m}^2$$

$$q = (0.26 \times 24) = 6.24 \text{ kN/m}^2$$

$$F_t = (30 \times \operatorname{cosec} 45^\circ + 6.24 \cot 45^\circ) 9/2 = 219 \text{ kN}$$

$\therefore$ Maximum hoop tension occurs at 1 m above the base section

$\therefore$ Maximum hoop tension $F_t = 222 \text{ kN}$

$$\therefore A_{st} = \left(\frac{222 \times 10^3}{150} \right) = 1480 \text{ mm}^2$$

Provide 10 mm Φ bars at 100 mm centres on both faces.

$$(A_{st} = 1570 \text{ mm}^2)$$

$$\begin{aligned} \text{Maximum tensile stress} &= \left[\frac{222 \times 10^3}{(300 \times 1000) + (13 \times 1570)} \right] \\ &= 0.69 \text{ N/mm}^2 < 1.2 \text{ N/mm}^2 \text{ (safe)} \end{aligned}$$

$$\text{Distribution steel} = \left(\frac{0.3 \times 300 \times 1000}{100} \right) = 900 \text{ mm}^2$$

Provide 10 mm Φ at 160 mm centres on both faces along the meridians.

7. Design of Bottom Dome and Internal Shaft

Diameter at base = 5 m

Rise of dome = 1 m

Thickness = $t = 150 \text{ mm}$

If R = Radius of the dome $(2R - 1) 1 = (5/2)^2 \quad \therefore R = 3.625 \text{ m}$

Self weight of dome slab = $(2 \times \pi \times 3.625 \times 1 \times 0.15 \times 24) = 82 \text{ kN}$

Internal diameter of vertical shaft = 800 mm

Thickness of walls = 100 mm

$\therefore$ External diameter = 1000 mm

Weight of water over bottom dome = $\pi(2.5^2 - 0.5^2) 4.5 \times 10 = 848 \text{ kN}$

Weight of vertical shaft = $(\pi \times 0.9 \times 0.1 \times 4 \times 24) = 28 \text{ kN}$

Total weight on dome = $(82 + 848 + 28) = 958 \text{ kN}$

$$\therefore \text{Load/unit area} = w = \left(\frac{985}{\pi \times 2.5^2} \right) = 49 \text{ kN/m}^2$$

$$\therefore \text{Meridional thrust} = T_1 = \left(\frac{wR}{1 + \cos \theta} \right)$$

$$\cos \theta = \left(\frac{2.5}{3.625} \right) = 0.68 \quad \therefore \theta = 47^\circ$$

$$\therefore T_1 = \left(\frac{49 \times 3.625}{1 + 0.68} \right) = 106 \text{ kN/m}$$

$$\begin{aligned} \therefore \text{Meridional stress} &= \left(\frac{106 \times 10^3}{150 \times 1000} \right) \\ &= 0.706 \text{ N/mm}^2 < 5 \text{ N/mm}^2 \text{ (safe)} \end{aligned}$$

$$\begin{aligned} \text{Circumferential force} &= wR \left[\cos \theta - \frac{1}{(1 + \cos \theta)} \right] \\ &= 49 \times 3.625 \left[0.68 - \frac{1}{1.68} \right] = 15 \text{ kN/m} \end{aligned}$$

$$\therefore \text{Hoop stress} = \left(\frac{15 \times 10^3}{150 \times 1000} \right) = 0.1 \text{ N/mm}^2 \text{ (safe)}$$

Provide nominal reinforcements of 0.3%

$$\therefore A_{st} = \left(\frac{0.3 \times 150 \times 1000}{100} \right) = 450 \text{ mm}^2$$

Adopt 10 mm Φ at 160 mm centres both radially and in circumferential direction.

$$\begin{aligned} \text{Maximum hoop compression in the internal shaft} &= \left(\frac{10 \times 4 \times 0.9}{2} \right) \\ &= 18 \text{ kN} \end{aligned}$$

Since hoop stresses are negligibly small provide nominal reinforcements of 8 mm Φ at 160 mm centres in both directions.

8. Design of Bottom Ring Beam

$$\begin{aligned} \text{Horizontal component of thrust from conical shell} &= H_1 = 460 \cos 45^\circ \\ &= 326 \text{ kN} \end{aligned}$$

$$\text{Horizontal thrust from bottom dome} = H_2 = 106 \cos 47^\circ = 72 \text{ kN}$$

$$\text{Net compressive force in ring beam} = (H_1 - H_2) = (326 - 72) = 254 \text{ kN}$$

$$\therefore \text{Hoop compression} = \left(\frac{254 \times 5}{2} \right) = 635 \text{ kN}$$

Assuming the ring beam size as 400 mm wide by 600 mm deep,

$$\text{Compressive stress} = \left(\frac{635 \times 10^3}{400 \times 600} \right) = 2.64 \text{ N/mm}^2 < 5 \text{ N/mm}^2 \text{ (safe)}$$

Provide minimum reinforcement of 0.3%

$$A_{st} = \left(\frac{0.3 \times 400 \times 600}{100} \right) = 720 \text{ mm}^2$$

Provide 8 bars of 12 mm Φ with 8 mm Φ - 2 legged stirrups at 200 mm centres. As the girder is supported by the shaft walls, there will be no shear in the ring beam.

9. Design of Supporting Cylindrical Shaft

Diameter of shaft = 5 m

Height of shaft (above G.L.) = 20 m

Thickness of shaft walls above G.L. = 150 mm

Height of shaft (below G.L.) = 1.5 m

Thickness of shaft below G.L. = 300 mm

Self weight of shaft = $\pi \times 5 \times 24[(20 \times 0.15) + (1.5 \times 0.3)] = 1300$ kN

The vertical shaft has an external diameter of 5.30 m and an internal diameter of 4.70 m, thickness of walls = 150 mm

Loads acting on Shaft at Ground Level

Load due to weight of	kN
(1) The dome	549
(2) Top ring beam	279
(3) Conical shell	1333
(4) Bottom spherical dome	82
(5) Internal shaft	28
(6) Bottom ring beam	90
(7) Supporting shaft	1300
(8) Plastering, pipes, Ladders Railings etc. (Lump sum)	339
Total dead loads	= 4000 kN
Weight of water	3880 kN

Wind Pressure

Vertically projected area of over head tank

$$= \left(\frac{1}{2} \times 15.4 \times 2 \right) + \frac{1}{2} (15.4 + 5.4) 5 = 70 \text{ m}^2$$

$\therefore$ Wind force on tank = $(0.7 \times 1.5 \times 70) = 73.5$ kN

acting at a distance of 24 m from base.

Wind force on shaft = $(0.7 \times 1.5 \times 5.3 \times 20) = 112$ kN

acting at 10 m from the base

Total moment due to wind about base = $M = (73.5 \times 24) + (112 \times 10)$
 $= 2884 \text{ kN} \cdot \text{m}$

Area of cross section = $A = \pi(2.65^2 - 2.35^2) = 4.71 \text{ m}^2$

Second moment of area = $I = \frac{\pi}{4}(2.65^4 - 2.35^4) = 14.92 \text{ m}^4$

Stresses at Base Section

Tank Empty Condition

$$P = 4000 \text{ kN} \quad D = 5.3 \text{ m} \quad M = 2884 \text{ kN} \cdot \text{m}$$

$$e = \left(\frac{M}{P} \right) = \left(\frac{2884}{4000} \right) = 0.72 \text{ m} < \left(\frac{D}{6} \right) = \left(\frac{5.3}{6} \right) = 0.88 \text{ m}$$

The section is under compression only .

$$\sigma_c = \left(\frac{P}{A} + \frac{My}{I} \right) = \left(\frac{4000 \times 10^3}{4.71 \times 10^6} + \frac{2884 \times 10^6 \times 2650}{14.92 \times 10^{12}} \right) = 1.361 \text{ N/mm}^2$$

(compression) $< 5 \text{ N/mm}^2$ (Hence safe)

Tank Full Condition

$$P = (4000 + 3880) = 7880 \text{ kN} \quad M = 2884 \text{ kN} \cdot \text{m}$$

$$\sigma_c = \left(\frac{7880 \times 10^3}{4.71 \times 10^6} + \frac{2884 \times 10^6 \times 2650}{14.92 \times 10^{12}} \right) = 2.185 \text{ N/mm}^2 < 4 \text{ N/mm}^2 \text{ (safe)}$$

Provide minimum reinforcement of 0.3%

$$A_{st} = \left(\frac{0.3 \times 150 \times 1000}{100} \right) = 450 \text{ mm}^2/\text{m}$$

Provide 8 mm Φ at 200 mm centres both vertically and circumferentially on both faces.

Check for Seismic Forces

$$\text{(Weight of tank portion + water)} W_1 = (2361 + 3880) = 6241 \text{ kN}$$

$$\text{(Weight of shaft + finishes etc.) } W_2 = (1300 + 339) = 1639 \text{ kN}$$

$$\text{Seismic coefficient} = \alpha_n = 0.015$$

Moment due to seismic force at ground level

$$M_a = \alpha_n (W_1 h_1 + W_2 h_2) = 0.015 (6241 \times 23 + 1639 \times 10) = 2399 \text{ kN} \cdot \text{m}$$

$$\therefore \text{Total load } P = 7880 \text{ kN} \quad M = 2399 \text{ kN} \cdot \text{m}$$

$$\sigma_c = \left(\frac{7880 \times 10^3}{4.71 \times 10^6} + \frac{2399 \times 10^6 \times 2650}{14.92 \times 10^{12}} \right) = 2.09 \text{ N/mm}^2 < 5.00 \text{ N/mm}^2 \text{ (safe)}$$

10. Design of Raft Foundations

$$\text{Total load from tank and shaft} = 7880 \text{ kN}$$

$$\text{Self weight of footing (10\%)} = 829 \text{ kN}$$

$$\text{Total load } W = 8700 \text{ kN}$$

$$\text{S.B.C. of the soil} = 200 \text{ kN/m}^2$$

If A = Area of footing

D = Diameter of the footing

$$\text{Direct load } W = 8700 \text{ kN}$$

$$\text{Moment } M = 2884 \text{ kN} \cdot \text{m}$$

$$\left(\frac{W}{A} + \frac{M}{Z} \right) = 200 \quad \left[\frac{W}{(\pi D^2/4)} + \frac{M}{(\pi D^3/32)} \right] = 200$$

$$\left[\frac{8700}{(\pi D^2/4)} + \frac{2884}{(\pi D^3/32)} \right] = 200 \quad \text{Solving } D = 8.54 \text{ m}$$

Adopt diameter of raft slab $D = 9$ m

$$\text{Intensity of soil pressure} = w = \left[\frac{8700}{(\pi \times 9^2)/4} \right] = 137 \text{ kN/m}^2$$

The loading on the base is taken as annular loading on the mean diameter of the shaft. The design procedure is similar to that described in the design Example 3.8 (refer chimney foundations).

Diameter of raft slab $= 2a = 9$ m

Diameter of the shaft $= 2b = 5$ m

Maximum bending moment in the section is governed by the radial moment.

Radial moment at centre of footing is given by

$$\begin{aligned} M_r &= \frac{W}{8\pi} \left[2 \log_e \left(\frac{a}{b} \right) + 1 - \left(\frac{b}{a} \right)^2 \right] - \frac{3}{16} w \cdot a^2 \\ &= \frac{8700}{8\pi} \left[2 \log_e \left(\frac{4.5}{2.5} \right) + 1 - \left(\frac{2.5}{4.5} \right)^2 \right] - \left[\frac{3}{16} \times 137 \times 4.5^2 \right] \\ &= 126 \text{ kN} \cdot \text{m/m} \end{aligned}$$

Moment at junction of footing and tank walls at a radius of 2.5 m is given by

$$\begin{aligned} M_r (\text{max}) &= \frac{W}{8\pi} \left[2 \log_e \left(\frac{a}{b} \right) + 1 - \left(\frac{b}{a} \right)^2 \right] - \frac{3}{16} w(a^2 - b^2) \\ &= \frac{8700}{8\pi} \left[2 \log_e \left(\frac{4.5}{2.5} \right) + 1 - \left(\frac{2.5}{4.5} \right)^2 \right] - \frac{3}{16} \times 137(4.5^2 - 2.5^2) \\ &= 287 \text{ kN} \cdot \text{m/m} \end{aligned}$$

Using M-20 grade concrete and Fe-415 grade tor steel.

$$\text{Effective depth } d = \sqrt{\frac{287 \times 10^6}{0.897 \times 10^3}} = 566 \text{ mm}$$

Adopt an effective depth of 570 mm and an over all depth of 650 mm.

$$A_{st} = \left(\frac{287 \times 10^6}{230 \times 0.9 \times 570} \right) = 2432 \text{ mm}^2$$

Adopt 25 mm Φ at 200 mm centres in perpendicular directions. both ways also provide 12 mm Φ - 200 c/c both ways at the top of the footing. The details of reinforcements in the tank, supporting shaft and foundations are shown in Figs. 6.14 and 6.15.

6.6 EXAMPLES FOR PRACTICE

1. Design an intz type water tank to store 1.5 million litres of water. The height of the tank above ground level is 20 m. The site has soil of safe bearing capacity $= 200 \text{ kN/m}^2$. Depth of foundations 1.5 m below ground level. Basic wind pressure $= 1.5 \text{ kN/m}^2$. Number of supporting columns

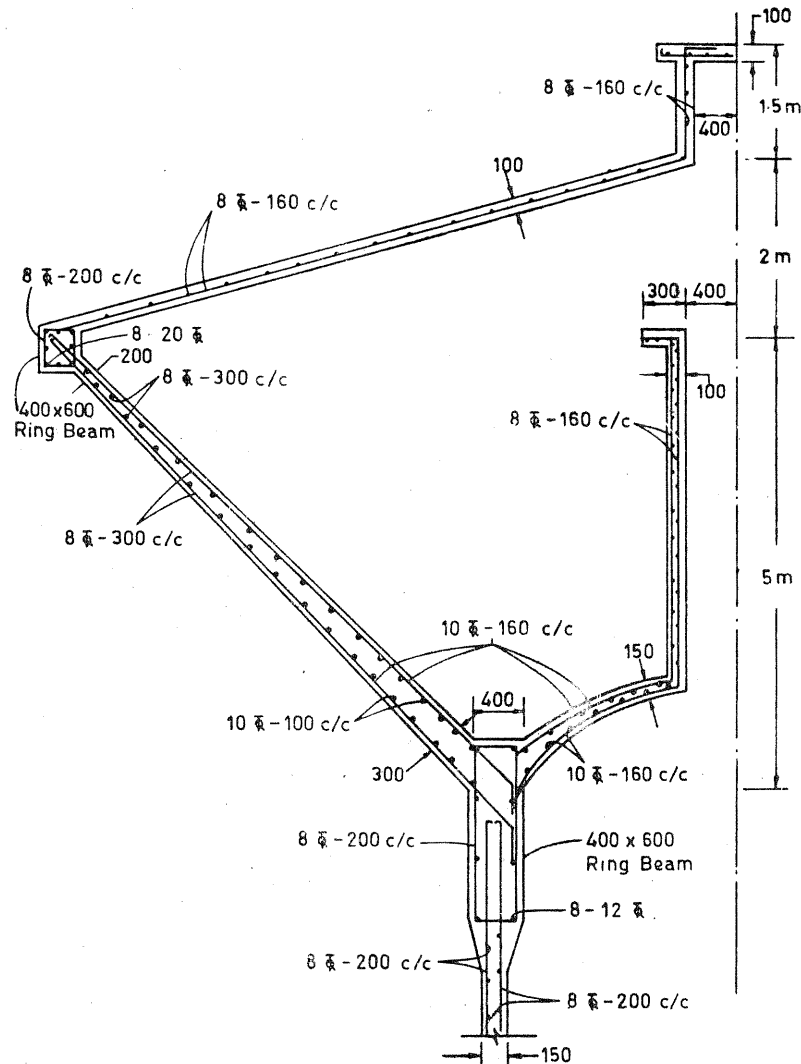


Fig. 6.14 Reinforcement details in funnel-shaped over head tank.

= 8. Adopt M-20 grade concrete and Fe-415 grade tor steel. The design of the tank should conform to the stresses specified in IS : 3370 and IS : 456 codes.

2. A reinforced concrete intz type water tank is required to store 250,000 liters of water. Height of staging is 12 m above ground level. The tank is supported on six columns. Safe bearing capacity of the soil = 150 kN/m². Basic wind pressure = 1.5 kN/m². Adopting M-20 grade concrete and Fe-415 grade tor steel, design the intz type tank and sketch the details of reinforcements in the various structural components of the tank.

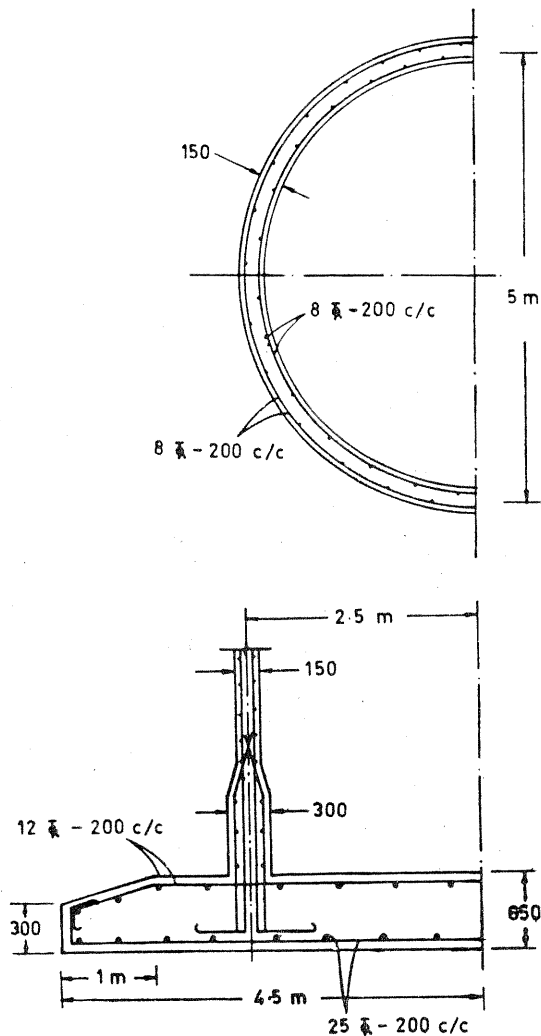


Fig. 6.15 Details of reinforcements in shaft and raft foundation

3. A funnel shaped over head water tank is required to store 500,000 litres of water with the height of supporting tower being 25 m above ground level.

Basic wind pressure = 1.5 kN/m^2

Depth of foundations = 2 m below G.L.

Safe bearing capacity of the soil = 200 kN/m^2

Adopt M-20 grade concrete and Fe-415 grade tor steel for all R.C.C. work.

Design the conical shaped tank, supporting cylindrical shaft and foundations for the tank. Sketch the details of reinforcements in the tank.

7

Box-Culverts

7.1 GENERAL ASPECTS

Box culverts consisting of two horizontal and two vertical slabs built monolithically are ideally suited for a road or a railway bridge crossing with high embankments crossing a stream with a limited flow. Reinforced concrete rigid frame box culverts with square or rectangular openings are used up to spans of 4 m. The height of the vent generally does not exceed 3 m.

Box culverts are economical due to their rigidity and monolithic action and separate foundations are not required since the bottom slab resting directly on the soil, serves as raft slab. For small discharges, single celled box culvert is used and for larger discharges, multicelled box culverts can be employed. The barrel of the box culvert should be of sufficient length to accommodate the carriage way and the kerbs.

7.2 DESIGN LOADS

The structural design of a reinforced concrete box culvert comprises the detailed analysis of the rigid frame for moments, shear forces and thrusts due to various types of loading conditions outlined below:

1. *Concentrated Loads*

In cases where the top slab forms the deck of the bridge, concentrated loads due to the wheel loads of the I.R.C. class AA or A type loading have to be considered.

If W = concentrated load on the slab

P = wheel load

I = impact factor

e = effective width of dispersion

Then $W = (P \cdot I/e)$

The soil reaction on the bottom slab is assumed to be uniform. The notations used for the box culvert and the type of loadings to be considered are shown in Fig. 7.1 (a) to (f).

2. *Uniform Distributed Load*

The weight of embankment, wearing coat and, deck slab and the track load are considered to be uniformly distributed loads on the top slab with the uniform soil reaction on the bottom slab.

3. *Weight of Side Walls*

The self weight of two side walls acting as concentrated loads are assumed to produce uniform soil reaction on the bottom slab.

4. *Water Pressure Inside Culvert*

When the culvert is full with water, the pressure distribution on side walls is assumed to be triangular with a maximum pressure intensity of $p = wh$ at the base where w = density of water and h is the depth of flow.

5. *Earth Pressure on Vertical Side Walls*

The earth pressure on the vertical side walls of the box culvert is computed according to the Coloumb's theory. The distribution of earth pressure on the side wall is shown in Fig. 7.1(e).

6. *Uniform Lateral Load on Side Walls*

Uniform lateral pressure on vertical side walls has to be considered due to the effect of live load surcharge. Also trapezoidal pressure distribution on side walls due to embankment loading can be obtained by combining the cases (5) and (6).

7. *Design Moments, Shears and Thrusts*

The box culvert is analysed for moments, shear forces and axial thrusts developed due to the various loading conditions by any of the classical methods such as moment distribution, slope deflection or column analogy procedures. Alternatively coefficients for moments, shears and thrusts compiled by Victor (Ref 22), are very useful in the computation of the various force components for the different loading conditions.

The fixed end moments developed for the six different loading cases are compiled in Table 7.1. The moment, shear and thrust coefficients for the various loading cases are shown in Table 7.2, for two different ratios of $(L/H) = 1$ and 1.5

where L = span of the culvert.

H = height of the culvert,

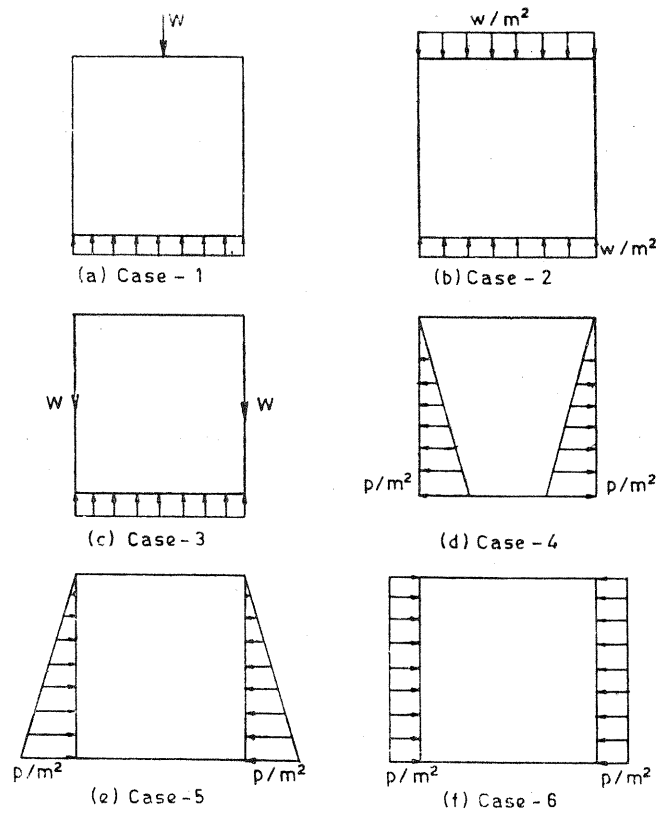
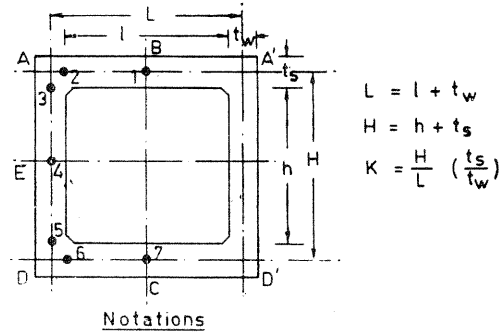


Fig. 7.1 Types of loading for box-culverts

7.3 DESIGN OF CRITICAL SECTIONS

The maximum design moments resulting from the combination of the various loading cases are determined. The moments at the centre of span of top and bottom slabs and the support sections and at the centre of the

TABLE 7.1 Fixed End Moments in Box Culvert

Loading Case	Fixed end Moments	
	$M_A = M_{A'}$	$M_D = M_{D'}$
1	$-\frac{WL}{12} \left[\frac{2k+4.5}{(k+3)(k+1)} \right]$	$-\frac{WL}{24} \left[\frac{k+6}{(k+3)(k+1)} \right]$
2	$-\frac{wL^2}{12(k+1)}$	$-\frac{wL^2}{12(k+1)}$
3	$+\frac{WL}{6} \left[\frac{k}{(k+3)(k+1)} \right]$	$-\frac{WL}{6} \left[\frac{3+2k}{(k+3)(k+1)} \right]$
4	$+\frac{pH^2}{60} \left[\frac{k(2k+7)}{(k+3)(k+1)} \right]$	$+\frac{pH^2}{60} \left[\frac{k(3k+8)}{(k+3)(k+1)} \right]$
5	$-\frac{pH^2}{60} \left[\frac{k(2k+7)}{(k+3)(k+1)} \right]$	$-\frac{pH^2}{60} \left[\frac{k(3k+8)}{(k+3)(k+1)} \right]$
6	$-\frac{p \cdot kH^2}{12(k+1)}$	$-\frac{p \cdot kH^2}{12(k+1)}$

Note: Positive moment indicates tension on inside face.

vertical walls are determined by suitably combining the different loading patterns. The maximum moments generally develop for the following loading conditions:

1. When the slab supports the dead and live loads and the culvert is empty.
2. When the top slab supports the dead and live loads and the culvert is running full.
3. When the sides of the culvert do not carry the live load and the culvert is running full.

The slabs of the box culvert is reinforced on both faces with fillets at the inside corners.

7.4 DESIGN EXAMPLE

Design a reinforced concrete box culvert having a clear vent way of 3 m by 3 m. The super imposed dead load on the culvert is 12.8 kN/m². The live load on the culvert is 50 kN/m². Density of soil at site is 18 kN/m². Angle of repose $\phi = 30^\circ$. Adopt M-20 grade concrete mix and Fe-415 grade tor steel. Sketch the details of reinforcements in the box culvert.

1. Data

Clear Span = $L = 3$ m
 Height of event = $h = 3$ m
 Dead load = 12.8 kN/m²
 Live load = 50 kN/m²

TABLE—7.2 Coefficients for Moment, Shear and Thrust

L:H Section	Factors For	Loading case					
		1	2	3	4	5	6
	<i>M</i>	<i>WL</i>	<i>wL</i> ²	<i>WL</i>	<i>pL</i> ²	<i>pL</i> ²	<i>pL</i> ²
	<i>N</i>	<i>W</i>	<i>wL</i>	<i>W</i>	<i>pL</i>	<i>pL</i>	<i>pL</i>
	<i>V</i>	<i>W</i>	<i>wL</i>	<i>W</i>	<i>pL</i>	<i>pL</i>	<i>pL</i>
1:1	B-1	<i>M</i>	+0.182	+0.083	+0.021	+0.019	-0.019
		<i>N</i>	0	0	-0.167	+0.167	+0.500
		<i>V</i>	0	0	0	0	0
	A-2	<i>M</i>	-0.068	-0.042	+0.021	+0.019	-0.019
		<i>N</i>	0	0	-0.167	+0.167	-0.500
		<i>V</i>	+0.500	+0.500	0	0	0
	A-3	<i>M</i>	-0.068	-0.042	+0.021	+0.019	-0.019
		<i>N</i>	+0.500	+0.500	0	0	0
		<i>V</i>	0	0	+0.167	-0.167	-0.500
	E-4	<i>M</i>	-0.052	-0.042	-0.042	-0.043	+0.043
		<i>N</i>	+0.500	+0.500	+0.500	0	0
		<i>V</i>	0	0	0	0	0
	D-5	<i>M</i>	-0.036	-0.042	-0.004	+0.023	-0.023
		<i>N</i>	+0.500	+0.500	+1.000	-0.333	+0.333
		<i>V</i>	0	0	0	0	0
	D-6	<i>M</i>	-0.036	-0.042	-0.104	+0.023	-0.023
		<i>N</i>	0	0	0	0	0
		<i>V</i>	-0.500	-0.500	-1.020	-0.333	+0.333
	C-7	<i>M</i>	+0.088	+0.083	+0.146	+0.023	-0.023
		<i>N</i>	0	0	0	-0.333	+0.333
		<i>V</i>	0	0	0	0	0
1.5:1	B-1	<i>M</i>	+0.170	+0.075	+0.018	+0.015	-0.015
		<i>N</i>	0	0	0	-0.167	+0.167
		<i>V</i>	0	0	0	0	0
	A-2	<i>M</i>	-0.079	-0.050	+0.018	+0.015	-0.015
		<i>N</i>	0	0	0	-0.167	+0.167
		<i>V</i>	+0.500	+0.500	0	0	0
	A-3	<i>M</i>	-0.079	-0.050	+0.018	+0.015	-0.015
		<i>N</i>	+0.500	+0.500	0	0	0
		<i>V</i>	0	0	0	+0.167	-0.167
	E-4	<i>M</i>	-0.062	-0.050	-0.050	-0.047	+0.047
		<i>N</i>	+0.500	+0.500	+0.500	0	0
		<i>V</i>	0	0	0	0	0
	D-5	<i>M</i>	-0.045	-0.050	-0.118	+0.018	-0.018
		<i>N</i>	+0.500	+0.500	+1.000	0	0
		<i>V</i>	0	0	0	-0.333	+0.333
	D-6	<i>M</i>	-0.045	-0.050	-0.118	+0.018	-0.018
		<i>N</i>	0	0	0	-0.333	+0.333
		<i>V</i>	-0.500	-0.500	-1.000	0	0
	C-7	<i>M</i>	+0.079	+0.075	+0.132	+0.018	-0.018
		<i>N</i>	0	0	0	-0.333	+0.333
		<i>V</i>	0	0	0	0	0

Refer Fig. 7.1 for details and notations.

- Note: 1. Positive moment indicates tension on inside face
 2. Positive shear indicates that the summation of force at the left of the section acts outward when viewed from within.
 3. Positive thrust indicates compression on the section.

Density of soil = 18 kN/m³

Angle of repose = $\phi = 30^\circ$

Concrete – M-20 grade, steel – Fe-415 tor steel.

2. Permissible Stresses

$$\sigma_{cc} = 5 \text{ N/mm}^2$$

$$m = 13$$

$$\sigma_{cb} = 7 \text{ N/mm}^2$$

$$J = 0.86$$

$$\sigma_{st} = 150 \text{ N/mm}^2 \text{ (water face)}$$

$$Q = 1.198$$

$$\sigma_{st} = 190 \text{ N/mm}^2 \text{ (Away from water face)}$$

3. Dimensions of Box Culvert

Adopting thickness of slab as 100 mm/metre span

Thickness = $t_s = t_w = 300 \text{ mm}$

Effective span = 3300 mm

4. Loads

Self weight of top = $(0.3 \times 24) = 7.2 \text{ kN/m}^2$

Super imposed dead load = 12.8

Live load = 50.0

Total load = $w = 70.0 \text{ kN/m}^2$

Weight of vertical side walls = $(0.3 \times 3.3 \times 24) = W = 24 \text{ kN}$

$$\text{Soil pressure} = p = wh \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)$$

At $h = 3.3 \text{ m}$

$$\phi = 30^\circ \quad p = (18 \times 3.3 \times 1/3) = 20 \text{ kN/m}^2$$

$$w = 18 \text{ kN/m}^3$$

Uniform lateral pressure due to the effect of superimposed dead load and live load surcharge is calculated as,

$$p = (50 + 12.8) \left[\frac{1 - \sin \phi}{1 + \sin \phi} \right] = (62.8 \times 1/3) = 21 \text{ kN/m}^2$$

Uniform lateral pressure due to the effect of super imposed dead load surcharge only is,

$$p = 12.8 \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right) = (12.8 \times 1/3) = 4.26 \text{ kN/m}^2$$

Intensity of water pressure is obtained as

$$p = wh = (10 \times 3.3) = 33 \text{ kN/m}^2$$

5. Analysis of Moments, Shears and Thrusts

The various loading patterns considered are shown in Fig. 7.2. The moments, shears and thrusts corresponding to the different cases of loading (case-1 to case-6), evaluated using the coefficients given in Table 7.2 are compiled in Table-7.3. The design forces resulting from the combination of the various cases yielding maximum moments and forces at the support and mid span sections are shown in Table 7.4.

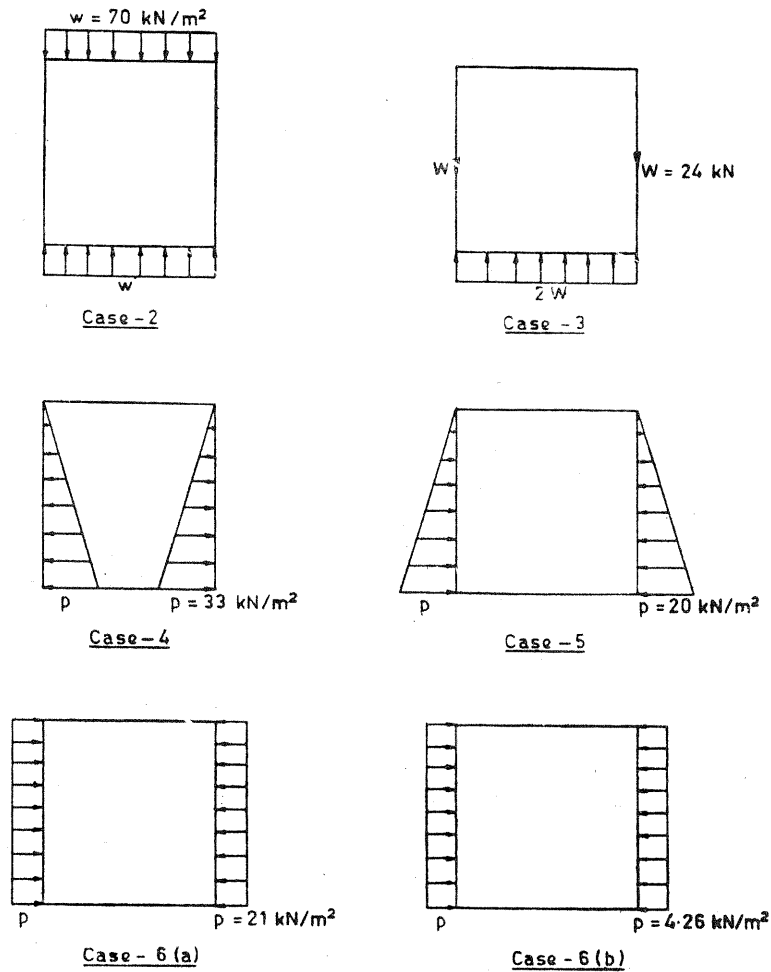


Fig. 7.2 Loading cases considered for box culvert

The maximum positive moments develop at the centre of bottom and top slab for the condition that the sides of the culvert not carrying the live load and the culvert is running full with water.

The maximum negative moments develop at the support sections of the bottom slab for the condition, culvert is empty and the top slab carries the dead and live load.

6. Design of Reinforcements

Section—C-7 (Mid span of bottom slab)

$$M = 76.10 \text{ kN}\cdot\text{m}$$

$$N = -7.43 \text{ kN (Tension)}$$

TABLE 7.3 Force Components for Different Cases of Loading
Loading Case

Section	Forces	Case-2	Case-3	Case-4	Case-5	Case-6(a)	Case-6(b)
B-1	M	63.2	1.66	6.82	-4.13	-9.6	-1.92
	N	0	0	-18.18	+11.0	+34.65	+6.93
A-2	M	-31.6	1.66	6.82	-4.13	-9.6	-1.92
	N	0	0	-18.18	+11.0	-34.65	-6.93
	V	115.5	0	0	0	0	0
A-3	M	-31.6	1.66	6.82	-4.13	-9.6	-1.92
	N	115.5	0	0	0	0	0
	V	0	0	18.18	-11.0	-34.65	-6.93
E-4	M	-31.6	-3.32	-15.45	-9.36	+19.2	+3.84
	N	115.5	+39.6	0	0	0	0
D-5	M	-31.6	-0.317	8.26	-5.00	-9.6	-1.92
	N	115.5	+79.2	-36.26	+21.9	0	0
	V	0	0	0	0	+34.65	+6.93
D-6	M	-31.6	-8.23	8.26	-5.00	-9.6	-1.92
	N	0	0	0	0	+34.65	+6.93
	V	-115.5	-79.2	-36.26	+21.9	0	0
C-7	M	63.2	11.56	8.26	-5.00	-9.60	-1.92
	N	0	0	-36.26	+21.9	+34.65	+6.93

Moments are in kN.m

Shear force and thrusts are in kN.

TABLE 7.4 Design Moments and Forces in Box Culvert

Section	Loading Combination cases	Moment M (kN·m)	Thrust N (kN)	Shear force V (kN)
D-6	2+3+5+6(a)	-54.43	+34.65	-172.8
A-2	2+3+5+6(a)	-43.67	-23.65	+115.5
B-1	2+3+4+5+6(b)	65.63	-1.25	0
C-7	2+3+4+5+6(b)	76.10	-7.43	0
E-4	2+3+4+5+6(b)	-55.89	+155.1	0

$$A_{st} = \left(\frac{76.10 \times 10^6}{150 \times 0.86 \times 270} \right) = 2234 \text{ mm}^2/\text{m}$$

Provide 20 Φ at 140 mm centres.

$$\text{Distribution steel} = \left(\frac{0.3 \times 300 \times 1000}{100} \right) = 900 \text{ mm}^2$$

Provide 10 Φ at 150 mm centres on both faces.

Section—D-6 (support section)

$$M = -54.43 \text{ kN}\cdot\text{m}$$

$$N = 34.65 \text{ kN}$$

$$A_{st} = \left(\frac{54.43 \times 10^6}{190 \times 0.86 \times 270} \right) = 1233 \text{ mm}^2/\text{m}$$

Provide 16Φ at 150 mm centres and distribution bars of 10Φ at 150 mm centres.

Section—E-4 (Vertical side wall)

$$M = -55.89 \text{ kN}\cdot\text{m} \quad \therefore M_u = (1.5 \times 55.89) = 83.83 \text{ kN}\cdot\text{m}$$

$$N = 155.1 \text{ kN} \quad N_u = (1.5 \times 155.1) = 232.5 \text{ kN}$$

$$\left(\frac{M_u}{f_{ck} \cdot b D^2} \right) = \left(\frac{83.83 \times 10^6}{20 \times 1000 \times 300^2} \right) = 0.046 \quad f_y = 415 \text{ N/mm}^2$$

$$\left(\frac{N_u}{f_{ck} \cdot b D} \right) = \left(\frac{232.5 \times 10^3}{20 \times 1000 \times 300} \right) = 0.0387 \quad \left(\frac{d'}{D} \right) = \left(\frac{30}{300} \right) = 0.10$$

Referring to interaction curve of SP-16 $\left(\frac{p}{f_{ck}} \right) = 0.02$

$$\text{Where } A_{sc} = A_{st} = 0.5(p b D 100) = 0.5(0.02 \times 20 \times 1000 \times 300)/100 \\ = 600 \text{ mm}^2 \quad \therefore A_s = 1200 \text{ mm}^2$$

But minimum reinforcement of 0.8% of cross section has to be provided

$$\therefore A_s = \left(\frac{0.8 \times 300 \times 1000}{100} \right) = 2400 \text{ mm}^2$$

Provide 16Φ —at 150 mm centres on both faces in the vertical side walls.
Distribution steel of 10Φ —at 150 mm centre provide on both faces.
The details of reinforcements in the box culvert is shown in Fig. 7.3.

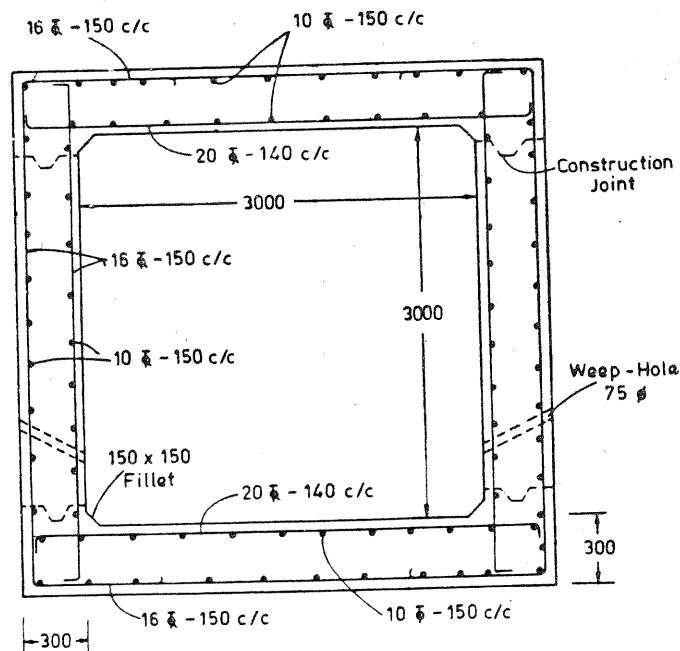


Fig. 7.3 Reinforcement details in box-culvert

7.5 EXAMPLES FOR PRACTICE

1. Design a reinforced concrete box culvert with inside dimensions of 3 m height and 4.5 width. The box culvert has to carry a super imposed dead load of 10 kN/m^2 and a live load of 50 kN/m^2 . The density of the earth is 18 kN/m^3 . Angle of repose of the soil is 30° . Adopt M-20 grade concrete and Fe-415 grade tor steel. Sketch the details of reinforcements in the box culvert.

(2) A reinforced concrete box culvert is required for a national high way crossing. The clear vent way of the box culvert is 4 m by 4 m. Design the box culvert assuming a super imposed dead load of 12 kN/m^2 and a live load of 50 kN/m^2 . The density of the soil is 16 kN/m^2 . Angle of repose of the soil is 30° . Adopt M-20 grade concrete and Fe-415 grade tor steel. Sketch the details of reinforcements in the box culvert.

3. A reinforced concrete box culvert of prismatic form with a clear vent-way of 3.5 by 3.5 m is required for a road crossing. The box culvert has to support a super imposed dead load of 8 kN/m^2 , and a live load of 50 kN/m^2 . Density of the soil is 18 kN/m^3 , and the angle of repose of the soil is 30° . Adopting M-20 grade concrete and Fe-415 grade tor steel, design the box culvert and sketch the details of reinforcements in the box culvert.

8

Portal Frames

8.1 INTRODUCTION

Reinforced concrete portal frames of single and multibay are commonly used in bridges, and industrial structures due to their fire resistance and negligible maintenance costs. A portal frame is a rigid frame having monolithic joints at the junction of columns and beams. The different types of portal frames used in the various types of structure are shown in Fig. 8.1. For work shops and storage sheds, portal frames with sloping roof is generally preferred. For high ways and buildings, portal frames with flat roof is used.

8.2 ANALYSIS AND DESIGN OF PORTAL FRAMES

In the case of buildings, the portal frames are generally spaced at intervals of 3 to 4 m with a reinforced concrete slab cast monolithically between the frames. Frames used for ware house sheds and work shop structures are provided with sloping roof comprising of purlins and asbestos sheet roofing between the portal frames.

The base of the columns of the portal frames are either fixed or hinged. If individual column footings are provided, the base is assumed to be hinged while raft or pile foundations are provided, the base is assumed to be fixed for purposes of structural analysis.

The portal frames are generally analysed by the various standard methods such as, moment distribution, slope deflection, column analogy or matrix methods. The roof slab between the frames is analysed as continuous slab. The mid span section of the frame behaves as a tee section while the support

section is designed as a rectangular section. Design aids like interaction diagrams are very useful in the design of beam sections and columns subjected to axial thrust and bending moment.

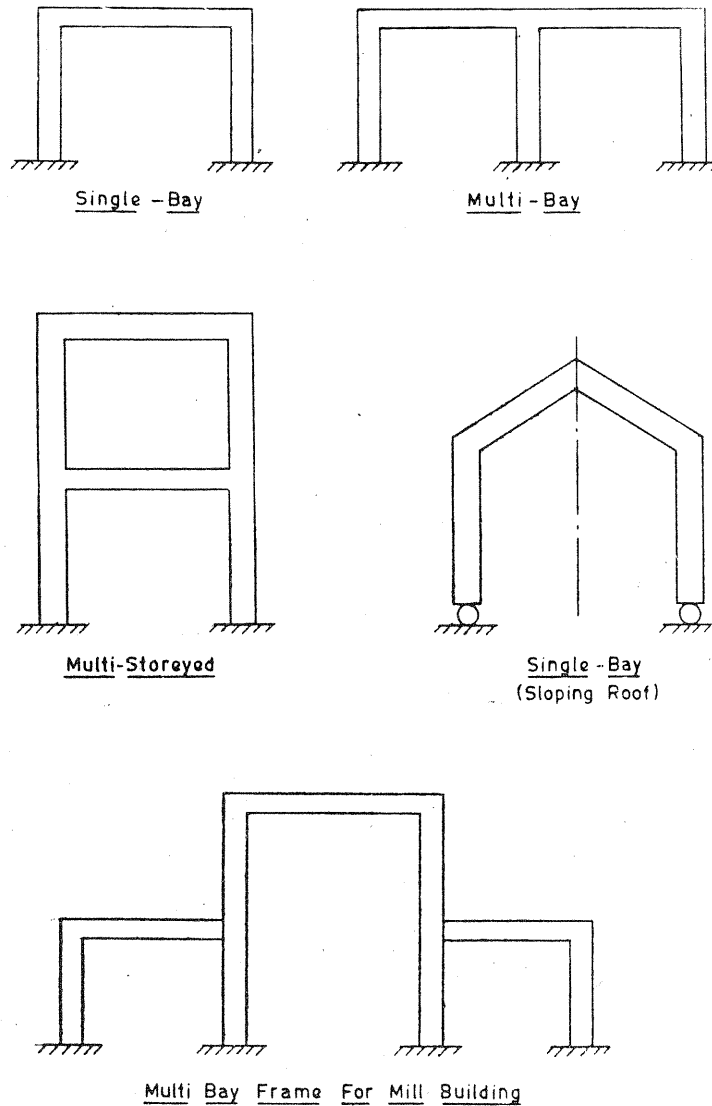


Fig. 8.1 Types of portal frames

Portal frames precast in the horizontal position on ground are economical in situations where a large number of similar frames are used in a building project and the precast units considerably reduce the construction time besides resulting in over all economy due to the better quality and

efficient utilisation of the materials. The analysis and design of the different types of portal frames are illustrated by the following design examples:

8.3 DESIGN EXAMPLE

Design a portal frame hinged at base to suit the following data:

spacings of portal frames = 4 m centres

Height of columns = 4 m

Distance between column centres = 10 m

Live load on roof = 1.5 kN/m²

R.C.C. slab continuous over portal frames. Safe bearing capacity of soil at site = 200 kN/m².

Adopt M-20 grade concrete and Fe-415 grade tor steel. Design the slab, portal frame and foundations and sketch the details of reinforcements.

1. Data

Spacings of portal frames = 4 m

Span of portal frame = 10 m

Height of columns = 4 m

Live load on roof = 1.5 kN/m²

M-20 grade concrete and Fe-415 grade tor steel.

2. Permissible Stresses

$$\sigma_{cb} = 7 \text{ N/mm}^2 \quad m = 13$$

$$\sigma_{st} = 230 \text{ N/mm}^2 \quad Q = 0.897$$

$$j = 0.906$$

3. Design of Continuous Slab

Since the slab serves as a roof, loading is light, hence assume an overall depth of slab of 120 mm.

$$\text{Dead load of slab} = (0.12 \times 24) = 2.88 \text{ kN/m}^2$$

$$\text{Roof finish} = 0.50$$

$$\text{Ceiling finish} = 0.25$$

$$\text{Dead load (g)} = 3.63 \text{ kN/m}^2$$

$$\text{Live load (q)} = 1.5 \text{ kN/m}^2$$

$$\text{Negative B.M. at interior support} = \left(\frac{gL^2}{12} + \frac{qL^2}{9} \right)$$

$$= \left(\frac{3.63 \times 4^2}{12} + \frac{1.5 \times 4^2}{9} \right) = 7.5 \text{ kN/m/m}$$

$$\text{Effective depth} = d = \sqrt{\frac{7.5 \times 10^6}{0.897 \times 1000}} = 91 \text{ mm}$$

Adopt $d = 100 \text{ mm}$ with a cover = 20 mm

$$A_{st} = \left(\frac{7.5 \times 10^6}{230 \times 0.9 \times 100} \right) = 363 \text{ mm}^2$$

Provide 10 mm Φ at 200 mm centres at supports. The same bars are provided at the centre of span section.

$$\text{Distribution steel} = \left(\frac{0.12 \times 120 \times 1000}{100} \right) = 144 \text{ mm}^2$$

Provide 6 mm Φ at 130 mm centres

4. Design of Portal Frame

Effective span of beam = 10 m

If d = effective depth

$$\left(\frac{\text{span}}{\text{effective depth}} \right) = \left(\frac{L}{d} \right) = 12 \text{ to } 15 \text{ (Heavy loading on beam)}$$

$$\therefore d = \left(\frac{10 \times 10^3}{12} \right) = 833 \text{ mm to } \left(\frac{10 \times 10^3}{15} \right) = 666 \text{ mm}$$

Adopt effective depth d = 700 mm

and over all depth D = 750 mm

Width of beam b = 450 mm

Column section is assumed as 450 mm by 600 mm.

(a) Loads on Frame

Load from slab = $(5.13 \times 4) = 20.52 \text{ kN/m}$

Self weight of beam = $(0.45 \times 0.63 \times 24) = 6.80$

Finishes on beam = 0.68

Total load $w = 28.00 \text{ kN/m}$

$$h = \text{height of centre line of beam above hinge} = \left(4.0 + 0.10 - \frac{0.75}{2} \right) = 3.72 \text{ m}$$

The moments in the portal frame are analysed by moment distribution

$$AB = 3.72 \text{ m}$$

$$BC = 10.00 \text{ m}$$

$$I_{AB} = \left(\frac{450 \times 600^3}{12} \right) \quad I_{BC} = \left(\frac{450 \times 750^3}{12} \right)$$

$$\therefore I_{AB} : I_{BC} = 1 : \left(\frac{750}{600} \right)^3 = 1 : 1.95$$

(b) Relative Stiffness

$$k_{BA} = \left(\frac{3}{4} \times \frac{I}{3.72} \right) = 0.20I$$

$$k_{BC} = \left(\frac{1.95I}{10} \right) = 0.195I = \text{or } 0.20I$$

(c) Distribution Factors

$$d_{BC} = d_{BA} = \left(\frac{0.20I}{0.20I + 0.20I} \right) = 0.5$$

(d) Fixed end Moments

$$F_{BC} = -\left(\frac{wL^2}{12}\right) = -\left(\frac{28 \times 10^2}{12}\right) = -234 \text{ kN}\cdot\text{m}$$

$$F_{CB} = +\left(\frac{wL^2}{12}\right) = +\left(\frac{28 \times 10^2}{12}\right) = 234 \text{ kN}\cdot\text{m}$$

The moment distribution is carried out to get the resultant moments for the frame loaded as shown in Fig. 8.2.

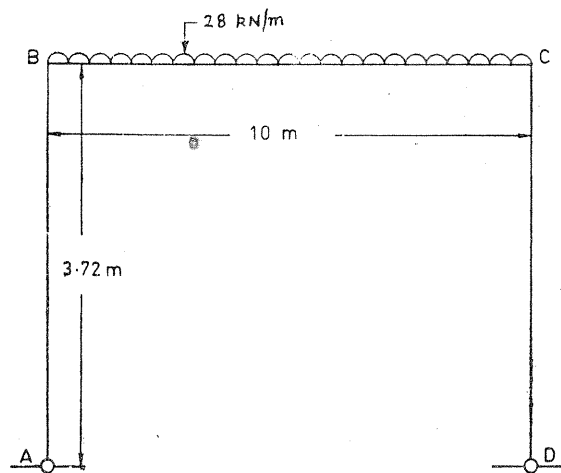


Fig. 8.2 Portal-frame

(e) Moment Distribution

A	B		C		D
	0.5	0.5	0.5	0.5	
		- 234	+ 234		
+ 117	+ 117		- 117	- 117	
	- 58.5		+ 58.5		
+ 29.25	+ 29.25		- 29.25	- 29.25	
	- 14.6		+ 14.6		
+ 7.3	+ 7.3		- 7.3	- 7.3	
	- 3.65		+ 1.83	- 1.83	
+ 1.83	+ 1.83		- 1.83	- 1.83	
	- 0.92		+ 0.92		
+ 0.46	+ 0.46		- 0.46	- 0.46	
0	+ 156	- 156	+ 156	- 156	0 kN.m

(f) Design Moments and Shear Forces

Maximum negative moment at support $B = 156 \text{ kN} \cdot \text{m}$

Maximum positive moment at centre of span

$$BC = \left(\frac{wL^2}{8} - 156 \right) = \left(\frac{28 \times 10^2}{8} - 156 \right) = 194 \text{ kN} \cdot \text{m}$$

$$\text{Maximum shear force at } B = V = \left(\frac{28 \times 10}{2} \right) = 140 \text{ kN}$$

$$\text{Shear force at the hinge } A = \left(\frac{194}{3.72} \right) = 52.15 \text{ kN}$$

(g) Design of Beam Sections

Mid span section

The mid span section is designed as a Tee-section

Design moment $M = 194 \text{ kN} \cdot \text{m}$

$$\text{Approximate lever arm} = \left(700 - \frac{120}{2} \right) = 640 \text{ mm}$$

$$A_{st} = \left(\frac{194 \times 10^6}{230 \times 640} \right) = 1318 \text{ mm}^2$$

Provide 4 bars of 22 mm Φ ($A_{st} = 1520 \text{ mm}^2$).

The stresses in steel and concrete are checked and shown to be within permissible limits

Support Section

Design moment $M = -156 \text{ kN} \cdot \text{m}$

The support section is designed as a rectangular section

$$\begin{aligned} \text{Moment of resistance of section} &= (0.897 \times 450 \times 700^2) \\ &= 197 \times 10^6 \text{ N} \cdot \text{mm} = 197 \text{ kN} \cdot \text{m} \end{aligned}$$

$$\therefore A_{st} = \left(\frac{156 \times 10^6}{230 \times 0.9 \times 700} \right) = 1077 \text{ mm}^2$$

Provide 4 bars of 22 mm Φ ($A_{st} = 1520 \text{ mm}^2$)

$$\text{Nominal shear stress} = \tau_v = \left(\frac{V}{bd} \right) = \left(\frac{140 \times 10^3}{450 \times 700} \right) = 0.44 \text{ N/mm}^2$$

$$\left(\frac{100A_{st}}{bd} \right) = \left(\frac{100 \times 1520}{450 \times 700} \right) = 0.48$$

From Table 17 (IS : 456), $\tau_c = 0.25 \text{ N/mm}^2$ Since $\tau_v > \tau_c$, shear reinforcements are to be designed.

$$\text{Balance shear} = \left[140 - \frac{0.25 \times 450 \times 700}{1000} \right] = 62 \text{ kN}$$

Using 6 mm Φ -2 legged stirrups, spacing is

$$s_v = \left(\frac{230 \times 2 \times 28 \times 700}{62 \times 10^3} \right) = 145 \text{ mm}$$

Adopt 140 mm spacing near supports gradually increasing to 400 mm towards the centre of span.

(h) *Design of Column Section*

The column section is subjected to a moment

$M = 156 \text{ kN}\cdot\text{m}$ and a Thrust $P = 140 \text{ kN}$.

Using a load factor of 1.5

$$M_u = (156 \times 1.5) = 234 \text{ kN}\cdot\text{m}$$

$$P_u = (140 \times 1.5) = 210 \text{ kN}$$

Section is of size $450 \times 600 \text{ mm}$

$$\therefore \begin{aligned} b &= 450 \text{ mm} & D &= 600 \text{ mm} & \left(\frac{d'}{D}\right) &= \left(\frac{50}{600}\right) = 0.10 \\ d &= 550 \text{ mm} & d' &= 50 \text{ mm} \end{aligned}$$

$$\left(\frac{M_u}{f_{ck} \cdot b \cdot D^2}\right) = \left(\frac{234 \times 10^6}{20 \times 450 \times 600^2}\right) = 0.07$$

$$\left(\frac{P_u}{f_{ck} \cdot b \cdot D}\right) = \left(\frac{210 \times 10^3}{20 \times 450 \times 600}\right) = 0.04$$

Using interaction curves of SP-16 we get

$$\left(\frac{P}{f_{ck}}\right) = 0.04 \quad \therefore p = (20 \times 0.04) = 0.8$$

$$\therefore A_s = (p \cdot b \cdot D / 100) = (0.8 \times 450 \times 600) / 100 = 2160 \text{ mm}^2$$

$$\text{But minimum area of steel} = \left(\frac{0.8 \times 450 \times 600}{100}\right) = 2160 \text{ mm}^2$$

Provide 4 bars of 20 mm Φ on each face, ($A_{st} = 2512 \text{ mm}^2$), and 8 mm Φ ties at 300 mm centres throughout the column.

(i) *Design of Hinge*

At the hinge portion concrete is under triaxial stress and can stand higher permissible stresses.

Permissible stress in concrete at hinge $= (0.5 \times 20) = 10 \text{ N/mm}^2$

$$\text{Area of hinge required} = \left(\frac{140 \times 10^3}{10}\right) = 14,000 \text{ mm}^2$$

Area provided = 200 mm by 100 mm

Shear at hinge = 52.15 kN

If A_{st} = Area of bars required to take the shear at hinge section

θ = Inclination of bars to vertical as shown in Fig. 8.3.

$$(A_{st} \cdot \sin \theta \times 230) = 52.15 \times 10^3$$

$$\therefore A_{st} = \left(\frac{52.15 \times 10^3}{230 \times \sin 31^\circ}\right) = 440 \text{ mm}^2$$

Provide 4 bars of 16 mm Φ ($A_{st} = 804 \text{ mm}^2$)

(j) *Design of Foundations*

Axial load on column = 140 kN

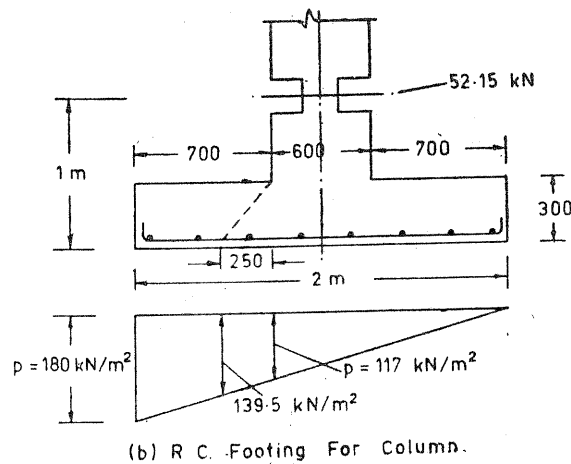
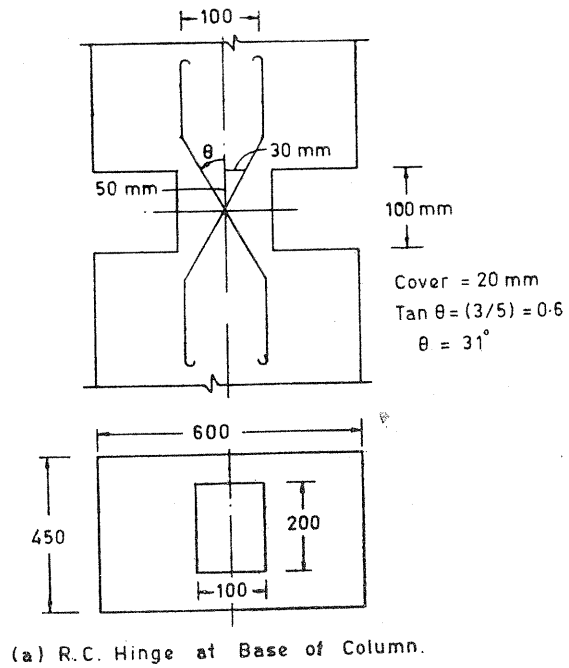


Fig. 8.3 Details of hinge and footing

Weight of column = $(0.45 \times 0.6 \times 3.72 \times 24) = 24 \text{ kN}$

Self weight of foundations at 10% = 16 kN

Total Load = 180 kN

Moment about base (Figs. 8.3(b)) = $(52.15 \times 1) = 52.15 \text{ kN} \cdot \text{m}$

$\therefore$ eccentricity $(e) = \left(\frac{52.15 \times 10^6}{180 \times 10^3} \right) = 290 \text{ mm}$

To avoid tension in the foundations,

Breadth $b = 6e = (6 \times 290) = 1740 \text{ mm}$

Provide a foundation area of $1 \text{ m} \times 2 \text{ m}$

$$\begin{aligned} \text{Intensity of maximum pressure } (p) &= \left(\frac{2 \times 180}{1 \times 2} \right) \\ &= 180 \text{ kN/m}^2 < 200 \text{ kN/m}^2 \text{ (safe)} \end{aligned}$$

$$p' = \left(\frac{180 \times 1.3}{2} \right) = 117 \text{ kN/m}^2$$

$$\text{Total pressure on the cantilever portion} = \left(\frac{180 + 117}{2} \right) 0.7$$

= 104 kN acting at 0.4 m from the column edge.

$$\therefore \text{B.M. at edge of column} = (104 \times 0.4) = 42 \text{ kN}\cdot\text{m}$$

$$\therefore \text{Effective depth of footing} = d = \sqrt{\frac{42 \times 10^6}{0.897 \times 100}} = 216 \text{ mm}$$

Provide effective depth $d = 250 \text{ mm}$

and over all depth = 300 mm

$$\therefore A_{st} = \left(\frac{42 \times 10^6}{230 \times 0.9 \times 250} \right) = 812 \text{ mm}^2$$

Adopt 16 mm Φ at 150 mm centres ($A_{st} = 1341 \text{ mm}^2$)

$$\text{Distribution bars} = \left(\frac{0.12 \times 300 \times 1000}{100} \right) = 360 \text{ mm}^2$$

Provide 10 mm Φ at 180 mm centres ($A_{st} = 436 \text{ mm}^2$)

Check for Shear

Shear force acting at a distance of 250 mm from the face of the column is given by

$$V = \left(\frac{180 + 139.5}{2} \right) \times 0.45 = 72 \text{ kN}$$

$$\therefore \tau_v = \left(\frac{72 \times 10^3}{1000 \times 250} \right) = 0.288 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd} \right) = \left(\frac{100 \times 1341}{1000 \times 250} \right) = 0.53 \quad \therefore \tau_c = 0.30 \text{ N/mm}^2$$

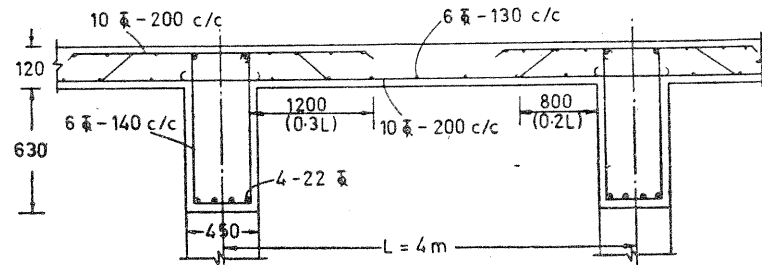
since $\tau_c > \tau_v$, no shear reinforcements are required. The details of reinforcements in the portal frame are shown in Fig. 8.4.

8.4 DESIGN EXAMPLE

The roof of a 8 m wide hall is supported on a portal frame spaced at 4 m intervals. The height of the portal frame is 4 m. The continuous slab is 120 mm thick. Live load on roof = 1.5 kN/m^2 . Bearing capacity of a soil 150 kN/m^2 . The columns are connected with a plinth beam and the base of

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the column may be assumed as fixed. Design the column and beam members and suitable foundation footing for the columns of the portal frame. Adopt M-20 grade concrete and Fe-415 grade tor steel.



(a) Section of Continuous Slab.

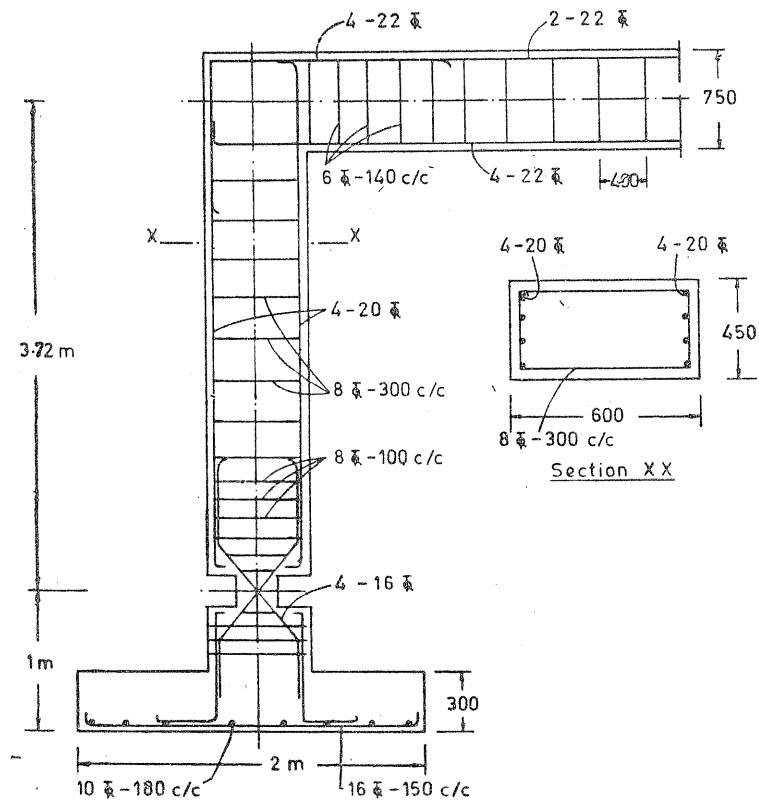


Fig. 8.4 Reinforcement details in portal frame

1. Data

- Spacing of portal frames = 4 m
- Span of portal frame = 8 m
- Height of columns = 4 m
- Live load on roof = 1.5 kN/m^2

M-20 grade concrete and Fe-415 grade tor steel.

2. *Permissible Stresses*

$$\begin{aligned}\sigma_{cb} &= 7 \text{ N/mm}^2 & m &= 13 \\ \sigma_{st} &= 230 \text{ N/mm}^2 & Q &= 0.897 \\ & & j &= 0.906\end{aligned}$$

3. *Design of Slab*

The slab design is similar to that presented in design Example 8.3. Provide 120 mm slab with 10 mm bars at 200 mm centres at supports and mid span sections. Distribution steel is 6 mm Φ at 130 mm centres.

4. *Design of Portal Frame*

Effective span of beam = 8 m

$$\text{Effective depth} = d = \left(\frac{8000}{12} \right) = 666 \text{ mm}$$

Adopt $d = 650$ mm and overall depth = 700 mm breadth = $b = 400$ mm
Column section is assumed as 400 mm by 600 mm

(a) *Loads on Frame*

$$\begin{aligned}\text{Self weight of slab} &= (0.12 \times 24) &= 2.88 \text{ kN/m}^2 \\ \text{Roof finish} &&= 0.50 \\ \text{Ceiling finish} &&= 0.25 \\ \text{Live load} &&= 1.50 \\ \text{Total load} &&= 5.13 \text{ kN/m}^2 \\ \text{Load from slab} &= (5.13 \times 4) &= 20.52 \text{ kN/m} \\ \text{Self weight of beam} &= (0.4 \times 0.58 \times 24) &= 5.56 \\ \text{Finishes of beam} &&= 0.92 \\ w &= 27.00 \text{ kN/m}\end{aligned}$$

The moments in the portal frame fixed at base and loaded as shown in Fig. 8.5 (a), are analysed by moment distribution.

$$AB = 4 \text{ m}, \quad BC = 8 \text{ m}$$

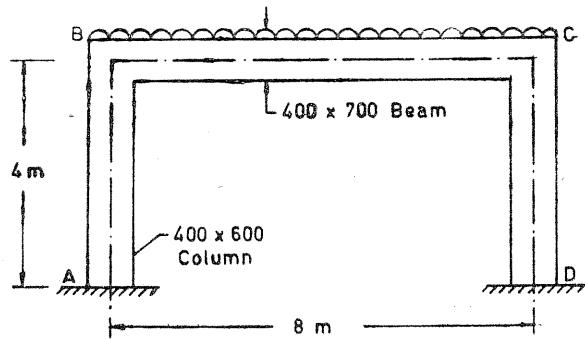
$$I_{AB} = \left(\frac{400 \times 600^3}{12} \right) \quad I_{BC} = \left(\frac{400 \times 700^3}{12} \right)$$

$$\therefore I_{AB} : I_{BC} = 1 : \left(\frac{700}{600} \right)^3 = 1 : 1.57$$

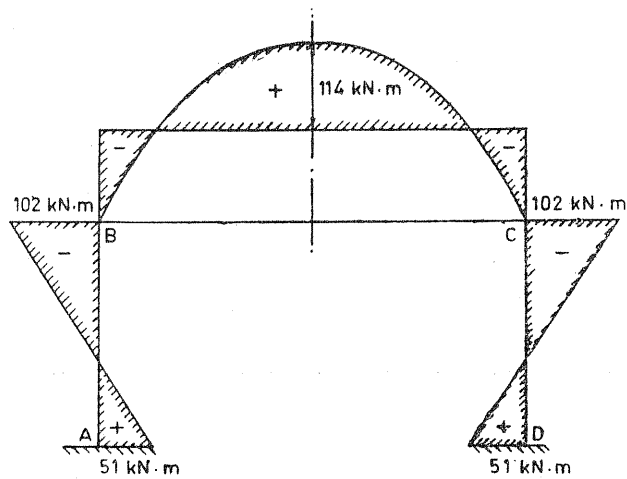
(b) *Relative Stiffness*

$$k_{BA} = \left(\frac{I}{4} \right) = 0.25I$$

$$k_{BC} = \left(\frac{1.57I}{8} \right) = 0.20I$$



(a) Portal-Frame



(b) Bending Moment Diagram

Fig. 8.5 Portal-frame (fixed at base)

(c) *Distribution Factors*

$$d_{BA} = \left(\frac{0.25I}{0.25I + 0.20I} \right) = 0.55$$

$$d_{BC} = \left(\frac{0.20I}{0.25I + 0.20I} \right) = 0.45$$

(d) *Fixed End Moments*

$$F_{BC} = - \left(\frac{wL^2}{12} \right) = - \left(\frac{27 \times 8^2}{12} \right) = -1444 \text{ kN} \cdot \text{m}$$

$$F_{CB} = + \left(\frac{wL^2}{12} \right) = + \left(\frac{27 \times 8^2}{12} \right) = +144 \text{ kN} \cdot \text{m}$$

$$F_{BA} = F_{AB} = 0$$

(e) Moment Distribution

A	B		C		D
	0.55	0.45	0.45	0.55	
0	0	-144	+144	0	0
39.5 ←	+79	+65	-65	-79 →	-39.5
		-32.5	32.5		
8.9 ←	+17.8	+14.7	-14.7	+17.8 →	-8.9
		-7.4	+7.4		
2.0 ←	+4.0	+3.4	-3.4	-4.0 →	-2.0
		-1.7	+1.7		
0.47	+0.93	0.77	-0.77	-0.93	-0.47
51	+102	-102	-102	+102	-51 kN.m

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(f) Design Moments and Shear Forces

$$M_B = 102 \text{ kN}\cdot\text{m} \quad M_A = 51 \text{ kN}\cdot\text{m}$$

$$\text{Maximum positive moment at centre of span } BC = \left(\frac{wL^2}{8} - 102 \right)$$

$$= \left(\frac{27 \times 8^2}{8} - 102 \right) = 114 \text{ kN}\cdot\text{m}$$

The bending moment diagram for the portal frame is shown in Fig. 8.5(b).

$$\text{Maximum shear force at } B = \left(\frac{27 \times 8}{2} \right) = 108 \text{ kN}$$

$$\text{Shear force at } A = \left(\frac{M_A + M_B}{4} \right) = \left(\frac{51 + 102}{4} \right) = 38.25 \text{ kN}$$

(g) Design of Beam Section

Mid span section

The mid span section is designed as a Tee-section

Design moment $M = 114 \text{ kN}\cdot\text{m}$

$$\text{Approximate lever arm} = \left(650 - \frac{120}{2} \right) = 590 \text{ mm}$$

$$\therefore A_{st} = \left(\frac{114 \times 10^6}{230 \times 590} \right) = 840 \text{ mm}^2$$

Provide 4 bars of 20 mm Φ ($A_{st} = 1256 \text{ mm}^2$)

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The stresses in steel and concrete are checked and shown to be within permissible limits.

Support section

Design moment $M = -102 \text{ kN}\cdot\text{m}$

The support section is designed as a rectangular section

$$\begin{aligned}\text{Moment of resistance of section} &= (0.897 \times 400 \times 650^2) \\ &= 151 \times 10^6 \text{ N}\cdot\text{mm} = 151 \text{ kN}\cdot\text{m}\end{aligned}$$

$$\therefore A_{st} = \left(\frac{102 \times 10^6}{230 \times 0.9 \times 650} \right) = 758 \text{ mm}^2$$

Provide 4 bars of 20 mm Φ ($A_{st} = 1256 \text{ mm}^2$)

$$\text{Nominal shear stress} = \tau_v = \left(\frac{108 \times 10^3}{400 \times 650} \right) = 0.415 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{b \cdot d} \right) = \left(\frac{100 \times 1256}{400 \times 650} \right) = 0.483 \quad \therefore \tau_c = 0.29 \text{ N/mm}^2$$

since $\tau_v < \tau_v$, shear reinforcements are required.

$$\text{Balance shear} = \left[108 - \frac{0.29 \times 45 \times 0.060}{1000} \right] = 33 \text{ kN}$$

Using 6 mm Φ - 2 legged stirrups, spacing is

$$s_v = \left(\frac{230 \times 2 \times 28 \times 650}{33 \times 10^3} \right) = 253 \text{ mm}$$

Adopt 250 mm spacing near supports, gradually increasing to 400 mm towards the centre of span.

(h) Design of Column Section

Section at Top (B)

Moment $M = 102 \text{ kN}\cdot\text{m}$

Thrust $P = 108 \text{ kN}$

Using a load factor of 1.5

$$M_u = (102 \times 1.5) = 153 \text{ kN}\cdot\text{m}$$

$$P_u = (108 \times 1.5) = 162 \text{ kN}$$

Column section is of size $400 \times 600 \text{ mm}$

$$\therefore b = 400 \text{ mm}$$

$$d = 550 \text{ mm}$$

$$D = 600 \text{ mm}$$

$$d' = 50 \text{ mm}$$

$$\left(\frac{d'}{D} \right) = 0.10$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$\left(\frac{M_u}{f_{ck} \cdot b \cdot D^2} \right) = \left(\frac{153 \times 10^6}{20 \times 400 \times 600^2} \right) = 0.053$$

$$\left(\frac{P_u}{f_{ck} \cdot b \cdot D} \right) = \left(\frac{162 \times 10^3}{20 \times 400 \times 600} \right) = 0.033$$

Using interaction curves of SP-16 we get

$$\left(\frac{p}{f_{ck}}\right) = 0.03 \quad \therefore p = (20 \times 0.03) = 0.6$$

$$\therefore A_s = (p \cdot b \cdot D)/100 = (0.6 \times 400 \times 600)/100 = 1440 \text{ mm}^2$$

$$\text{But minimum area of steel} = \left(\frac{0.8 \times 400 \times 600}{100}\right) = 1920 \text{ mm}^2$$

Provide 4 bars of 20 mm Φ on each face ($A_{st} = 2512 \text{ mm}^2$).
and 8 mm Φ ties at 300 mm centres throughout the column.
Same reinforcements are provided at the base section of the column.

(i) *Design of Foundations*

Axial load on column = 108 kN

Self weight of column

$$(0.4 \times 0.6 \times 4 \times 24) = 23$$

Self weight of foundation

$$\text{at } 10\% = 14$$

$$\text{Total} = 145 \text{ kN}$$

Moment at base = $M = 51 \text{ kN}\cdot\text{m}$

$$\therefore \text{eccentricity} = e = \left(\frac{M}{P}\right) = \left(\frac{51 \times 10^6}{145 \times 10^3}\right) = 350 \text{ mm}$$

To avoid tension in the foundations

Breadth $b = 6e = (6 \times 350) = 2100 \text{ mm}$

Adopt a foundation area of 1 m by 2.1 m

$$\text{Intensity of maximum pressure} = p = \left(\frac{2 \times 108}{1 \times 2.1}\right) = 103 \text{ kN/m}^2$$

$$< 150 \text{ kN/m}^2 \text{ (safe)}$$

$$p' = \left(\frac{103 \times 1.35}{2.1}\right) = 66 \text{ kN/m}^2$$

$$\text{Total pressure on the cantilever portion} = \left(\frac{103 + 66}{2}\right) 0.75 = 64 \text{ kN}$$

acting at 0.4 m from the column edge

$$\therefore \text{B.M. at edge of column} = (64 \times 0.4) = 25.6 \text{ kN}\cdot\text{m}$$

$$\therefore \text{Effective depth of footing} = d = \sqrt{\frac{25.6 \times 10^6}{0.897 \times 1000}} = 169 \text{ mm}$$

Provide effective depth $d = 250 \text{ mm}$ and over all depth = 300 mm
(Increased depth required to limit the shear stress within permissible limits).

$$A_{st} = \left(\frac{25.6 \times 10^6}{230 \times 0.9 \times 250}\right) = 494 \text{ mm}^2$$

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Provide 12 mm Φ at 150 mm centres both ways. ($A_{st} = 754 \text{ mm}^2$)

Check for shear

Shear force acting at a distance of 250 mm from face of column is given by

$$V = \left(\frac{103 \times 79}{2} \right) 0.5 = 45.5 \text{ kN}$$

$$\tau_v = \left(\frac{45.5 \times 10^3}{1000 \times 250} \right) = 0.182 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd} \right) = \left(\frac{100 \times 754}{1000 \times 250} \right) = 0.30 \quad \therefore \tau_c = 0.23 \text{ N/mm}^2$$

since $\tau_c > \tau_v$, no shear reinforcements are required.

The details of reinforcements in the portal frame is shown in Fig. 8.6.

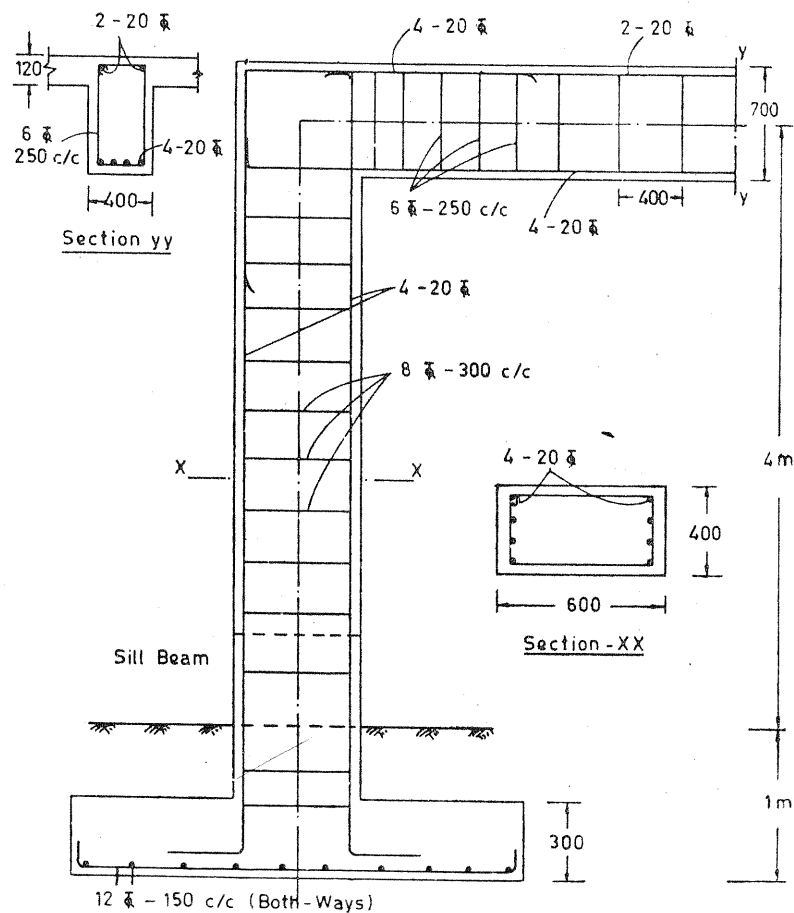


Fig. 8.6 Reinforcement details in portal-frame

8.5 EXAMPLE FOR PRACTICE

1. A portal frame $ABCD$ has fixed supports at A and D . The columns AB and CD are 5 m in height while the transom BC is 10 m in length. The frames are spaced at 3.5 m intervals. The live load on the roof slab which is 10 cm thick may be taken as 1.5 kN/m^2 . Design the transom BC and sketch the details of reinforcements. (Bangalore University, Jan. 1979).

2. A hall 50 m long and 10 m wide has to be covered by a continuous R.C.C slab over portal frames spaced at 3 m intervals. The height of the hall is 7 m. Design the slab and one intermediate portal frame. Adopt M-20 grade concrete and Fe-415 grade for steel. Sketch the details of reinforcements.

3. The roof of an assembly hall 30 m long and 12 m wide between centres of columns, consists of a continuous reinforced concrete slab over rectangular portal frames spaced 3 m apart. The columns are provided with independent footings and are hinged at the bottom. The ceiling height is 3.5 m above the hinge level. Adopting M-20 grade concrete and Fe-415 grade for steel, design the continuous roof slab and the portal frame and foundation footing for the columns. Assume safe bearing capacity of the soil as 150 kN/m^2 . Sketch the details of reinforcements in the portal frame.

4. A reinforced concrete two bay portal frame is used for covering a hall 20 m wide by 40 m length. Distance between centre line of columns is 10 m. The portals are spaced at 4 m intervals. The height of the columns is 5 m. Design the continuous slab, and one of the intermediate portal frames. Assume hinged condition at the base of the columns. Assume safe bearing capacity of the soil as 200 kN/m^2 . Sketch the details of reinforcements in the portal frame.

9

Multi Storey Building Frames

9.1 ANALYSIS OF MULTISTOREY FRAMES

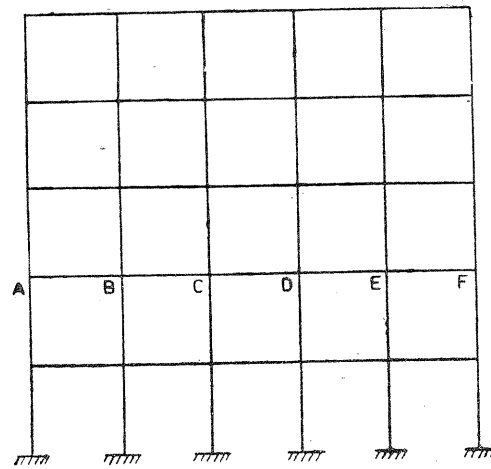
Reinforced concrete multistorey building frames are statically indeterminate structures and their rigorous analysis for vertical and horizontal loads involves lengthy computations. There are several methods of analysis such as moment distribution, Kani's rotation contribution, Takabeya's method and Matrix methods. The classical methods like moment distribution and Kani's rotation contribution are suitable for small frames but involve lengthy computations when used for solving multistorey and multibay frames.

Matrix methods with the use of computers are ideally suited for the analysis of multistorey frames with large number of redundants. However certain reasonable assumptions will result in approximate methods which are simpler in computational effort, although resulting in conservative magnitudes of the redundant forces. The most commonly used method for the analysis of vertical loads comprising of dead and live loads is the substitute frame method. For the analysis of wind loads, the portal and cantilever methods are generally used.

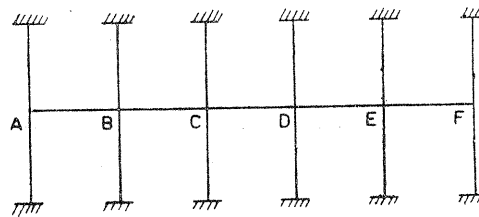
9.2 METHOD OF SUBSTITUTE FRAMES

A substitute frame consists of a small portion of the multistorey, multibay frame generally comprising of the floor beams, with the columns above and below the floor assumed to be fixed at the far ends as shown in Fig. 9.1.

It is sufficient to consider the loads on the two nearest spans on each side of the joint under consideration. The continuous beam is analysed for vertical loads by moment distribution to compute the maximum span and support moments using the following criterion:



(a) Multi Storey - Multi Bay Building Frame

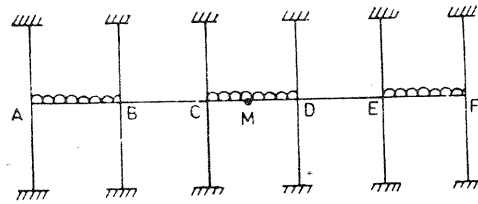


(b) Substitute - Frame

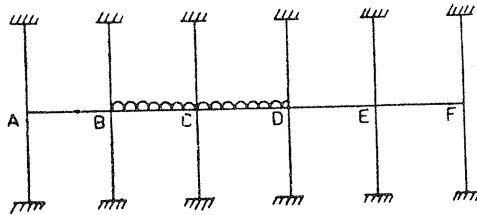
Fig. 9.1 Multistorey-multibay building frame

- (a) The maximum positive bending moment at mid point of any particular span develops when the load is placed on the span under consideration and on the alternate span as shown in Fig. 9.2(a).
- (b) The maximum negative bending moment at any particular support develops when the loads are placed on two spans adjacent to the support under consideration as shown in Fig. 9.2(b).
- (c) The maximum negative bending moment at mid point of any particular span develops when the loads are placed on the spans adjacent to the span under consideration as shown in Fig. 9.2(c).

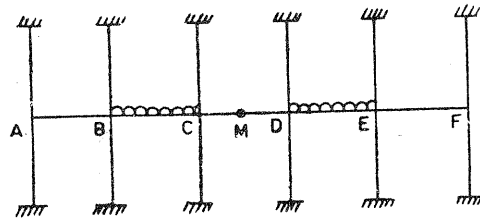
The computation of moments in beams and columns by using a substitute frame is illustrated by the following example.



(a) Loading for Maximum Positive B.M at M



(b) Loading for Maximum Negative B.M at C



(c) Loading for Maximum Negative B.M at M

Fig. 9.2 Loading pattern for maximum moments

9.3 DESIGN EXAMPLE

The substitute frame shown in Fig. 9.3 has to be analysed for maximum positive and negative moments in the beams AB , BC and CD . Use the following data to estimate the maximum moments in beams and columns. The beams are spaced at 3 m intervals.

Thickness of floor slab	= 100 mm
Live load (Residential flats)	= 2 kN/m^2
Floor finish	= 0.6 kN/m^2
Size of beams	= 200 mm by 400 mm
Size of columns	= 200 mm by 400 mm

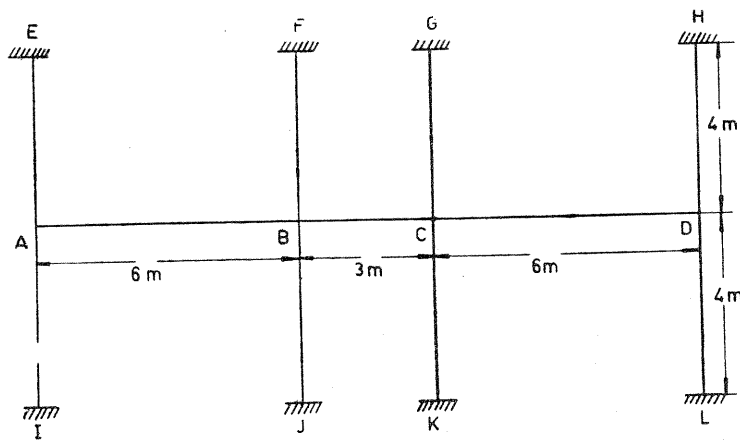


Fig. 9.3 Substitute frame for analysis of moments

1. *Loads on Beams*

$$\text{Self weight of slab} = (0.1 \times 24) = 2.4 \text{ kN/m}^2$$

$$\text{Floor finish} = 0.6$$

$$\text{Dead load} = 3.0 \text{ kN/m}^2$$

$$\text{Live load} = q = 2 \text{ kN/m}^2$$

$$\text{Self weight of beam} = (0.2 \times 0.4 \times 24) = 2 \text{ kN/m}$$

$$\text{Dead load from slab} = (3 \times 3) = 9 \text{ kN/m}$$

$$\text{Live load} = (2 \times 3) = 6 \text{ kN/m}$$

$$\therefore \text{Total D.L. on beam} = (9 + 2) = 11 \text{ kN/m}$$

$$\text{Total L.L. on beam} = 6 \text{ kN/m}$$

2. *Stiffness and Distribution Factors*

Since the beam and column section is same (200 mm by 400 mm),

$$I_{\text{beam}} = I_{\text{column}}$$

$$k_{AB} = k_{CD} = \left(\frac{I}{6}\right), \quad k_{AE} = \left(\frac{I}{4}\right) = k_{AI} \text{ and } k_{BC} = \left(\frac{I}{3}\right)$$

$$\therefore d_{AB} = \left[\frac{I/6}{\frac{I}{6} + \frac{I}{4} + \frac{I}{4}} \right] = 0.25$$

$$d_{AE} = d_{AI} = \left[\frac{I/4}{\frac{I}{6} + \frac{I}{4} + \frac{I}{4}} \right] = 0.375$$

$$d_{BA} = \left[\frac{I/4}{\frac{I}{6} + \frac{I}{3} + \frac{I}{4} + \frac{I}{4}} \right] = 0.166$$

$$d_{BC} = \left[\frac{I/3}{\frac{I}{6} + \frac{I}{3} + \frac{I}{4} + \frac{I}{4}} \right] = 0.333$$

$$d_{BF} = d_{BJ} = \left(\frac{1/4}{1.0} \right) = 0.25$$

3. Fixed End Moments

Dead load F.E.M.

$$F_{AB} = - \left(\frac{11 \times 6^2}{12} \right) = -33 \text{ kN}\cdot\text{m}$$

$$F_{BC} = - \left(\frac{11 \times 3^2}{12} \right) = -8.25 \text{ kN}\cdot\text{m}$$

Live load F.E.M.

$$F_{AB} = - \left(\frac{6 \times 6^2}{12} \right) = -18 \text{ kN}\cdot\text{m}$$

$$F_{BC} = - \left(\frac{6 \times 3^2}{12} \right) = -4.5 \text{ kN}\cdot\text{m}$$

For span *AB*, positive bending moment at mid span

$$= \left(\frac{wL^2}{8} \right) = \left(\frac{17 \times 6^2}{8} \right) = 76.5 \text{ kN}\cdot\text{m}$$

For span *BC*, positive bending moment at mid span

$$= \left(\frac{wL^2}{8} \right) = \left(\frac{17 \times 3^2}{8} \right) = 19.12 \text{ kN}\cdot\text{m}$$

4. Moments in Beams

Case 1. Maximum positive B.M. at mid span of *AB*. The continuous beam *ABCD* is loaded as shown in Fig. 9.4 and moment distribution is carried out to determine the support moments.

Negative moments at supports *A* and *B* are

$$M_{AB} = 41 \text{ kN}\cdot\text{m}$$

$$M_{BA} = 48 \text{ kN}\cdot\text{m}$$

Positive B.M. ordinate at mid span of *AB*

$$= \left(\frac{17 \times 6^2}{8} \right) = 76.5 \text{ kN}\cdot\text{m}$$

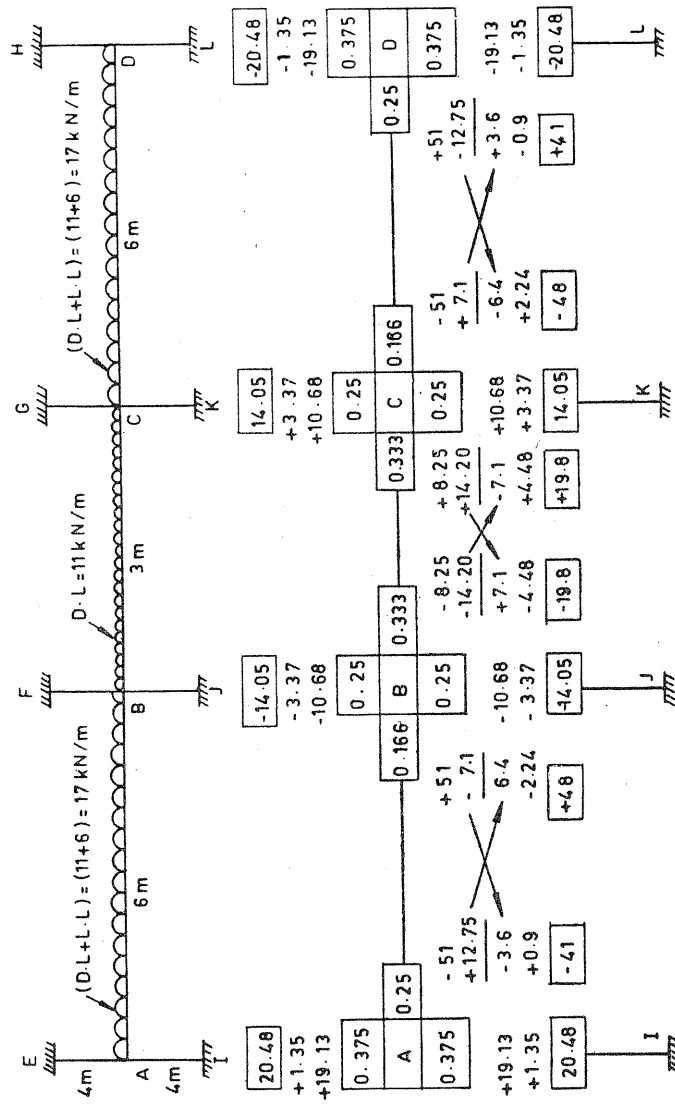
Maximum positive B.M. at mid span of *AB*

$$= \left[76.5 - \left(\frac{41 + 48}{2} \right) \right] = 32 \text{ kN}\cdot\text{m}$$

Case 2. Maximum positive B.M. at mid span of *BC*

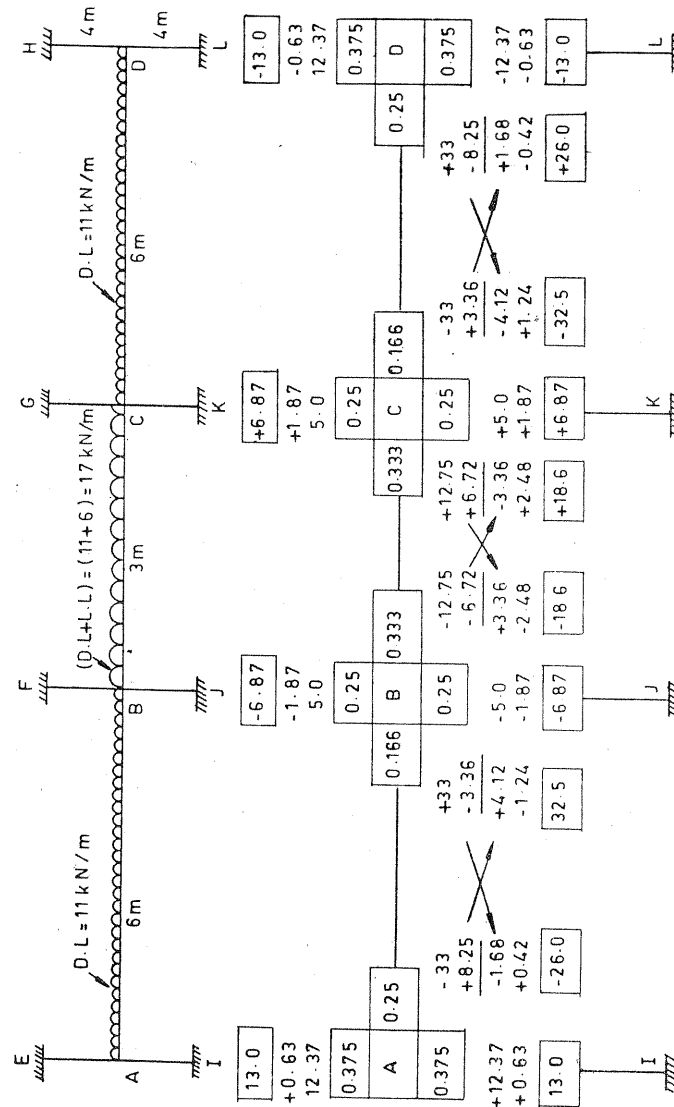
Referring to Fig. 9.5, the support moments obtained from moment distribution are given by,

$$M_{BC} = M_{CB} = 18.6 \text{ kN}\cdot\text{m}$$



Case -1. Maximum Positive B.M at Mid Span of AB (Moments in kN.m)
D.L on ABCD L.L on AB and CD

Fig. 9.4 Moment distribution of substitute frame



Case-2 Maximum Positive B.M at Mid Span of BC (Moments in kNm)

D.L on ABCD L.L on BC

Fig. 9.5 Moment distribution of substitute frame

Positive B.M. ordinate at mid span of BC

$$= \left(\frac{17 \times 3^2}{8} \right) = 19.1 \text{ kN} \cdot \text{m}$$

$\therefore$ Maximum positive B.M. at mid span of BC

$$= (19.1 - 18.6) = 0.5 \text{ kN} \cdot \text{m}$$

Case 3. Maximum negative B.M. and S.F. at A

Referring to Fig. 9.6, the maximum negative B.M. at support section A of beam AB is obtained as

$$M_{AB} = -41 \text{ kN} \cdot \text{m}$$

By analysing the free body diagram, the shear force at support A is computed as

$$V_A = \left(\frac{17 \times 6}{2} \right) + \left(\frac{41 - 48.5}{6} \right) = 49.75 \text{ kN}$$

Case 4. Maximum Negative B.M and S.F at B

Referring to Fig. 9.7, the maximum negative B.M. at support section B of beam BA is obtained as

$$M_{BA} = 49.4 \text{ kN} \cdot \text{m}$$

By analysing the free body diagram, the maximum shear force at support B of beam BA is computed as

$$V_B = \left(\frac{49.4 - 40.6}{6} \right) + \left(\frac{17 \times 6}{2} \right) = 55.46 \text{ kN}$$

5. Moments and Axial Thrust in Columns

(a) *Exterior Column AI*

From Fig. 9.6

Moment at top of column $AI = 20.5 \text{ kN} \cdot \text{m}$

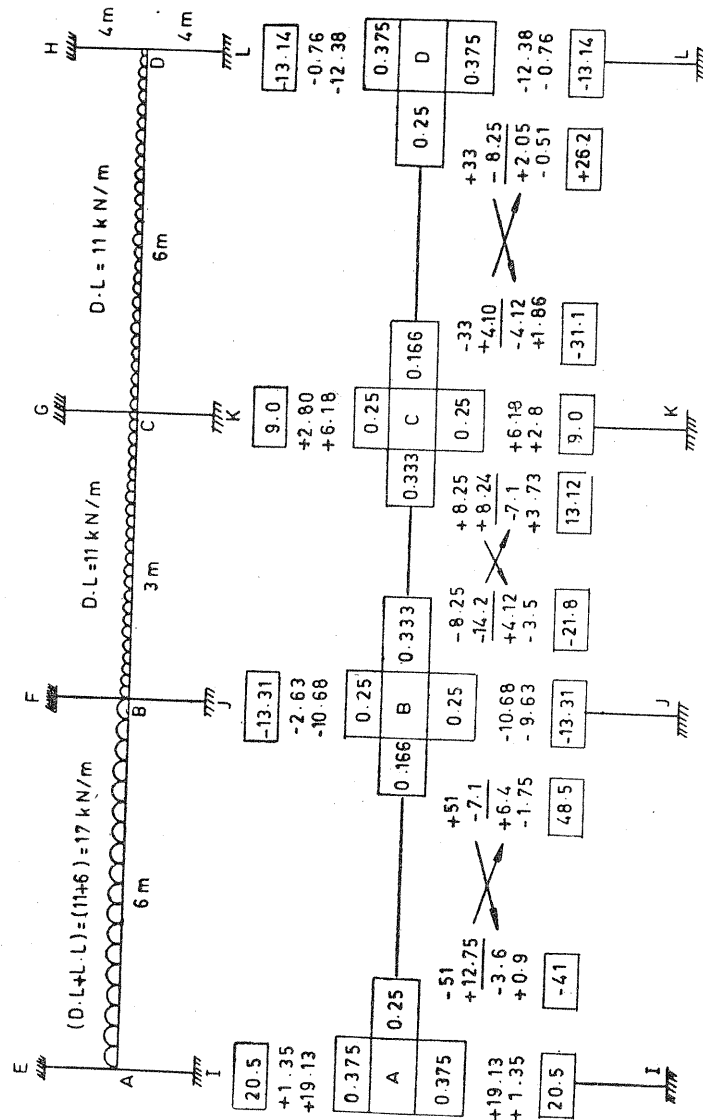
Axial thrust $= V_A = 49.75 \text{ kN}$

(b) *Interior Column BJ*

From Fig. 9.7

Moment at top of column $BJ = 12.01 \text{ kN} \cdot \text{m}$

$$\begin{aligned} \text{Axial thrust} &= \left[\left(\frac{49.4 - 40.6}{6} \right) + \left(\frac{17 \times 6}{2} \right) \right] + \left[\left(\frac{25.2 - 16.6}{3} \right) \right. \\ &\quad \left. + \left(\frac{17 \times 3}{2} \right) \right] = 84 \text{ kN} \end{aligned}$$



Case-3 Maximum Negative B.M. at A (Moments in kN.m)
D.L. on ABCD L.L. on AB

Fig. 9.6 Moment distribution of substitute frame

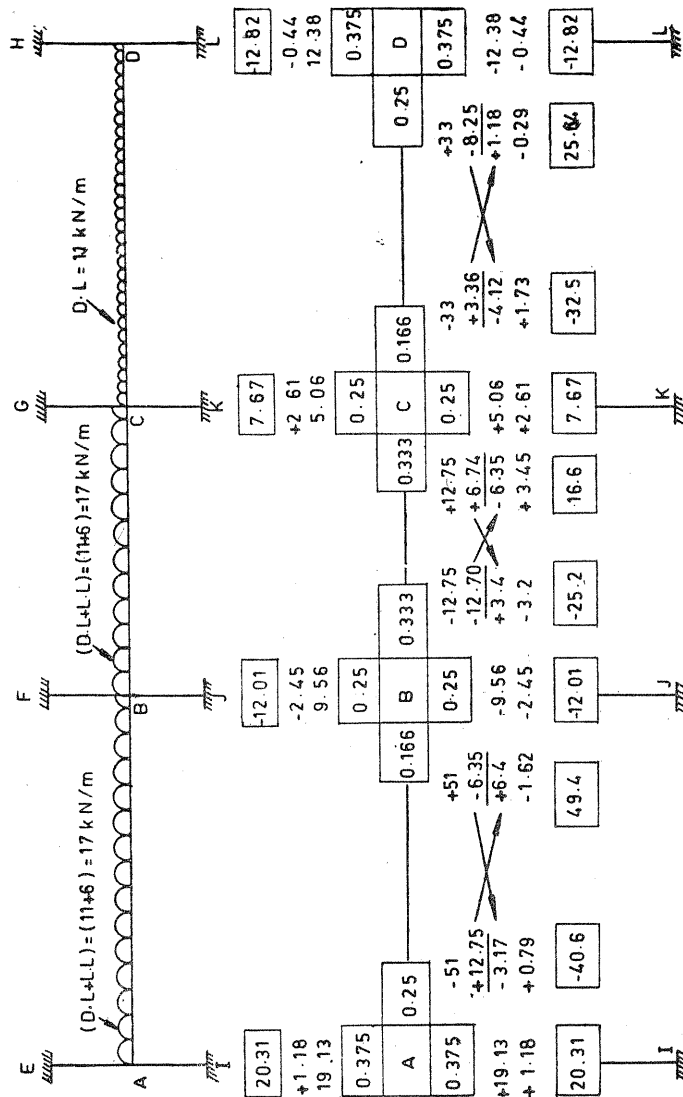


Fig. 9.7 Moment distribution of substitute frame

9.4 BENDING MOMENTS IN COLUMNS

Bending moments developed in the columns of a multistorey, single and multibay frames can be evaluated by an approximate method which takes into account the stiffness of the columns and beams in the frame. This method gives reasonably good results and is sufficiently accurate for general designs.

The moments in columns of single and multibay are computed by the expressions given in Table 9.1.

TABLE 9.1 Bending Moments in Columns

	Moments for Frames of one bay	Moments for Frames of two or more bays
1. External Columns		
(a) Moment at foot of upper column	$M_e \left(\frac{k_u}{k_u + k_L + 0.5k_b} \right)$	$M_e \left(\frac{k_u}{k_u + k_L + k_b} \right)$
(b) Moment at head of lower column	$M_e \left(\frac{k_L}{k_u + k_L + 0.5k_b} \right)$	$M_e \left(\frac{k_L}{k_u + k_L + k_b} \right)$
2. Internal Columns		
(a) Moment at foot of upper column	—	$M_{es} \left(\frac{k_u}{k_L + k_u + k_{b1} + k_{b2}} \right)$
(b) Moment at head of lower column	—	$M_{es} \left(\frac{k_L}{k_L + k_u + k_{b1} + k_{b2}} \right)$

M_e = Bending moment at the end of the beam framing into the column assuming fixity at the connection;

M_{es} = Maximum difference between the moments at the ends of the two beams framing into opposite sides of the column each calculated on the assumption that the ends of the beams are fixed and assuming one of the beams unloaded.

k_u = Stiffness factor of the upper column

k_L = Stiffness factor of the lower column

k_{b1} = Stiffness of the beam on one of the side of the column

k_{b2} = Stiffness of the beam on other side of the column.

The application of this method to compute moments in the exterior columns of design Example 9.3, indicates that this methods results in moments about 6 percent less than those obtained by the substitute frame method. However the method is simpler and yields reasonably accurate values of moments for design purposes.

9.5 ANALYSIS OF MULTISTOREY FRAMES SUBJECTED TO HORIZONTAL FORCES

Multistorey building frames with the ratio of height to the least lateral dimension greater than 2, have to be analysed for moments developed in the members due to the effect of horizontal wind forces acting on the building. The horizontal forces due to wind are assumed to act at each of the floor levels and they induce axial forces in the columns and bending moments in all the members of the frame. These moments and forces can be analysed by the following approximate methods.

(a) Portal Method

(b) Cantilever Method

(a) Portal Method

The portal method is based on the following assumptions:

1. Points of contraflexure are assumed to develop at the mid point of beams and columns.
2. The interior columns are assumed to resist double the shear force taken by each of the external columns.

The method is illustrated with reference to the multistorey, multibay frame shown in Fig. 9.8.

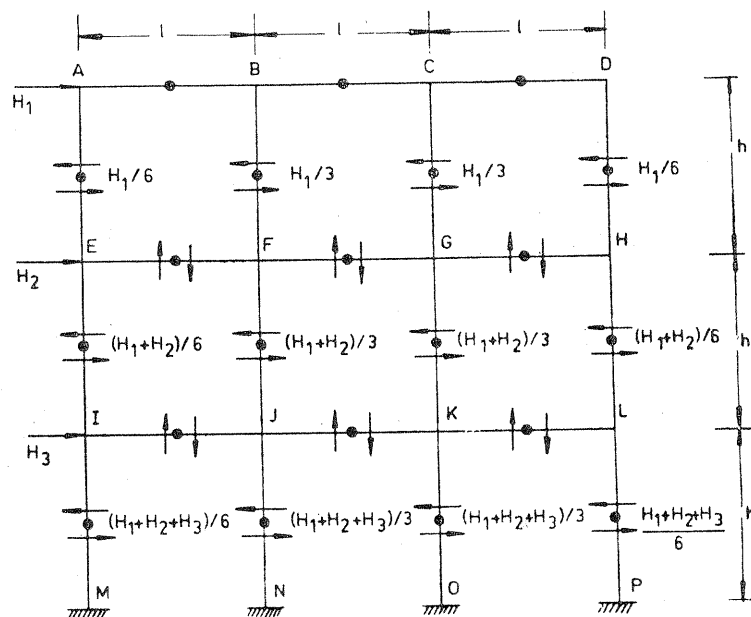


Fig. 9.8 Portal method of analysis for wind loads

The horizontal shear forces resisted by the columns in each of the floors are shown above and below the contraflexure points in the Fig. 9.8.

The moments in the columns are computed as shown below:

$$M_{EA} = \left(\frac{H_1}{6} \times \frac{h}{2} \right) = \left(\frac{H_1 h}{12} \right)$$

$$M_{EI} = \left(\frac{H_1 + H_2}{6} \right) \times \left(\frac{h}{2} \right) = \frac{(H_1 + H_2)h}{12}$$

$$M_{MI} = \frac{(H_1 + H_2 + H_3)}{6} \times \left(\frac{h}{2} \right) = \frac{(H_1 + H_2 + H_3)h}{12}$$

$$M_{FB} = \left(\frac{H_1}{3} \times \frac{h}{2} \right) = \left(\frac{H_1 h}{6} \right)$$

$$M_{FI} = \left(\frac{H_1 + H_2}{3} \right) \times \left(\frac{h}{2} \right) = \frac{(H_1 + H_2)h}{6}$$

$$M_{NJ} = \left(\frac{H_1 + H_2 + H_3}{3} \right) \times \left(\frac{h}{2} \right) = \frac{(H_1 + H_2 + H_3)h}{6}$$

The moments in the beams are computed by using the free body diagrams shown in Fig. 9.9.

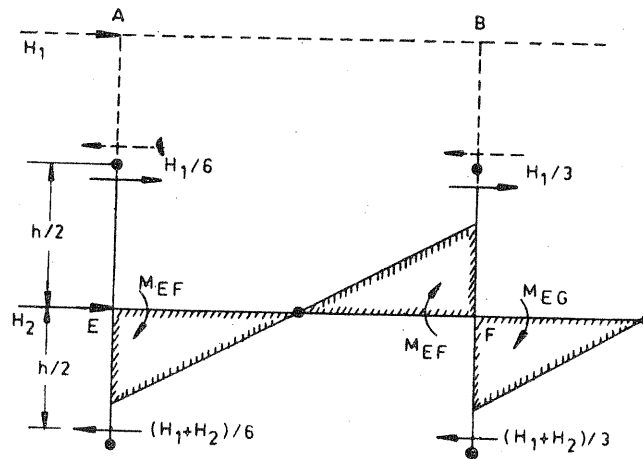


Fig. 9.9 Moments in beams

Moment in column just above $E = \left(\frac{H_1}{6} \cdot \frac{h}{2} \right)$

Moment in column just below $E = \left(\frac{H_1 + H_2}{6} \right) \times \left(\frac{h}{2} \right)$

Therefore for equilibrium of moments at joint E , the beam is subjected to a moment

$$M_{EF} = \left[\frac{H_1 h}{12} + \frac{(H_1 + H_2)h}{12} \right]$$

Similarly at joint F , the clockwise moment is given by,

$$M_F = \left[\left(\frac{H_1}{3} \right) \left(\frac{h}{2} \right) + \frac{(H_1 + H_2)}{3} \times \frac{h}{2} \right]$$

This moment is sheared equally by the beams FE and FG .

$$\therefore M_{FE} = M_{FG} = \frac{1}{2} \left[\frac{H_1 h}{6} + \frac{(H_1 + H_2)h}{6} \right] = \left[\frac{H_1 h}{12} + \frac{(H_1 + H_2)h}{12} \right]$$

Vertical reactions in columns develop only for the exterior columns. If M is the moment at the ends of beams and L = length of beam, the Windward column is subjected to a pull of $(2M/L)$ and the Leeward column is subjected to a push of $(2M/L)$. The reactions neutralize for interior columns.

(b) *Cantilever Method*

The following assumptions are made in the cantilever method.

1. Points of contraflexure occur at the middle points of the various members of the frame.
2. Direct stresses in the columns are proportional to their distances from the centroidal vertical axis of the frame.

Referring to the multistorey frame shown in Fig. 9.10. Let V_1, V_2, V_3 and V_4 are the vertical reactions developed in the top storey of the frame due to the horizontal force H_1 acting at the joint A_1 .

If the cross sectional area of the columns are the same, then the reactive forces in the columns are proportional to their distances from the centroidal axis of the frame. Let P_1, P_2, P_3 and P_4 be the horizontal shears in the top storey. Then using the assumptions specified, the following equations can be obtained.

$$\left(\frac{V_1}{x_1} \right) = \left(\frac{V_2}{x_2} \right) = \left(\frac{V_3}{x_3} \right) = \left(\frac{V_4}{x_4} \right) \quad (1)$$

$$V_1 x_1 + V_2 x_2 + V_3 x_3 + V_4 x_4 = \left(\frac{H_1 h}{2} \right) \quad (2)$$

$$(P_1 + P_2 + P_3 + P_4) \frac{h}{2} = \left(\frac{H_1 h}{2} \right) \quad (3)$$

From Eqs. (1) and (2), the vertical reaction components V_1, V_2, V_3 and V_4 can be computed.

By taking moments about the point of contraflexure in beam AB (refer Fig. 9.11) we have.

$$\left(\frac{P_1 h}{2} \right) = \left(\frac{V_1 L}{2} \right) \quad \therefore \quad P_1 = \left(\frac{L}{h} \cdot V_1 \right) \quad (4)$$

By taking moments about the contraflexure point at the mid point R of BC , we have,

$$(P_1 + P_2) \frac{h}{2} = V_1 (L + L/2) + V_2 \left(\frac{L}{2} \right) = V_1 L + (V_1 + V_2) \frac{L}{2}$$

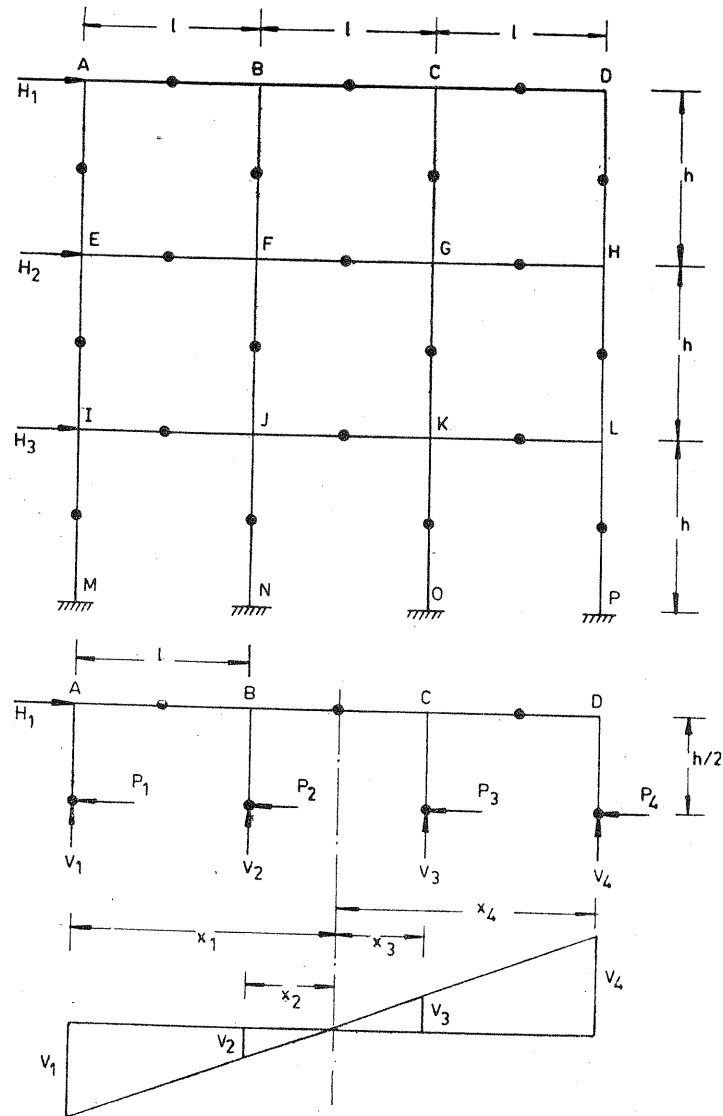


Fig. 9.10 Cantilever method of analysis for wind loads

$$\therefore (P_1 + P_2) = \left[\frac{2V_1L + (V_1 + V_2)L/2}{h} \right] \quad (5)$$

Since P_1 is evaluated from Eq. 4, P_2 can be obtained from Eq. 5. Similarly by taking moments about the contraflexure points, the horizontal shears P_3 and P_4 can be determined. The analysis is repeated for different storeys. Knowing the horizontal shears, the moments in the columns and beams can be computed in every floor of the frame.

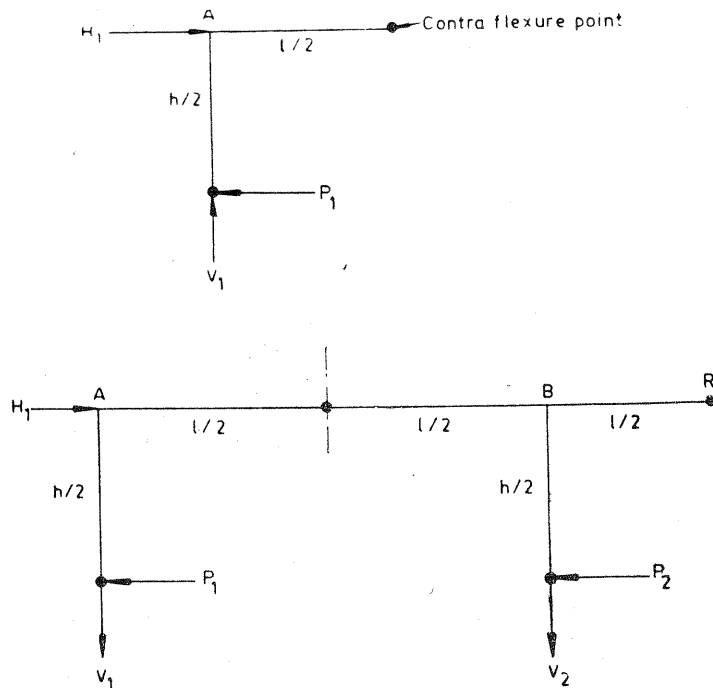


Fig. 9.11 Free body diagrams

9.6 DESIGN EXAMPLE

Analyse the multistorey frame shown in Fig. 9.8 for moments in the ground floor columns *IM*, *JN*, *KO* and *LP* and the beams *IJ*, *JK* and *KL*. Adopt the following data: $L = 6$ m, $h = 3$ m, Wind loads are $H_1 = 6$ kN, $H_2 = 12$ kN, $H_3 = 12$ kN. The columns have the same cross section. Compare the results of the portal and cantilever methods of analysis.

(a) Portal Method

Referring to Fig. 9.8, the horizontal shears in the first storey are computed as,

$$\left(\frac{H_1 + H_2 + H_3}{6} \right) = \left(\frac{6 + 12 + 12}{6} \right) = 5 \text{ kN}$$

$$\left(\frac{H_1 + H_2 + H_3}{3} \right) = \left(\frac{6 + 12 + 12}{3} \right) = 10 \text{ kN}$$

The moments in columns are obtained as

$$M_{IM} = M_{MI} = (5 \times 1.5) = 7.5 \text{ kN} \cdot \text{m} = M_{LP}$$

$$M_{JN} = M_{NJ} = (10 \times 1.5) = 15.0 \text{ kN} \cdot \text{m} = M_{KO}$$

The moments in the beams are computed as follows:

$$\begin{aligned}
 M_{IJ} &= \left[\frac{H_1 + H_2}{6} \times \frac{h}{2} \right] + \left[\frac{H_1 + H_2 + H_3}{6} \times \frac{h}{2} \right] \\
 &= \left[\left(\frac{6+12}{6} \times \frac{3}{2} \right) + \left(\frac{6+12+12}{6} \times \frac{3}{2} \right) \right] = 12 \text{ kN}\cdot\text{m} \\
 M_{JI} = M_{JK} &= \frac{1}{2} \left[\left(\frac{H_1 + H_2}{3} \times \frac{h}{2} \right) + \left(\frac{H_1 + H_2 + H_3}{3} \times \frac{h}{2} \right) \right] \\
 &= \frac{1}{2} \left[\left(\frac{6+12}{3} \times \frac{3}{2} \right) + \left(\frac{6+12+12}{3} \times \frac{3}{2} \right) \right] = 12 \text{ kN}\cdot\text{m}
 \end{aligned}$$

(b) *Cantilever Method*

Referring to Fig. 9.10. If V_1, V_2, V_3 and V_4 are the reactions developed in the columns, we have by symmetry $V_1 = V_4$ and $V_2 = V_3$.

Also $\left(\frac{V_1}{x_1} \right) = \left(\frac{V_2}{x_2} \right) \quad \therefore \quad V_2 = V_1 \left(\frac{x_2}{x_1} \right) = V_1 \left(\frac{3}{9} \right) = \frac{V_1}{3}$

Let P_1, P_2, P_3 and P_4 are the horizontal shears, in the second storey.

Then $(P_1 + P_2 + P_3 + P_4) = (H_1 + H_2) = (6 + 12) = 18 \text{ kN}$

Also $(P_1 + P_2 + P_3 + P_4)h/2 = V_1x_1 + V_2x_2 + V_3x_3 + V_4x_4$

$$\begin{aligned}
 \therefore \quad \left(\frac{18 \times 3}{2} \right) &= [V_1 \cdot 9 + (V_1/3) \cdot 3 + (V_1/3) \cdot 3 + V_1 \cdot 9] \\
 27 &= 20V_1
 \end{aligned}$$

$$\therefore \quad V_1 = 1.35 \text{ kN} = V_4, \quad V_2 = \left(\frac{1.35}{3} \right) = 0.45 \text{ kN} = V_3$$

$$P_1 = \left(\frac{L}{h} \right) V_1 = (6/3) \cdot 1.35 = 2.70 \text{ kN}$$

$$\begin{aligned}
 P_1 + P_2 &= 2 \left[\frac{V_1 L + (V_1 + V_2)L/2}{h} \right] \\
 &= 2 \left[\frac{(1.35 \times 6) + (1.35 + 0.45)3}{3} \right] = 9
 \end{aligned}$$

$$\therefore \quad P_2 = (9 - 2.70) = 6.30 \text{ kN}$$

$$\therefore \quad P_1 = P_4 = 2.70 \text{ kN}, \quad P_2 = P_3 = 6.30 \text{ kN}$$

Similarly considering the first storey (Ground floor). If P_1, P_2, P_3 and P_4 are the horizontal shears at the contraflexure points, we have

$$(P_1 + P_2 + P_3 + P_4) = (H_1 + H_2 + H_3) = (6 + 12 + 12) = 30 \text{ kN}$$

Also $[(P_1 + P_2 + P_3 + P_4)h/2] = [V_1x_1 + V_2x_2 + V_3x_3 + V_4x_4]$

$$\begin{aligned}
 \therefore \quad \left(\frac{30 \times 3}{2} \right) &= V_1 \cdot 9 + \left(\frac{V_1}{3} \right) \cdot 3 + \left(\frac{V_1}{3} \right) \cdot 3 + V_1 \cdot 9 \\
 45 &= 20V_1
 \end{aligned}$$

$$\therefore V_1 = 2.25 \text{ kN} = V_4 \quad \therefore V_2 = \left(\frac{2.25}{3}\right) = 0.75 \text{ kN} = V_3$$

$$P_1 = (L/h)V_1 = (6/3)2.25 = 4.5 \text{ kN}$$

$$(P_1 + P_2) = 2 \left[\frac{V_1 L + (V_1 + V_2)L/2}{h} \right] = 2 \left[\frac{2.25 \times 6 + (2.25 + 0.75)3}{3} \right] \\ = 15.00$$

$$\therefore P_2 = (15 - 4.5) = 10.5 \text{ kN}$$

Column Moments

$$M_{IE} = M_{LH} = (2.70 \times 1.5) = 4.05 \text{ kN} \cdot \text{m}$$

$$M_{JF} = M_{KG} = (6.3 \times 1.5) = 9.45 \text{ kN} \cdot \text{m}$$

$$M_{IM} = M_{LP} = (4.5 \times 1.5) = 6.75 \text{ kN} \cdot \text{m}$$

$$M_{JN} = M_{KO} = (10.5 \times 1.5) = 15.75 \text{ kN} \cdot \text{m}$$

Beam moments

$$M_{IJ} = (M_{IE} + M_{IM}) = (4.05 + 6.75) = 10.80 \text{ kN} \cdot \text{m}$$

$$M_{JI} = M_{JK} = \left[\frac{(M_{JF} + M_{JN})}{2} \right] = \left[\frac{(9.45 + 15.75)}{2} \right] = 12.6 \text{ kN} \cdot \text{m}$$

The portal method results in slightly higher values of moments in the exterior columns where as the cantilever method gives higher values of moments for the interior columns. Also the shears resisted by the interior columns are higher according to the cantilever method of analysis.

9.7 EXAMPLES FOR PRACTICE

1. The substitute frame of a multistoreyed building having 3 bays has a continuous beam $ABCD$ with $AB = 4.0 \text{ m}$, $BC = 2.5 \text{ m}$, and $CD = 4.0 \text{ m}$. The beams are spaced at 3 m intervals. Thickness for floor slab = 120 mm. Live load (office floor) = 4 kN/m^2 . Floor finish = 0.6 kN/m^2 . Size of beams = 250 mm by 400 mm. Size of columns = 250 mm by 400 mm. Height between floors = 4 m. Analyse the substitute frame and estimate the maximum design moments in the beams and columns.

2. A four bay multistoreyed frame has the following details. Continuous beam $ABCDE$ with $AB = BC = CD = DE = 4 \text{ m}$. Height between floors = 4 m. Size of beams = 300 mm by 500 mm. Size of columns = 300 mm by 400 mm. Thickness of floor slabs = 150 mm. Floor finish = 1 kN/m^2 . Live load = 2 kN/m^2 . Estimate the maximum design moments in the beams and columns.

3. A four storeyed multistorey building frame has four equal bays of 4 m each and the height between floors is 4 m. The wind loads acting at roof level and various floor levels are,

$$H_1 = 5 \text{ kN}, \quad H_2 = 10 \text{ kN}, \quad H_3 = 10 \text{ kN} \quad \text{and} \quad H_4 = 10 \text{ kN}.$$

The columns have the same cross section. Estimate the moments in the columns and beams using.

(a) Portal method (b) Cantilever method.

10

Shells

10.1 INTRODUCTION

Shells or skin space roofs are preferable to plane roofs since they can be used to cover large floor spaces with economical use of materials of construction. The use of curved space roofs requires 25 to 40% less materials than that of the plane elements. Structurally the shell roofs are superior since the whole cross section is uniformly stressed due to the direct forces with negligible bending effects. Due to this aspect the thickness of shells is usually very small in the range of 75 mm to 150 mm.

Shell roofs are generally adopted for hangers, sports auditoriums, exhibition halls, industrial buildings and a variety of other large span structures where uninterrupted floor space is required. Shell roofs are architecturally very expressive and have been used for domes by Romans. Recent advances include the construction of shell structures using prefabricated shell elements.

10.2 SHELL TERMINOLOGY

The following terms are generally used in the study of shell structures according to the Indian Standard Code IS : 2210.

Shell: A curved surface having small thickness compared to the radius and other dimensions.

Shell or Revolution: These are obtained when a plane curve is rotated about the axis of symmetry. The common examples of shells of revolution are the circular dome, the cone and the paraboloid of revolution shown in Fig. 10.1.

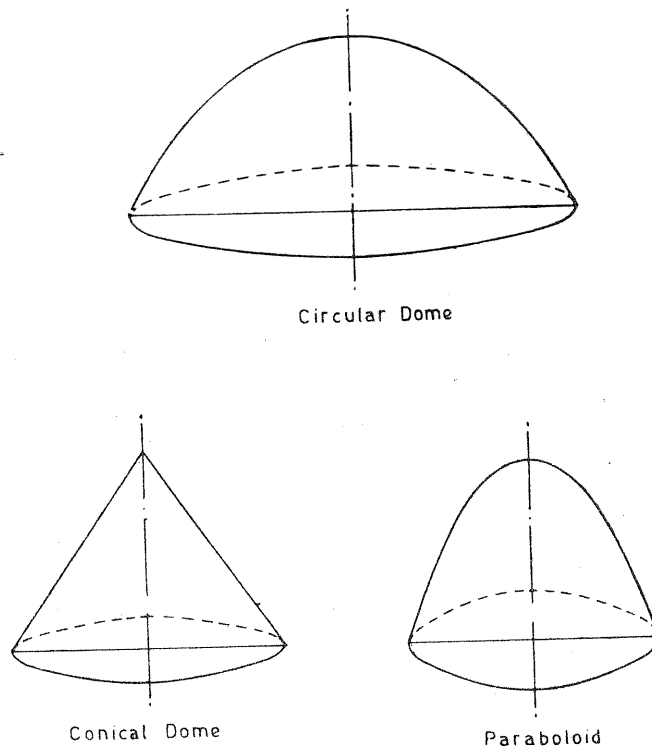


Fig. 10.1 Shells of revolution

Shells of Translation: Shells of revolution are formed when one curve moves parallel to itself along another curve, the planes of the two curves being at right angles to each other. Elliptic paraboloid and hyperbolic paraboloid are common examples of shells of translation. These are shown in Fig. 10.2.

Cylindrical Shells: These are shells in which the generatrix (moving curve) or the directrix (stationary curve) is a straight line. For cylindrical

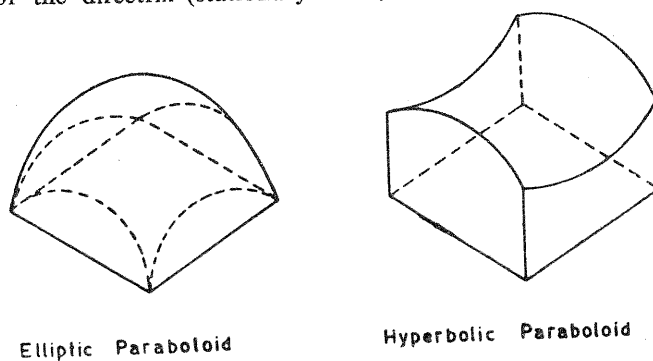


Fig. 10.2 Shells of translation

shells, the common curves used are the arc of a circle, semi-ellipse, parabola or catenary. The various structural components of a cylindrical shell are the thin shell, edge beam and end frame or traverse as shown in Fig. 10.3.

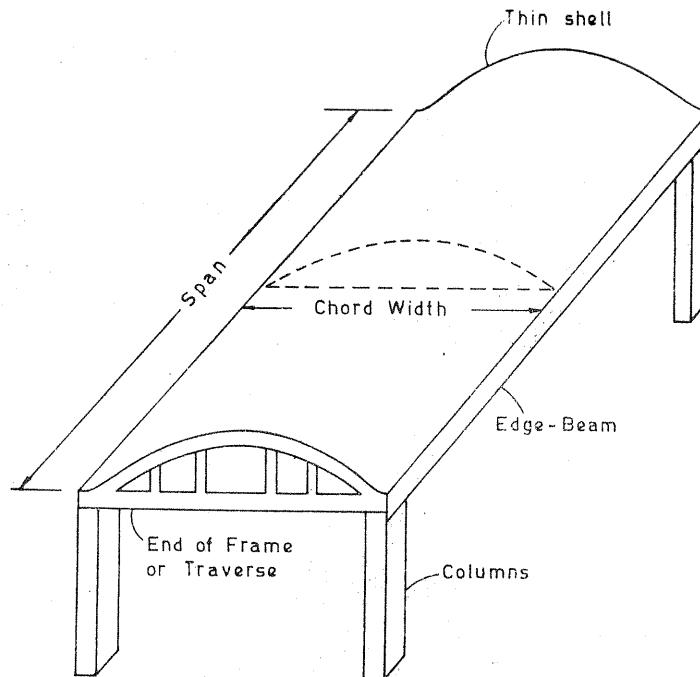


Fig. 10.3 Cylindrical shell

Multiple Cylindrical Shells: A series of parallel cylindrical shells which are transversely continuous are termed as multiple cylindrical shells. Generally used for hangers, Ware houses and factory buildings. A typical multiple cylindrical shell is shown in Fig. 10.4.

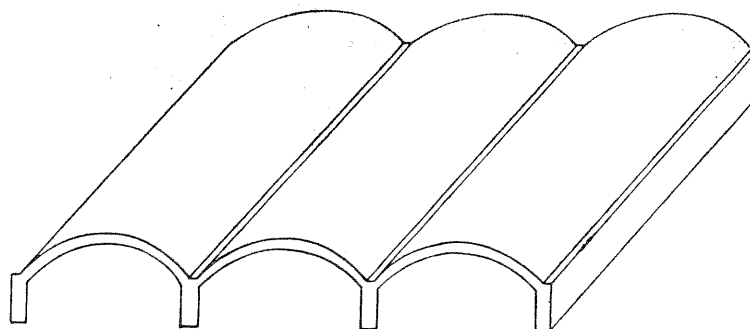


Fig. 10.4 Multiple cylindrical shells

Continuous Cylindrical Shells: These are cylindrical shells which are longitudinally continuous over the traverses. Multiple and continuous cylindrical shells are widely used for market hall and ware house roofs. Cylindrical shells which are symmetrical about the crown are termed as *Barrel shells*.

North Light Shells: North light shells comprise of cylindrical shells with two springings at different levels and built in single or multiple bays and have provisions for north light glazing. North light shells are commonly used for factory shed roofs. A typical north light shell of multibay is shown in Fig. 13.5.

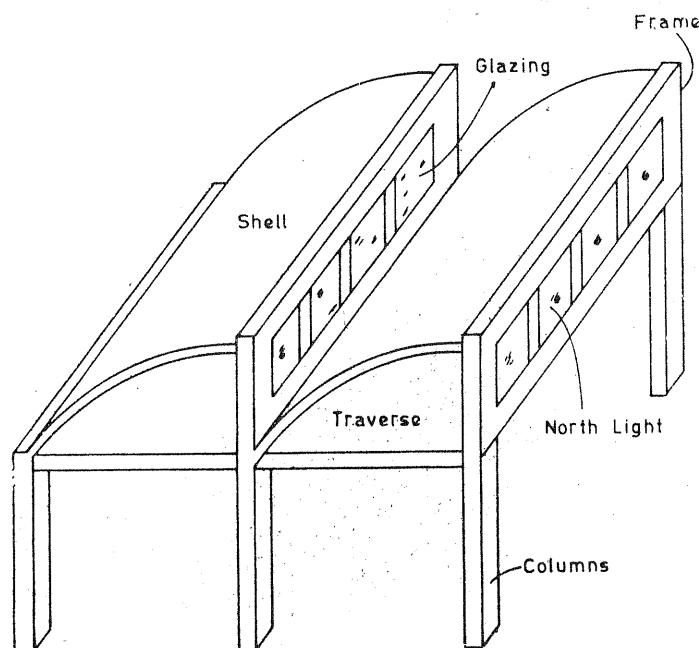


Fig. 10.5 North light shells

Butterfly Shells: A butterfly shell is formed when two parts of a cylindrical shell are joined together at their lower edges as shown in Fig. 10.6. This type of shell is commonly used for railway platforms and bus shelters.

Ruled Surface: Are surfaces which can be generated entirely by straight lines. If at every point, a single straight line can be ruled, the surface generated is termed as singly ruled surface. Common examples of singly ruled surfaces are cylindrical shells and conical shells.

If at every point, two straight lines can be ruled, the surface generated is termed as doubly ruled surface. Common examples of doubly ruled surfaces are hyperbolic paraboloid and hyperboloid of revolution of one sheet.

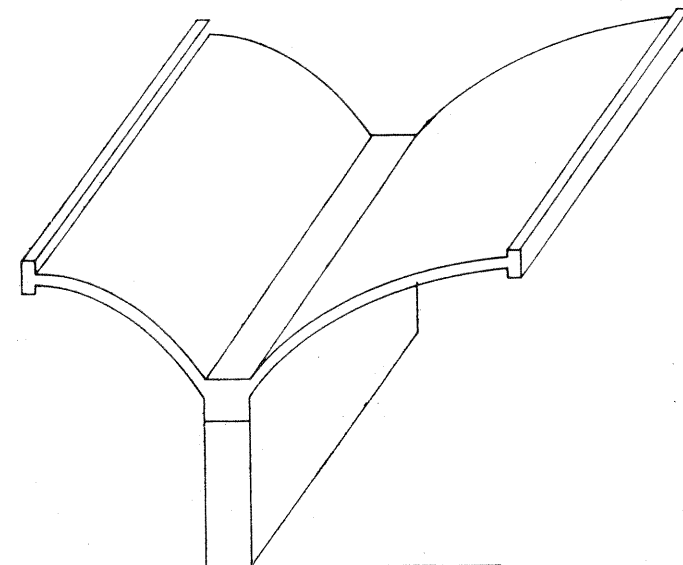


Fig. 10.6 Butterfly shell

Radius of the Shell: The general expression for the radius at any point of a shell surface is given by,

$$R = R_0 \cos^n \phi$$

where R = Radius at any point

R_0 = Radius at crown

ϕ = Slope of the tangent to the curve at that point

n = A constant depending upon the type of curve

$n = 0$ for a circle

$= 1$ for cycloid

$= -2$ for catenary

$= -3$ for parabola

For an ellipse,

$$R = \left[\frac{a^2 b^2}{(a^2 \sin^2 \phi + b^2 \cos^2 \phi)^{3/2}} \right]$$

where a = Semi major axis

b = Semi minor axis

ϕ = Slope of tangent at that point

Gauss - Curvature

Gauss curvature is the product of the two principal curvatures $1/R_1$ and $1/R_2$ at any point on the surface of the shell. For singly curved developable shells, Gauss curvature is zero. For doubly curved non-developable shells, Gauss curvature is positive for synclastic shells and negative for Anticlastic shells.

Span of Shell: The distance between the two adjacent end frames or traverses is termed as the span of the shell.

Chord Width: Horizontal projection of the arc of the shell.

Rise: Vertical distance between the apex of the curve and the springing.

End Frames or Traverses: Frames provided at the ends to support and preserve the geometry of the shell.

Edge Member or Beam: The horizontal beam supported on columns and supporting the longitudinal edges of the shell.

10.3 CLASSIFICATION OF SHELLS

Shells are broadly classified into two major groups as,

- (a) Singly curved shells which are developable
- (b) Doubly curved shells which are non-developable

Under the singly curved shells we have the conical and cylindrical shells. The common examples of doubly curved shells are the circular domes, paraboloid, ellipsoid, hyperbolic paraboloid and elliptic paraboloid. These shells are generally grouped under the three categories designated as,

- (a) Shells of revolution
- (b) Shells of translation
- (c) Ruled surfaces

Figure 10.7 shows the general classification of shells with examples.

Shells are also classified as thin and thick shells. A shell can be considered as thin if the ratio of the radius to the thickness of the shell is greater than 20. In general, most of the shells used in practice come under the category of thin shells.

Thin cylindrical shells are generally classified in two groups,

- (a) Long shells
- (b) Short shells

According to the A.S.C.E. manual on design of cylindrical concrete shell roofs, shells having ratios of radius to span (R/L) less than 0.6 are classified as long shells. If the ratio exceeds this value, the shells are considered as short barrels.

10.4 GENERAL SPECIFICATIONS OF SHELLS

The salient dimensions of the shells have to be selected before a rigorous mathematical analysis is carried out for the computation of membrane

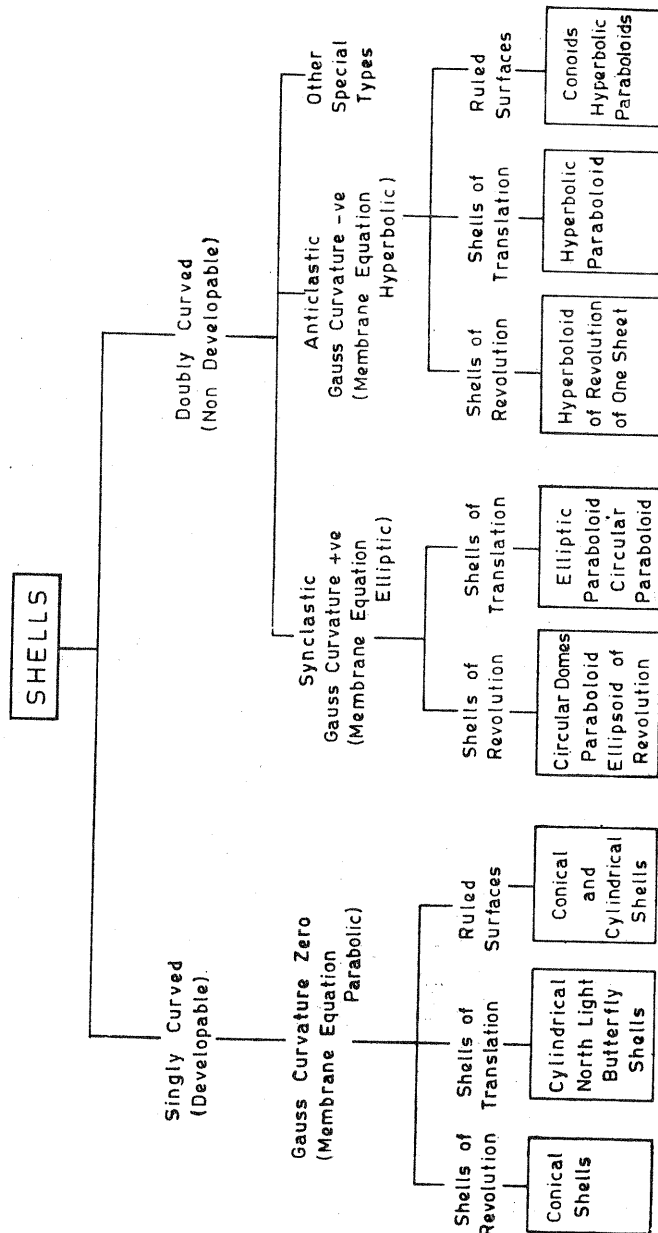


Fig. 10.7 General classification of shells

forces in the shell. The general guide lines followed for selecting the dimensions of the various structural components of the shell are detailed below:

1. *Thickness*

The overall thickness of a reinforced concrete shell should not be less than 50 mm for singly curved shells, 40 mm for doubly curved shells and 25 mm for precast shells. Generally the thickness is in the range of 80 mm to 120 mm for most of the shells based on practical considerations.

At the junction of the edge beams, the thickness of the shell is increased by 30 percent over a length of $0.38\sqrt{Rd}$ to $0.76\sqrt{Rd}$ for singly curved shells, where R = radius of curvature of the shells and d = overall thickness of the shell. For doubly curved shells, this distance depends upon the geometry of the shell and boundary conditions.

2. *Span and Chord Width*

The span of reinforced concrete shells should not be greater than 30 m to limit the size and reinforcement within practicable limits in the edge beams. For longer spans, prestressed edge beams can be used. The width of the edge member is limited to 2 to 3 times the thickness of the shell.

In shells with chord widths much larger than the span, the chord width shall be preferably be 3 to 6 times the span.

3. *Depth of Shells*

For large span shells, $\text{depth} = 1/6$ to $1/12$ span, larger figures are applicable to small spans. For shells without edge members, depth

$\leq \left(\frac{\text{span}}{10} \right)$. For shells with chord width much larger than the span,

$\text{Depth} \leq 1/10$ chord width.

4. *Semi Central Angle*

The semi central angle should be in the range of 30 to 45 degrees. If the angle is less than 45° , the effect of wind load may be ignored. For larger angles with steep slopes, back forms may be necessary for casting.

5. *Reinforcements in Shell*

The diameter of reinforcements should not exceed 10 mm for 50 mm thick shells and 12 mm for 65 mm thick shell and 16 mm for shells having thickness greater than 65 mm. In the junction zones where the shell is thickened, larger diameter bars are permissible. The spacing of the bars should not be more than five times the thickness of the shell. Minimum clear cover must be 12 mm or the nominal size of the reinforcing bar. Generally a minimum reinforcement of

of 0.15 percent of the gross cross section in the principal direction is recommended for thin shell structures.

6. Shell Joints

In the case of shells of long lengths exceeding 40 m, expansion joints have to be provided. The construction joints are provided along the curved lengths of the shell where the shear forces are minimum. Shells have to be well cured for a minimum period of two weeks before decentering. The end beams and diaphragms have to be cured for 3 to 4 weeks before decentering.

10.5 ANALYSIS OF SHELLS

10.5.1 Membrane Theory

The membrane theory was formulated by Dischinger with the assumptions that the shell is regarded as a perfectly flexible membrane of infinite extent, carrying direct forces in its plane only. Over a limited zone at sufficient distances away from the boundaries, the stresses in the shell slab approach a distribution which is statically determinate and may be found by the membrane theory. This procedure is applicable to shells whose span to radius ratio is less than 0.5.

The equilibrium of the shell element shown in Fig. 10.8 is examined under the given set of direct forces with the following notations.

X —Direction of generatrix

Y —Direction of tangent to directrix at A

Z —Direction of the outward normal at A

T_x , T_y and S are the forces per unit length, R —Radius of the singly curved shell X , Y and Z are the components of external loads per unit area on the element.

$$dy = R \cdot d\phi$$

Equations of Equilibrium

$$\sum X = 0$$

$$\sum Y = 0$$

$$\sum Z = 0$$

(a) Forces in X -direction

$$\frac{\partial T_x}{\partial x} \cdot R \cdot d\phi \cdot dx + \frac{\partial S}{R \cdot \partial \phi} \cdot R \cdot d\phi \cdot dx + X \cdot dx \cdot R \cdot d\phi = 0$$

$$\therefore \frac{\partial T_x}{\partial x} + \frac{\partial S}{R \cdot \partial \phi} + X = 0 \quad (1)$$

(b) Forces in Y -direction

$$\frac{\partial T_y}{R \cdot \partial \phi} \cdot R \cdot d\phi \cdot dx + \frac{\partial S}{\partial x} \cdot dx \cdot R \cdot d\phi + Y \cdot dx \cdot R \cdot d\phi = 0$$

$$\therefore \frac{\partial T_y}{R \cdot \partial \phi} + \frac{\partial S}{\partial x} + Y = 0 \quad (2)$$

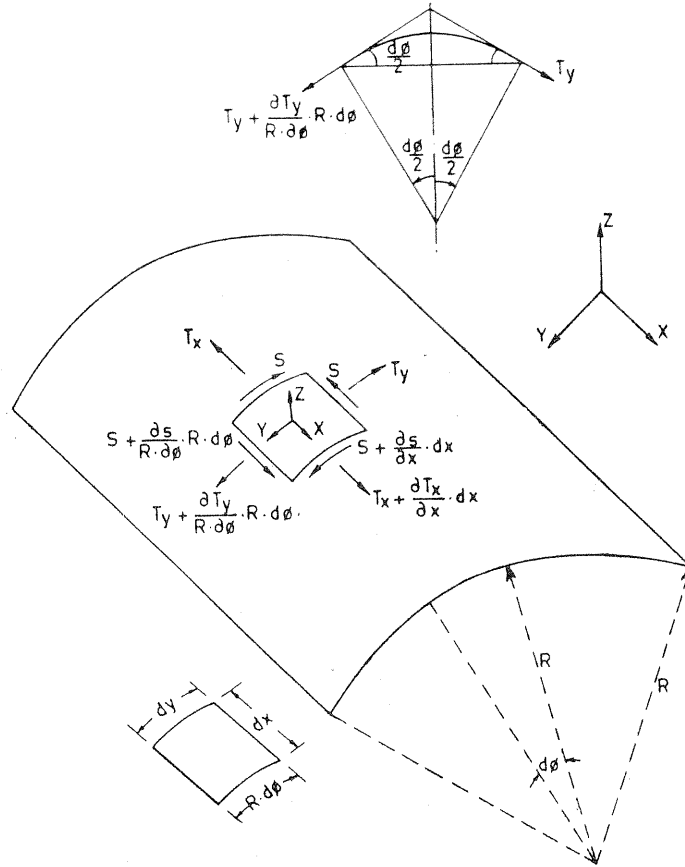


Fig. 10.8 Equilibrium of shell element

(c) Forces in Z-direction

$$2T_y \cdot dx \cdot \sin \frac{d\phi}{2} + \frac{\partial T_y}{R \cdot \partial \phi} R \cdot d\phi \cdot dx \cdot \sin \frac{d\phi}{2} - Z \cdot dx \cdot R \cdot d\phi = 0$$

since $d\phi$ is small $\sin \frac{d\phi}{2} = \frac{d\phi}{2}$

$$\text{and} \quad \frac{\partial T_y}{R \cdot \partial \phi} R \cdot d\phi \cdot dx \cdot \sin \frac{d\phi}{2} = 0$$

$$\therefore T_y - R \cdot Z = 0 \quad (3)$$

The values of T_x and S are to be evaluated by integrating equations (1) and (2)

$$\therefore S = - \int \frac{\partial T_y}{R \cdot \partial \phi} \cdot dx - \int Y \cdot dx + F_1(\phi)$$

$$T_x = - \int \frac{\partial S}{R \cdot \partial \phi} \cdot dx - \int X \cdot dx + F_2(\phi)$$

where $F_1(\phi)$ and $F_2(\phi)$ are functions of ϕ only which have to be determined from boundary conditions.

In most cases in practice X , Y , and Z are functions of ϕ only and do not vary along X . Then T_y is a function of ϕ only and does not depend on X .

Hence

$$\begin{aligned} S &= -x \left(\frac{T_y}{R \cdot \partial \phi} + Y \right) + F_1(\phi) \\ &= -x \cdot K + F_1(\phi) \end{aligned}$$

$$T_x = \frac{x^2}{2} \left(\frac{\partial K}{R \cdot \partial \phi} \right) - \frac{x \cdot \partial [F_1(\phi)]}{R \cdot \partial \phi} - x \cdot X + F_2(\phi)$$

For a shell, simply supported at the end frames and symmetrically loaded, shear $S=0$ at $x=0$

$$\therefore F_1(\phi) = 0$$

The traverses at ends cannot withstand forces normal to their planes. Hence $T_x=0$ at $x=\pm L$ and also in most cases $X=0$

$$\therefore F_2(\phi) = - \left(\frac{L^2}{2} \right) \left(\frac{\partial K}{R \cdot \partial \phi} \right)$$

$$\therefore T_y = ZR$$

$$S = -x \left(\frac{\partial T_y}{R \cdot \partial \phi} + Y \right) = x \cdot K$$

$$T_x = - \left(\frac{L^2 - x^2}{2} \right) \left(\frac{\partial K}{R \cdot \partial \phi} \right)$$

These three equations can be used to calculate membrane stresses for any type of directrix.

Equations for Membrane stresses under various types of loads

(a) *Self Weight* (Refer Fig. 10.9(a))

$$Y = g \sin \phi$$

$$Z = -g \cos \phi$$

$$X = 0$$

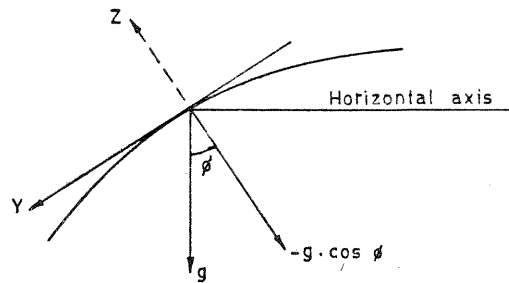
$$\therefore T_y = -g \cdot R \cos \phi$$

Hence

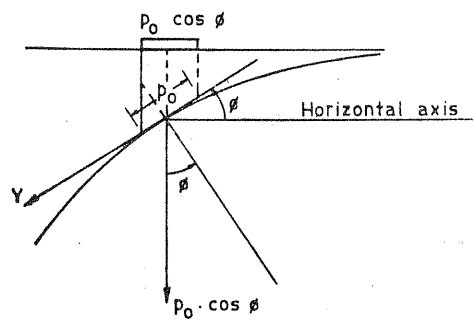
$$K = \left(Y + \frac{\partial T_y}{R \partial \phi} \right)$$

$$= g \cdot \sin \phi + 1/R \left(gR \cdot \sin \phi - g \cdot \cos \phi \frac{dR}{d\phi} \right)$$

$$= 2g \cdot \sin \phi - \left(\frac{g \cdot \cos \phi}{R} \right) \cdot \left(\frac{dR}{d\phi} \right)$$



(a) Shell Weight



(b) Snow Load

Fig. 10.9 Types of loads on membrane

(b) *Snow load* [Refer Fig. 10.9(b)]

Considering the snow load to be uniform per unit length of projection

$$Y = p_0 \cos \phi \cdot \sin \phi$$

$$Z = -p_0 \cos^2 \phi$$

$$X = 0$$

$$\therefore T_y = ZR = -p_0 R \cdot \cos^2 \phi$$

$$K = \left(Y + \frac{\partial T_y}{R \partial \phi} \right)$$

$$= 3p_0 \sin \phi \cos \phi - p_0 \frac{\cos^2 \phi}{R} \left(\frac{dR}{d\phi} \right)$$

Membrane stresses for various directrices

General equation for the family of curves is

$$R = R_0 \cos^n \phi$$

where R_0 = Radius of curvature at crown

R = Radius of curvature at any point

ϕ = Angle made by the tangent to the curve at any point with the horizontal.

n = constant having values of

$n = 0$ for circle

$n = 1$ for cycloid

$n = -2$ for catenary

$n = -3$ for parabola

Substituting the values of R in the membrane equations, we have

$$T_y = -gR \cdot \cos \phi = -g \cdot R_0 \cos^n \phi \cdot \cos \phi = -g \cdot R_0 \cdot \cos^{n+1} \phi$$

$$R = R_0 \cdot \cos^n \phi$$

$$\therefore \frac{dR}{d\phi} = -n \cdot R_0 \cos^{n-1} \phi \cdot \sin \phi$$

$$K = \left(Y + \frac{\partial T_y}{R \partial \phi} \right) = g \cdot \sin \phi + (n+1)g \cdot \sin \phi = (n+2)g \cdot \sin \phi$$

$$\frac{\partial K}{R \cdot \partial \phi} = 1/R(n+2)g \cdot \cos \phi = \left[\frac{(n+2)g}{R_0 \cos^{n-1} \phi} \right]$$

Similary the membrane forces can be calculated. The membrane forces due to self weight (g) and due to snow load ($p_0 \cos \phi$) are compiled in Table 10.1.

TABLE 10.1 Membrane Forces due to self weight and snow loads

Membrane force	Due to Self weight (g)	Due to Snow load ($p_0 \cos \phi$)
T_x	$-\frac{(n+2)g \cdot (L^2 - x^2)}{2R_0 \cos^{n-1} \phi}$	$-\frac{(n+3)p_0(L^2 - x^2)(\cos^2 \phi - \sin^2 \phi)}{2R_0 \cdot \cos^n \phi}$
T_y	$-g \cdot R_0 \cos^{n+1} \phi$	$-p_0 R_0 \cos^{n+2} \phi$
S	$-(n+2)g \cdot x \cdot \sin \phi$	$-(n+3)p_0 x \cdot \sin \phi \cdot \cos \phi$

The membrane forces for circular cylindrical shells ($n = 0$) are compiled in Table 10.2.

10.5.2 Design Example

A reinforced concrete shell having semicircular directrix is freely supported at the ends. Given the data that radius of the shell = $R = 8$ m

TABLE 10.2 Membrane Forces in Circular Cylindrical Shells

Membrane force	Due to Self weight (g)	Due to snow load ($p_0 \cos \phi$)
T_x	$-\frac{g(L^2 - x^2) \cos \phi}{R}$	$-\frac{3p_0(L^2 - x^2)(\cos^2 \phi - \sin^2 \phi)}{2R}$
T_y	$-gR \cdot \cos \phi$	$-p_0R \cdot \cos^2 \phi$
S	$-2g \cdot x \cdot \sin \phi$	$-3p_0 \cdot x \cdot \sin \phi \cdot \cos \phi$

Length of shell = $2L = 36$ m

Thickness of shell = $t = 60$ mm

Calculate the membrane forces at $x = 0, 9$ m and 18 m. and $\phi = 0, 30^\circ, 60^\circ$ and 90° under its own self weight.

Density of concrete = 25 kN/m³

The self weight per unit area of shell = $g = (0.06 \times 25) = 1.5$ kN/m²

For a circle $R = R_0 = 8$ since $n = 0$

The membrane forces are given by the equations:

$$T_x = - \left[\frac{g(L^2 + x^2) \cos \phi}{R} \right] = - \left[\frac{1.5(18^2 - x^2) \cos \phi}{8} \right]$$

$$= -(60.75 - 0.1875x^2) \cos \phi$$

$$T_y = -g \cdot R \cos \phi = -1.5 \times 8 \times \cos \phi = -12 \cos \phi$$

$$S = -2gx \cdot \sin \phi = -2 \times 1.5 \times x \times \sin \phi = -3x \cdot \sin \phi$$

Using these equations, the membrane forces are computed for different values of

$\phi = 0^\circ, 30^\circ, 60^\circ$ and 90° and $x = 0, 9$ m and 18 m.

The values are compiled in Table 10.3.

TABLE 10.3 Membrane forces in semi-circular shell

ϕ	Membrane forces kN/m						(-ve) compression					
	0°			30°			60°			90°		
x	T_x	T_y	S	T_x	T_y	S	T_x	T_y	S	T_x	T_y	S
0	60.75	12.00	0	52.61	10.39	0	30.37	0	6.00	0	0	0
9 m	45.56	12.00	0	39.46	10.39	13.50	22.78	6.00	23.38	0	0	27.00
18 m	0	12.00	0	0	10.39	27.00	0	6.00	46.77	0	0	54.00

Forces T_x and T_y are compressive.

Maximum unit stress occurs at $x = 0$ and $\phi = 0$ i.e. at centre of span and crown.

$$T_x = 60.75 \text{ kN/m}$$

$$\text{Maximum compressive stress} = \left(\frac{60.75 \times 10^3}{1000 \times 60} \right) = 1.01 \text{ N/mm}^2$$

10.5.3 Design Example

A reinforced concrete shell with circular directrix has the following dimensions.

$$\begin{aligned} R &= 6 \text{ m} \\ 2L &= 24 \text{ m} \quad \text{Self weight } g = (0.05 \times 25) = 1.25 \text{ kN/m}^2 \\ t &= 50 \text{ mm} \\ \phi &= 60^\circ \end{aligned}$$

Calculate

- (1) The maximum stress in the shell
- (2) The maximum bending moment and tension developed in the edge beams.

Maximum stress is developed at the crown and centre of span for values of $x=0$ and $\phi=0$

$$\therefore T_x = -\left(\frac{gL^2}{R}\right) = -\left(\frac{1.25 \times 12^2}{6}\right) = -30 \text{ kN/m}$$

$\therefore$ Maximum compressive stress is given by

$$\sigma_{\max} = \left\{ \frac{30 \times 10^3}{1000 \times 50} \right\} = 0.6 \text{ N/mm}^2 \text{ (compression)}$$

Forces acting on edge beam T_y at $\phi = 60^\circ$ is given by

$$T_y = -g \cdot R \cos \phi = -1.25 \times 6 \times \cos 60^\circ = -3.75 \text{ kN/m}$$

$\therefore$ Load acting on the edge beam is 3.75 kN/m over a span of 24 m.

$\therefore$ Maximum bending moment at centre of span of edge beam of length $L=24$ m, is

$$M_{\max} = \left\{ \frac{3.75 \times 24^2}{8} \right\} = 270 \text{ kN}\cdot\text{m}$$

Tension in the edge beam (axial force) at a distance x from the centre is

$$= -\int_x^L S \cdot dx = -\int_x^L -2g \cdot x \sin \phi = g(L^2 - x^2) \sin \phi$$

Maximum tension occurs at $x=0$ (centre of span section)

$$\therefore N_{\max} = gL^2 \sin \phi = (1.25 \times 12^2 \times \sin 60^\circ) = 156 \text{ kN}$$

10.5.4 Design Example

A reinforced concrete shell of circular directrix has the following parameters.

$$\begin{aligned} \phi &= \text{Semicentral angle} = 60^\circ \\ 2L &= \text{span} = 24 \text{ m} \\ R &= \text{Radius} = 6 \text{ m} \\ t &= \text{Thickness} = 50 \text{ mm} \end{aligned}$$

The shell is subjected to the action of snow load only of intensity 1 kN/m^2 per unit length of curved surface of shell.

1. Determine the maximum stresses in the shell.
2. Calculate also the maximum bending moment and tension in edge beams.

Given data :

$$\begin{aligned}\phi &= 60^\circ \\ 2L &= 24 \text{ m} \\ R &= 6 \text{ m} \\ t &= 50 \text{ mm} \\ p_0 &= 1 \text{ kN/m}^2\end{aligned}$$

Membrane forces are given by

$$\begin{aligned}T_x &= -\left[\frac{3p_0(L^2 - x^2)(\cos^2 \phi - \sin^2 \phi)}{2R} \right] = -\left[\frac{3 \times 1(12^2 - x^2) \cos 2\phi}{2 \times 6} \right] \\ &= -(36 - 0.25x^2) \\ T_y &= -p_0 R \cdot \cos^2 \phi = -1 \times 6 \times \cos^2 \phi = -6 \cos^2 \phi \\ S &= -3p_0 x \cdot \sin \phi \cos \phi = -3 \times 1 \times x \times \frac{\sin 2\phi}{2} \\ &= -1.5x \sin 2\phi\end{aligned}$$

Maximum compressive stress occurs at $x=0$ and $\phi=0$

$$\begin{aligned}\therefore T_x &= -36 \text{ kN/m} \\ T_y &= -6 \text{ kN/m} \\ \therefore \sigma_{\max} &= \left(\frac{36 \times 10^3}{1000 \times 50} \right) = 0.72 \text{ N/mm}^2\end{aligned}$$

At the edges, $\phi = 60^\circ$

$$\begin{aligned}\therefore T_y &= -6 \cos^2 60^\circ = -1.5 \text{ kN/m} \\ \therefore \text{B.M. in edge beam} &= \left(\frac{1.5 \times 24^2}{8} \right) = 108 \text{ kN}\cdot\text{m}\end{aligned}$$

Tension in the edge beam at a distance x from the centre is

$$\begin{aligned}&= -\int_x^L S \cdot dx = -\int_x^L -3p_0 \cdot x \cdot \sin \phi \cdot \cos \phi \\ &= 3p_0 \sin \phi \cos \phi \left(\frac{L^2 - x^2}{2} \right)\end{aligned}$$

Maximum tension occurs at $x=0$ and $\phi = 60^\circ$

$$\therefore N_{\max} = \left(3 \times 1 \times \sin 60^\circ \times \cos 60^\circ \times \frac{12^2}{2} \right) = 94 \text{ kN}$$

10.5.5 Beam Theory

In the beam theory developed by Lundgren the shell is analysed as a beam of curved cross section spanning between the end frames or traverses. In the case of long shells, the longitudinal force components are predominant and hence the beam theory is ideally suited for the analysis. The beam theory is generally applicable for cylindrical shells of (L/R) ratio exceeding the value of π . In the beam theory the cross section of the shell is assumed with or without edge beams and the sectional properties are determined and the stresses are computed using the beam theory. An arch analysis is also conducted to determine the transverse moments and thrusts so that suitable reinforcements are designed.

Referring to Fig. 10.10

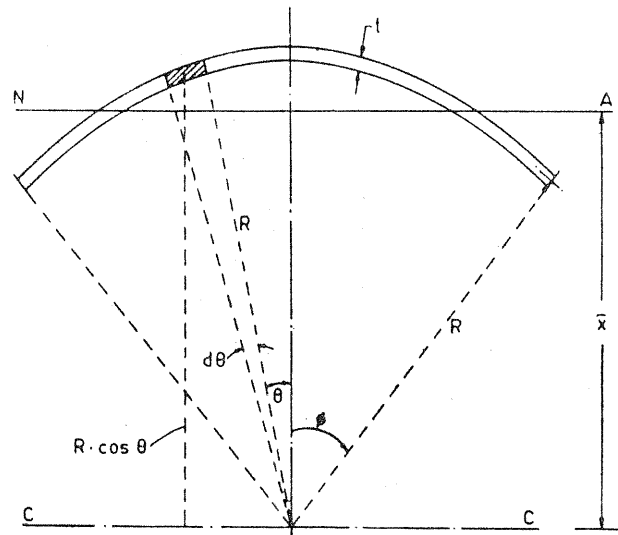


Fig. 10.10 Shell-properties

$$\bar{x} = \frac{\sum A \bar{x}}{\sum A} = \frac{2 \int_0^{\pi} R \cdot d\theta \cdot t \cdot R \cos \theta}{2 \int_0^{\pi} R \cdot d\theta \cdot t}$$

where R = Radius of the shell and
 t = thickness of the shell

$$\therefore \bar{x} = \frac{\int_0^{\pi} R^2 \cdot d\theta \cdot t \cdot \cos \theta}{\int_0^{\pi} R \cdot d\theta \cdot t}$$

Second Moment of area

$$I_{CC} = 2 \int_0^{\phi} R \cdot d\theta \cdot t \cdot (R \cdot \cos \theta)^2 = 2R^3 t \int_0^{\phi} \frac{(1 + \cos 2\theta)}{2} d\theta$$

$$= 2R^3 t \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\phi}$$

$$\text{Area of segment of shell} = \sum A = 2 \int_0^{\phi} R \cdot d\theta \cdot t$$

$$\therefore I_{NA} = [I_{CC} - A \cdot \bar{x}^2]$$

$$I_{NA} = 2R^3 t \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\phi} - \left(2 \int_0^{\phi} R \cdot d\theta \cdot t \right) \bar{x}^2$$

The use of these equations are illustrated with the help of the following example.

10.5.6 Design Example

A reinforced concrete circular shell has the following particulars:

Radius = $R = 6$ m

Span = $2L = 24$ m

Semicentral angle = $\phi = 60^\circ$

Thickness = $t = 50$ mm

Calculate the maximum stress due to self weight only in the shell by beam theory and compare the values with the results of membrane theory.

Rise of shell = $(R - R \cos 60^\circ) = 6(1 - \cos 60^\circ) = 3$ m

$$\text{Area of segment} = 2 \int_0^{\pi/3} (6 \cdot d\theta)(0.05)$$

$$\therefore \sum A = (2 \times \pi/3 \times 6 \times 0.05) = 0.628 \text{ m}^2$$

$$\bar{x} = \frac{2 \int_0^{\pi/3} R^2 \cdot d\theta \cdot t \cdot \cos \theta}{2 \int_0^{\pi/3} R \cdot d\theta \cdot t} = \left[\frac{2 \times 6^2 \times 0.05 \times \sin 60^\circ}{0.628} \right] = 4.97 \text{ m}$$

$$I_{CC} = 2R^3 \cdot t \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\phi} = 2 \times 6^3 \times 0.05 \left[\frac{\pi}{6} + \frac{\sin 120}{4} \right] = 15.99 \text{ m}^4$$

$$\therefore I_{NA} = [I_{CC} - A \cdot \bar{x}^2] = [15.99 - (0.628)(4.97)^2] = 0.50 \text{ m}^4$$

$$\text{Weight/metre run of shell} = (0.628 \times 25) = 15.71 \text{ kN/m}$$

$$\therefore \text{Maximum B.M.} = \left(\frac{15.71 \times 24^2}{8} \right) = 1131 \text{ kN}\cdot\text{m}$$

Extreme fibre stress at crown

$$\sigma_c = \left[\frac{1131 \times 10^6 \times 1.03 \times 10^3}{0.50 \times 10^{12}} \right] = 2.33 \text{ N/mm}^2$$

The value of compressive stress σ_c obtained by membrane theory in design Example 10.5.3 is 0.6 N/mm². This indicates that when membrane theory is applied for long shells, the stresses are under estimated.

10.5.7 Design Example

A circular cylindrical shell with edge beams has the following particulars:

Radius of the shell (R)	= 10.10 m
Central rise	= 2.35 m
Chord width	= 13.00 m
Span	= 30.00 m
Thickness of shell	= 80 mm
Semi-central angle	= 40°
Edge beam size	= 200 mm by 1880 mm
Reinforcements in edge beam	= 16 bars of 32 diameter.
Width of edge beams	= 200 mm
Modular ratio	= 13
Effective cover	= 300 mm

Analyse the shell for stress in concrete and steel if the live load on the shell is 1 kN/m².

The shell with edge beams is shown in Fig. 10.11.

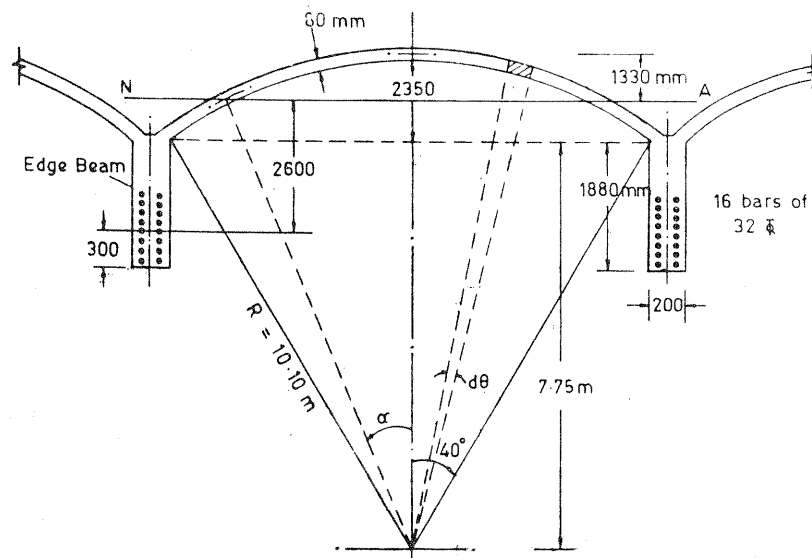


Fig. 10.11 Cylindrical shell with edge beams

Let the neutral axis cut the shell at an angle α . Taking moments of effective areas about neutral axis, we have

$$2 \int_0^\alpha R \cdot d\theta \cdot t (R \cos \theta - R \cos \alpha) = m \cdot 2A_t(1.58 + R \cdot \cos \alpha - 7.75)$$

Putting $m = 13$

$$R = 10.10$$

$$A_t = (8 \times 8.04) = 64.32 \text{ cm}^2 = 0.0064 \text{ m}^2$$

$$R^2 t (\sin \alpha - \alpha \cos \alpha) = m \cdot A_t (R \cdot \cos \alpha - 6.17)$$

$$10.10^2 \times 0.08 (\sin \alpha - \alpha \cos \alpha) = 13 \times 0.0064 (10.10 \cos \alpha - 6.17)$$

$$(\sin \alpha + 0.063) = \cos \alpha (\alpha + 0.103)$$

$$\alpha = 26^\circ - 15'$$

$$I_{NA} = 2 \int_0^\alpha R \cdot d\theta \cdot t \cdot R^2 (\cos \theta - \cos \alpha)^2$$

$$+ m \cdot 2 \cdot A_t (1.58 + R \cos \alpha - 7.75)^2 = 1.835 \text{ m}^4$$

$$\text{Self weight of shell} = (0.08 \times 25) = 1.92 \text{ kN/m}^2$$

$$\text{Water proofing and live load} = 1.00 \text{ kN/m}^2$$

$$\text{Total load} = \underline{2.92 \text{ kN/m}^2}$$

$$\therefore \text{Total weight per meter run} = 2 \int_0^{40} 2.92 (R \cdot d\theta) = 40 \text{ kN/m}$$

$$\text{Weight of edge beam} = 2(0.1 \times 1.88 \times 24) = 9 \text{ kN/m}$$

$$\text{Weight of filling in the vally portion} = 1.5 \text{ kN/m}$$

$$\therefore \text{Total load} = (40 + 10.50) = 50.5 \text{ kN/m}$$

$$\text{Maximum bending moment} = \left(\frac{50.5 \times 30^2}{8} \right) = 5700 \text{ kN} \cdot \text{m}$$

$$\text{Maximum shear force} = (0.5 \times 50.5 \times 30) = 757.5 \text{ kN}$$

Maximum compressive stress at crown

$$\sigma_c = \left(\frac{5700 \times 10^6 \times 1330}{1.835 \times 10^{12}} \right) = 3.52 \text{ N/mm}^2$$

Maximum tensile stress at centre of gravity of steel is given by

$$\sigma_t = \left(\frac{3.52 \times 2600 \times 23}{1330} \right) = 89.5 \text{ N/mm}^2$$

$$\text{Shear stress} \quad \tau = \frac{V}{I \cdot b} (A \bar{y})$$

$$A \cdot \bar{y} = (16 \times 804 \times 13 \times 2600) = 435 \times 10^6 \text{ mm}^3$$

$\therefore$ Maximum horizontal shear stress at neutral axis

$$\tau_v = \left(\frac{757.5 \times 10^3 \times 435 \times 10^6}{1.835 \times 10^{12} \times 2 \times 80} \right) = 1.12 \text{ N/mm}^2$$

Using 8 mm diameter bars inclined at 45° to the longitudinal axis of the shell, spacing is given by

$$s_v = \left(\frac{\sqrt{2} \times 50 \times 230}{80 \times 1.12} \right) = 180 \text{ mm near supports}$$

Towards the centre of span where shear stress is less, adopt 6 mm diameter bars at 250 mm centres.

10.5.8 Arch Analysis

Transverse moments M_s and thrust N_s are obtained by treating the shell as an arch fixed between the longitudinal beams. The given shell arch is divided into a suitable number of parts at 5 to 6 degree intervals as shown in Fig. 10.12(a). In the present example, the shell arch is divided into 16 parts at 5° intervals.

$$\text{Area of each element} = (t \cdot R \cdot d\theta) = \left(\frac{0.08 \times 10.1 \times 5 \times 2\pi}{360} \right) = 0.0705 \text{ m}^2$$

$$\text{Area of edge beam at each end} = (1.58 \times 0.10) = 0.158 \text{ m}^2$$

Specific shears acting on the different elements determined by shear analysis. Shear stress at each element is given by

$$q = \left(\frac{\partial F}{\partial t \cdot I} \right) A \cdot \bar{Z}$$

In this example :

$$\partial F = \text{Load on one metre length of shell}$$

$$= 50.5 \text{ kN}$$

$$t = 80 \text{ mm}$$

$$I = 183.5 \times 10^{12} \text{ mm}^4 = 1.835 \text{ m}^4$$

$$A \cdot \bar{Z} = 2 \int_0^\theta (r \cdot d\theta) t \cdot Z$$

$$\text{where } Z = R (\cos \theta - \cos 26^\circ - 15')$$

$$\therefore A \bar{Z} = 2 \int_0^\theta R \cdot d\theta \cdot t \cdot R (\cos \theta - \cos 26^\circ - 15')$$

$$= 2R^2 \cdot t (\cos \theta - 0.8970\theta)$$

$$\therefore q = \left(\frac{50.5}{2 \times 0.08 \times 1.835} \right) (2 \times 10.1^2 \times 0.08) (\sin \theta - 0.8970\theta)$$

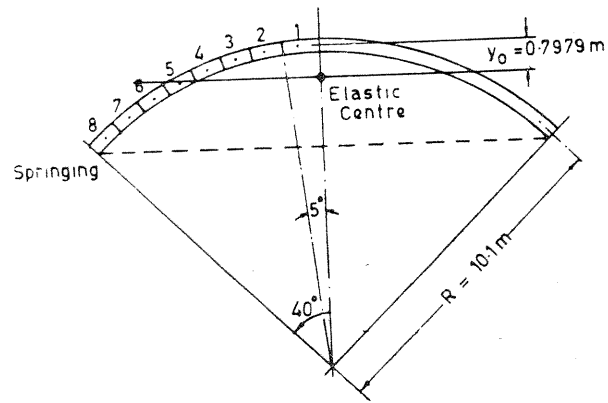
$$q = 2800 (\sin \theta - 0.8970\theta)$$

Using this equation, shear stress at each section is calculated

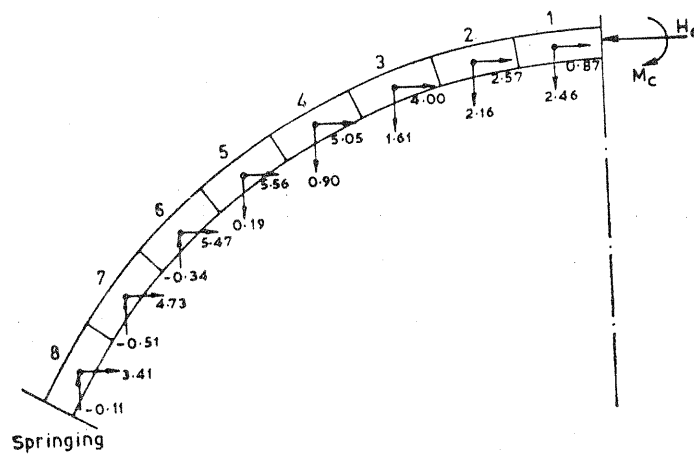
The computations are shown in Table 10.4.

The specific shear force is given by

$$T = (q \times A)$$



(a) Elastic - Centre



(b) Forces acting on Elements

Fig. 10.12 Arch-analysis of shell

where A = Area of each element

Net vertical force $= V = (W - T \cdot \sin \theta)$

Net horizontal force $= H = T \cos \theta$

The thrusts at the various sections are resolved into vertical and horizontal components. The vertical component with the dead load at each section gives the net vertical force at each section.

W = self weight of each element

TABLE 10.4 Specific Shear at Different Sections

1	2	3	4	5	6	7	8	9	10	11	
Section	θ		$\sin \theta$	0.8970θ	$q=2800$ [3-4] (kN/m ²)	$T=(q \times A)$ ($q \times 0.0705$) (kN)	W (kN)	$T \cdot \sin \theta$ (kN)	$\cos \theta$	$H=T \cdot \cos \theta$ (kN)	$V=(W-T \cdot \sin \theta)$ (kN)
	Degree	Radians									
1	2.5	0.0436	0.0436	0.03910	12.60	0.88	2.5	0.0383	0.9990	0.8791	2.4617
2	7.5	0.1308	0.1300	0.1173	35.56	2.60	2.5	0.3380	0.9914	2.5776	2.1620
3	12.5	.02181	0.2164	0.1956	58.24	4.10	2.5	0.8872	0.9763	4.0028	1.6128
4	17.5	0.3054	0.3007	0.2739	75.04	5.30	2.5	1.5930	0.9537	5.0546	0.9070
5	22.5	0.3926	0.3826	0.3521	85.40	6.02	2.5	2.3032	0.9239	5.5618	0.1968
6	27.5	0.4800	0.4617	0.4305	87.64	6.17	2.5	2.8486	0.8870	5.4727	-0.3486
7	32.5	0.5672	0.5372	0.5087	79.80	5.62	2.5	3.0190	0.8433	4.7393	-0.5190
8	37.5	0.6544	0.6087	0.5869	61.14	4.30	2.5	2.6174	0.7933	3.4111	-0.1174

Total weight of shell per metre run = 40 kN

$$W = \left(\frac{40}{16}\right) = 2.5 \text{ kN}$$

The force acting on the elements is shown in Fig. 10.12(b). The elastic centre of the arch shell is determined as shown in Table 10.5.

TABLE 10.5 Determination of Elastic Centre

Section	θ	$\sin \theta$	$\cos \theta$	$(1 - \cos \theta)$	$x = R \sin \theta$ (m)	$y = R(1 - \cos \theta)$ (m)
1	2.5	0.0436	0.9990	0.0010	0.4403	0.0101
2	7.5	0.1300	0.9914	0.0086	1.3130	0.0868
3	12.5	0.2164	0.9763	0.0237	2.1856	0.2393
4	17.5	0.3007	0.9537	0.0463	3.0370	0.4676
5	22.5	0.3826	0.9239	0.0761	3.8642	0.7686
6	27.5	0.4617	0.8870	0.1130	4.6631	1.1413
7	32.5	0.5372	0.8433	0.1567	5.4257	1.5826
8	37.5	0.6087	0.7933	0.2067	6.1478	2.0876
						$\Sigma y = 6.3839$
springing	40.0	0.6427	0.7660	0.2340	6.4912	2.36

$$\text{Elastic centre} = y_0 = \left(\frac{\Sigma y}{N}\right) = \left(\frac{6.3839}{8}\right) = 0.7979 \text{ m}$$

If M_c = Moment at crown

H_c = Thrust at crown

m = Cantilever moment about the point due to applied loads V and H between the point and the crowns.

y_0 = position of the elastic centre from crown

y_1 = Vertical distance of the point from the elastic centre

Then the horizontal thrust at crown is given by

$$H_c = \left(\frac{\Sigma m y_1}{\Sigma y^2}\right)$$

and the moment at crown M_c is given by

$$M_c = \left(\frac{\Sigma m}{N} - H_c \cdot y_c\right)$$

where N = Number elements

The net moment M at any section is $M = M_c + H_c \cdot y - m$

The net thrust at any section is $P = \Sigma V \cdot \sin \theta + (H_c - \Sigma H) \cos \theta$

The computations of moment at various sections is shown in Table 10.6.

Maximum transverse moment of -3.102 kNm, occurs at springing. In general the magnitude of transverse moments are small in long shells. Using M-20 grade concrete and Fe-415 grade—tor steel. The effective depth of shell is given by

TABLE 10.6 Computation of

1	2	3	4	5	6	7	8	9	10
Section	x (m)	Δx	V (kN)	ΣV	m_v $= \Delta x \cdot \Sigma v$	y	Δy	H (kN)	ΣH
Crown	0	0	0	—	—	—	—	—	—
1	0.44	0	2.46	—	—	0.010	—	0.87	—
2	1.31	0.87	2.16	2.46	2.14	0.086	0.076	2.57	0.87
3	2.18	0.87	1.61	4.62	4.01	0.239	0.153	4.00	3.44
4	3.03	0.85	0.90	6.23	5.29	0.467	0.228	5.05	7.44
5	3.86	0.83	0.19	7.13	5.91	0.768	0.301	5.56	12.49
6	4.66	0.80	-0.34	7.32	5.85	1.141	0.373	5.47	18.05
7	5.42	0.76	-0.51	6.98	5.30	1.582	0.441	4.73	23.52
8	6.14	0.72	-0.11	6.47	4.65	2.087	0.505	3.41	28.25
Springing	6.49	0.35	—	6.36	2.23	2.360	0.273	—	31.66

$$H_C = \frac{\Sigma my_1}{\Sigma y_1^2} = \frac{133.21}{3.94} = 33.80 \text{ kN}$$

Transverse Moments and Thrusts

11	12	13	14	15	16	17	18
m_H $= \Delta y \cdot \Sigma H$	m_V $+ m_H$	m $= \Sigma(m_V$ $+ m_H)$	$y_1 = y -$ $y_0 = y -$ 0.798	my_1	y_1^2	Net Moment $M = Mc +$ $Hcy - m$ (kN·m)	Net Thrust $= \Sigma V \sin \theta$ $+ (H_c - H)$ $\cos \theta$ (kN)
—	—	—	—	—	—	-1.430	33.80
—	—	—	-0.788	—	0.620	-1.092	—
0.066	2.206	2.206	-0.712	-1.570	0.506	-0.729	—
0.526	4.536	6.742	-0.559	-3.768	0.312	-0.093	—
1.696	6.986	13.728	-0.331	-4.543	0.109	+0.626	—
3.759	9.669	23.397	-0.030	-0.701	0.0009	+1.131	—
6.732	12.582	35.979	0.343	12.340	0.177	+1.157	—
10.372	15.672	51.651	0.784	40.494	0.614	+0.390	—
14.266	18.916	70.567	1.289	90.960	1.661	-1.456	—
$\Sigma m = 204.27$			$\Sigma my_1 = 133.21$		$\Sigma y_1^2 = 3.94$		
8.643	10.873	81.440	—	—	—	-3.102	5.726

$$M_C \left[\frac{\Sigma m}{N} - H_C \cdot Y_0 \right] = \left[\frac{204.27}{8} - 33.8 \times 0.798 \right] = -1.43 \text{ kN} \cdot \text{m}$$

$$d = \sqrt{\frac{3.102 \times 10^6}{0.897 \times 10^3}} = 58.80 \text{ mm}$$

Thickness of shell adopted = 80 mm

Using an effective thickness of 60 mm

$$A_{st} = \left(\frac{3.102 \times 10^6}{230 \times 0.906 \times 60} \right) = 248 \text{ mm}^2$$

Adopt 8 mm Φ at 200 mm centres in the transverse direction. The details of reinforcements in the shell are shown in Fig. 10.13.

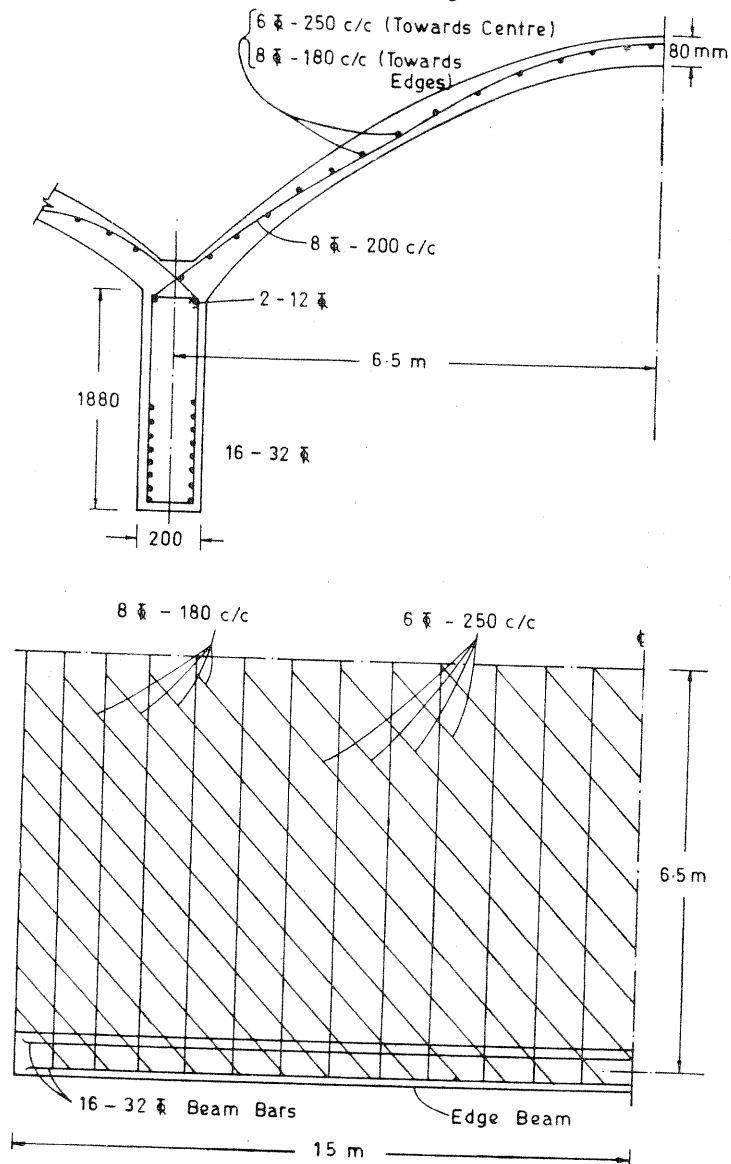


Fig. 10.13 Reinforcements in shell

10.5.9 Rigorous Theories of Analysis

The rigorous mathematical analysis of thin curved shells is governed by a differential equation of eighth order involving the main shell parameters and deformations. Solutions based on simplifying approximations have been developed by several investigators and a comparative analysis of various analytical approximations is reported by McNamee. It is pertinent to note that an exact mathematical solution of the eighth order differential equation, would by itself be no more than an approximate solution due to the inelastic properties of concrete.

The Donnel-Karman-Jenkins theory is considered to be the simplest among the rigorous theories which is applicable for both long and short shells. The A.S.C.E. method associated with the design tables is immensely useful in the analysis and design of cylindrical shells with any ratio of width to length. The Schorer-Tottenham formulation is by far the simplest among the various theories and sufficiently accurate for purposes of design in the range of medium to long shells. The rigorous methods generally involve lengthy computations to estimate the redundant reactions between the shell and edge beam.

A.S.C.E manual 31 presents design coefficients of forces and deformations of the shell for different types of load conditions. These coefficients are useful in the design of cylindrical shells.

10.6 EXAMPLES FOR PRACTICE

1. A reinforced concrete shell with a circular directrix has the following dimensions.

Distance between the traverses = 30 m

Radius of the shell = 8 m

Thickness of shell = 60 mm

Semi-central angle = 60°

If the water proofing course and occasional live load is 1 kN/m^2 of roof surface calculate.

- (a) The maximum compressive stress in the shell
- (b) Maximum bending moment and tension in the longitudinal edge of the shell.

2. A reinforced concrete shell having a semicircular directrix is freely supported between the tranverses separated by a distance of 35 m. If the radius and thickness of shell are 10 m and 60 mm respectively, calculate the membrane forces at the crown and edges due to its own self weight and a snow load of 1 kN/m^2 . Also calculate the maximum compressive stress in concrete and the maximum tension in the edge members.

3. A circular cylindrical reinforced concrete shell spanning 25 m, has a radius of 8 m, with the central angle being 90 degrees. The thickness of the shell is 70 mm and the density of concrete may be assumed as 24 kN/m^3 . If the weight of water proofing and occasional live load on the shell may be taken 1.2 kN/m^2 of the surface of shell, calculate the maximum compressive stress in concrete at the crown of the shell using beam theory.

4. A cylindrical shell roof consists of ten parallel shells, each shell having a chord width of 10 m and a span of 25 m. The central angle is 80° and the thickness of the shell is 70 mm. The intermediate edge beams are 180 mm thick and 1.5 m in depth. The beams are reinforced with 20 bars of 28 mm diameter with their effective depth 30 cm from the soffit of the beams. The super imposed load due to water proofing cover and occasional live load may be taken as equivalent to 1.2 kN/m^2 of the surface of the shell. If M-20 grade concrete is used with modular ratio = 13, calculate the maximum stresses in concrete and steel and also the maximum shear stress. Design suitable shear reinforcements. Also determine the transverse moments by arch analysis and design suitable reinforcements in the shell.

11

Hyperbolic Paraboloid Shells

11.1 GENERAL FEATURES

Hyperbolic paraboloid generally known as hypar shell is a doubly curved anticlastic shell of translation. Hypar shell is widely used for a variety of structures like exhibition halls, swimming pools, cycle stands in modern monumental and industrial architecture. The main advantage of the hypar shell is that its surface can be generated by two systems of straight lines so that the form work is provided by straight boards, warped only slightly over their length. An added advantage of the hypar shell being that it can be used for roofs as well as foundations where the safe bearing capacity of the soil is very low.

11.2 GEOMETRY OF HYPAR SHELL

The generation of hypar shell surface is shown with the aid of Fig. 11.1. Consider a plane rectangle $OXBY$. If B is now elevated by an amount $BB' = h$, the warped surface $OXB'Y$ is an hyperbolic paraboloid. Each pair of opposite sides are divided into equal parts and the corresponding points joined by straight lines to obtain the hypar surface.

The surface between X and Y is parabolic (convex parabola) and is well suited to resist compressive forces similar to that of arch elements. The surface between O and B' is an inverted parabola (concave parabola or catenary) and is well suited to take tensile forces much similar to that of a catenary.

The hypar surface combining these shapes has a great stiffness and resistance to buckling and except for secondary bending effects, normal applied

- (a) Central supporting column
- (b) Edge beams
- (c) Sloping ribs connecting the column
- (d) Shell surface.

The edge beams FE , ED , DA , AC , CB , BH , HG and GF are subjected to tensile force. The tensile forces are zero at the corners D , F , H and C and maximum at A , E , G and B .

The sloping ribs GO , EO , AO and BO are subjected to compression, varying from zero at the edges A , E , G and B to a maximum at the junction O .

11.5 DESIGN EXAMPLE

Design a hypar shell roof of the inverted umbrella type to suit the following data:

Area to be covered in plan = 12 m by 12 m

M-20 grade concrete and Fe-415 grade tor steel.

Sketch the details of reinforcements in the shell and edge beams Referring to Fig. 11.3

$$\text{Central dip} = \left(\frac{a}{5}\right) = \left(\frac{6}{5}\right) = 1.2 \text{ m}$$

Let the thickness of shell = $t = 70 \text{ mm}$

Loads

$$\text{Dead load} = (0.7 \times 24) = 1.68 \text{ kN/m}^2$$

$$\text{Live load} = 0.50$$

Load due to edge beams

$$\text{water proofing etc.} = 0.22$$

$$\text{Total load} = w = 2.40 \text{ kN/m}^2$$

$$\therefore w = 2.40 \text{ kN/m}^2$$

$$a = b = 6 \text{ m}$$

$$h = 1.2 \text{ m}$$

$$\text{Membrane shear force} = \left(\frac{wab}{2h}\right) = \left(\frac{2.4 \times 6 \times 6}{2 \times 1.2}\right) = 36 \text{ kN/m}$$

$$\text{shear stress} = \left(\frac{36 \times 10^3}{1000 \times 70}\right) = 0.51 \text{ N/mm}^2$$

$$\therefore \text{Principal stress} = 0.51 \text{ N/mm}^2$$

Using nominal reinforcement for temperature and shrinkage at 0.2% of gross cross section,

$$A_{st} = \left(\frac{0.2}{100} \times 1000 \times 70\right) = 140 \text{ mm}^2/\text{m}$$

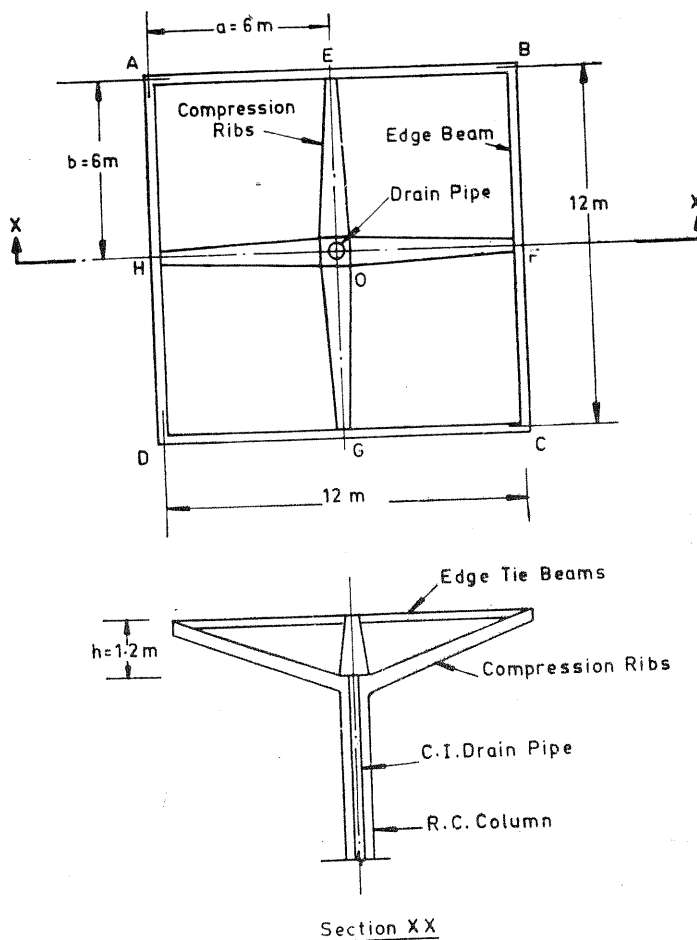


Fig. 11.3 Inverted umbrella roof

But $A_{st} = \left(\frac{36 \times 10^3}{230} \right) = 157 \text{ mm}^2/\text{m}$

Using 8 mm Φ at 300 mm centres both ways. $A_{st} \text{ provided} = 167 \text{ mm}^2$

$\therefore \text{ Tensile stress} = \left[\frac{36 \times 10^3}{(1000 \times 70) + (13 - 1)167} \right] = 0.50 \text{ N/mm}^2$
 $< 2.8 \text{ N/mm}^2$

Sloping Compression Ribs

Length of member $OF = OE = OH = OG = \sqrt{6^2 + 1.2^2} = 6.12 \text{ m}$

Maximum compression in sloping rib at $O = (2 \times 36 \times 6.12) = 441 \text{ kN}$

Length of compression member = 6.12 m

Assuming least lateral dimension of $(70 + 130) = 200 \text{ mm}$, $b = 0.2 \text{ m}$

Since $\left(\frac{L_{ef}}{b} \right) = \left(\frac{6.12}{0.2} \right) = 30.6 > 12$

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Hence the compression member should be designed as long column.

$$\text{Reduction coefficient } C_r = \left[1.25 - \frac{L_{ef}}{48b} \right] = \left[1.25 - \frac{30.6}{48} \right] = 0.612$$

∴ Using 1% steel in the compression member

$$P = (\sigma_{cc} A_c + \sigma_{sc} \cdot A_{sc})$$

$$441 \times 10^3 = 0.612 [5 \times 0.99 A_c + 190 \times 0.01 A_c]$$

$$\therefore A_c = 105,195 \text{ mm}^2$$

$$\therefore \text{Width of beam} = \left(\frac{105,195}{200} \right) = 525 \text{ mm}$$

Provide 600 mm wide by 200 mm deep compression ribs

$$A_{st} = (0.01 \times 600 \times 200) = 1200 \text{ mm}^2$$

Provide 4 bars of 20 mm Φ ($A_{st} = 1256 \text{ mm}^2$), as main steel and nominal binders of 6 mm Φ at 200 mm centres.

Edge beams (Tension members)

The tension members are *AE*, *EB*, *BF*, *FC*, *CG*, *GD*, *DH* and *HA*.

Maximum tension develops at the centre of the beam (At point *E*, *H*, *G* and *F*).

$$\text{Maximum tension} = (36 \times 6) = 216 \text{ kN}$$

$$\therefore A_{st} = \left(\frac{216 \times 10^3}{230} \right) = 939 \text{ mm}^2$$

Using 4 bars of 20 mm, $A_{st} = 1256 \text{ mm}^2$

For M-20 grade concrete, permissible tensile stress = 2.8 N/mm²

If A_c = Area of concrete

$$\left[\frac{216 \times 10^3}{A_c + (13 - 1)1256} \right] = 2.8$$

$$\therefore A_c = 62,071 \text{ mm}^2$$

Adopting a depth of 200 mm,

$$\text{Width of beam} = \left(\frac{62,071}{200} \right) = 310 \text{ mm}$$

Adopt edge beams of size 320 mm by 200 mm. The details of reinforcements in the hypar shell is shown in Fig. 11.4.

11.6 DESIGN EXAMPLE

Design a hypar shell roof of tilted inverted umbrella type to cover an area of 24.4 m by 24.4 m. The edges *AHD* and *BFC* are 3.66 and 2.44 m respectively.

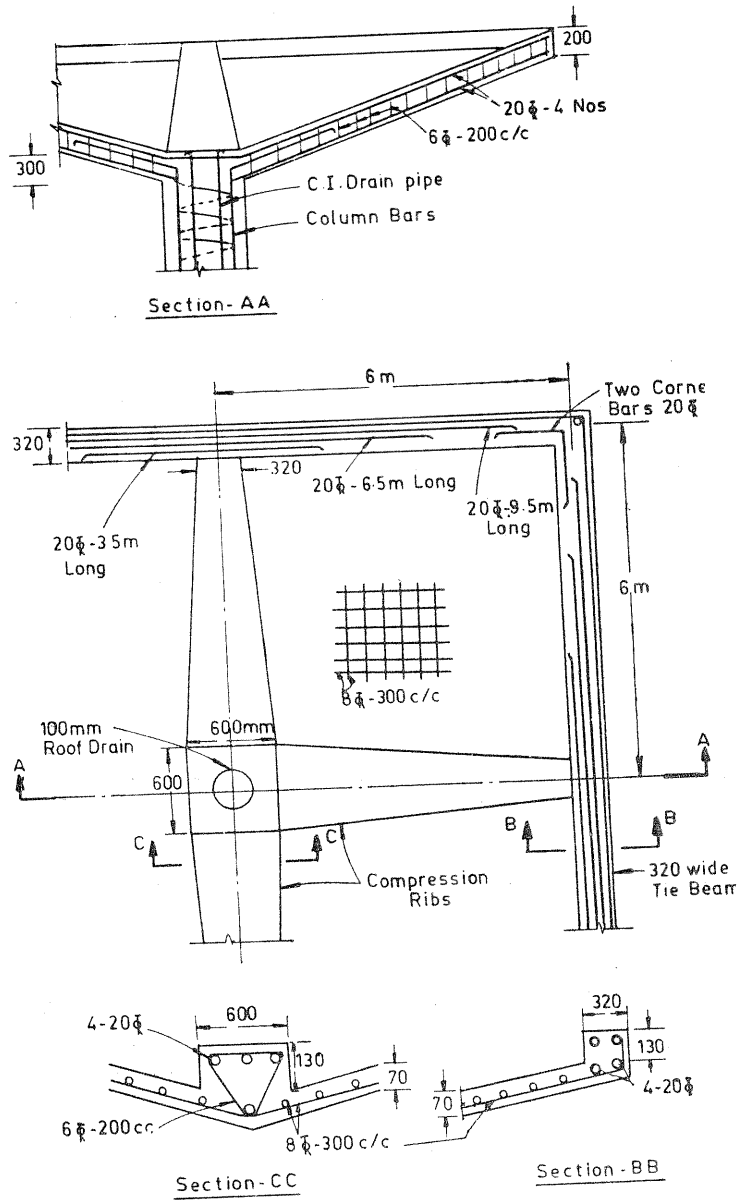


Fig. 11.4 Reinforcement details in hypar shell

vely above the central valley point O as shown in Fig. 11.5. Adopting M-20 grade concrete and Fe-415 tor steel design the various structural components of the hypar shell roof.

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1. Data

$$\begin{aligned} a &= 12.2 \text{ m} \\ b &= 12.2 \text{ m} \\ h_{\min} &= 2.44 \text{ m} \\ h_{\max} &= 3.66 \text{ m} \end{aligned} \quad \begin{array}{l} \text{M-20 grade concrete} \\ \text{Fe-415 grade tor steel} \end{array}$$

2. Shell Surface

Assume thickness of shell $t = 80 \text{ mm}$

Self weight of shell $= (0.08 \times 24) = 1.92 \text{ kN/m}^2$

Live load $= 0.30$

Water proofing, weight of ribs etc. $= 0.20$

$$w = 2.42 \text{ kN/m}^2$$

$$\text{Maximum shear force} = \left(\frac{wab}{2h_{\min}} \right) = \left(\frac{2.42 \times 12.2 \times 12.2}{2 \times 2.44} \right) = 72 \text{ kN/m}$$

$$\therefore \text{Maximum shear stress} = \left(\frac{72 \times 10^3}{1000 \times 80} \right) = 0.9 \text{ N/mm}^2$$

$$\therefore \text{Principal stress} = 0.9 \text{ N/mm}^2$$

Using nominal reinforcement for temperature and shrinkage of 0.2% of gross area

$$A_{st} = \left(\frac{0.2}{100} \times 1000 \times 80 \right) = 160 \text{ mm}^2/\text{m}$$

$$\text{But } A_{st} = \left(\frac{72 \times 10^3}{230} \right) = 313 \text{ mm}^2/\text{m}$$

Provide 10Φ at 250 mm centres both ways

3. Sloping Compression Ribs

The lengths of the various sloping ribs are computed as shown in Fig. 11.5.

$$OH = \sqrt{3.66^2 + 12.2^2} = 12.7 \text{ m}$$

$$OF = \sqrt{2.44^2 + 12.2^2} = 12.45 \text{ m}$$

$$OG = \sqrt{3.05^2 + 12.2^2} = 12.6 \text{ m}$$

$$GD = \sqrt{12.2^2 + 0.61^2} = 12.25 \text{ m}$$

Shear force for left half of structure

$$= \left(\frac{wab}{2h_{\max}} \right) = \left(\frac{2.42 \times 12.2 \times 12.2}{2 \times 3.66} \right) = 49 \text{ kN/m}$$

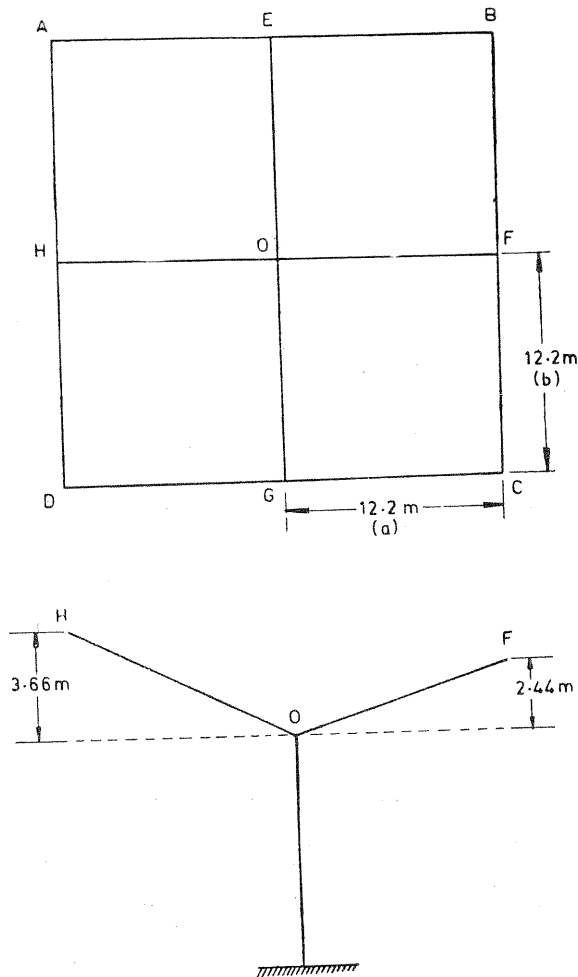


Fig. 11.5 Tilted inverted umbrella type roof

shear force in the right half panel = 72 kN/m

$$\therefore \text{Average shear force at junction} = \left(\frac{2.42 \times 12.2 \times 12.2}{2 \times 3.05} \right) \\ = 59.2 \text{ kN/m}$$

4. Forces in Compression and Tension Members

$$\left. \begin{aligned} OH &= 2(12.7 \times 49) = 1244 \text{ kN} \\ OF &= 2(12.45 \times 72) = 1790 \text{ kN} \\ OG &= 2(12.60 \times 59.2) = 1498 \text{ kN} \end{aligned} \right\} \text{Compression}$$

$$\left. \begin{aligned} GD &= (12.25 \times 72) = 880 \text{ kN} \\ GC &= (12.25 \times 49) = 600 \text{ kN} \\ AH &= (12.20 \times 72) = 878 \text{ kN} \\ BF &= (12.20 \times 49) = 597 \text{ kN} \end{aligned} \right\} \text{ Tension}$$

5. The tension and compression members are designed for the forces according to the method explained in design Example 11.5.

11.7 TYPES OF HYPERBOLIC PARABOLOID ROOFS

- (a) The hipped type roof consisting of 4 units of hyper shells as shown in Fig. 11.6, are ideally suited for covering large rectangular grids for market halls or storage sheds. In this type of roof the edge members BE and BF are in compression. The supports are located at A , B , C and D . The members OE , OF , OG and OH are subjected to tension.

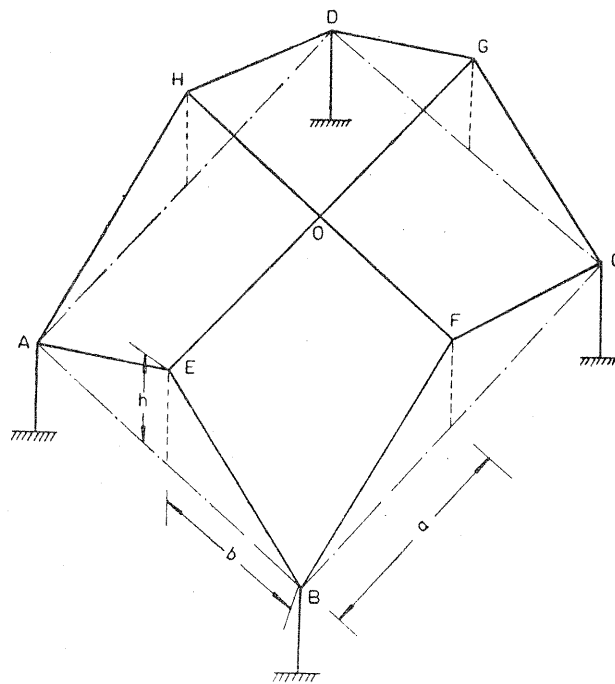


Fig. 11.6 Hipped type roof

- (b) The turkey shed shown in Fig. 11.7 is a combination of four basic hyper units with the supports located at the mid points of the sides.

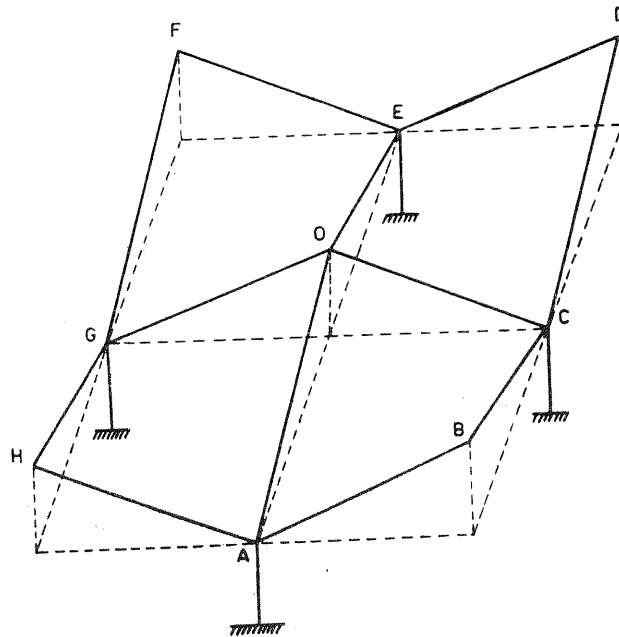


Fig. 11.7 Turkey shed using hypar units

In this type of roof the edge members are in tension and the interior members OA , OE , OC and OG are in compression. Ties are required along AE and GC to take up the thrust from the arch elements AOE and GOC .

- (c) Hypar shell roofs formed by depressing the two opposite corners of a square grid and elevating the remaining two opposite corners is widely popular for swimming pools. The sides are covered by glazing. A typical hypar shell of this type is shown in Fig. 11.8. A tie is required connecting the two low corners, A and C .

11.8 EXAMPLES FOR PRACTICE

1. Design a hypar shell roof of inverted umbrella type to suit the following data; Area to be covered in plan = 20 m by 20 m M-20 grade concrete and Fe-415 grade tor steel are available for use. Sketch the details of reinforcements in the shell, edge tie beams and compression ribs.
2. A hypar shell roof of tilted inverted umbrella type of cover an area of 20 m by 20 m is required. The opposite edges of the roof are 3 and 2 m respectively above the central valley point. Adopting M-20 grade concrete and Fe-415 grade to steel, design the various structural elements of the shell.

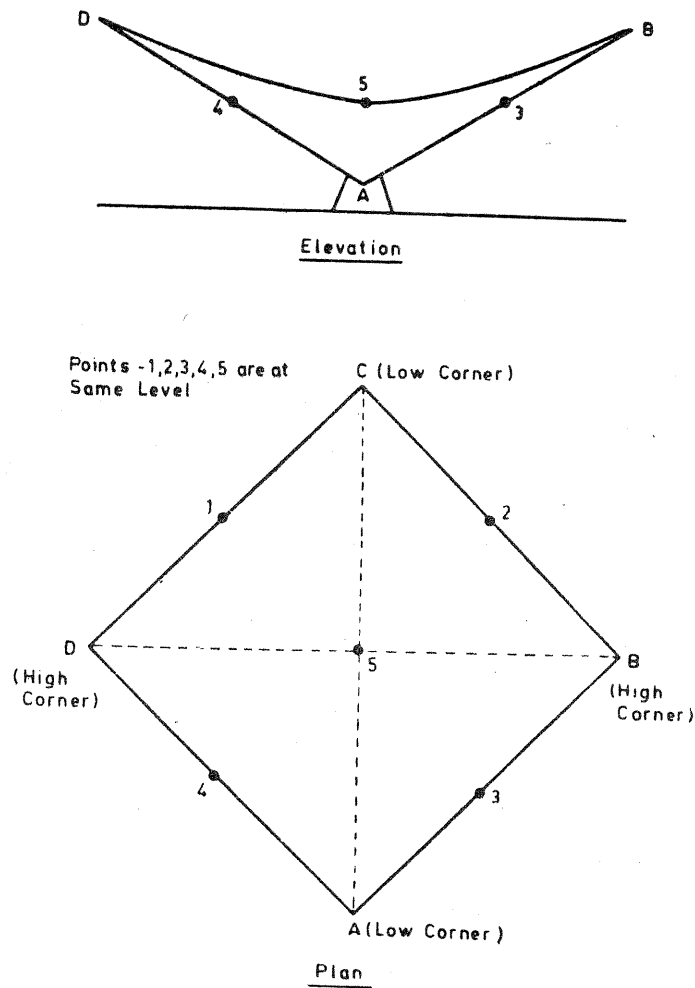


Fig. 11.8 Hypar shell roof

3. A market hall measuring 20 m by 30 m is to be covered by a turkey shed using hypar shell units. Design the hypar shell roof using the supports at the mid points of the sides of the grid. Adopt M-20 grade concrete and Fe-415 grade tor steel.

12

Hyperbolic Cooling Towers

12.1 GENERAL ASPECTS

Hyperbolic reinforced concrete cooling towers are widely used for cooling large quantities of water in thermal power stations, refineries, atomic power plants, steel plants, air conditioning and other industrial plants. A cooling tower incorporates a draft tower, water distributing and spraying devices and a cold water basin. A typical section of a hyperbolic cooling tower of height 100 m is shown in Fig. 12.1.

The draft is induced naturally or mechanically. Hot water is pumped to a certain height and then distributed through a piping system of nozzles, where it splashes over a system called "Filling or stacking". The filling may be of wood or transite material placed in several layers. The water splashes drips and flows through the layers of filling. The air which is forced upward either mechanically or by atmospheric pressure difference rises through the driplets or films of the hot water and consequently cools the water through evaporation and convection. The cooled water is collected in a pond at the bottom of the towers and then recirculated for industrial use.

12.2 HYPERBOLOID OF REVOLUTION

The hyperboloid of revolution of one sheet is generally used for cooling towers of thermal stations. The great advantage of this type of shell is that it is generated by two families of intersecting straight lines and the form work can be achieved by straight boards warped only slightly over the lengths.

The intersecting grid of straight lines form rhombuses of intersection. The shell surface can also be built of precast rhomic elements which are

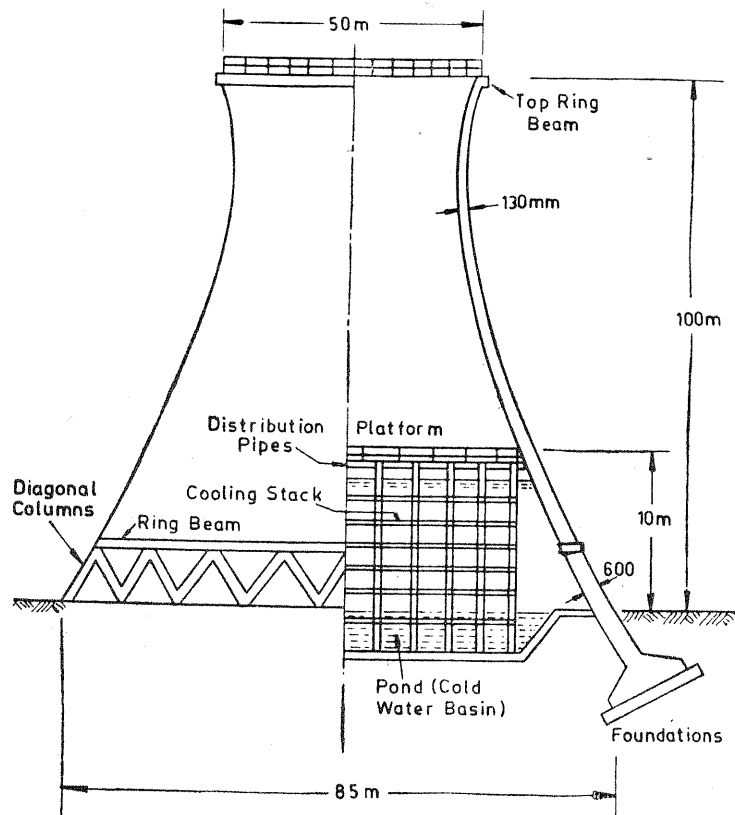


Fig. 12.1 Hyperbolic cooling tower

repeated along the complete circumferences at fixed heights. The generation of the hyperboloid of revolution by intersecting straight lines is shown in Fig. 12.2.

12.3 ANALYSIS OF MEMBRANE FORCES

The equation of the hyperboloid is given by

$$\left[\frac{r_0^2}{a^2} - \frac{Z^2}{b^2} \right] = 1 \quad (1)$$

Principal radii of curvature r_1 and r_2 are given by the equations

$$r_1 = -a^2 b^2 \left(\frac{r_0^2}{a^4} + \frac{Z^2}{b^4} \right) \quad (2)$$

$$r_2 = a \left[1 + \frac{Z^2}{a^2} \left\{ \frac{a^2}{b^2} + \left(\frac{a^2}{b^2} \right)^2 \right\} \right]^{1/2} \quad (3)$$

Also

$$\alpha = \left(\frac{a^2}{b^2} \right)$$

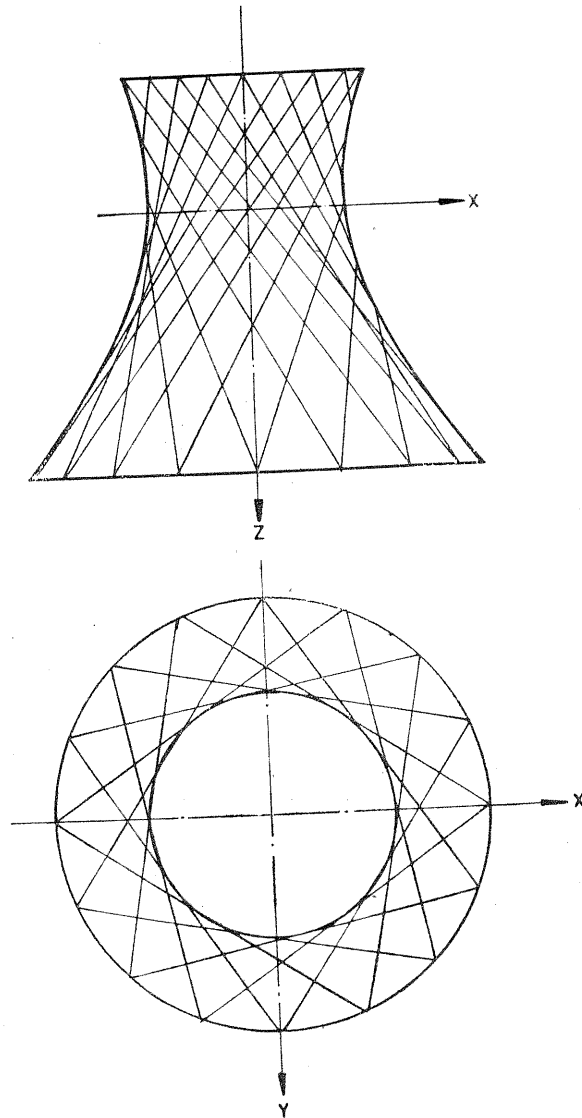


Fig. 12.2 Hyperboloid of revolution

The geometry of the hyperboloid is shown in Fig. 12.3.
Relation between r_1 and r_2 is given by

$$r_1 = - \left(\frac{r_2^3}{\alpha a^2} \right) \quad (4)$$

Also from (1)

$$Z = \pm b/a \sqrt{r_0^2 - a^2}$$

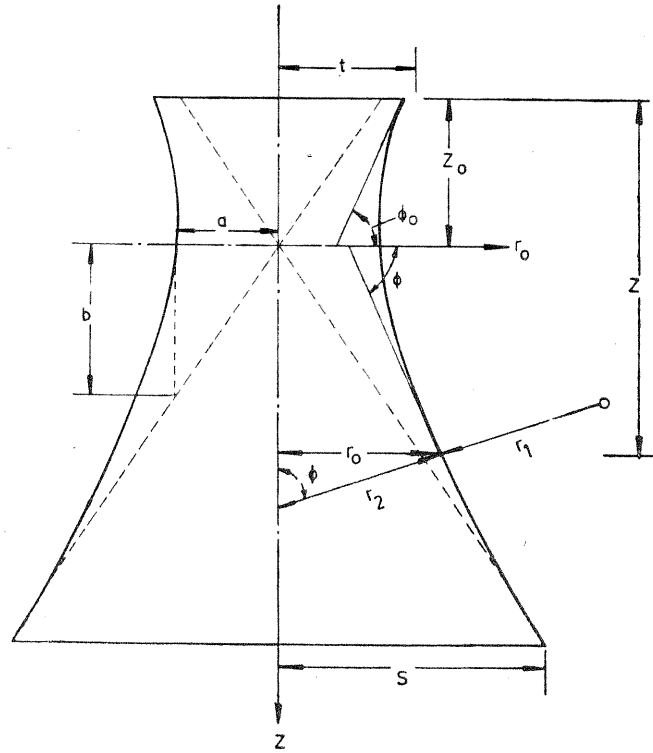


Fig. 12.3 Geometry of hyperboloid

$$\therefore \frac{dZ}{dr_0} = \tan \phi = +b/a \sqrt{\frac{r_0^2}{r_0^2 - a^2}}$$

$$\therefore \tan^2 \phi = \frac{b^2}{a^2} \left(\frac{r_0^2}{r_0^2 - a^2} \right)$$

$$\cot^2 \phi = \frac{a^2}{b^2} \left(1 - \frac{a^2}{r_0^2} \right)$$

Simplifying

$$r_0 = \frac{a^2 \sin \phi}{(a^2 \sin^2 \phi - b^2 \cos^2 \phi)^{1/2}} \quad (5)$$

Similarly

$$Z = \frac{b^2 \cos^2 \phi}{(a^2 \sin^2 \phi - b^2 \cos^2 \phi)^{1/2}} \quad (6)$$

$$r_2 = \left(\frac{r_0}{\sin \phi} \right) = \frac{a^2}{(a^2 \sin^2 \phi - b^2 \cos^2 \phi)^{1/2}} \quad (7)$$

From Eq. 4

$$r_1 = \frac{-a^2 b^2}{(a^2 \sin^2 \phi - b^2 \cos^2 \phi)^{3/2}} \quad (8)$$

If W = Total vertical load above the level ϕ and

g = density of material

Then

$$W = g \int 2\pi r_0 r_1 d\phi$$

$$= 2\pi g \cdot a^4 \cdot b^2 \int_{\phi}^{\phi_0} \frac{\sin \phi d\phi}{(a^2 \sin^2 \phi - b^2 \cos^2 \phi)^2} \quad (9)$$

where ϕ corresponds to height Z

ϕ_0 corresponds to height Z_0

For purposes of integration

Substitute $\cos \phi = \left[\frac{a}{\sqrt{a^2 + b^2}} \cdot \xi \right] \quad (10)$

Then,

$$W = -\frac{2\pi g \cdot ab^2}{\sqrt{a^2 + b^2}} \int_{\xi}^{\xi_0} \frac{d\xi}{(1 - \xi^2)^2} = \frac{\pi g}{2} \cdot \frac{ab^2}{\sqrt{a^2 + b^2}} \left[\frac{2\xi}{1 - \xi^2} + \log \left(\frac{1 - \xi}{1 + \xi} \right) \right]_{\xi}^{\xi_0} \quad (11)$$

For different values of ξ , the function is defined as

$$f(\xi) = \left[\frac{2\xi}{1 - \xi^2} + \log \left(\frac{1 - \xi}{1 + \xi} \right) \right]$$

The values of $f(\xi)$ are compiled in Table 12.1.

TABLE 12.1 Values of Function $f(\xi)$

ξ	$f(\xi)$	ξ	$f(\xi)$
0.000	0.275	0.275	1.159
0.010	0.040	0.300	1.278
0.020	0.080	0.325	1.401
0.030	0.120	0.350	1.529
0.040	0.160	0.375	1.661
0.050	0.200	0.400	1.800
0.060	0.241	0.425	1.945
0.070	0.281	0.450	2.111
0.080	0.321	0.475	2.261
0.090	0.362	0.500	2.432
0.100	0.403	0.550	2.814
0.110	0.444	0.600	3.261
0.120	0.485	0.650	3.802
0.130	0.526	0.700	4.480
0.140	0.567	0.750	5.374
0.150	0.609	0.800	6.642
0.175	0.725	0.850	8.638
0.200	0.822	0.900	12.418
0.225	0.932	0.950	23.151
0.250	1.044	1.000	∞

$$f(\xi) = \left[\frac{2\xi}{1 - \xi^2} + \log \left(\frac{1 - \xi}{1 + \xi} \right) \right]$$

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If N_ϕ = Meridional thrust at level ϕ

W = Total load above level ϕ

Then
$$N_\phi \cdot \sin \phi = \left(\frac{W}{2\pi r_2 \cdot \sin \phi} \right) \quad \therefore \quad N_\phi = - \left(\frac{W}{2\pi r_2 \cdot \sin^2 \phi} \right)$$

The negative value indicates compression. Substituting the value of W from Eq. 11

$$N_\phi = -\frac{g}{4} \cdot b^2 \sqrt{a^2 + b^2} \left\{ \frac{\sqrt{1 - \xi^2}}{a^2 + b^2 - a^2 \xi^2} \right\} [f(\xi) - f(\xi_0)] \quad (12)$$

The circumferential thrust N_θ is given by

$$\left[\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} \right] = -g \cdot \cos \phi$$

Substituting the values of N_ϕ from Eq. 12

$$N_\theta = -\frac{ga^2}{\sqrt{a^2 + b^2}} \cdot \left(\frac{\xi}{1 - \xi^2} \right) + N_\phi \cdot \frac{a^2}{b^2} (1 - \xi^2) \quad (13)$$

12.4 DESIGN EXAMPLE

A hyperbolic cooling tower of height 84 m has the following data:

Top diameter = 45 m

Throat diameter = 42 m

Density of concrete = 24 kN/m³

Analyse for membrane forces at the base section of the tower and design suitable thickness and reinforcements for the bottom section. Referring to Fig. 12.4.

$$t = 22.5 \text{ m}$$

$$Z_t = 20 \text{ m}$$

$$a = 21.0 \text{ m}$$

$$Z_b = 64 \text{ m}$$

From the relation $\left(\frac{r_0^2}{a^2} - \frac{Z^2}{b^2} \right) = 1$

$$\left(\frac{t^2}{a^2} - \frac{Z^2}{b^2} \right) = 1 \quad \text{where} \quad r_0 = \text{radius at any section}$$

$$\therefore \quad b = \left[\frac{aZ_t}{\sqrt{t^2 - a^2}} \right] \quad \therefore \quad b = \left(\frac{21 \times 20}{\sqrt{22.5^2 - 21^2}} \right) = 53 \text{ m}$$

If S = radius of base section

$$S = a \sqrt{1 + \frac{Z_b^2}{b^2}} = 21 \sqrt{1 + \frac{64^2}{53^2}} = 32.5 \text{ m}$$

$\therefore$ Diameter at base = 65 m

For base section, we have

$$\tan \phi = \frac{b}{a} \sqrt{\frac{r_0^2}{r_0^2 - a^2}} = \frac{53}{21} \sqrt{\frac{32.5^2}{32.5^2 - 21^2}} = 3.32$$

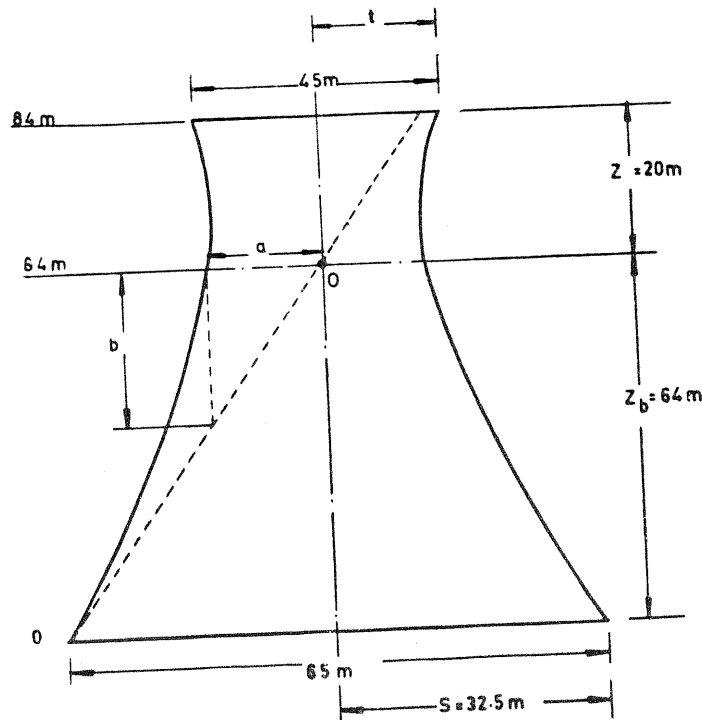


Fig. 12.4 Geometry of cooling tower

$$\cos \phi = \left(\frac{1}{\sqrt{1 + \tan^2 \phi}} \right) = 0.28$$

From Eq. 10

$$\cos \phi = \frac{a}{\sqrt{a^2 + b^2}} \cdot \xi$$

$$\therefore \xi = \left[\frac{\cos \phi \cdot \sqrt{a^2 + b^2}}{a} \right] = \left[\frac{0.28 \sqrt{21^2 + 53^2}}{21} \right] = 0.75$$

From Table 12.1, for this value $\xi = 0.75$, $f(\xi) = 5.374$.

For the top section

$$t = \sqrt{1 + \frac{Z^2}{b^2}} = 21 \sqrt{1 + \frac{20^2}{53^2}} = 22.5 \text{ m}$$

$$\tan \phi = \frac{b}{a} \sqrt{\frac{r_0^2}{r_0^2 - a^2}} = \frac{53}{21} \sqrt{\frac{22.5^2}{22.5^2 - 21^2}} = 7.1$$

$$\therefore \cos \phi = 0.14$$

$$\therefore \xi_0 = \left[\frac{0.14 \sqrt{21^2 + 53^2}}{21} \right] = 0.375$$

$\therefore$ From Table 12.1, for this value of $\xi_0 = 0.375$

$$(\xi_0) = 1.661$$

Membrane Forces

At top of section, $N_\theta = 0$ because of $f(\xi_0) - f(\xi_0) = 0$ and

$$N_\theta = - \left(\frac{ga^2}{\sqrt{a^2 + b^2}} \right) \left(\frac{\xi_0}{\sqrt{1 - \xi_0^2}} \right) = - \left(\frac{24 \times 21^2}{21^2 + 53^2} \right) \left(\frac{0.375}{1 - 0.375^2} \right) \\ = - 74 \text{ kN/m}$$

At base section,

$$N_\theta = - \frac{g}{4} \cdot b^2 \sqrt{a^2 + b^2} \left\{ \frac{1 - \xi^2}{a^2 + b^2 - a^2 \xi^2} \right\} [f(\xi) - f(\xi_0)]$$

where $g = 24 \text{ kN/m}^3$

$$b = 21 \text{ m}$$

$$b = 53 \text{ m}$$

$$\xi = 0.75$$

$$f(\xi) = 5.374$$

$$f(\xi_0) = 1.661$$

$$\therefore N_\theta = - \left(\frac{24}{4} \right) \cdot 53^2 \sqrt{21^2 + 53^2} \left\{ \frac{\sqrt{1 - 0.75^2}}{21^2 + 53^2 - (21^2 \times 0.75^2)} \right\} [5.374 - 1.661] \\ = - 810 \text{ kN/m (Compression)}$$

$$N_\theta = - \left(\frac{g \cdot a^2}{\sqrt{a^2 + b^2}} \right) \cdot \left(\frac{\xi}{1 - \xi^2} \right) + N_\phi \left(\frac{a^2}{b^2} \right) (1 - \xi^2) \\ = - \left(\frac{24 \times 21^2}{\sqrt{21^2 + 53^2}} \right) \left(\frac{0.75}{\sqrt{1 - 0.75^2}} \right) + \left[- 810 \times \frac{21^2}{53^2} (1 - 0.75^2) \right] \\ = - 264 \text{ kN/m (Compression)}$$

Design of shell section and Reinforcement

At base section $N_\phi = 810 \text{ kN/m}$

Using M-20 grade concrete and Fe-415 grade Tor steel permissible stresses $= \sigma_{cc} = (0.5 \times 5) = 2.5 \text{ N/mm}^2$

If t = thickness of the concrete shell at base.

$$\text{Then} \quad \left(\frac{N_\phi}{1000t} \right) = \sigma_{cc}$$

$$\therefore t = \left(\frac{810 \times 10^3}{1000 \times 2.5} \right) = 324 \text{ mm}$$

Adopt a shell thickness of 350 mm at base gradually reducing to 150 mm at the top.

Minimum reinforcement = 1% of cross sectional area

$$= \left(\frac{1}{100} \times 1000 \times 350 \right) = 3500 \text{ mm}^2$$

Provide 20 mm Φ bars at 175 mm centres on both faces in meridional direction ($A_{st} = 3600 \text{ mm}^2$)

Minimum reinforcement in the circumferential direction

$$= 0.15\% \text{ of gross area} = \left(\frac{0.15}{100} \times 350 \times 1000 \right) = 525 \text{ mm}^2$$

Provide 10 mm Φ bars at 300 mm centre on both faces in the circumferential direction as hoops.

The reinforcements can be decreased towards the top of the tower.

12.5 ANALYSIS OF HYPERBOLIC COOLING TOWERS FOR WIND LOADS

Hyperbolic cooling towers being tall structures are subjected to wind loads, which result in meridional and circumferential forces in towers. Based on wind tunnel experiments Rish and steel (Refer the paper "Design and selection of Hyperbolic cooling towers", Journal of power division, A.S.C.E. proceedings, Vol. 85, October 1959, pp. 89-117), have recommended the following type of wind pressure distribution in hyperbolic cooling towers, as detailed Fig. 12.5.

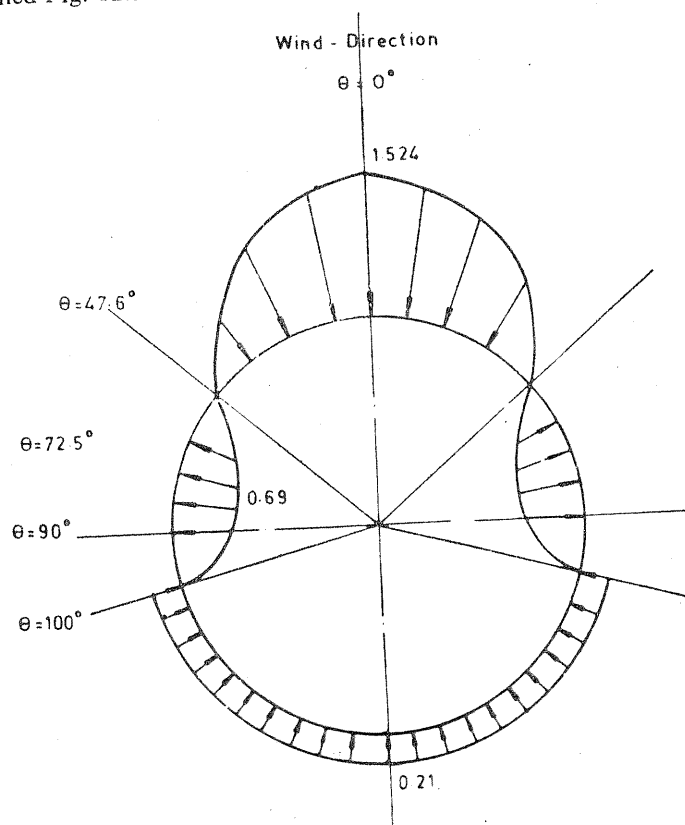


Fig 12.5 Wind pressure distribution

The wind ward side of the tower is subjected to positive wind pressure over an area bounded by an angle of 94.12° . The Leeward side is subjected to suction pressure of intensities as shown in figure. Due to the action of wind, membrane forces N_ϕ , N_θ and $N_{\theta\phi}$ develop with maximum values at the base section.

Typical distribution of the various force components at the base of a hyperbolic cooling tower are shown in Figs. 12.6, 12.7 and 12.8, for the following tower parameters.

$$\left(\frac{a}{t}\right) = 0.90, \quad \left(\frac{a}{s}\right) = 0.55$$

$$K^2 = \left(1 + \frac{a^2}{b^2}\right) = 1.18 \quad \Phi = 1.0$$

P = Wind intensity

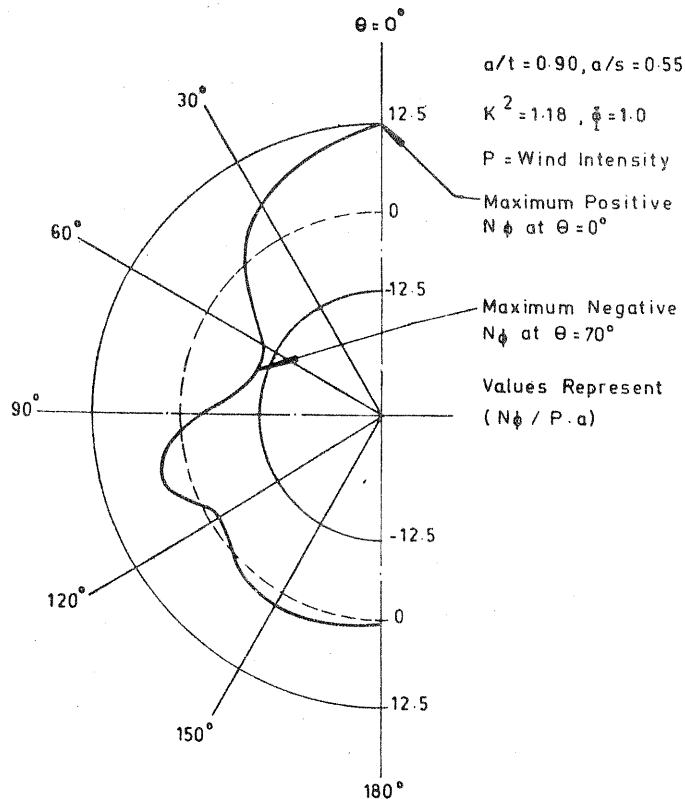


Fig. 12.6 Meridional thrust

The force distribution analysis indicates that the maximum tensile meridional force develops at $\theta = 0$ and compressive force at $\theta = 70^\circ$. The maximum hoop tension develops at $\theta = 70^\circ$ while the maximum hoop compression develops at an angle of $\theta = 0^\circ$. The maximum shear force develop at angles of $\theta = 90^\circ$ and $\theta = 50^\circ$.

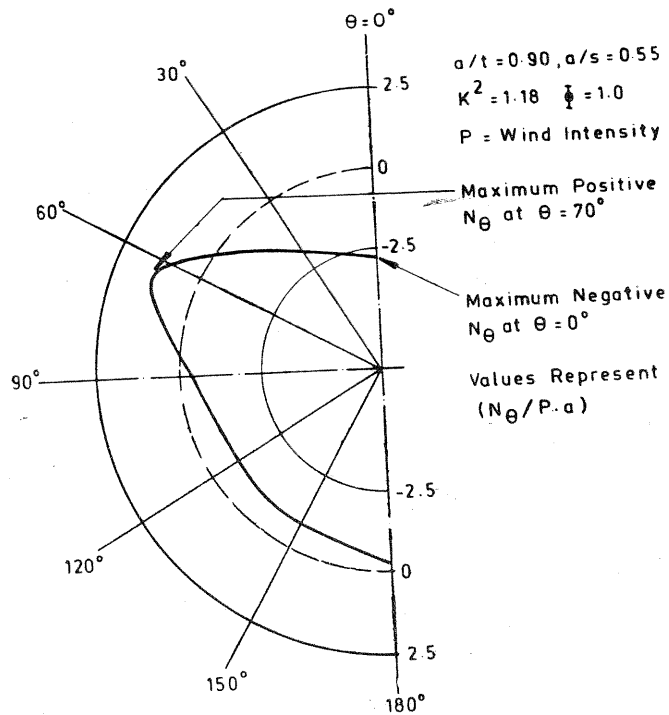


Fig. 12.7 Circumferential-thrust

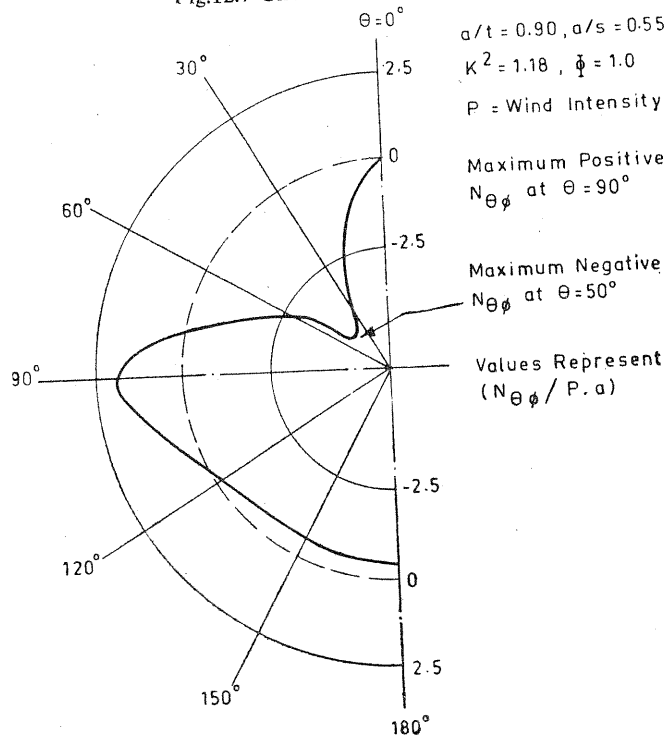


Fig. 12.8 Shear-Forces

Based on membrane analysis Gould and Lee (Refer paper "Hyperbolic cooling towers under wind load", Journal of structural division, A.S.C.E., Proceedings, Vol. 93, October 1967, pp. 487-514) have developed design tables which give the meridional force (N_ϕ) and circumferential force (N_θ) in the cooling towers subjected to the action of wind loads. The design tables have been presented in the form of

$\left(\frac{N_\phi}{P \cdot a}\right)$, $\left(\frac{N_\theta}{P \cdot a}\right)$ and $\left(\frac{N_{\phi\theta}}{P \cdot a}\right)$ for different values of the variables, (a/s) , (a/t) , K^2 , θ and Φ , as shown in Fig. 12.9. The range of values curved in the tables are as follows:

(a/s) from 0.45 to 0.65

(a/t) from 0.85 to 0.95

K^2 from 1.05 to 1.50

θ at 0° to 70°

Φ from 0 to 1.0

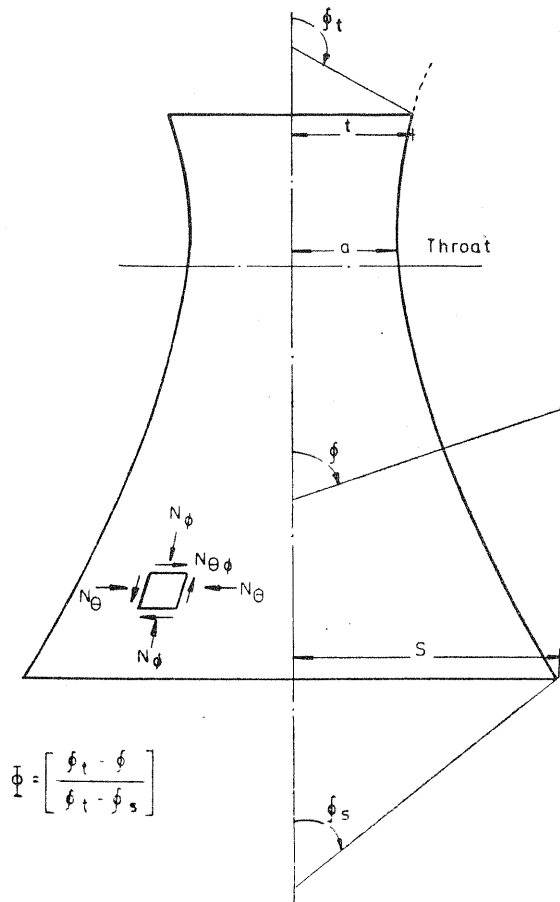


Fig. 12.9 Cooling tower parameters

12.6 DESIGN EXAMPLE

A hyperbolic cooling tower has the following parameters

Throat diameter = 42 m

Base diameter = 65 m

$b = 53$ m

Top diameter = 42 m

Wind intensity = $P = 1$ kN/m².

Compute the maximum design membrane forces at the base section due to the action of wind and self weight of shell.

The salient design parameters are

$a = 21$ m $P = 1$ kN/m²

$b = 53$ m $a/s = 0.65$

$s = 32.5$ m $a/t = 0.95$

$t = 22.5$ m $K^2 = \left(1 + \frac{a^2}{b^2}\right) = 1.155$

Referring to design tables for maximum values of N_ϕ , N_θ , we have for $\theta = 0$ and $\Phi = 1.00$ (base section)

$$\left(\frac{N_\phi}{P \cdot a}\right) = 12.35$$

$$\therefore N_\phi = (12.35 \times 1 \times 21) = 260 \text{ kN/m}$$

$$\left(\frac{N_\theta}{P \cdot a}\right) = -1.914$$

$$\therefore N_\theta = (-1.914 \times 1 \times 21) = -40 \text{ kN/m}$$

Maximum negative value of N_ϕ occurs at $\theta = 70^\circ$ and $\Phi = 1.00$ (base section)

$$\left(\frac{N_\phi}{P \cdot a}\right) = -11.202$$

$$\therefore N_\phi = (-11.202 \times 1 \times 21) = -235 \text{ kN/m}$$

Maximum positive value of N_θ occurs at $\theta = 70^\circ$ and $\Phi = 1.00$ (base section)

$$\left(\frac{N_\theta}{P \cdot a}\right) = 0.60$$

$$\therefore N_\theta = (0.6 \times 1 \times 21) = 12.6 \text{ kN/m}$$

The forces N_ϕ and N_θ due to self weight for the shell are computed as shown in design Example 12.4.

Hence the design forces are shown in table given below.

Force	Due to D.L	Due to W.L	Maximum Design force
N_ϕ	- 810	- 235	- 1045 kN/m
N_θ	- 264	- 40	- 304 kN/m

The base section is designed for the maximum meridional and circumferential forces.

12.7 EXAMPLES FOR PRACTICE

1. A hyperbolic cooling tower of over all height 100 m has a throat radius of 23 m. The throat section is located 25 m from the top of the shell. The diameter of the top section is 50 m. Analyse for membrane forces at the base section of the tower and design suitable concrete section and reinforcements. Adopt M-20 grade concrete and Fe-415 grade tor steel.
2. A hyperbolic reinforced concrete cooling tower has the following parameters:

Top diameter = 50 m

Throat diameter = 46 m

Base diameter = 68 m

Height of throat section = Z_t = 25 m

Total height of tower = 100 m.

The tower is subjected to a wind pressure of 1.5 kN/m². Estimate the maximum membrane forces in the shell due to self weight and wind loads and design the section. Adopt M-30 grade concrete and Fe-415 grade tor steel.

13

Folded Plates

13.1 GENERAL FEATURES

Folded plates of Faltwerke or hipped plates are ideally suited for a variety of structures such as factory buildings, assembly halls, godowns, auditoriums and gymnasia, requiring large column free area. Folded plates were first used for large coal bunkers by G. Ehlers of Germany in 1924-25. The structural behaviour of folded plates resembles that of shells and they can be considered as examples of stressed skin construction. The different types of folded plates used for the various types of structures are shown in Fig. 13.1.

In comparison with the shells, folded plates have certain advantages as detailed below:

1. Simple shuttering involving only straight planks are required.
2. Moveable form work can be used resulting in speedy construction.
3. Simpler diaphragms are used in place of complicated traverses required for shells.
4. The design computations are simpler in comparison with shells.
5. Folded plates consume slightly higher quantities of concrete and steel, but the increased cost on this account is more than off set by the lower shuttering costs. Hence folded plates are competitive with shells for a wide range of spans.
6. With their light reflecting geometry and pleasing outlines, folded plates are aesthetically comparable with shells.

13.2 ANALYSIS OF FOLDED PLATES

G. Ehlers of Germany in his first paper on folded plates published in

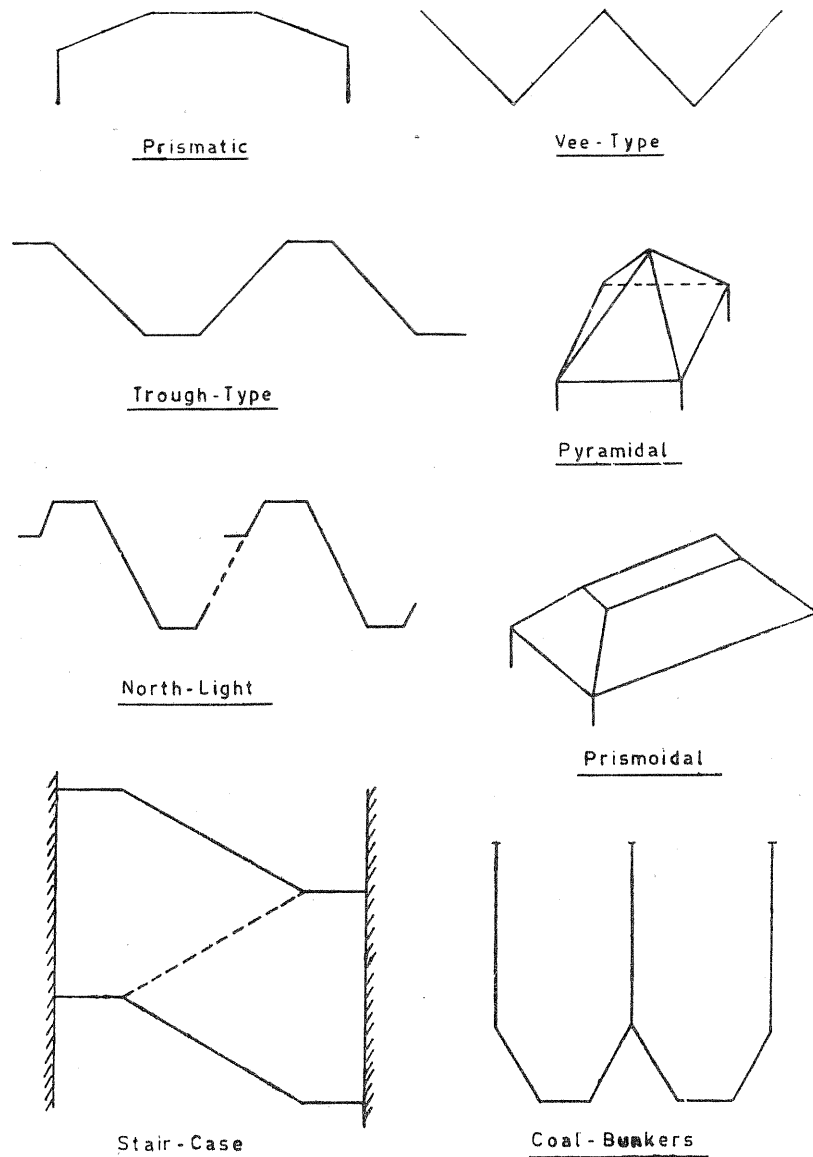


Fig. 13.1 Types of folded plates

1930 assumed the longitudinal joints to be hinged, neglecting the transverse moments at the junction of the plates. The displacements of the joints were also ignored. In 1932 Gruber improved the analysis by considering transverse continuity and joint displacements. He developed a solution in the form of simultaneous differential equations of the fourth order which were solved by the use of rapidly converging series. This approach involves

$(7n+2)$ unknowns for $(n+1)$ plates. The analysis indicated that assumption of hinged or rigid joints would considerably influence the resulting moments and stresses in the plates.

Craemer published a paper in 1953, setting out rough limits for classification of folded plates as long and short based on the length to width ratio of the individual plates. Significant improvements were made in the analysis of folded plates by Winter and Pei in 1947, who reduced the algebraic solution into a stress distribution procedure which is very much similar to the traditional moment distribution method. But this method neglects joint displacements and is useful only for the analysis of short folded plates. For long folded plates, the joint displacements cannot be ignored. In 1948, Girkmann, improved the analytical procedures by considering joint displacements and treating transverse moments at the joints as unknowns and formulated conditions for the compatibility of longitudinal stresses and displacements at joints. This method results in the solution of as many simultaneous equations as the unknown transverse moments. In 1953 Gaafar published a modification of the Winter and Pei method including the effect of joint displacements. Whitney in 1959 presented a simplified version of the modified Girkman's method. In 1958, Simpson published a new method involving a number of moment and stress distributions.

A comparative analysis of the various methods indicate that Winter and Pei's method is the simplest but it is applicable only to short folded plates for which the joint displacements can be ignored without appreciable error. In general Whitney and Simpson's methods are applicable to folded plates of all proportions and are ideally suited for use in design office. For long folded plates with a span to depth ratio of more than 10, Whitney has shown that a simplified analysis can be used based on beam theory assuming a straight line distribution of longitudinal stress. The iteration method developed by Brielamaier offers a simple means of analyzing symmetrical V-type folded plates. However this method is not universally applicable to folded plates of all types. The analysis of the folded plates is based on the following assumptions:

1. The structure is monolithic with rigid joints.
2. The material is homogeneous, elastic and isotropic.
3. The length of each plate is more than twice its width.
4. Plane sections remain plane after deformation in all the plates.

13.3 STRUCTURAL BEHAVIOUR OF FOLDED PLATES

Folded plates resist the system of transverse loads by 'plate' and 'slab' action. The loads acting normal to each plate causes transverse bending between the junctions of the plates, which can be considered as imaginary supports of a continuous slab. This transverse bending is termed as '*Slab action*'. The transverse moments developed in the plates can be determined by a continuous beam analysis assuming the supports to be at the junctions of the plates.

The plates being supported at their ends on transverses, bend under the action of loads in their own plane as shown in Fig. 13.2. The longitudinal bending of the plates in their own plane is termed as '*Plate*' action'. The bending stresses resulting from plate action may be considered to have a linear distribution across each plate, with maximum intensity at the centre of span section. The salient features of Whitney and Simpson's method of analysis of folded plates are outlined in the following sections.

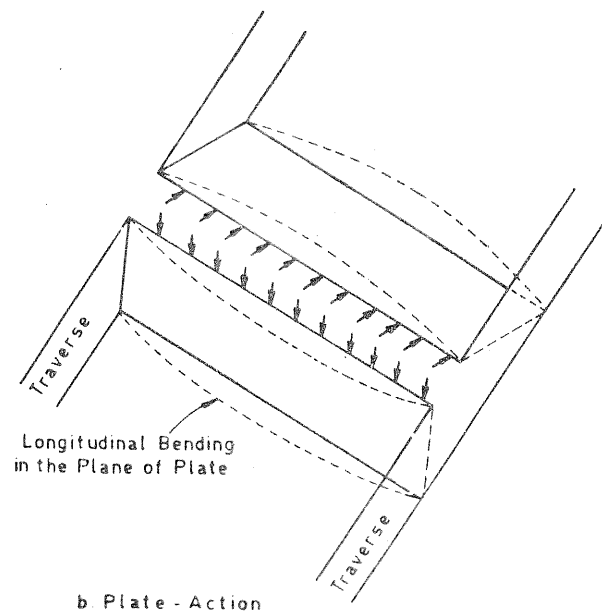
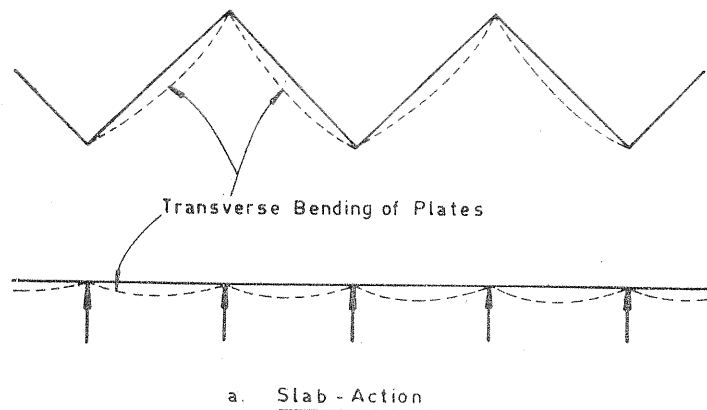


Fig. 13.2 Structural-behaviour of folded plates

13.4 WHITNEY'S METHOD

The Whitney method is applicable for folded plates with width and thickness of the plates and the intensity of loading uniform along the length of the plate. Mathematical computations are greatly simplified by replacing the uniform load by Fourier loading and considering only the first term of the series. The plate moments, stresses and deformations therefore vary as sine functions along the length, having maximum values at mid span. The Salient steps of the method are as follows:

1. The joint loads are computed by assuming each plate to be simply supported spanning between the adjacent plates. The joint loads are replaced by their components in the plane of the plates. These form the initial plate loads.
2. The continuity of the plates due to the rigidity of the joints induces transverse moments causing additional joint loads in terms of unknown transverse moments and hence additional plate loads which are added to the initial plate loads.
3. Under the action of plates loads, each plate bends in the longitudinal direction in its own plane between the traverses. The bending moments and the longitudinal stresses are computed at the common edges. It will be observed that the longitudinal stresses at the common edges of the adjacent plates will not be equal. Longitudinal edge shear forces are now introduced to establish the compatibility of stresses at the junctions.
4. The individual plate deflections caused by the transverse loads and the edge shear forces are calculated in terms of the applied loads and the unknown transverse moments.
5. For each of the rigid joints, the condition of zero angle change is written down in terms of the applied loads, transverse joint moments and plate deflections resulting in n equations for n plates.
6. The equations are solved for the unknown transverse moments and other stress resultants are computed.

13.5 SIMPSON'S METHOD

The prominent steps to be followed in Simpson's method of analysis of folded plates are outlined below:

1. A transverse section of the folded plate of unit width is treated as a continuous beam on rigid supports and the joint loads obtained are resolved into components in the planes of the plates.
2. The plates are assumed to bend independently between the diaphragms due to plate loads and the corresponding moments and stresses are calculated.
3. Compatibility conditions require that the longitudinal stresses at common edges of adjacent plates should be equal. The stresses are

corrected by distribution procedure. (At this stage, the results obtained will be identical to those obtained by Winter and Pei solution).

4. The stresses obtained are now further corrected to allow for the joint displacements. An arbitrary rotation is given to each joint in turn at mid span, the rotation along the span varying as a sine curve. The joint loads and the corresponding longitudinal stresses are calculated in each case and the compatibility of the stresses established by a distribution procedure. This procedure is repeated for each joint and the joint deflections are calculated.
5. The final deflections are then expressed as the sum of the deflections due to the no-rotation solutions each multiplied by an unknown factor α_n . The final plate rotations are equal to the values for the no rotation solution plus α_n times the arbitrarily assumed rotations.
6. The total angle changes at the junctions are equated to zero resulting in a set of simultaneous equations which are solved for α_n .
7. The final moments and stresses in the plates are obtained as the sum of the no rotation solution and the rotation solutions each multiplied by its corresponding α_n .

13.6 ITERATION METHOD

The iteration method is applicable for symmetrical V-type folded plates with simpler computational effort in comparison with the Whitney and Simpson's methods. However this method is not applicable for all types of folded plates. The following steps are involved in the analysis of folded plates by iteration method.

1. The folded plate is analysed for slab action as a continuous beam and the ridge loads are computed from the reactions. The longitudinal stresses developed in the plates are computed assuming the plates to bend in their own plane due to the action of plate loads. The stresses at the junctions are computed and the no rotation solution obtained by the stress distribution procedure as in the case of Simpson's method.
2. The plate deflections corresponding to the no rotation solution is computed and from these the relative displacements of the joints can be obtained.
3. The transverse moments developed due to the relative joint displacements are computed and the moments are distributed to estimate the ridge loads and plate loads. The longitudinal stresses due to the plate loads are computed and corrected by stress distribution procedure. The resulting stresses are compared with those obtained from the no rotation solution. If the corrections are relatively small, the iteration process is stopped at this stage and the final moments and stresses are obtained by adding the results of the no rotation solution and those due to the first cycle of iteration.

4. If the corrections are not small, the deflections caused by correction stresses are computed and the cycle of iteration repeated until the desired results are obtained.

The convergence of the iteration method depends upon the relative rigidities of longitudinal plate action and transverse slab action and on the geometry of the structure. The iteration method has been found to converge rapidly for symmetrical V-type folded plates while it diverges when applied to north light folded plates.

13.7 BEAM METHOD

The beam method is based on the linear stress variation across the section and the folded plate is assumed to bend longitudinally as a beam between the transverse. This method is applicable for long folded plates with span to depth ratios exceeding 10. The beam method is by far the simplest due to the minimum computational effort involved when compared with the various other methods.

13.8 WINTER AND PEI'S METHOD

For short folded plates having span to depth ratio less than 4, Winter and Pei's method can be adopted. In the case of short folded plates, the displacements of junctions of hipped plates can be ignored without any appreciable error. In this method the folded plate is treated as a continuous beam on rigid supports and the joint loads obtained are resolved into plate loads. Under the action of the plate loads, the individual plates are assumed to bend in their own plane in the longitudinal direction and the corresponding moments and stresses are computed in the plates. Compatibility conditions require that the longitudinal stresses at common edges of adjacent plates should be equal. The stresses are corrected by a stress distribution procedure.

13.9 EQUATION OF THREE SHEARS

Consider a folded plate of two-folds only having depths of plates as h_1 and h_2 and the cross sectional area of the plates being A_1 and A_2 .

Z_1 and Z_2 are the section moduli of the two plates.

M_1 and M_2 are the moments acting on the plates computed from the plate loads.

σ_A , σ_B and σ_C are the edge stresses. T_A , T_B and T_C are the shear forces at the junctions A, B, and C, considered as positive when acting in opposition to beam moments M_1 and M_2 .

Moments are considered as positive when they develop tension at bottom and compression at top.

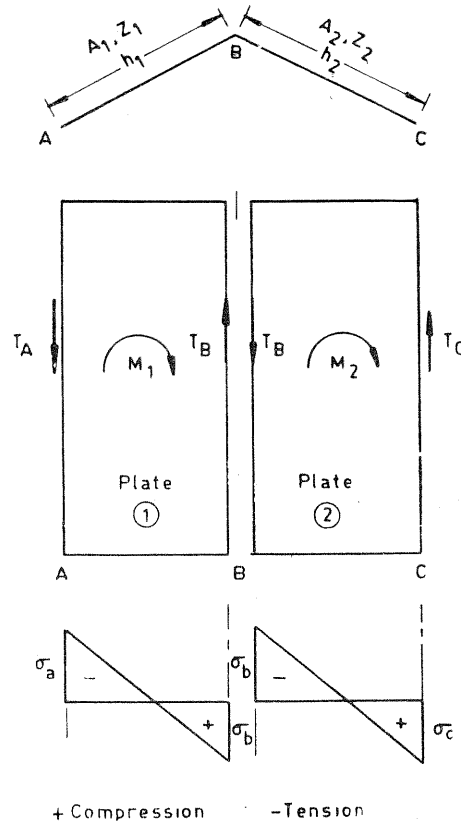


Fig. 13.3 Force components in folded plates

The stresses developed in plates 1 and 2 can be written as:
Stress at edge B in plate 1 is

$$\sigma_b = \left[\frac{M_1}{Z_1} + \frac{T_B}{A_1} + \frac{T_B \cdot h_1}{2Z_1} - \frac{T_A}{A_1} + \frac{T_A \cdot h_1}{2Z_1} \right] \quad (1)$$

Stress at edge B in plate 2 is

$$\sigma_b = \left[-\frac{M_2}{Z_2} - \frac{T_B \cdot h_2}{2Z_2} - \frac{T_B}{A_2} + \frac{T_C}{A_2} - \frac{T_C \cdot h_2}{2 \cdot Z_2} \right] \quad (2)$$

If t_1 = Thickness of plate-1

t_2 = Thickness of plate-2

$$A_1 = t_1 \cdot h_1 \quad Z_1 = (t_1 h_1^2)/6$$

$$A_2 = t_2 h_2 \quad Z_2 = (t_2 h_2^2)/6$$

$$\left(\frac{T_A h_1}{2Z_1} \right) = \frac{T_A \cdot h_1}{2 \left(\frac{t_1 h_1^2}{6} \right)} = \frac{3T_A}{t_1 h_1} = \frac{3T_A}{A_1}$$

Similarly

$$\frac{T_B h_1}{2Z_1} = \frac{3T_B}{A_1}, \quad \frac{T_B \cdot h_2}{2Z_2} = \frac{3T_B}{A_2}, \quad \frac{T_C \cdot h_2}{2Z_2} = \frac{3T_C}{A_2}$$

Substituting these values in equations (1) and (2) and equating them we have the condition,

$$\left[\frac{M_1}{Z_1} + \frac{4T_B}{A_1} + \frac{2T_A}{A_1} \right] = \left[-\frac{M_2}{Z_2} - \frac{4T_B}{A_2} - \frac{2T_C}{A_2} \right]$$

$$\left[\frac{2T_A}{A_1} + \frac{4T_B}{A_1} + \frac{4T_B}{A_2} + \frac{2T_C}{A_2} \right] = - \left[\frac{M_1}{Z_1} + \frac{M_2}{Z_2} \right]$$

The equation of three shears is written as:

$$\frac{T_A}{A_1} + 2T_B \left[\frac{1}{A_1} + \frac{1}{A_2} \right] + \frac{T_C}{A_2} = - \frac{1}{2} \left[\frac{M_1}{Z_1} + \frac{M_2}{Z_2} \right]$$

In a given problem, the equation of three shears is applied to two adjacent folded plates at a time and the algebraic equations thus obtained are solved for the unknown shears. Once the shears are computed, the stresses in the plates can be computed. This is illustrated in the following example.

13.10 DESIGN EXAMPLE

A folded plate with two folds AB and BC is subjected to moments in the plane of the plates. Using the following data, calculate the stress in the folded plate.

1. Data

Thickness of plates = 100 mm

Depth of plates, $h_1 = h_2 = 2$ m

Moment in plates, $M_1 = M_2 = 333$ kN·m

$$Z_1 = Z_2 = \left(\frac{100 \times 2000^2}{6} \right) = 66.66 \times 10^6 \text{ mm}^3$$

$$A_1 = A_2 = (100 \times 2000) = 2 \times 10^5 \text{ mm}^2$$

$$\left(\frac{M_1}{Z_1} \right) = \left(\frac{M_2}{Z_2} \right) = \left(\frac{333 \times 10^6}{66.66 \times 10^6} \right) = 5 \text{ N/mm}^2$$

2. Edge Shear Forces

Let T_A , T_B and T_C be the edge shear forces since edges A and C are free edges,

$$T_A = T_C = 0$$

Using the equation of three shears

$$2T_B \left[\frac{1}{A_1} + \frac{1}{A_2} \right] = - \frac{1}{2} \left[\frac{M_1}{Z_1} + \frac{M_2}{Z_2} \right]$$

$$2T_B \left[\frac{1}{2 \times 10^5} + \frac{1}{2 \times 10^5} \right] = -\frac{1}{2}(5+5)$$

$$\therefore T_B = -250000 \text{ N}$$

3. Resultant Stresses

The resultant stresses at plate edges are obtained from equations 1 or 2.

$$\sigma_B = \left[\frac{M_1}{Z_1} + \frac{T_B}{A_1} + \frac{T_B \cdot h_1}{2Z_1} \right] = 5 + \left(\frac{-250000}{2 \times 10^5} \right) + \left(\frac{-250000 \times 2000}{2 \times 66.66 \times 10^6} \right) = 0$$

$$\sigma_A = \left[-\frac{M_1}{Z_1} + \frac{T_B}{A_1} - \frac{T_B \cdot h_1}{2Z_1} \right] = -5 - 1.25 + 3.75 = -2.5 \text{ N/mm}^2 \text{ (Tension)}$$

$$\sigma_C = \left[\frac{M_2}{Z_2} - \frac{T_B}{A_2} + \frac{T_B \cdot h_2}{2Z_2} \right] = 5 + 1.25 - 3.75 = +2.5 \text{ N/mm}^2$$

(Compression)

The resultant stress distribution with the force components are shown in Fig. 13.4.

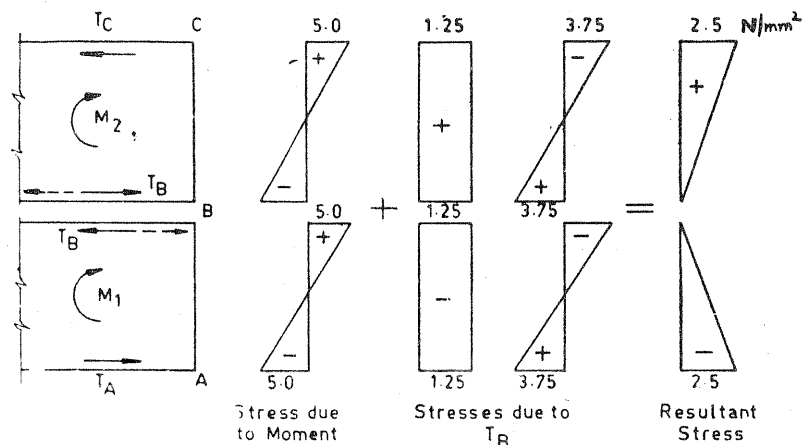


Fig. 13.4 Stresses in folded plate

13.11 STRESS CONDITIONS IN HIPPED PLATES

The nature of stresses developed in hipped plate depend upon the sign of moments in the individual plates, the different combination of moments such as sagging or hogging results in different types of stress distribution in individual and hipped plate structure as shown in Fig. 13.5. The nature of moments in the plates depends upon whether the junction of the two plates form a ridge or a valley. If the two plates meet at a ridge, the moments developed are such that compressive stresses develop at the ridge. Alternatively if the two plates meet in a valley, the moments develop tensile stresses in the valley.

STRESSES

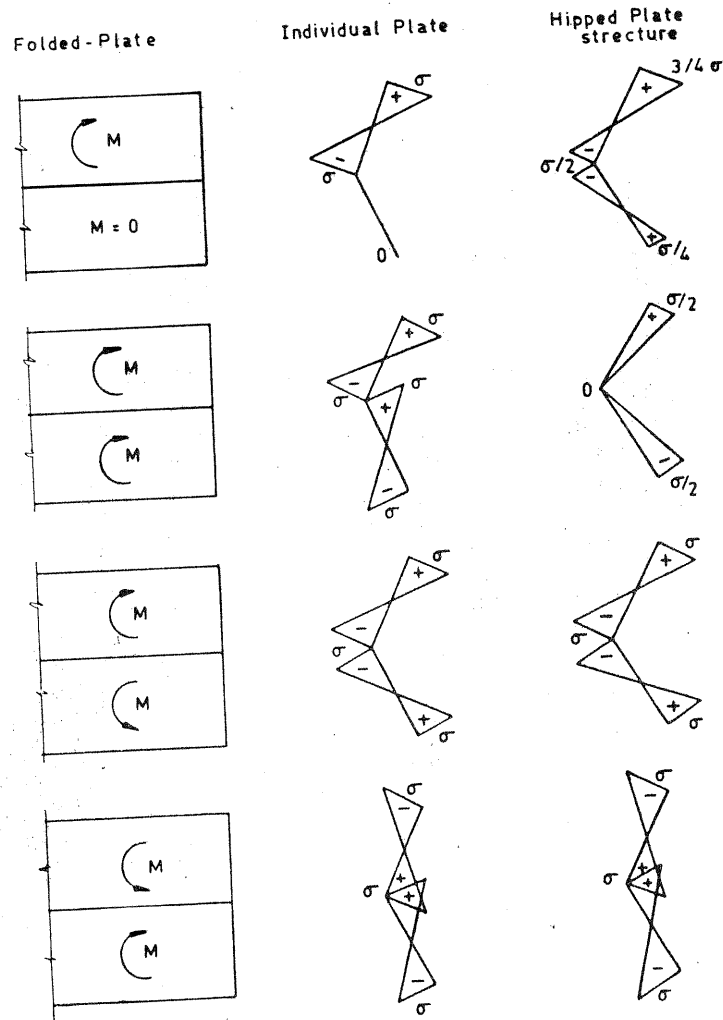


Fig. 13.5 Stress conditions in hipped plate structures

13.12 DESIGN EXAMPLE

A prismatic folded plate $ABCDE$ shown in Fig. 13.6, supports a live load of 0.4 kN/m^2 in addition to the self weight. Estimate the stresses developed in the plate at mid span section if the plates BC , CD and DE are 120 mm thick and plates AB and EF are 250 mm thick. Span of the folded plate $= 8 \text{ m}$.

1. Loads

Self weight of plates BC , CD and $DE = (0.12 \times 24) = 2.88 \text{ kN/m}^2$

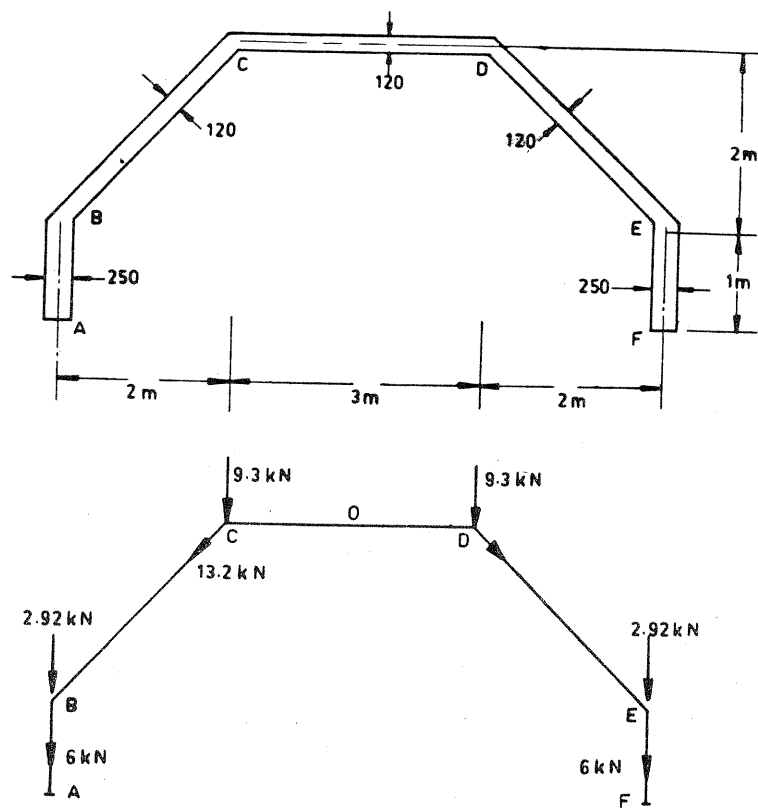


Fig. 13.6 Forces acting on folded plates

Self weight of plates AB and $EF = (0.25 \times 24) = 6.00 \text{ kN/m}^2$
 Live load $= 0.40 \text{ kN/m}^2$

$\therefore$ Total load/m on $CD = (2.88 + 0.4) = 3.28 \text{ kN/m}$

Total load/m on BC and $DE = \left(\frac{2.88 \times 2.25}{2} \right) + 0.4 = 3.65 \text{ kN/m}$

2. Reactions

The approximate value of reactions at B , C , D and E , treating the folded plate as a continuous beam $BCDE$ are,

$$R_B = R_E = (0.4 \times 3.65 \times 2) = 2.92 \text{ kN}$$

$$R_C = R_D = (0.6 \times 3.65 \times 2) + (0.5 \times 3.28 \times 3) = 9.30 \text{ kN}$$

The plate loads are shown in Fig. 13.6.

3. Section Properties

Plate-1 (AB) area $A_1 = (0.25 \times 1) = 0.25 \text{ m}^2$

Plate-2 (BC), area $A_2 = (2.84 \times 0.12) = 0.34 \text{ m}^2$

Plate-3 (*CD*), area $A_3 = (3 \times 0.12) = 0.36 \text{ m}^2$

$$Z_1 = \left(\frac{0.25 \times 1^2}{6} \right) = 0.041 \text{ m}^3$$

$$Z_2 = \left(\frac{0.12 \times 2.84^2}{6} \right) = 0.162 \text{ m}^3$$

$$Z_3 = \left(\frac{0.12 \times 3^2}{6} \right) = 0.18 \text{ m}^3$$

4. Moments in Plates

$$M_1 = \left(\frac{8.92 \times 8^2}{8} \right) = 71.36 \text{ kN} \cdot \text{m}$$

$$M_2 = \left(\frac{13.20 \times 8^2}{8} \right) = 105.6 \text{ kN} \cdot \text{m}$$

$$M_3 = 0$$

5. Stresses in Plates

$$\left(\frac{M_1}{Z_1} \right) = \left(\frac{71.36 \times 10^6}{0.041 \times 10^9} \right) = 1.74 \text{ N/mm}^2$$

$$\left(\frac{M_2}{Z_2} \right) = \left(\frac{105.6 \times 10^6}{0.162 \times 10^9} \right) = 0.65 \text{ N/mm}^2$$

6. Equation of Three Shears

Let T_B and T_C be the unknown shears at junctions *B* and *C*.

$$T_A = T_F = 0 \quad T_C = -T_D$$

Applying the equation of three shears to plates *AB* and *BD*, we have

$$\left(\frac{T_A}{0.25} \right) + 2T_B \left[\frac{1}{0.25} + \frac{1}{0.34} \right] + \frac{T_C}{0.34} = - \left(\frac{1.74 + 0.65}{2} \right) 10^3$$

For plates *BC* and *CD*

$$\left(\frac{T_B}{0.34} \right) + 2T_C \left[\frac{1}{0.34} + \frac{1}{0.36} \right] + \frac{T_D}{0.36} = - \left(\frac{0.65 + 0}{2} \right) 10^3$$

Also $T_C = -T_D$

Solving the two equations, the unknown shears are

$$T_C = -8.7 \text{ kN}$$

$$T_B = -84.0 \text{ kN}$$

7. Stresses in Plates Due to Shear Forces

Plate-1

$$\sigma_A = \left[\frac{84 \times 10^6 \times 0.5}{0.041 \times 10^9} - \frac{84 \times 10^3}{0.25 \times 10^6} \right] = 0.684 \text{ N/mm}^2$$

$$\sigma_B = \left[-\frac{84 \times 10^6 \times 0.5}{0.041 \times 10^9} - \frac{84 \times 10^3}{0.25 \times 10^6} \right] = -1.356 \text{ N/mm}^2$$

Plate-2

$$\sigma_B = \left[\frac{(84 + 8.7)10^6 \times 2.84}{2 \times 0.162 \times 10^9} + \frac{(84 - 8.7)(10^3)}{0.34 \times 10^6} \right] = 1.037 \text{ N/mm}^2$$

$$\sigma_C = \left[-\frac{(84 + 8.7)10^6 \times 2.84}{2 \times 0.162 \times 10^9} + \frac{(84 - 8.7)(10^3)}{0.34 \times 10^6} \right] = -0.594 \text{ N/mm}^2$$

Plate-3

$$\sigma_C = \left[\frac{8.7 \times 2 \times 10^3}{0.36 \times 10^6} \right] = 0.049 \text{ N/mm}^2$$

The stresses due to shears in various plates are shown in Fig. 13.7.

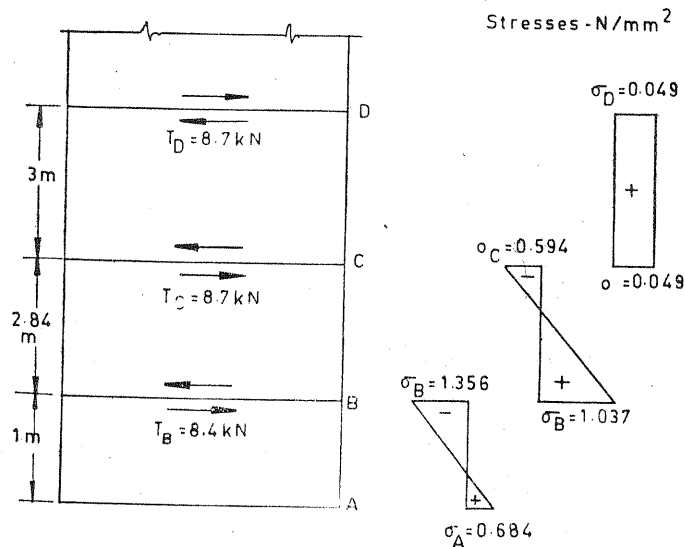


Fig. 13.7 Stresses in folded plates due to joint shear forces

8. Resultant Stresses

The resultant stresses in plates are obtained by super posing the stresses due to plate moments and joint shears.

Plate-1

$$A = (-1.74 + 0.684) = -1.056 \text{ N/mm}^2$$

$$B = (1.74 - 1.356) = +0.384 \text{ N/mm}^2$$

Plate-2

$$B = (-0.65 + 1.037) = +0.387 \text{ N/mm}^2$$

$$C = (+0.65 - 0.594) = +0.056 \text{ N/mm}^2$$

Plate-3

$$C = (0 + 0.049) = +0.049 \text{ N/mm}^2$$

The resultant stresses are shown in Fig. 13.8.

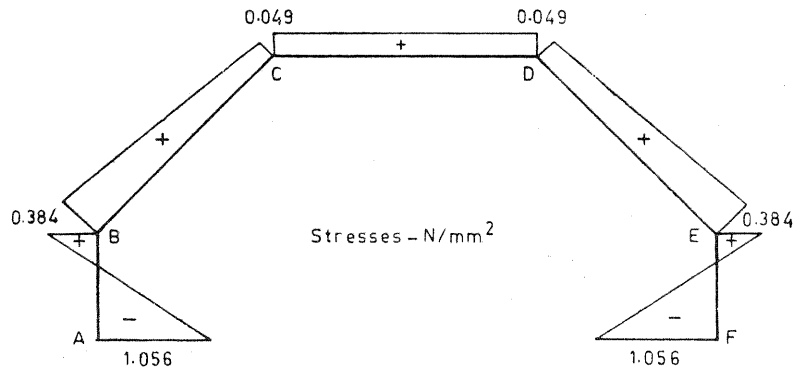


Fig. 13.8 Resultant final stresses in hipped plate

13.13 DESIGN EXAMPLE

Analyse the symmetrical *V*-shaped folded plate shown in Fig. 13.9 and design reinforcements using the following data:

Span of the folded plate = 20 m

Thickness of plates = 100 m

Live load = 0.6 kN/m²

Concrete = M-15 grade

Steel = Tor steel ($f_y = 415 \text{ N/mm}^2$)

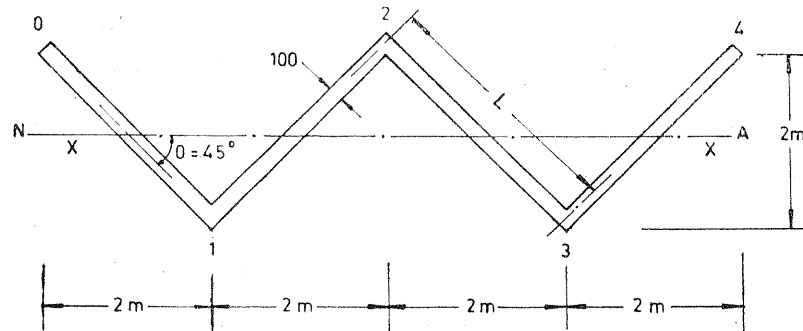


Fig. 13.9 *V*-shaped folded plate

The symmetrical *V*-shaped plate is analysed by using iteration method.

1. Loads

$$\text{Self weight} = (0.1 \times 24) = 2.4 \text{ kN/m}^2$$

$$\text{Live load} = 0.6 \text{ kN/m}^2$$

$$\text{Total load} = w = 3.0 \text{ kN/m}^2$$

2. Fixed End Moments

Considering the transverse section of the folded plate as a continuous beam on rigid supports as shown in Fig. 13.10, the fixed end moments are computed and tabulated as shown below:

Plate	Load (kN/m)	Fixed end Moment (kN/m/m)	Remarks
0-1	3.0	$\frac{wL^3}{2} = 6.00$	Cantilever
1-2	4.0	$\frac{wL^2}{12} = 1.00$	Fixed beam
2-3	3.0	$\frac{wL^2}{12} = 1.00$	Fixed beam
3-4	3.0	$\frac{wL^2}{2} = 6.00$	Cantilever

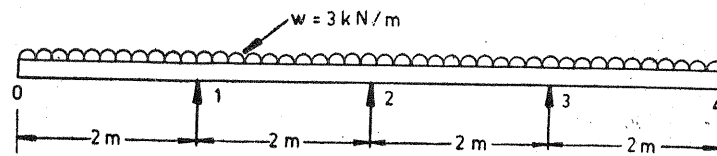


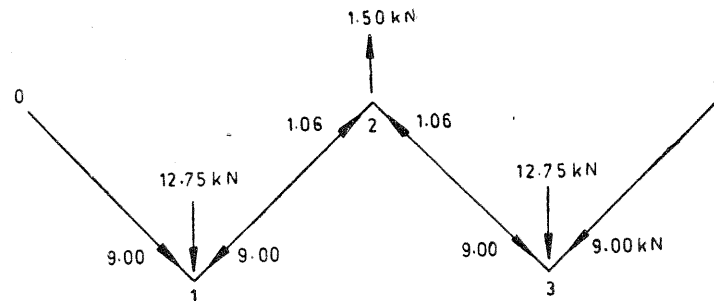
Fig. 13.10 Analysis for slab action

3. Moment Distribution (No-Rotation Solution)

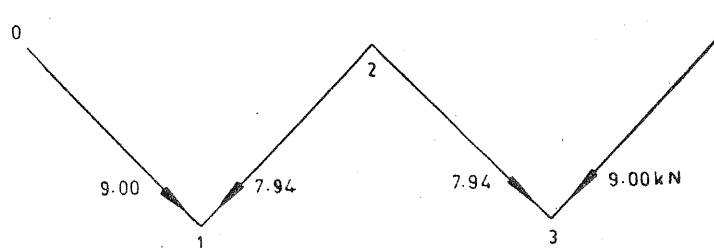
	0	1	0.5	0.5	1	0
Initial Moments (kN.m)	+6.00	-1.00	+1.00	-1.00	+1.00	-6.00
		-5.00 →	-2.50	+2.50 ←	+5.00	
Final Moments (kN.m)	+6.00	-6.00	-1.50	+1.50	+6.00	-6.00
Reaction due to Moments (kN)		↑ 3.75	3.75 ↓	↓ 3.75	3.75 ↑	
Reaction due to Loads	6.00 ↑	↑ 3.00	3.00 ↑	↑ 3.00	3.00 ↑	↑ 6.00
Total reactions		↑ 12.75		↓ 1.50		↑ 12.75
Joint Loads (kN)		↓ 12.75		↑ 1.50		↓ 12.75

4. Plate Loads

The joint loads are resolved into components along the directions of the plates resulting in plate loads as shown in Fig. 13.11.



a) JOINT LOADS



b) PLATE LOADS

Fig. 13.11 Joint loads and plate loads

5. Moments and Stresses in Plates

The longitudinal moments developed in the plates due to the plate loads are computed assuming the plates to bend longitudinally in their own plane between the traverses. The stresses developed at the edges of the plates are compiled in Table 13.1.

Table - 13.1. Longitudinal Stresses

Plate	Load (kN/m) R	Moment (kN.m) (RL^2/δ)	Section Modulus (m^3)	Stresses (N/mm^2)	Free edge stresses (N/mm^2)
0 20m 1 ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓	9.00	450	0.133	3.40	3.4 3.4
1 ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑	7.94	397	0.133	3.00	3.0 3.0
2 3 ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓	7.94	397	0.133	3.00	3.0 3.0
3 ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑	9.00	450	0.133	3.40	3.4 3.4
4					
+ Compression - Tension					

6. Stress Distribution for No-rotation Solution shown in Table 13.2.

TABLE 13.2 Stress Distribution (No-Rotation)

	0	1	2	3	4
Distribution factors		0.5	0.5	0.5	0.5
Free edge	+3.4	-3.4	-3.0	+3.0	+3.0
Stresses (N/mm ²)	-0.1	+0.2	-0.2	+0.1	+0.1
$(\sigma_b - \sigma_t)$	+3.3	-3.2	-3.2	+3.1	+3.1
(N/mm ²)	6.5		6.3	6.3	6.5

7. Plate Deflections due to No-Rotation Solution

The maximum deflection of a plate of length 'L' and depth 'h' can be computed as detailed below:

$$\begin{aligned}
 \text{Deflection } \delta &= \frac{5}{384} \left(\frac{wL^4}{EI} \right) = \frac{5}{48} \left(\frac{wL^2}{8} \right) \frac{L^2}{EI} \\
 &= \frac{5}{48} \left(\frac{\sigma_b - \sigma_t}{2} \right) \cdot Z \cdot \frac{L^2}{EI} \\
 &= \frac{5}{48} \left(\frac{\sigma_b - \sigma_t}{2} \right) \left(\frac{I}{h/2} \right) \left(\frac{L^2}{EI} \right) \\
 &= \frac{5}{48} \frac{(\sigma_b - \sigma_t)L^2}{Eh}
 \end{aligned}$$

The plate deflections and relative displacements of joints are shown in Fig. 13.12.

$$V_1 = V_4 = \left(\frac{5}{48} \right) \frac{(6.5 \times 10^6) 20^2}{2\sqrt{2} \cdot E} = \left(\frac{1 \times 10^8}{E} \right) \text{m}$$

$$V_2 = V_3 = \left(\frac{5}{48} \right) \frac{(6.3 \times 10^6) 20^2}{2\sqrt{2} \cdot E} = \left(\frac{0.9 \times 10^8}{E} \right) \text{m}$$

8. Joint Displacements

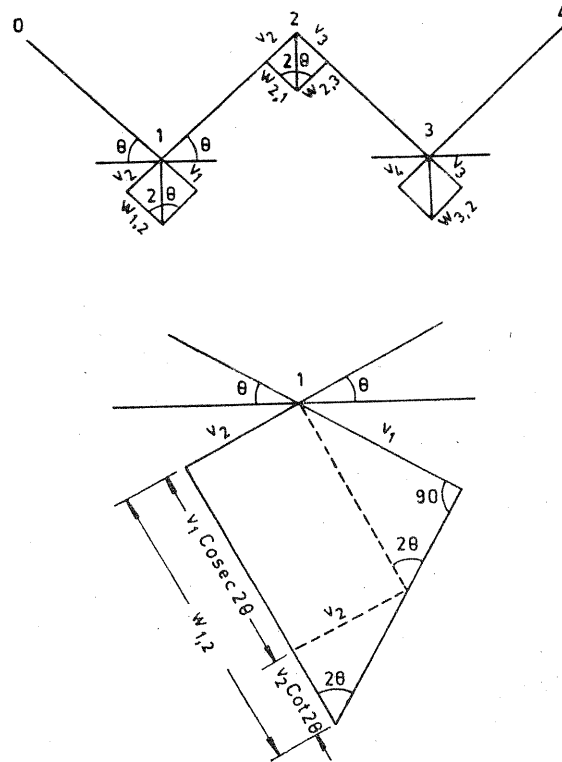
$$w_{1,2} = V_1 \operatorname{cosec} 2\theta + V_2 \cot 2\theta$$

$$\text{since } \theta = 45^\circ \quad w_{1,2} = \left(\frac{1 \times 10^8}{E} \right) \text{m}$$

$$w_{2,1} = V_3 \operatorname{cosec} 2\theta + V_2 \cot 2\theta$$

$$\text{since } V_2 = V_3 \quad \text{and } \theta = 45^\circ \quad w_{2,1} = \left(\frac{0.9 \times 10^8}{E} \right) \text{m}$$

$$\therefore \text{ Transverse Moment} = \left(\frac{6EI_2 \Delta_2}{h_2^2} \right)$$



$$w_{1,2} = v_1 \operatorname{Cosec} 2\theta + v_2 \cdot \cot 2\theta$$

$$w_{2,1} = v_3 \operatorname{Cosec} 2\theta + v_2 \cdot \cot 2\theta$$

Fig. 13.12 Relative displacements of joints

$$\Delta_2 = (w_{1,2} - w_{2,1}) = \left(\frac{0.1 \times 10^8}{E} \right) \text{ m}$$

For plate-2, Transverse moment is given by

$$\left(\frac{6EI_2 \Delta_2}{h_2^2} \right) = \left[\frac{6 \times E \times 1 \frac{(0.1)^3}{12} \times \frac{0.1 \times 10^8}{E}}{(2\sqrt{2})^2} \right] = 625 \text{ N} \cdot \text{m} = 0.625 \text{ kN} \cdot \text{m}$$

Similarly for plate-3, by symmetry

$$\text{The transverse moment} = \left(\frac{6EI_3 \Delta_3}{h_3^2} \right)$$

$$\text{where } \Delta_3 = (w_{2,3} - w_{3,2}) = - \left(\frac{0.1 \times 10^8}{E} \right) \text{ m}$$

$$\therefore \text{ Transverse Moment} = -0.625 \text{ kN} \cdot \text{m}$$

9. Determination of Joint Loads

Using these moments, we can determine the joint loads developed by the distribution of moments as shown in Table 13.3.

TABLE 13.3 Distribution of Moment caused by joint Displacements
(First cycle of Iteration)

Joints	1	2	3
Distribution factor	0.1	0.2	0.2
Initial moments (kN·m)	+0.625	+0.625	-0.625
Resultant moments	-0.625	-0.312	+0.625
Reactions	0	+0.312	-0.312
Loads (kN)	0.156	0.156	0.156

The joint loads and plate loads are shown in Fig. 13.13.

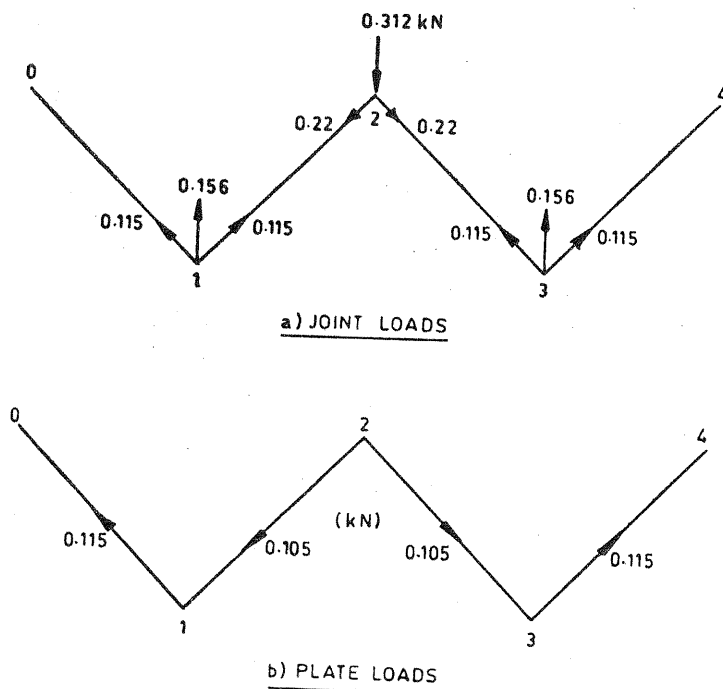


Fig. 13.13 Plate loads due to joint displacements

10. Longitudinal Stresses

The longitudinal stresses developed due to these plate loads are shown in Table 13.4.

TABLE 13.4 Longitudinal Stresses

Plate	Load (kN/m) (R)	Moment (kN.m) ($RL^2/8$)	Section Modulus (m^3)	Stresses (N/mm^2)	Free edge stresses (N/mm^2)
0					
1	0.115	5.75	0.133	0.043	0.043
1					0.043
2	0.105	5.25	0.133	0.039	0.039
2					0.039
3	0.105	5.25	0.133	0.039	0.039
3					0.039
4	0.115	5.75	0.133	0.043	0.043
4					0.043
+ Compression - Tension					

11. Stress Distribution

The stresses obtained in Table 13.4, are distributed as shown in Table 13.5.

TABLE 13.5 Stresses-Distribution

Joints	0	1	2	3	4
Free edge Stresses (N/mm^2)	-0.043	+0.043	-0.039	+0.039	-0.043
	+0.020 ←	-0.041	+0.020 ←	-0.020	+0.02
Resultant stresses	-0.023	+0.002	+0.002	+0.019	+0.019
				+0.002	+0.002
					-0.023

12. Final Moments and Stresses

The final moments and stresses are compiled in Tables 13.6 and 13.7 respectively.

TABLE 13.6 Final Transverse Moments (Values in kN.m)

Joint	No rotation	First cycle of Iteration	Final moments
0	0	0	0
1	+6.00	0	+6.000
2	-1.50	+0.312	-1.188
3	+6.00	0	+6.00
4	0	0	0

TABLE 13.7 Final Longitudinal Stresses
(Values in N/mm²) + Compression – Tension

Joint	No rotation	First cycle of Iteration	Final stresses
0	+3.3	-0.023	+3.277
1	-3.2	+0.002	-3.198
2	+3.1	+0.019	+3.119
3	-3.2	+0.002	-3.198
4	+3.3	-0.023	+3.277

13. Design of Reinforcements

The design of reinforcements in the folded plate should conform to the recommendations made in IS: 2210-1962. The transverse reinforcement is designed to resist the moments in the transverse direction computed at mid span section. For greater economy, the spacing of the bars may be increased from mid span to the support section based on the magnitude of the transverse bending moment.

The longitudinal reinforcements in the direction of the span are designed to resist the total tensile force developed in the tension zone. However minimum percentage of reinforcements as recommended in the codes should be provided at all cross sections.

The shear reinforcements are designed to resist the principal tensile stresses. When principal tensile stress exceeds $1.7\sqrt{f_{ck}}$, reinforcement will be necessary to take up the entire tension; elsewhere, nominal reinforcement at spacing not exceeding five times the thickness of the plate is to be provided. The maximum shear stresses are likely to develop at quarter span sections.

(a) Transverse Reinforcement

At joints 1 and 3, the transverse moments cause tension at top of slabs.

Thickness of folded plate slab = 100 mm

Assuming effective cover = 25 mm

Effective depth $d = 75$ mm

Maximum moment = 6 kN·m

$$\therefore A_{st} = \left(\frac{6 \times 10^6}{230 \times 0.9 \times 75} \right) = 386 \text{ mm}^2$$

Provide 10 mm diameter bars at 200 mm centres.

At joint-2, the transverse moment causes tension at bottom of slab

$$\therefore A_{st} = \left(\frac{1.5 \times 10^6}{200 \times 0.9 \times 75} \right) = 97 \text{ mm}^2$$

But minimum percentage of steel is

$$\therefore 0.3\% = \left(\frac{0.3}{100} \times 1000 \times 100 \right) = 300 \text{ mm}^2$$

Provide 10 mm diameters at 200 mm centres.

(b) *Longitudinal Reinforcement*

The maximum tensile stresses develop at joints 1 and 3. Hence considering plate-1 as shown in Fig. 13.14.

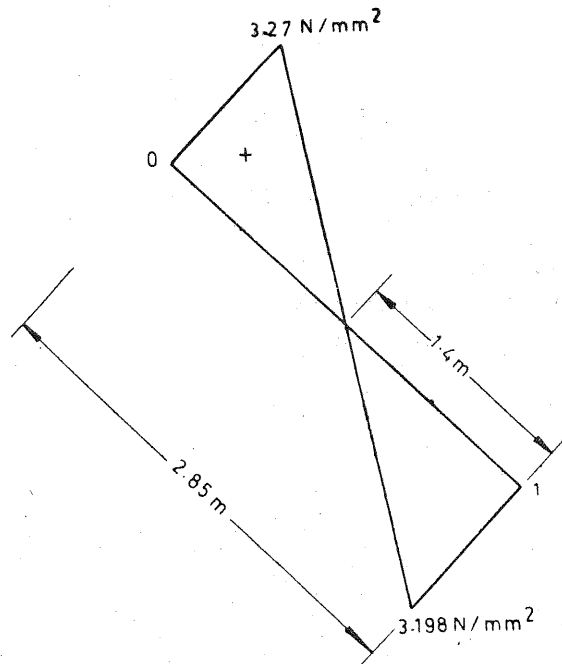


Fig. 13.14 Stress distribution in folded plate

Reinforcement in the tension zone is

$$A_{st} = \left[\frac{(1.4 \times 10^3 \times 100) \times \left(\frac{3.198}{2} \right)}{230} \right] = 973 \text{ mm}^2$$

Provide 5 bars of 16 mm diameter in the tension zone as detailed in Fig. 13.15. For the compression zone portion, provide 10 mm diameter bars at 200 mm centres.

13.14 DESIGN EXAMPLE

Analyse the symmetrical V-shaped folded plate of 20 m span given in design Example 13.13 by the beam method and compare the results of stresses obtained by the iteration and beam methods.

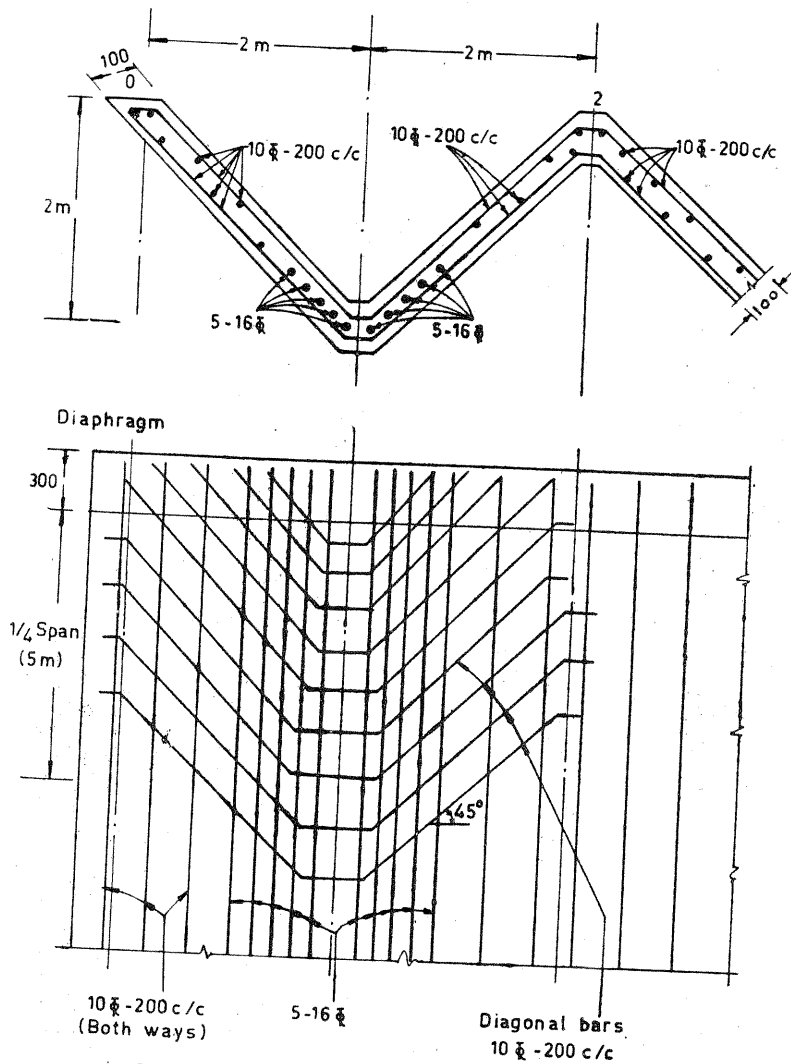


Fig. 13.15 Details of reinforcements in folded plate

1. Data

Span = $L = 20$ m

Depth of plate = $2\sqrt{2}$ m

Thickness = $t = 100$ mm

Live load = 0.6 kN/m²

2. Section Properties

For the cross section of the V-shaped folded plate shown in Fig. 13.16, the second moment of area is calculated as shown below.

For each plate inclined at an angle θ to axis $x-x$

$$I_{xx} = \left[\frac{2 \cdot t \sin^2 \theta (L/2)^3}{3} \right]$$

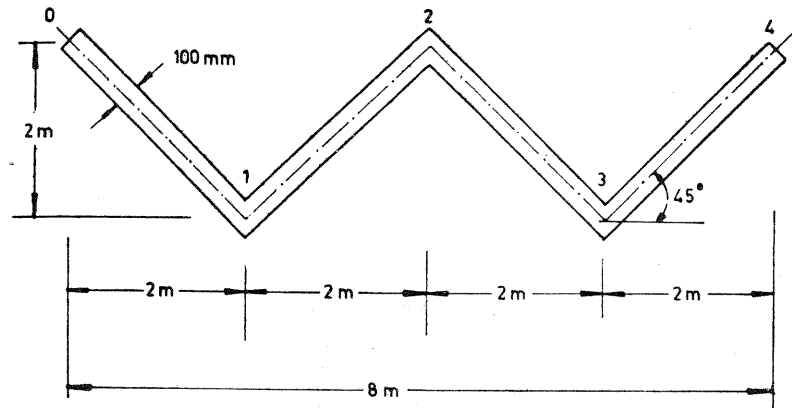


Fig. 13.16 Cross-section of folded plate

where t = thickness of the plate

L = Length of the plate

θ = Angle made by the plate with the horizontal axis $x-x$

The second moment of area of the cross section with four plates is given by

$$I_{xx} = 4 \left[\frac{2 \times 0.1 \times \sin^2 45^\circ \times \frac{(2\sqrt{2})^3}{2}}{3} \right] = 0.38 \text{ m}^4$$

3. Loads

$$\text{Dead load} = (0.1 \times 24 \times 4 \times 2\sqrt{2}) = 27.2 \text{ kN/m}$$

$$\text{Live load} = (4 \times 2 \times 0.6) = 4.8$$

$$\text{Total load} \quad w = 32.0 \text{ kN/m}$$

3. Bending Moment

$$M_{\max} = \left(\frac{wL^2}{8} \right) = \left(\frac{32 \times 20^2}{8} \right) = 1600 \text{ kN} \cdot \text{m}$$

4. Longitudinal Stresses

Bending stress

$$\sigma = \left(\frac{M \cdot y}{I} \right) = \left(\frac{1600 \times 10^6 \times 1000}{0.38 \times 10^{12}} \right) = 4.2 \text{ N/mm}^2$$

The maximum stresses developed in the plate are 4.2 N/mm². (Compression at points 0, 2 and 4 and tension at points 1 and 3).

5. Comparison of Stresses

(stresses in N/mm^2) + Compression – Tension)

Joint	Iteration Method	Beam method
0	+3.277	+4.20
1	-3.198	-4.20
2	+3.119	+4.20
3	-3.198	-4.20
4	+3.277	+4.20

The beam method results in prediction of stress values nearly 30 per cent higher than those obtained by the iteration method.

13.15 EXAMPLES FOR PRACTICE

- Analyse the folded plate *ABCDEF* using the theorem of the three edge shears. The plates *AB* and *EF* are vertical, while the plate *CD* is horizontal. The plates *BC* and *DE* are inclined at 45° to the horizontal. All the plates are 12 cm thick and their widths are as follows:
 AB and $EF = 1.25$ m
 BC and $DE = 3.90$ m
 $CD = 3.00$ m
 Live load = 0.6 kN/m^2 of covered area
 Distance between the traverses = 10 m.
 Draw the stress distribution diagram for the plates. In what way the stresses differ if the hipped plate structure is analysed as a beam spanning horizontally between the traverses.
- A *V*-shaped folded plate *ABCDE* spans over 12 m between the traverses and the folds have a uniform thickness of 100 mm. The plates are inclined at 45° to the horizontal and vertical and horizontal projections of each of the plates is 3 m. Edges *B* and *D* are the ridges and the edge *C* form the valley portion. The folded plate is to be analysed for incidental live loads of 0.75 kN/m^2 of covered area. Analyse the stress distribution for the section at centre of span using the following methods.
 (a) Theorem of three edge shears
 (b) Stresses distribution procedure (No rotation solution)
 Draw the stress diagrams.
- Design a folded plate roof with *V*-shaped units to cover a store house 9 m wide by 20 m long. Six plates, each having a thickness of 120 mm may be used with plates inclined at 45° to the horizontal. The vertical and horizontal projections of the plates = 1.5 m. Live load = 0.75 kN/m^2 . Analyse the folded plate using the iteration method and design the longitudinal and transverse reinforcements in the plate. Adopt M-20 grade concrete and tor steel reinforcements. Compare the stresses developed at the centre of span with those resulting from the beam method.

4. Analyse the hipped plate roof of a small single storey industrial building shown in Fig. 13.17, using the theorem of three edge shears.

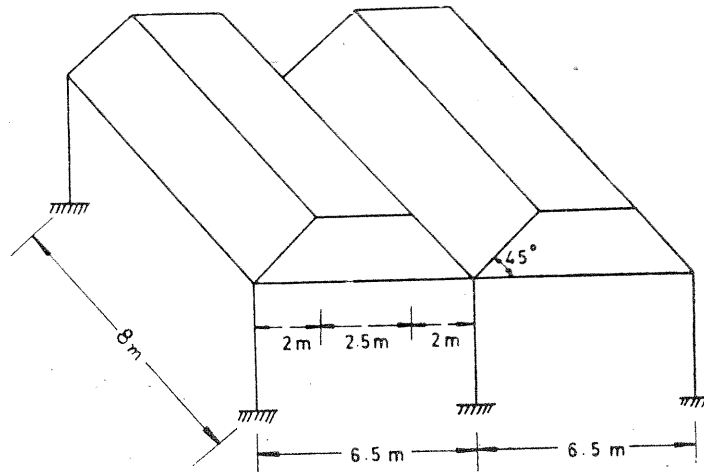


Fig. 13.17 Hipped plate roof

Thickness of plates = 125 mm

Live load = 0.75 kN/m^2

Design suitable reinforcements and sketch the details of transverse and longitudinal reinforcements in the folded plate roof. Adopt M-20 grade concrete and tor steel reinforcements.

5. Design the folded plate roof shown in Fig. 13.18, using Simpson's method using the following data.

Span = 20 m

Live load = 0.75 kN/m^2

Adopt M-20 grade concrete and Fe-415 grade tor steel. Sketch the details of reinforcements in the folded plate.

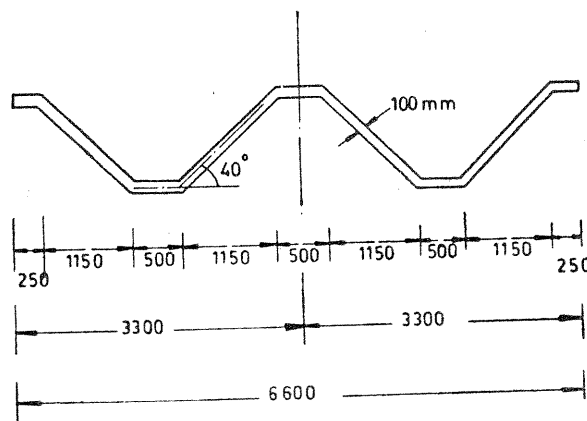


Fig. 13.18 Trough shaped folded plate

14

Grid or Coffered Floors

14.1 GENERAL FEATURES

Grid or coffered floor systems consisting of beams spaced at regular intervals in perpendicular directions, monolithic with a slab are generally employed for architectural reasons for large rooms such as auditoriums, vestibules, theatre halls, show rooms of shops where column free space is often the main requirement.

The rectangular or square voids formed in the ceiling is advantageously utilised for concealed architectural lighting. The sizes of the beams running in perpendicular directions is generally kept the same. Instead of rectangular beam grid, a diagonal grid can also be used with the beams inclined at 45° to the sides. The different types of grid floors which are commonly used are shown in Fig. 14.1.

14.2 ANALYSIS OF GRID FLOORS

(a) *Approximate Methods*

According to the Indian Standard Code IS: 456-1978, the ribbed slab system can be analysed as a solid slab, if the following requirements regarding spacing of beams, thickness of slab and edge beams are satisfied.

1. The spacing of the ribs should not be greater than 1.5 m and it should not be greater than 12 times the flange thickness.
2. In situ ribs should not be less than 65 mm wide.
3. The ribs should be formed along each edge parallel to the span, having a width equal to that of the bearing.

The moments and shears per unit width of grid are determined from Table 22 of IS : 456 code and the reinforcements are designed in the ribs.

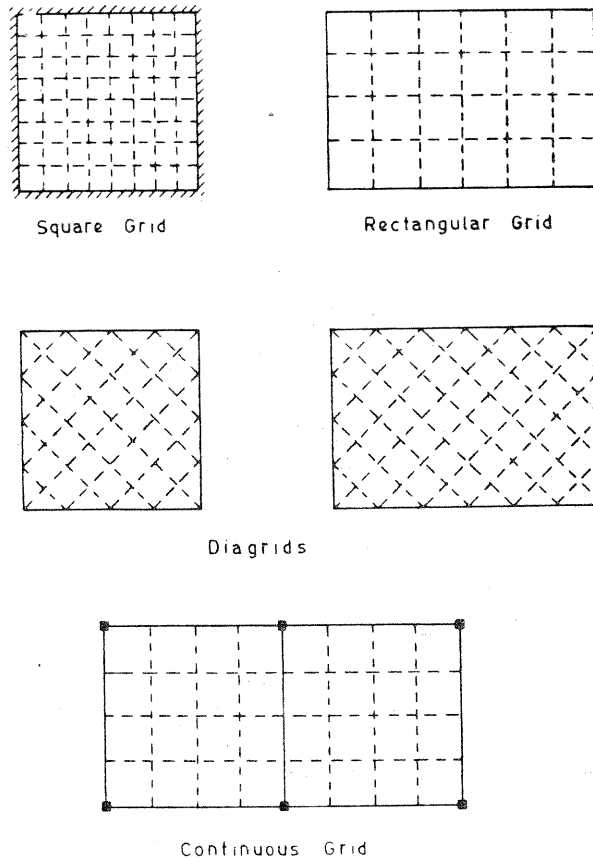


Fig. 14.1 Types of grid-floors

The slab reinforcement generally consists of a mesh or fabric. A second approximate method which is applicable to the grid floor system is the Rankine Grashoff theory of equating deflections at the junctions of ribs.

Consider a grid floor shown in Fig. 14.2 in which the spacings of the ribs are a_1 and b_1 in the x and y directions respectively.

Let q = Total load per unit area.

q_1 and q_2 = The loads shared in the x and y directions

a = Shorter dimension of grid

b = Longer dimension of grid

The deflections of the ribs AB and CD at the junction O must be the same and by equating the deflections, we have

$$\delta = \left(\frac{5q_1a^4}{384EI} \right) = \left(\frac{5q_2b^4}{384EI} \right)$$

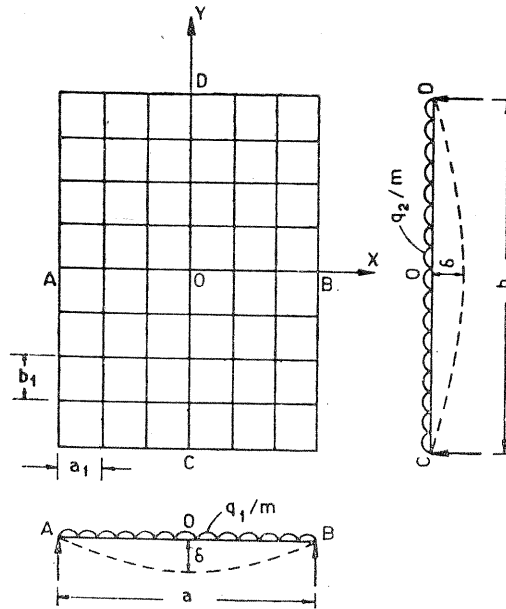


Fig. 14.2 Deflections of ribbed slabs

$$\therefore q_1 a^4 = q_2 b^4 \quad (1)$$

$$\text{Also } q = q_1 + q_2 \quad (2)$$

Solving Eqs. (1) and (2), we have

$$q_1 = q \left(\frac{b^4}{a^4 + b^4} \right) \text{ and } q_2 = q \left(\frac{a^4}{a^4 + b^4} \right)$$

The bending moments for the central ribs are given by

$$M_{AB} = \left(\frac{q_1 b_1 a^2}{8} \right) \quad M_{BD} = \left(\frac{q_2 a_1 b^2}{8} \right)$$

The bending moments in the other ribs can also be determined in direct proportion to their distances from the centre. The ribs are designed as flanged sections to resist the moments and shears. However the approximate methods do not yield the twisting moments in the beams. For small span grids with spacings of ribs not exceeding 1.5 metres, approximate methods can be used, but for grids of larger spans with spacings of ribs exceeding 1.5 m, a rigorous analysis based on orthotropic plate theory is generally used.

(b) Analysis of Grid Floor by Plate Theory

A reinforced concrete grid floor with ribs at close intervals in two mutually perpendicular directions connected by slab in between in the ribs can be considered as an orthotropic plate freely supported on four sides.

Timoshenko's analysis may be used to evaluate the moments and shears in the grid which depend upon the deflection surface.

The vertical deflection 'a' at any point of the grid shown in Fig. 14.3 is expressed as,

$$a = \frac{16q}{\pi^6} \left[\frac{\sin\left(\frac{\pi x}{a_x}\right) \sin\left(\frac{\pi y}{b_y}\right)}{\frac{D_x}{a_x^4} + \frac{2H}{a_x^2 b_y^2} + \frac{D_y}{b_y^4}} \right]$$

where q = total uniformly distributed load per unit area

a_x, b_y = Length of plate in x and y directions respectively

D_x, D_y = Flexural rigidity per unit length of plate along x and y direction

C_x, C_y = Torsional rigidity per unit length of plate along x and y direction.

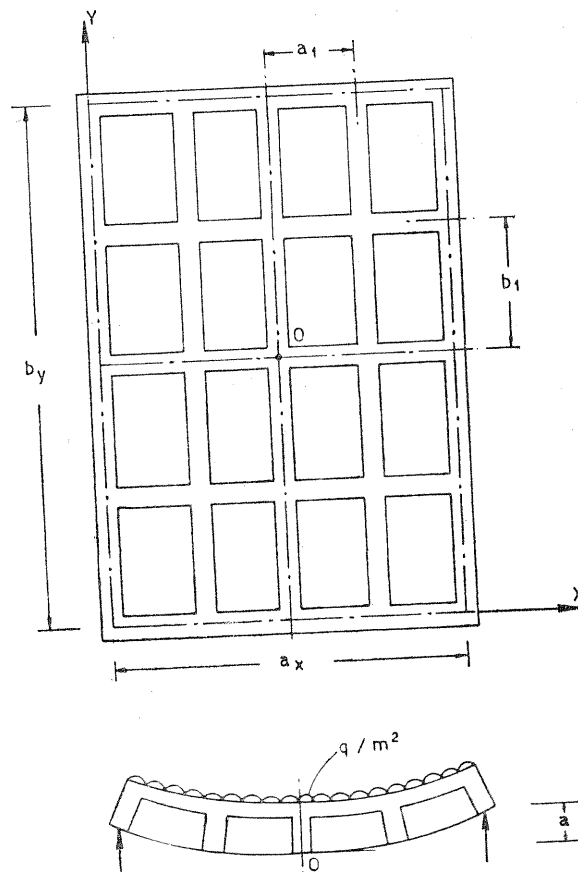


Fig. 14.3 Deflection characteristics of grid floors

If a_1 and b_1 are the spacings of the ribs in x and y directions respectively, then we have the relations,

$$D_x = (EI_1/b_1) \quad C_x = (C_1/b_1)$$

$$D_y = (EI_2/a_1) \quad C_y = (C_2/a_1)$$

where EI_1 , EI_2 , C_1 and C_2 are the flexural and torsional rigidities of the effective section in x and y directions. The moments and shears are computed using the following expressions:

$$M_x = -D_x \left(\frac{\partial^2 a}{\partial x^2} \right) \quad M_y = -D_y \left(\frac{\partial^2 a}{\partial y^2} \right)$$

$$T_{xy} = -\frac{C_1}{b_1} \left(\frac{\partial^2 a}{\partial x \cdot \partial y} \right) \quad T_{yx} = -\frac{C_2}{a_1} \left(\frac{\partial^2 a}{\partial x \cdot \partial y} \right)$$

$$Q_x = -\frac{\partial}{\partial x} \left[D_x \left(\frac{\partial^2 a}{\partial x^2} \right) + \frac{C_2}{a_1} \left(\frac{\partial^2 a}{\partial x \cdot \partial y} \right) \right]$$

$$Q_y = -\frac{\partial}{\partial y} \left[D_y \left(\frac{\partial^2 a}{\partial y^2} \right) + \frac{C_1}{b_1} \left(\frac{\partial^2 a}{\partial x \cdot \partial y} \right) \right]$$

Maximum bending moments develop at centre of span while maximum torsional moments are generated at the corners of the grid and maximum shear forces develop at mid points of longer side supports.

14.3 DESIGN EXAMPLE

A reinforced concrete grid floor is to be designed to cover a floor area of size 12 m by 16 m. The spacings of the ribs in mutually perpendicular directions being 2 m c/c. Live load = 1.5 kN/m². Adopt M-20 grade concrete and tor steel. Analyse the grid floor for moments and shears using.

- (a) Approximate method (Rankine Grashoff theory)
- (a) Rigorous analysis (plate theory)

Design suitable reinforcements at critical sections.

1. Data

Size of grid = 12 m by 16 m
 Spacings of ribs = 2 m c/c
 Concrete = M-20 grade
 Steel = Ribbed tor steel Fe-415.

2. Dimensions of Slab and Beams

Adopt thickness of slab = 100 mm

Depth of ribs based on $\left(\frac{\text{span}}{\text{depth}} \right) = 20$ is obtained as depth

$$= \left(\frac{12 \times 10^3}{20} \right) = 600 \text{ mm}$$

Width of rib = 200 mm

Adopt over all depth of ribs = 600 mm

Figure 14.4, shows the details of the grid and the section of the ribs in both directions.

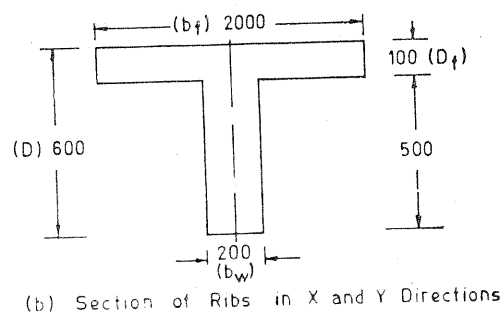
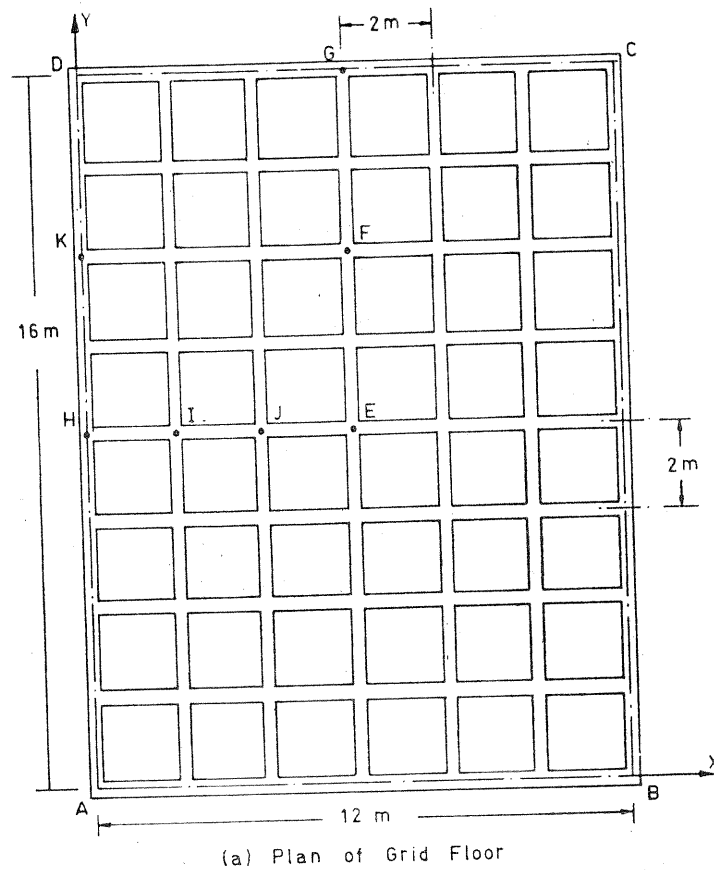


Fig. 14.4 Reinforced concrete grid floor

3. Loads

$$\text{Weight of slab} = (0.1 \times 24) = 2.4 \text{ kN/m}^2$$

$$\text{Total load of slab} = (2.4 \times 16 \times 12) = 460.8 \text{ kN}$$

$$\text{Weight of ribs} = (0.2 \times 0.5 \times 24) = 2.4 \text{ kN/m}$$

$$\text{Total weight of beams (x-direction)} = (7 \times 2.4 \times 12) = 201.6 \text{ kN}$$

$$\text{Total weight of beams (y-direction)} = (5 \times 2.4 \times 14.6) = 175.2 \text{ kN}$$

$$\text{Total weight of floor finish} = (0.6 \times 12 \times 16) = 115.2 \text{ kN}$$

$$\text{Total live load} = (1.5 \times 12 \times 16) = 288 \text{ kN}$$

$\therefore$ Total dead and live loads on grid floor

$$= (460.8 + 201.6 + 175.2 + 115.2 + 288.0) = 1240.8 \text{ kN}$$

$$\text{Load per } m^2 = q = \left(\frac{1240.8}{16 \times 12} \right) = 6.5 \text{ kN/m}^2$$

4. Approximate Method (Moments)

If q_1 and q_2 are the loads shared in the x and y directions.

$$q_1 = q \left(\frac{b_y^4}{a_x^4 + b_y^4} \right) = 6.5 \left(\frac{16^4}{12^4 + 16^4} \right) = 5 \text{ kN/m}^2$$

$$q_2 = q \left(\frac{a_x^4}{a_x^4 + b_y^4} \right) = 6.5 \left(\frac{12^4}{12^4 + 16^4} \right) = 1.5 \text{ kN/m}^2$$

Moments in x and y directions at centre of grid for 2 m width is taken as,

$$M_x = \left(\frac{q_1 b_1 a^2}{8} \right) = \left(\frac{5 \times 2 \times 12^2}{8} \right) = 180 \text{ kN} \cdot \text{m}$$

$$M_y = \left(\frac{q_2 a_1 b^2}{8} \right) = \left(\frac{1.5 \times 2 \times 16^2}{8} \right) = 96 \text{ kN} \cdot \text{m}$$

$$Q_x = \left(\frac{5 \times 2 \times 12}{2} \right) = 60 \text{ kN}$$

$$Q_y = \left(\frac{1.5 \times 2 \times 16}{2} \right) = 24 \text{ kN}$$

The approximate methods neglect the twisting moments in the grid

5. Rigorous Method (Plate theory)

(a) Section Properties

$$\left(\frac{D_f}{D} \right) = \left(\frac{100}{600} \right) = 0.166$$

$$\left(\frac{b_w}{b_f} \right) = \left(\frac{200}{2000} \right) = 0.10$$

From design table (Reynolds R.C. designers hand book)

$$I = C \cdot b_w D^3$$

The value of constant $C = 0.191$

$$\therefore I = (0.191 \times 200 \times 600^3) = 83 \times 10^8 \text{ mm}^4$$

If I_1 and I_2 are the second moment of area of the tee-sections about their centroidal axis in the x and y directions respectively,

$$D_1 = EI_1$$

$$D_2 = EI_2$$

In the present case $I_1 = I_2 = I$, and $a_1 = b_1 = 2 \text{ m}$

$$\therefore D_x = D_y = \left(\frac{83 \times 10^8 \times E}{2 \times 10^{12}} \right) = 0.00415E$$

The torsional rigidity in the x and y directions are given by,

$$C_1 = C_2 = k_1 G \cdot (2a)^3 2b$$

where $2a = 200 \text{ mm}$

$2b = 600 \text{ mm}$

$$\text{For } (b/a) = \left(\frac{300}{100} \right) = 3, \quad k_2 = 0.263$$

(From Tables in Timoshenko theory of elasticity)

$$\begin{aligned} \therefore C_1 = C_2 &= k_1 \left[\frac{E}{2(1+\mu)} \right] (2a)^3 2b \\ &= \left[\frac{0.263 \times E \times (200)^3 (600)}{2(1+0.15)} \right] = 5.488 \times 10^8 E \text{ mm}^4 = 0.0005488 E \text{ m}^4 \\ 2H \left[\frac{C_1}{b_1} + \frac{C_2}{a_1} \right] &= E \left[\frac{0.0005488}{2} + \frac{0.0005488}{2} \right] 0.0005488 E \end{aligned}$$

(b) Deflections at Centre of Span

$$q = 6.5 \text{ kN/m}^2, \quad E = 5700 \sqrt{f_{ck}} = 25.49 \times 10^6 \text{ kN/m}^2$$

$$\left(\frac{D_x}{a_x^4} \right) = \left(\frac{0.00415E}{12^4} \right) = \left(\frac{0.00415 \times 25.49 \times 10^6}{12^4} \right) = 5.089$$

$$\left(\frac{D_y}{a_y^4} \right) = \left(\frac{0.00415E}{16^4} \right) = \left(\frac{0.00415 \times 25.49 \times 10^6}{16^4} \right) = 1.61$$

$$\left(\frac{2H}{a_x^2 b_y^2} \right) = \left(\frac{0.0005488E}{12^2 \times 16^2} \right) = \left(\frac{0.0005488 \times 25.49 \times 10^6}{12^2 \times 16^2} \right) = 0.3794$$

The deflection at the centre of the plate is given by the equation,

$$a = \frac{16q}{\pi^6} \left[\frac{\sin \frac{\pi x}{a} \sin \frac{\pi y}{b}}{\frac{D_x}{a_x^4} + \frac{2H}{a_x^2 b_y^2} + \frac{D_y}{b^4}} \right]$$

$$= \left(\frac{16 \times 6.5}{963} \right) \left[\frac{1}{5.089 + 0.3794 + 1.61} \right] = 0.015 \text{ m}$$

Assuming a creep coefficient $\theta = 2$

The modified modulus of elasticity

$$E_{ce} = \left(\frac{E_c}{1 + \theta} \right) = \left(\frac{E_c}{1 + 2} \right) = \left(\frac{E_c}{3} \right)$$

$$\therefore \text{Long term deflection} = (0.015 \times 3) = 0.045 \text{ m}$$

According to IS: 456-1978, the long term deflection should not exceed $\left(\frac{\text{span}}{250} \right)$

$$\begin{aligned} \text{Hence maximum permissible long term deflection} &= \left(\frac{12}{250} \right) \\ &= 0.048 \text{ m} > 0.045 \text{ m} \end{aligned}$$

The maximum deflection including long term effects is within permissible limits.

(c) Design Moments and Shears

The bending moments, torsional moments and shears at various salient points E, F, G, H, I, J, K and D marked in Fig. 14.4, are computed using the equations given below:

$$\begin{aligned} M_x &= -D_x \left(\frac{\partial^2 a}{\partial x^2} \right) = D_x \left(\frac{\pi}{a_x} \right)^2 \frac{16q}{\pi^6} \left[\frac{\sin \frac{\pi x}{a_x} \sin \frac{\pi y}{b_y}}{\frac{D_x}{a_x^4} + \frac{2H}{a_x^2 b_y^2} + \frac{D_y}{b_y^4}} \right] \\ &= 0.00415 \times 25.49 \times 10^6 \left(\frac{\pi}{12} \right)^2 (0.015) \sin \frac{\pi x}{a_x} \sin \frac{\pi y}{b_y} \\ &= 108 \left(\sin \frac{\pi x}{a_x} \right) \left(\sin \frac{\pi y}{b_y} \right) \end{aligned}$$

$$\begin{aligned} M_y &= -D_y \left(\frac{\partial^2 a}{\partial y^2} \right) = D_y \left(\frac{\pi}{b_y} \right)^2 \frac{16q}{\pi^6} \left[\frac{\sin \frac{\pi x}{a_x} \sin \frac{\pi y}{b_y}}{\frac{D_x}{a_x^4} + \frac{2H}{a_x^2 b_y^2} + \frac{D_y}{b_y^4}} \right] \\ &= 0.00415 \times 25.49 \times 10^6 \left(\frac{\pi}{16} \right)^2 (0.015) \left(\sin \frac{\pi x}{a_x} \right) \left(\sin \frac{\pi y}{b_y} \right) \\ &= 61 \left(\sin \frac{\pi x}{a_x} \right) \left(\sin \frac{\pi y}{b_y} \right) \end{aligned}$$

$$M_{xy} = - \left(\frac{C_1}{b_1} \right) \cdot \left(\frac{\partial^2 a}{\partial x \cdot \partial y} \right) = - \frac{C_1 (\pi^2)}{b_1 (ab)} \frac{16q}{\pi^6} \left[\frac{\cos \frac{\pi x}{a_x} \cos \frac{\pi y}{b_y}}{\frac{D_x}{a_x^4} + \frac{2H}{a_x^2 b_y^2} + \frac{D_y}{b_y^4}} \right]$$

$$\begin{aligned}
&= -0.0002744 \times 25.49 \times \frac{10^6 \times \pi^2}{12 \times 16} \times 0.015 \left(\cos \frac{\pi x}{a_x} \right) \left(\cos \frac{\pi y}{b_y} \right) \\
&= -5.3 \left(\cos \frac{\pi x}{a_x} \right) \left(\cos \frac{\pi y}{b_y} \right) \\
M_{yx} &= - \left(\frac{C_2}{a_1} \right) \left(\frac{\partial^2 a}{\partial x \cdot \partial y} \right) \\
&= - \frac{C_2}{a_1} \left(\frac{\pi^2}{ab} \right) \frac{16q}{\pi^6} \left[\frac{\cos \frac{\pi x}{a_x} \cos \frac{\pi y}{b_y}}{\frac{D_x}{a_x^4} + \frac{2H}{a_x^2 b_y^2} + \frac{D_y}{b_y^4}} \right] \\
&= -0.000274 \times 25.49 \times 10^6 \times \frac{\pi^2}{12 \times 16} (0.015) \left(\cos \frac{\pi x}{a_x} \right) \left(\cos \frac{\pi y}{b_y} \right) \\
&= -5.3 \left(\cos \frac{\pi x}{a_x} \right) \left(\cos \frac{\pi y}{b_y} \right) \\
Q_x &= - \frac{\partial}{\partial x} \left[D_x \frac{\partial^2 a}{\partial x^2} + \frac{C_2}{a_1} \frac{\partial^2 a}{\partial y^2} \right] \\
&= - \frac{16q}{\pi^6} \left[\frac{\cos \frac{\pi x}{a_x} \sin \frac{\pi y}{b_y}}{\frac{D_x}{a_x^4} + \frac{2H}{a_x^2 b_y^2} + \frac{D_y}{b_y^4}} \right] \left[D_x \frac{\pi^3}{a_x^3} + \frac{C_2}{a_1} \frac{\pi^3}{a_x b_y^2} \right] \\
&= -0.015 \left[(0.00415 \times 25.49 \times 10^6 \times \frac{\pi^3}{12^3} \right. \\
&\quad \left. + (0.0002744 \times 25.49 \times 10^6 \times \frac{\pi^3}{12 \times 16^2}) \right] \left(\cos \frac{\pi x}{a_x} \right) \left(\sin \frac{\pi y}{b_y} \right) \\
&= -10.107 \left(\cos \frac{\pi x}{a_x} \right) \left(\sin \frac{\pi y}{b_y} \right) \\
Q_y &= - \frac{\partial}{\partial y} \left[D_y \frac{\partial^2 a}{\partial y^2} + \frac{C_1}{b_1} \frac{\partial^2 a}{\partial x^2} \right] \\
&= - \frac{16q}{\pi^6} \left[\left(\sin \frac{\pi x}{a_x} \right) \left(\cos \frac{\pi y}{b_y} \right) \right] \left[D_y \frac{\pi^3}{b_y^3} + \frac{C_1}{b_1} \frac{\pi^3}{b_y a_x^2} \right] \\
&= -0.015 \left[(0.00415 \times 25.49 \times 10^6 \times \frac{\pi^3}{16^3} \right. \\
&\quad \left. + (0.0002744 \times 25.49 \times 10^6 \times \frac{\pi^3}{16 \times 12^2}) \right] \left(\sin \frac{\pi x}{a_x} \right) \left(\cos \frac{\pi y}{b_y} \right) \\
&= -13.387 \left(\sin \frac{\pi x}{a_x} \right) \left(\cos \frac{\pi y}{b_y} \right)
\end{aligned}$$

Using these moment and shear equations, the values of bending moment, twisting moment and shear forces are computed at the various salient points of the grid and tabulated as shown in Table 14.1.

TABLE 14.1 Moments and Shear Forces Per Metre Width of Grid

Point	x (m)	y (m)	M_x (kN·m)	M_y (kN·m)	M_{xy} (kN·m)	M_{yx} (kN·m)	Q_x (kN)	Q_y (kN)
E	6	8	108	61	0	0	0	0
F	6	12	77	43	0	0	0	9.4
G	6	16	0	0	0	0	0	13.4
H	0	8	0	0	0	0	10.1	0
I	2	8	54	30.5	0	0	8.74	0
J	4	8	94	53	0	0	5.05	0
K	0	12	0	0	3.74	3.74	7.14	0
D	0	16	0	0	5.30	5.30	0	0

6. Comparison of Moments

The moments per metre width computed by the rigorous and approximate methods are shown in Table 14.2.

TABLE 14.2 Comparison of Maximum Moments in Grid

Method	Moments per metre width (kN·m)	
	M_x	M_y
Approximate method (Rankine Grashoff theory)	90	48
Rigorous analysis (plate theory)	108	77

The approximate method under estimates the bending moments, developed in x - and y -directions, to the extent of 17% and 38% respectively. The moments are very much under estimated in the long span direction.

7. Design of Reinforcements

Maximum working moment $M_w = 108 \text{ kN·m/m}$

Moment resisted by central rib in x -direction over 2 m width

$$= (2 \times 108) = 216 \text{ kN·m}$$

Ultimate moment $M_u = (1.5 \times 216) = 324 \text{ kN·m}$

Moment capacity of flange section

$$\begin{aligned}
 M_{uf} &= 0.36 f_{ck} \cdot b_f \cdot D_f (d - 0.42 D_f) \\
 &= (0.36 \times 20 \times 2000 \times 100) \times (550 - 0.42 \times 100) \\
 &= 731 \times 10^6 \text{ N·mm} = 731 \text{ kN·m}
 \end{aligned}$$

Since $M_u < M_{uf}$

Neutral axis falls within the flange

$$M_u = 0.87 f_y \cdot A_{st} \cdot d \left[1 - \frac{A_{st} \cdot f_y}{b \cdot d f_{ck}} \right]$$

$$324 \times 10^6 = 0.87 \times 415 \times A_{st} \times 550 \left[1 - \frac{A_{st} \times 415}{2000 \times 550 \times 20} \right]$$

Solving $A_{st} = 1700 \text{ mm}^2$

Provide 4 bars of 25 mm diameter ($A_{st} = 1964 \text{ mm}^2$)

Maximum ultimate shear $= (1.5 \times 2 \times 13.4) = 40.2 \text{ kN}$

$$\therefore \tau_v = \left(\frac{V_u}{bd} \right) = \left(\frac{40.2 \times 10^3}{200 \times 550} \right) = 0.379 \text{ N/mm}^2$$

Assuming two bars to be bent up near supports,

A_{st} at support $= 982 \text{ mm}^2$

$$\left[\frac{100 A_{st}}{bd} \right] = \left[\frac{100 \times 982}{200 \times 550} \right] = 0.89 \quad \tau_c = 0.57 \text{ N/mm}^2$$

since $\tau_v < \tau_c$, provide nominal shear reinforcements, using 6 mm diameter, two legged stirrups, spacing

$$s_v = \left(\frac{A_{sv} f_y}{0.4b} \right) = \left(\frac{2 \times 28 \times 415}{0.4 \times 200} \right) = 290 \text{ mm}$$

Provide 6 mm diameter 2 legged stirrups at 250 mm c/c at supports and the spacing gradually increased to 400 mm towards the centre of span. Maximum moment in the central rib in the y -direction (long span) is,

$$M_u = (2 \times 77 \times 1.5) = 231 \text{ kN}\cdot\text{m}$$

Provide 4 bars of 20 mm Φ ($A_{st} = 1256 \text{ mm}^2$) and 6 mm Φ , 2 legged stirrups at 250 mm centres near the supports. The beam section at D is checked for torsion and suitably reinforced. The moments in the slab being small, mesh reinforcement consisting of 6 mm Φ at 200 mm centres provided both ways for positive and negative moments in the slab. The reinforcement details in the typical sections of the grid is shown in Fig. 14.5.

14.4 EXAMPLES FOR PRACTICE

1. A two way slab 5 m by 5 m size with ribs at 1 m intervals is to be designed to support a live load of 4 kN/m^2 . Adopting M-15 grade concrete and Fe-415 grade for steel, design a suitable grid floor and sketch details of reinforcements in ribs.
2. A reinforced concrete grid floor of size 9 m by 12 m is required for an assembly hall. Assuming rib spacing of 1.5 m in the short span direction and 2 m in the long span direction, design the grid floor. Adopt M-20 grade concrete and Fe-415 grade for steel. Live load may be assumed as 4 kN/m^2 .

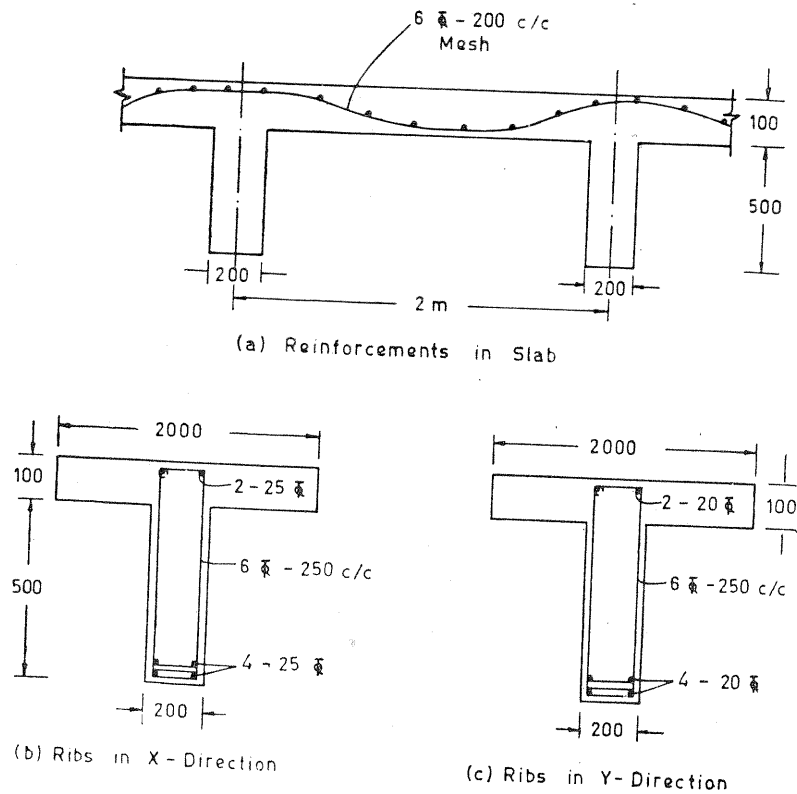


Fig. 14.5 Reinforcement details

3. An orthotropic reinforced concrete grid 16 m by 20 m is required for the roof of an auditorium. The ribs are spaced at 2 m intervals. Live load on roof = 1.5 kN/m^2 . Adopt M-20 grade concrete and Fe-415 grade tor steel. Design suitable reinforcements in the grid beams and sketch the details of reinforcements.

15

Virendeel Girders

15.1 GENERAL FEATURES

Professor Arthur Virendeel in 1896, developed a rigid frame with open web girders having rigid joints, comprising of a top and bottom chord with vertical members between the top and bottom booms. This type of open frame is popularly known as Virendeel girder. The prominent feature of the Virendeel girder being the absence of diagonal members and the frame depends on the rigidity of the joints for stability.

Virendeel girders have been widely used in Europe and particularly in Belgium where the pioneering work on this type of girders were made. Virendeel girders are used in structures where free unobstructed space is required between the top and bottom chords such as in clerestory lighting in churches and also for main girders in factories and ware house sheds. The various types of Virendeel girders used in practice are shown in Fig. 15.1.

15.2 ANALYSIS OF VIRENDEEL GIRDERS

Due to the rigid joints at the junctions of the members, Virendeel girder is a statically indeterminate structure. Several methods have been developed for the analysis of Virendeel girders such as,

- (a) Statically determinate analysis
- (b) Naylor's moment distribution
- (c) Modified moment distribution
- (d) Computer analysis based on generalised slope deflection equations

The first two methods will be used for the analysis and design of reinforced concrete Virendeel girders in the following sections.

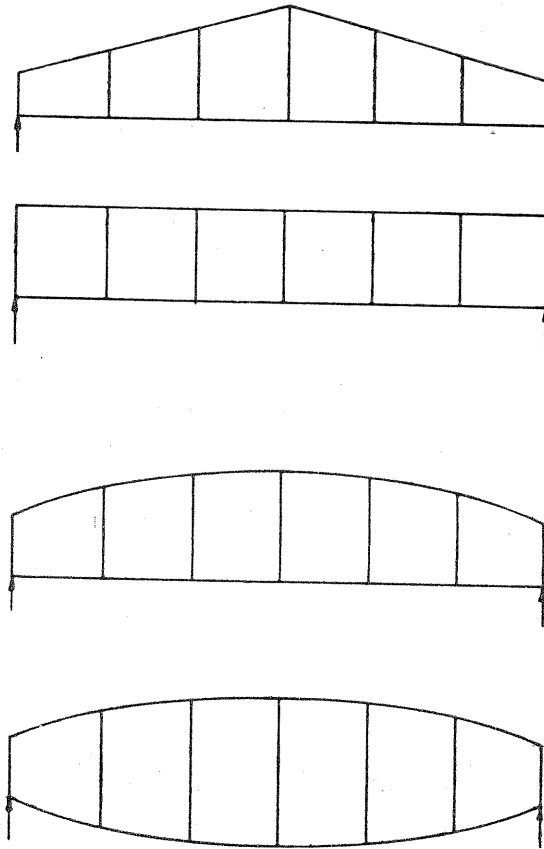


Fig. 15.1 Types of virendeel girders

Statically Determinate Analysis

In the statically determinate analysis, hinges are assumed at the mid span of the chords and mid height of the verticals as shown in Fig. 15.2.

Let W = Loads acting on the joints of the top boom.

The vertical shears acting at the mid sections of each of the panels of the Vierendeel girder is determined and this is equally divided between the top and bottom chords. Accordingly the shear at section xx is $V = W$

$$\text{Shear at hinges} = \left(\frac{W}{2} \right)$$

The chord bending moments are obtained by multiplying the shears at the hinges by half the length of the panel. The shear in the verticals is obtained by dividing the moment in the verticals at the joints by half height. The axial forces in the chords are obtained by summing up the horizontal shears. The simplified method of statically determinate analysis is applied to the following example.

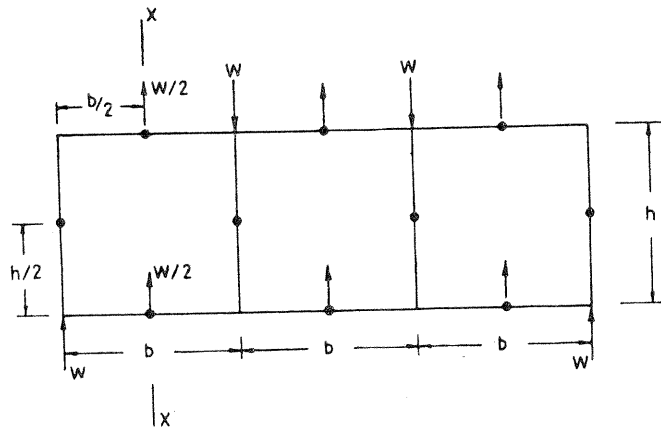


Fig. 15.2 Analysis of Vierendeel-girder

15.3 EXAMPLE

A Vierendeel girder spanning over 12 m has 6 bays of 2 m each. The verticals are 2 m in height. The loads acting on the top boom of the Vierendeel girder are 10 kN at each of the nodes.

Analyse the forces in the members of the Vierendeel girder using the statically determinate analysis.

Referring to Fig. 15.3, considering half the frame, the shear forces in the panels are computed as shown below:

Shear in panel 1 – 2 = 25 kN

Shear at pains of top and bottom chords = $\left(\frac{25}{2}\right) = 12.5$ kN

Shear in panel 2 – 3 = $(25 - 10) = 15$ kN

Shear at pins = $\left(\frac{15}{2}\right) = 7.5$ kN

Shear in panel 3 – 4 = $(25 - 10 - 10) = 5$ kN

Shear at pins = $\left(\frac{5}{2}\right) = 2.5$ kN

The chord moments are obtained by multiplying the pin shears by half the length of the panel.

$$M_{1-2} = M_{8-7} = (12.5 \times 1) = 12.5 \text{ kN} \cdot \text{m}$$

$$M_{2-3} = M_{7-6} = (7.5 \times 1) = 7.5 \text{ kN} \cdot \text{m}$$

$$M_{3-4} = M_{6-5} = (2.5 \times 1) = 2.5 \text{ kN} \cdot \text{m}$$

Considering the equilibrium of joints we have,

$$M_{1-8} = 12.5 \text{ kN} \cdot \text{m}$$

$$M_{2-7} = (12.5 + 7.5) = 20 \text{ kN} \cdot \text{m}$$

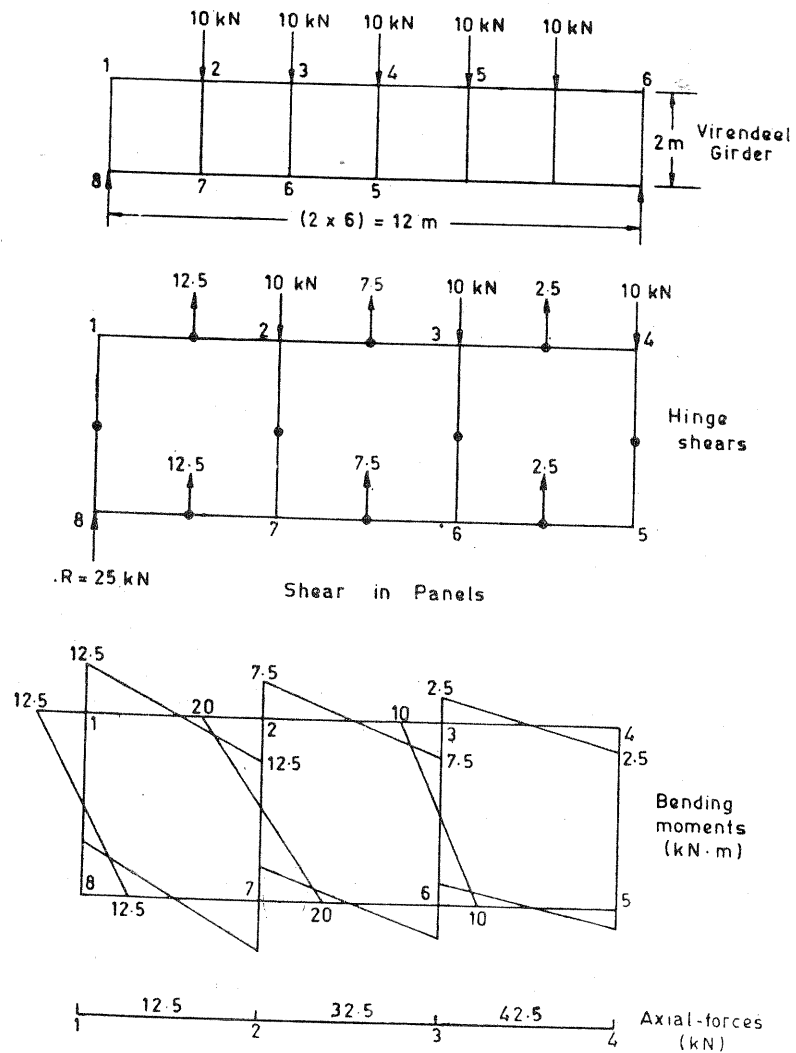


Fig. 15.3 Analysis of forces in Vierendeel girders

$$M_{3-6} = (7.5 + 2.5) = 10 \text{ kN} \cdot \text{m}$$

$$M_{4-5} = 0$$

The shear in the verticals are obtained by dividing the moment in the verticals at the joints by half their height.

$$\text{For member 1-8, shear} = V_{1-8} = \left(\frac{12.5 \times 2}{2} \right) = 12.5 \text{ kN}$$

$$\text{For member 2-7, shear} = V_{2-7} = \left(\frac{20 \times 2}{2} \right) = 20 \text{ kN}$$

$$\text{For member 3-6, shear} = V_{3-6} = \left(\frac{10 \times 2}{2} \right) = 10 \text{ kN}$$

The bending moment diagram for the Vierendeel frame is shown in Fig. 15.3. The axial forces in the chords are obtained by summing the horizontal shear in the vertical members.

Axial forces in members are,

$$H_{1-2} = 12.5 \text{ kN (Compression)}$$

$$H_{2-3} = (12.5 + 20) = 32.5 \text{ kN (Compression)}$$

$$H_{3-4} = (12.5 + 20 + 10) = 42.5 \text{ kN (Compression)}$$

The axial forces in the bottom boom have the same magnitude as their top counterparts, but they will be in tension.

15.4 EXAMPLE

Analyse the Vierendeel girder of Example 15.3, by Naylor's moment distribution procedure and evaluate the forces in the members.

In the case of Vierendeel girders having the same section for the top and bottom chords, Naylor's modified moment distribution procedure can be advantageously used for the evaluation of forces in the members. In the given example the stiffness values k of the chords and verticals are assumed to be the same. However in the Naylor's method only half the frame is considered with hinges assumed at mid heights of verticals of the Vierendeel girder as shown in Fig. 15.4.

The following salient points of Naylor's method are noteworthy:

1. Since only half the length of vertical members are considered with hinge at mid height, the modified stiffness of verticals are obtained as $6k$, where k is the original stiffness of the vertical members.
2. The carry over factor is -1
3. The fixed end moments in each panel is obtained from the shears in the panels.

The distribution factors at each of the joints are computed using the modified stiffness values of the members and the moment distribution is carried out as shown in Table 15.1.

The shears in each panel, fixed end moment and bending moment diagram for the Vierendeel girder is shown in Fig. 15.4.

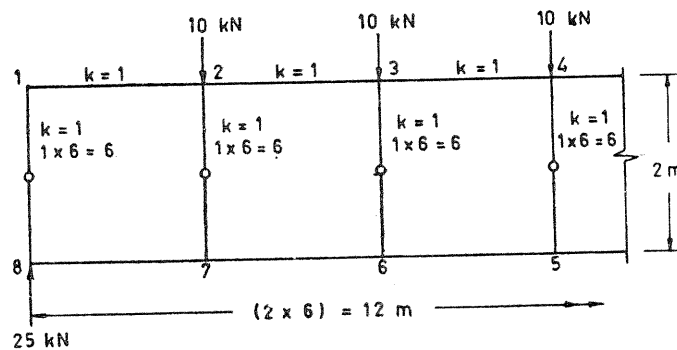
A comparative analysis of the moments obtained by the statically determinate analysis and Naylor's moment distribution procedure indicates that the latter method results in higher moments in end verticals and lower moments in interior vertical members.

15.5 DESIGN OF MEMBERS OF VIRENDEEL GIRDER

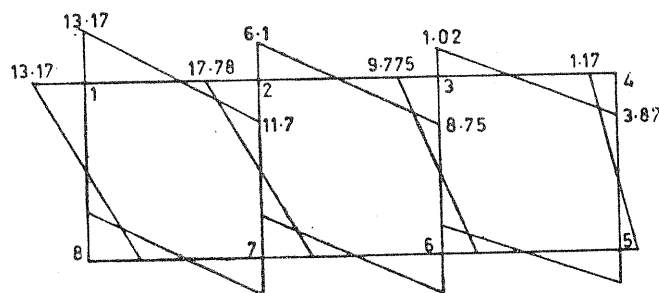
The top boom members of the Vierendeel girder is subjected to axial compression and bending moments while the bottom boom members are subjected to axial tension and bending moment. The vertical members are under

TABLE 15.1 NAYLOR'S Moment Distribution for Vierendeel Girder

D.F.	$\frac{6}{7}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{6}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{6}{8}$	$\frac{1}{8}$
F.E.M	-12.50	-12.50	-7.50	-7.50	-2.50	-2.50	-2.50	+2.50
Dist	$\frac{+10.70+1.80}{-}$	$\frac{-12.50}{+2.50+15.0+2.50}$	$\frac{-7.50}{+2.50+15.0+2.50}$	$\frac{-7.50}{+1.25+7.50+1.25}$	$\frac{-2.50}{+1.25+7.50+1.25}$	$\frac{-2.50}{+1.25+7.50+1.25}$	$\frac{-2.50}{+1.25+7.50+1.25}$	$\frac{-2.50}{+1.25+7.50+1.25}$
C.O.	-2.50	-1.80	-1.25	-2.50	-2.50	-2.50	-2.50	1.25
Dist	$\frac{+2.14+0.36}{-}$	$\frac{+0.38+2.28+0.38}{+0.38+2.28+0.38}$	$\frac{-1.25}{+0.38+2.28+0.38}$	$\frac{-2.50}{+0.31+1.875+0.31}$	$\frac{-2.50}{+0.31+1.875+0.31}$	$\frac{-2.50}{+0.31+1.875+0.31}$	$\frac{-2.50}{+0.31+1.875+0.31}$	$\frac{+0.15+0.94+0.15}{+0.31+1.875+0.31}$
Dist	$\frac{+0.33+0.05}{-}$	$\frac{-0.38}{+0.08+0.50+0.08}$	$\frac{-0.31}{+0.08+0.50+0.08}$	$\frac{-0.38}{+0.07+0.40+0.07}$	$\frac{-0.15}{+0.07+0.40+0.07}$	$\frac{-0.15}{+0.07+0.40+0.07}$	$\frac{-0.31}{+0.07+0.40+0.07}$	$\frac{+0.04+0.23+0.04}{+0.07+0.40+0.07}$
Final Moments (R.N.m)	+13.17-13.17	-11.70+17.18-6.10	-8.75+9.775-1.02	-3.87+1.17+2.69				



Shear :- 25 kN	15 kN	5 kN
F.E.M. $-(\frac{25 \times 2}{2 \times 2})$	$-(\frac{15 \times 2}{2 \times 2})$	$-(\frac{5 \times 2}{2 \times 2})$
$= -12.5$	-7.5	$-2.5 \text{ kN} \cdot \text{m}$



Bending Moment Diagram

Fig. 15.4 Naylor's moment distribution for Vierendeel girders

axial thrust and bending moments. In addition, all the members are subjected to shear forces to a varying degree.

The top boom members and the verticals are designed for combined compression and moment using interaction curves while the bottom boom members are designed for axial tension and moment in a manner similar to that of rectangular tank walls. The design of various members of the Vierendeel girder is illustrated in the following example.

15.6 DESIGN EXAMPLE

Design the following members of the Vierendeel girder subjected to maximum design force components shown in Table 15.2. Adopt M-20 grade concrete and Fe-415 grade tor steel. Assume suitable load factors wherever necessary. All the members are 2 m long.

TABLE 15.2 Design Force Components

Member Type	Maximum design working forces			
	Axial compression (kN)	Axial tension (kN)	Bending moment (kN·m)	Shear force (kN)
Top boom	12.5	—	12.5	12.5
Bottom boom	—	12.5	12.5	12.5
Verticals	10.0	—	20.0	20.0

(a) Top Boom Member

Top boom member is subjected to compression and moment. If the ratio of effective length to least lateral dimension is restricted to 12, we have

$$\left(\frac{Le}{D}\right) = 12 \quad \therefore D = \left(\frac{Le}{12}\right) = \left(\frac{2000}{12}\right) = 166 \text{ mm}$$

$\therefore$ Adopt a section 200 mm by 200 mm with effective depth of 170 mm.

Using load factor of 1.5 we have the ultimate moments, thrusts shears as,

$$M_u = (12.5 \times 1.5) = 18.75 \text{ kN·m}$$

$$P_u = (12.5 \times 1.5) = 18.75 \text{ kN}$$

$$V_u = (12.5 \times 1.5) = 18.75 \text{ kN}$$

$$\left(\frac{M_u}{f_{ck} \cdot b \cdot D^2}\right) = \left(\frac{18.75 \times 10^6}{20 \times 200 \times 200^2}\right) = 0.117$$

$$\left(\frac{P_u}{f_{ck} \cdot b \cdot D}\right) = \left(\frac{18.75 \times 10^3}{20 \times 200 \times 200}\right) = 0.023$$

From interaction curves of SP-16, using the parameters, $f_y = 415 \text{ N/mm}^2$

$$\left(\frac{d'}{D}\right) = \left(\frac{30}{200}\right) = 0.15, \quad \left(\frac{p}{f_{ck}}\right) = 0.085$$

$$\therefore p = (0.085 \times 20) = 1.7$$

$$\therefore A_s = \left(\frac{pbD}{100}\right) = \left(\frac{1.7 \times 200 \times 200}{100}\right) = 680 \text{ mm}^2$$

Use 4 bars of 16 mm Φ ($A_{st} = 804 \text{ mm}^2$)

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{12.5 \times 10^3}{200 \times 170}\right) = 0.36 \text{ N/mm}^2$$

$$\left(\frac{100A_s}{bd}\right) = \left(\frac{100 \times 340}{200 \times 170}\right) = 1.00$$

From Table 13 of IS : 456, $\tau_c = 0.62 \text{ N/mm}^2$

since $\tau_v < \tau_c$, only nominal shear reinforcements are provided. Using 6 mm Φ 2 legged stirrups, spacing is given by

$$s_v = \left(\frac{A_{sv}f_y}{0.4b}\right) = \left(\frac{2 \times 28 \times 415}{0.4 \times 200}\right) = 290 \text{ mm}$$

Adopt 6 mm Φ -2 legged stirrups at a uniform spacing of 250 mm centres.

(b) *Bottom Boom Member*

The bottom boom member is subjected to axial tension and bending moment.

$$M = 12.5 \text{ kN} \cdot \text{m}$$

$$P = -12.5 \text{ kN (Tension)}$$

$$V = 12.5 \text{ kN}$$

Section adopted is 200 mm by 200 mm cover = $d^1 = 30 \text{ mm}$, $D = b = 200 \text{ mm}$
The ultimate moments and forces are,

$$M_u = (12.5 \times 1.5) = 18.75 \text{ kN} \cdot \text{m}$$

$$P_u = -(12.5 \times 1.5) = -18.75 \text{ kN}$$

$$\left(\frac{d^1}{D}\right) = \left(\frac{30}{200}\right) = 0.15$$

If the reinforcement is distributed equally on four sides.

Referring Chart 80 of SP.-16, we have

$$\left(\frac{P_u}{f_{ck} \cdot b \cdot D}\right) = -\left(\frac{18.75 \times 10^3}{20 \times 200 \times 200}\right) = -0.023$$

$$\left(\frac{M_u}{f_{ck} \cdot b \cdot D^2}\right) = \left(\frac{18.75 \times 10^6}{20 \times 200 \times 200^2}\right) = 0.00011$$

$$\left(\frac{p}{f_{ck}}\right) < 0.02 \quad \therefore \quad p = (20 \times 0.02) = 0.4$$

$$\therefore \quad A_s = \left(\frac{pbD}{100}\right) = \left(\frac{0.4 \times 200 \times 200}{100}\right) = 160 \text{ mm}^2$$

Alternatively using the working stress method effective depth = $d = 170 \text{ mm}$

Distance of tensile steel from centre of section = $(x) = (100 - 30) = 70 \text{ mm}$

$$A_{st} = \left[\frac{M - P \cdot x}{\sigma_{st} \cdot j \cdot d}\right] + \left[\frac{P}{\sigma_{st}}\right] = \left[\frac{12.5 \times 10^6 - (12.5 \times 10^3 \times 70)}{230 \times 0.9 \times 170}\right] + \left[\frac{12.5 \times 10^3}{230}\right]$$

$$= 385 \text{ mm}^2$$

Use 4 bars of 12 mm Φ ($A_{st} = 452 \text{ mm}^2$) and nominal shear reinforcements of 6 mm Φ - 2 legged stirrups at 250 mm centres.

(c) *Vertical Members*

$$M_u = (20 \times 1.5) = 35 \text{ kN} \cdot \text{m}$$

$$P_u = (10 \times 1.5) = 15 \text{ kN}$$

$$V_u = (20 \times 1.5) = 35 \text{ kN}$$

Using a section 200 mm by 200 mm

$$\left(\frac{P_u}{f_{ck} \cdot b \cdot D}\right) = \left(\frac{15 \times 10^3}{20 \times 200 \times 200}\right) = 0.018$$

$$\left(\frac{M_u}{f_{ck} \cdot b \cdot D^2}\right) = \left(\frac{20 \times 10^6}{20 \times 200 \times 200^2}\right) = 0.125$$

$$\left(\frac{d^1}{D}\right) = \left(\frac{30}{200}\right) = 0.15 \quad \therefore \quad \text{From SP-16, Chart 45, we have}$$

$$\left(\frac{p}{f_{ck}}\right) = 0.09 \quad \therefore \quad p = (0.09 \times 20) = 1.8$$

$$\therefore \quad A_s = \left(\frac{pbD}{100}\right) = \left(\frac{1.8 \times 200 \times 200}{100}\right) = 720 \text{ mm}^2$$

Use 4 bars of 16 mm Φ ($A_{st} = 804 \text{ mm}^2$)

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{20 \times 10^3}{200 \times 170}\right) = 0.59 \text{ N/mm}^2, \quad \left(\frac{100A_s}{bd}\right) = \left(\frac{100 \times 340}{200 \times 170}\right) = 1.00$$

$$\tau_c = 0.62 \text{ N/mm}^2$$

since $\tau_v < \tau_c$, provide nominal shear reinforcements of 6 mm, Φ 2 legged stirrups at 250 mm centres.

15.7 EXAMPLES FOR PRACTICE

1. A Vierendeel girder spanning over 15 m has 10 bays of 1.5 m each. The verticals are 1.5 m long. The girder supports vertical loads of 15 kN each at all the node points of the top boom. Assuming the stiffness to be constant for all the members, analysis for moments, axial forces and shears in the Vierendeel girder using the statically determinate analysis. Also design the end vertical and top bottom boom members using M-20 grade concrete and Fe-415 grade tor steel.
2. A Vierendeel girder of 9 m span has 3 bays of 3 m each. The height of the verticals is 3 m. The girder supports concentrated loads of 100 kN and 50 kN at the interior node points of the top boom. Assuming constant stiffness for the members, investigate the moments and forces in the girder by,
 - (a) Statically determinate analysis
 - (b) Naylor's moment distribution procedure.
3. A Vierendeel girder spans over 20 m and has 10 bays of 2m each. The height of the verticals is 2 m. The Vierendeel girder supports loads of 10 kN each at all the node points of the bottom boom except the left and right hand supports. Assuming constant stiffness, analyse the Vierendeel girder by any suitable procedure and design a typical top boom, bottom boom and vertical member subjected to maximum moments. Adopt M-20 grade concrete and Fe-415 grade tor steel.

16

Trusses

16.1 GENERAL FEATURES

For spans exceeding 18 m, reinforced concrete trusses are more economical and are widely used for industrial buildings, warehouses, aircraft hangers, where column free space is desired. The configuration of the truss depends on the shape of the roof and its general lay out. In U.S.S.R., trusses are generally adopted for industrial buildings having spans more than 18 m. The most favourable configuration of the top chord is obtained in the bow string truss, the web or diagonal members are subjected to forces of very low magnitude and the bottom boom member, subjected to very heavy tension is generally prestressed. The different types of truss configurations used for span ranges from 18 to 30 m are shown in Fig. 16.1.

16.2 DIMENSIONS OF TRUSSES

The height of a reinforced concrete truss at mid span is in the range of $1/7$ to $1/9$ of its span length. The span of the trusses range from 18 to 30 m and more. The width of the various compression and tension members is kept constant at 200 to 350 mm depending upon the span of the truss. The depth of top boom members which are in compression generally is in the range of 200 to 300 mm. The bottom tie member should be of sufficient size to house the pretensioned wires or post tensioned cables. The depth is around 200 mm for spans of 15 m increasing to 300 mm for spans of 30 m. The depth of diagonal web members which are in compression and tension generally vary in the narrow range from 100 to 150 mm.

The use of concrete trusses with modular co-ordination for spans of 6, 9, 12, 15, 18, 24, 30 and 36 m and with a base module of 3 m is most common for industrial buildings of East European Countries such as

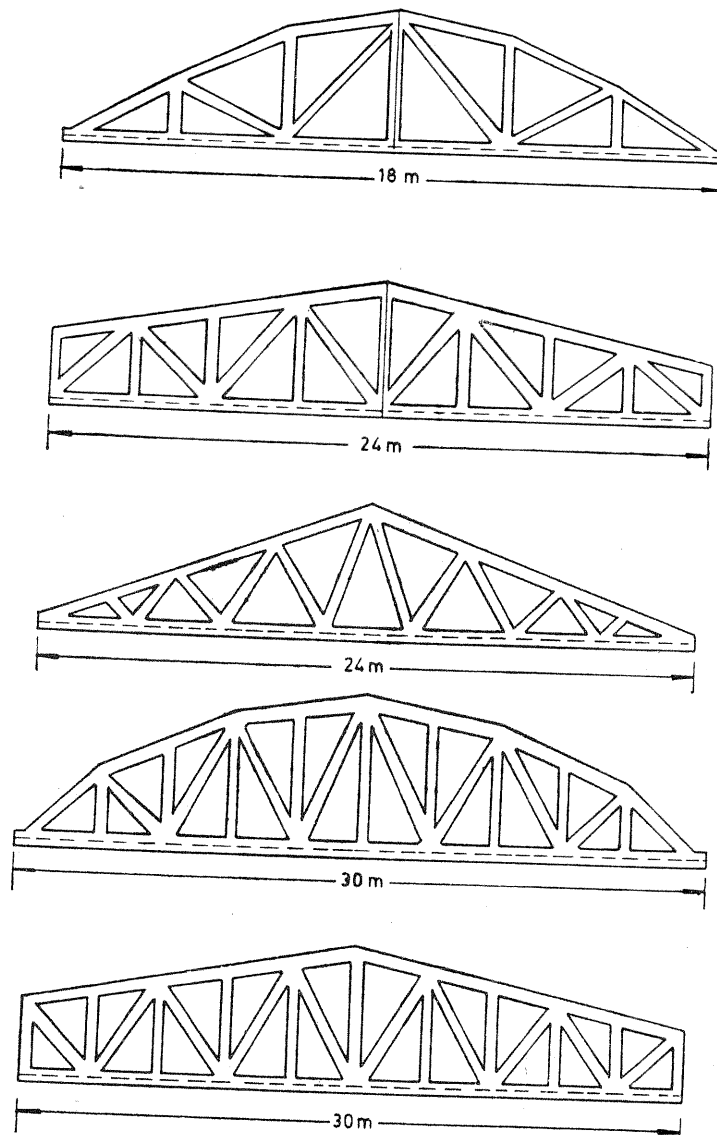


Fig. 16.1 Types of trusses

U.S.S.R., Poland, Yugoslavia and East Germany. Branko Zezelji has reported the construction of reinforced concrete trusses with prestressed tie members for spans up to 60 m in Yugoslavia.

16.3 MATERIAL REQUIREMENTS

Concrete used in stresses is normally of grade ranging from M-30 to M-50 which can be considered as high strength concrete. The reinforcements

consists of the mild steel or deformed tor steel together with high tensile steel wires used in the bottom tie member. The material requirements per truss varies with the span and spacings of trusses. Table 16.1, shows the requirements of materials for precast reinforced concrete trusses designed for roof loads of 3.5 to 5.3 kN/m².

TABLE 16.1 Material Requirements for Precast Roof Trusses

Types of Truss	Weight of Truss (kN)	Grade of Concrete (N/mm ²)	Material requirements per truss	
			Steel (kg)	Concrete (m ³)
1. Truss spacing=6 m				
Prestressed bow string truss with cable reinforcement span,				
(a) 18 m	4.3-4.8	30	338-433	1.72-1.9
(b) 24 m	8.8-10.0	30-40	621-689	3.50-4.0
(c) 30 m	15.2-17.0	30-40	1041-1219	6.08-6.8
Polygonal, built up from blocks with prestressed bottom chord having wire cablereinforcement span,				
(a) 18 m	6.58	40	514-529	2.63
(b) 24 m	9.60	40	744-765	3.85
(c) 30 m	13.20	40	1135-1186	5.28
2. Truss spacing=12 m				
Prestressed bow string of linear elements with wire reinforcement span				
(a) 18 m	7.6-9.1	30-40	491-759	3.06-3.63
(b) 24 m	14.9-17.4	30-50	1018-1367	5.95-6.96
(c) 30 m	25.5-29.8	30-50	1422-2213	10.20-11.90
Prestressed bow string with bar reinforcement span				
(a) 18 m	7.6-9.1	30-40	563-962	3.06-3.63
(b) 24 m	14.9-17.4	30-50	1238-1822	5.95-6.96
(c) 30 m	25.5-29.8	30-50	1778-2981	10.20-11.90

16.4 CONSTRUCTIONAL FEATURES

The chords and struts of trusses are designed to have the same width for convenience in fabricating in a horizontal position. If precast roof slabs are used for roof covering, the upper chord panels are made equal to the width of precast slabs which is usually about 3 m. The lower tension chord is prestressed with the use of bunched high strength wire or cables running in

preformed holes. For spans in the range of 18 to 24 m, the trusses are made in one piece but when spans run from 24 to 30 m, they are made in two pieces with the joint in mid span.

Polygonal trusses with inclined top chords are generally made of 6 m blocks or half trusses with 3 m panels. Due to higher tensions developed in the diagonal members of large span trusses, prestressing them becomes inevitable. In general polygonal trusses are less economical than the bow type with regard to material and labour costs. At the ends of trusses near the supports, 10 to 12 mm steel bearing plates are anchored and embedded while casting and these serve as bearings and for fixing of trusses to the columns.

16.5 ANALYSIS OF TRUSSES

Based on theoretical and experimental work, it is established that the stresses in the various members of the truss are analysed on the assumption of hinged joints in steel, reinforced concrete and prestressed trusses. The rigidity of the joints does not significantly affect the forces developed in the members, as such the analysis is generally carried out on the assumption of hinged joints.

The trusses are analysed for dead, wind and snow loads applied to the joints of the top chord. The load of suspended mechanical handling facilities is applied at the panel points of the bottom chord. When loads are applied to the chords of a truss between the panel points, the bending moments are determined by assuming the chord as a continuous beam with spans equal to the distance between the points.

Trusses must also be analysed for forces developed during fabrication and erection. Initial stresses developed due to the prestressing of the bottom chord member should also be considered in the design of trusses.

If Δ = Contraction of the bottom chord due to precompression

P = Prestressing force

L = Length of the bottom chord

A = Cross sectional area of bottom chord

E_c = Modulus of elasticity of concrete

Then we have the relation

$$\Delta = \left(\frac{PL}{AE_c} \right)$$

The displacements of the ends of truss members in a direction perpendicular to their longitudinal axis caused by the elastic contraction Δ , is determined with the aid of Williot diagram.

If δ = displacement at the ends of members

M = Bending moment developed in the members

The moments in the members are obtained by the relation,

$$M = \left(\frac{6EI\delta}{L^2} \right)$$

In the case of large span trusses with larger magnitudes of prestressing forces in the bottom chord, the secondary stresses developed in the various members due to the contraction of the bottom chord should be investigated and considered in the design of the members.

16.6 DESIGN EXAMPLE

A reinforced concrete truss is to be designed for a warehouse shed to suit the following data:

Span of truss (centre to centre of bearings) = 25 m

Spacings of trusses = 5 m

Central rise of truss = 4.13 m

The fink type truss shown in Fig. 16.2, supports reinforced concrete purlins at intervals of 1.35 m and the roof is covered by Asbestos sheets. The bottom chord member is to be prestressed. The maximum design forces in the various members are shown in Table 16.2.

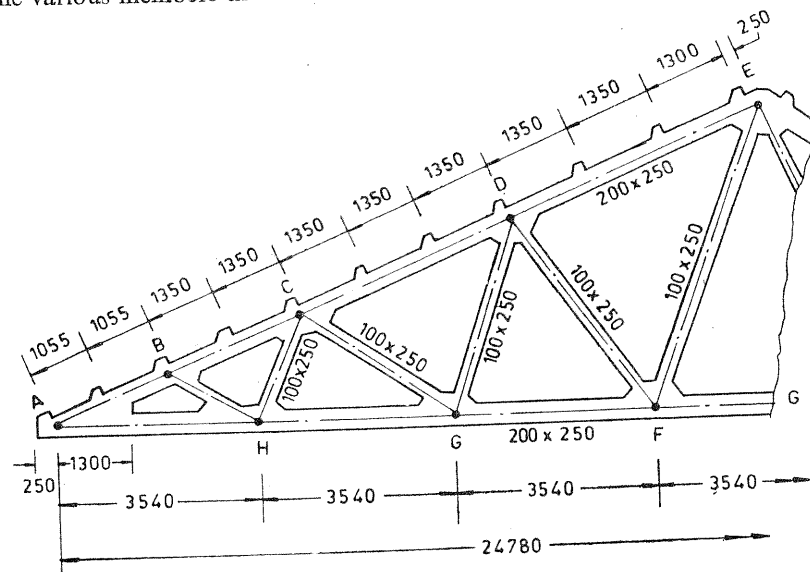


Fig. 16.2 Precast reinforced concrete truss for 25 span

Design suitable reinforcements in the truss members. Adopt M-35 grade concrete and Fe-415 grade tor steel for main reinforcements and Fe-250 grade mild steel for secondary reinforcements. Sketch the typical details of reinforcements and high tensile wire cables in the members of the truss.

TABLE 16.2 Forces in Truss Members

Member	Cross sectional Dimensions (mm × mm)	Direct force		Bending moment (kN · m)
		Compression (kN)	Tension (kN)	
AB	200 × 250	395	—	4.3
BC	200 × 250	364	—	12.0
CD	200 × 250	297	—	16.0
DE	200 × 250	236	—	16.0
BH	100 × 250	36	—	—
CG	100 × 250	62	—	—
DF	100 × 250	72	—	—
AH	200 × 250	—	377	—
HG	200 × 250	—	335	—
GF	200 × 250	—	268	—
FG	200 × 250	—	225	—
CH	100 × 250	—	18	—
DG	100 × 250	—	35	—
EF	100 × 250	—	59	—

1. Design of Compression Members

(a) Member AB

$$P_u = (1.5 \times 395) = 592.5 \text{ kN} \quad b = 250 \text{ mm}$$

$$M_u = (1.5 \times 4.3) = 6.45 \text{ kN} \cdot \text{m} \quad D = 200 \text{ mm}$$

$$\text{Length } L = 2.11 \text{ m} \quad f_{ck} = 35 \text{ N/mm}^2$$

$$\text{Effective length} = L_e = (0.65 \times 2.11) = 1.37 \text{ m}$$

$$\left(\frac{L_e}{D}\right) = \left(\frac{1.37}{0.2}\right) = 6.85 < 12$$

$$\left(\frac{P_u}{f_{ck} \cdot b \cdot D}\right) = \left(\frac{592.5 \times 10^3}{35 \times 200 \times 200}\right) = 0.338$$

$$\left(\frac{M_u}{f_{ck} \cdot b \cdot D^2}\right) = \left(\frac{6.45 \times 10^6}{35 \times 200 \times 200^2}\right) = 0.018$$

If cover $d' = 40 \text{ mm}$

$$\left(\frac{d'}{D}\right) = \left(\frac{40}{200}\right) = 0.2$$

Referring to Chart 46 of SP 16,

$\left(\frac{p}{f_{ck}}\right) = 0 \quad \therefore$ Provide minimum reinforcement of 0.8 percent in the section

$$A_s = \left(\frac{0.8 \times 200 \times 250}{100}\right) = 400 \text{ mm}^2$$

Provide 4 bars of 12 mm Φ ($A_s = 452 \text{ mm}^2$) and 6 mm ties at 200 mm centres.

(b) Member BC

Length $L = 2.70$ mEffective length $= L_e = (0.65 \times 2.70) = 1.755$ m

$$\left(\frac{L_e}{D}\right) = \left(\frac{1.755}{0.2}\right) = 8.77 < 12$$

$$P_u = (1.5 \times 364) = 546 \text{ kN}$$

$$M_u = (1.5 \times 12) = 18.0 \text{ kN}\cdot\text{m}$$

$$\left(\frac{P_u}{f_{ck} \cdot b \cdot D}\right) = \left(\frac{546 \times 10^3}{35 \times 250 \times 200}\right) = 0.312$$

$$\left(\frac{M_u}{f_{ck} \cdot b \cdot D^2}\right) = \left(\frac{18 \times 10^6}{35 \times 250 \times 200^2}\right) = 0.051$$

From design Chart 46 of SP. 16,

$$\left(\frac{p}{f_{ck}}\right) = 0.01 \quad \therefore \quad p = (35 \times 0.01) = 0.35$$

Provide minimum reinforcement of 0.8 percent $A_s = 400 \text{ mm}^2$ Provide 4 bars of 12 mm Φ ($A_s = 452 \text{ mm}^2$) and 6 mm Φ ties at 200 mm centres.

(c) Member CD

$$P_u = (1.5 \times 297) = 445.5 \text{ kN}$$

$$M_u = (1.5 \times 16) = 24 \text{ kN}\cdot\text{m}$$

$$\left(\frac{L_e}{D}\right) = \left(\frac{0.65 \times 4.05}{0.2}\right) = 13.16 > 12$$

Slenderness effects have to be considered additional moments to be considered are

$$M_{ux} = \left(\frac{P_u \cdot D}{2000}\right) \left(\frac{L_e}{D}\right)^2 = \left(\frac{445.5 \times 0.2}{2000}\right) (13.16)^2 = 7.71 \text{ kN}\cdot\text{m}$$

$$\therefore \text{Total moment } M_u = (24 + 7.71) = 31.71 \text{ kN}\cdot\text{m}$$

$$P_u = 445.5 \text{ kN}$$

$$\left(\frac{P_u}{f_{ck} \cdot b \cdot D}\right) = \left(\frac{445.5 \times 10^3}{35 \times 250 \times 200}\right) = 0.254$$

$$\left(\frac{M_u}{f_{ck} \cdot b \cdot D^2}\right) = \left(\frac{31.71 \times 10^6}{35 \times 250 \times 200^2}\right) = 0.09$$

From Chart 46 of SP-16 we have

$$\left(\frac{p}{f_{ck}}\right) = 0.06 \quad \therefore \quad p = (35 \times 0.06) = 2.10$$

$$\therefore \quad A_s = \left(\frac{2.10 \times 250 \times 200}{100}\right) = 1050 \text{ mm}^2$$

Provide 4 bars of 16 mm Φ and 4 bars of 10 mm Φ . ($A_s = 1120 \text{ mm}^2$)
 Adopt 6 mm Φ ties at 200 mm centres.

(d) Member DE

$$P_u = (1.5 \times 236) = 354 \text{ kN}$$

$$M_u = (1.5 \times 16) = 24 \text{ kN}\cdot\text{m}$$

$$\left(\frac{L_e}{D}\right) = \left(\frac{0.65 \times 4.2}{0.2}\right) = 13.65 > 12$$

$$\therefore M_{ux} = \left(\frac{354 \times 0.2}{2000}\right) (13.65)^2 = 6.6 \text{ kN}\cdot\text{m}$$

$$\therefore \text{Total moment } M_u = (24 + 6.6) = 30.6 \text{ kN}\cdot\text{m}$$

$$\left(\frac{P_u}{f_{ck} \cdot b \cdot D}\right) = \left(\frac{354 \times 10^3}{35 \times 250 \times 200}\right) = 0.202$$

$$\left(\frac{M_u}{f_{ck} \cdot b \cdot D^2}\right) = \left(\frac{30.6 \times 10^6}{35 \times 250 \times 200^2}\right) = 0.087$$

From Chart 46 of SP-16,

$$\left(\frac{p}{f_{ck}}\right) = 0.055 \quad \therefore p = (35 \times 0.055) = 1.925$$

$$\therefore A_s = \left(\frac{1.925 \times 250 \times 200}{100}\right) = 963 \text{ mm}^2$$

Adopt 4 bars of 16mm Φ and 4 bars of 10 mm Φ ($A_s = 1120 \text{ mm}^2$).
 Use 6 mm Φ ties at 200 mm centres.

(e) Member DF

$$P_u = (1.5 \times 72) = 108 \text{ kN section used is } 100 \times 250 \text{ mm.}$$

$$\left(\frac{L_e}{D}\right) = \left(\frac{0.65 \times 4.1}{0.1}\right) = 26.65 > 12$$

$\therefore$ Additional moments to be considered is,

$$M_{ux} = \left(\frac{108 \times 0.1}{2000}\right) (26.65)^2 = 3.83 \text{ kN}\cdot\text{m}$$

Since the moments and forces are of small magnitude, provide minimum reinforcement of 0.8 percent in the section.

$$A_s = \left(\frac{0.8 \times 100 \times 250}{100}\right) = 200 \text{ mm}^2$$

Provide 2 bars of 12 Φ ($A_s = 226 \text{ mm}^2$).

Adopt 6 mm Φ ties at 200 mm centres.

2. Design of Tension Members

(a) Bottom Chord Member

Maximum design tensile force in the bottom chord member

$$AH = 376 \text{ kN} \quad \therefore N_d = 377 \text{ kN}$$

Permissible compressive stress in concrete of M-35 grade at transfer
 $= \sigma_{ct} = 15 \text{ N/mm}^2$.

Permissible tensile stress at working loads $\sigma_{tw} = 0$ loss ratio $= \eta = 0.8$
 $\therefore$ Area of concrete section

$$= \left(\frac{N_d}{\eta f_{ct}} \right) = \left(\frac{377 \times 10^3}{0.8 \times 15} \right) = 31,416 \text{ mm}^2$$

Section adopted $= (200 \times 250) = 50,000 \text{ mm}^2$

$\therefore$ Compressive prestress

$$= \left(\frac{377 \times 10^3}{0.8 \times 50,000} \right) = 9.43 \text{ N/mm}^2$$

Prestressing force

$$= P = \left(\frac{9.43 \times 50,000}{1000} \right) = 471.5 \text{ kN}$$

Using 7 mm diameter high tensile wires initially stressed to 1100 N/mm^2 and having an ultimate tensile strength of 1500 N/mm^2 , the number of wires required is given by

$$n = \left(\frac{471.5 \times 10^3}{38.5 \times 1100} \right) = 11.13$$

Use 1 Freyssinet cable containing 12 wires of 7 mm with suitable end anchorages.

Ultimate tensile strength of tie

$$= \left(\frac{12 \times 38.5 \times 0.87 \times 1500}{1000} \right) = 603 \text{ kN}$$

Load factor against collapse

$$= \left(\frac{603}{377} \right) = 1.599 > 1.50$$

Assuming direct tensile strength of concrete as 4 N/mm^2

$$\text{Cracking load} = \left[\frac{50,000(0.8 \times 9.43 + 4.0)}{1000} \right] = 577.2 \text{ kN}$$

$\therefore$ Load factor against cracking

$$= \left(\frac{577.2}{377} \right) = 1.53 > 1.25$$

Provide 4 bars of 12 mm Φ and 4 bars of 10 mm Φ as untensioned reinforcement with 6 mm Φ ties at 200 mm centres.

(b) *Member EF*

Section adopted is $100 \times 250 \text{ mm}$

Maximum design tensile force $N_d = 59 \text{ kN}$

$$\therefore A_s = \left(\frac{59 \times 10^3}{230} \right) = 257 \text{ mm}^2$$

modular ratio for M-35 grade concrete is

$$m = \left(\frac{280}{3 \times 11.5} \right) = 8.1$$

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Using 2 bars of 16 mm Φ ($A_s = 404 \text{ mm}^2$)

$$\text{Tensile stress} = \left[\frac{N_d}{A_c + (m-1)A_{st}} \right] = \left[\frac{59 \times 10^3}{(250 \times 100) + (8.1-1)404} \right]$$

$$= 2.11 \text{ N/mm}^2 < 4 \text{ N/mm}^2$$

Provide single legged ties of 6 mm Φ at 200 mm centres.
Typical reinforcement details are shown in Fig. 16.3.

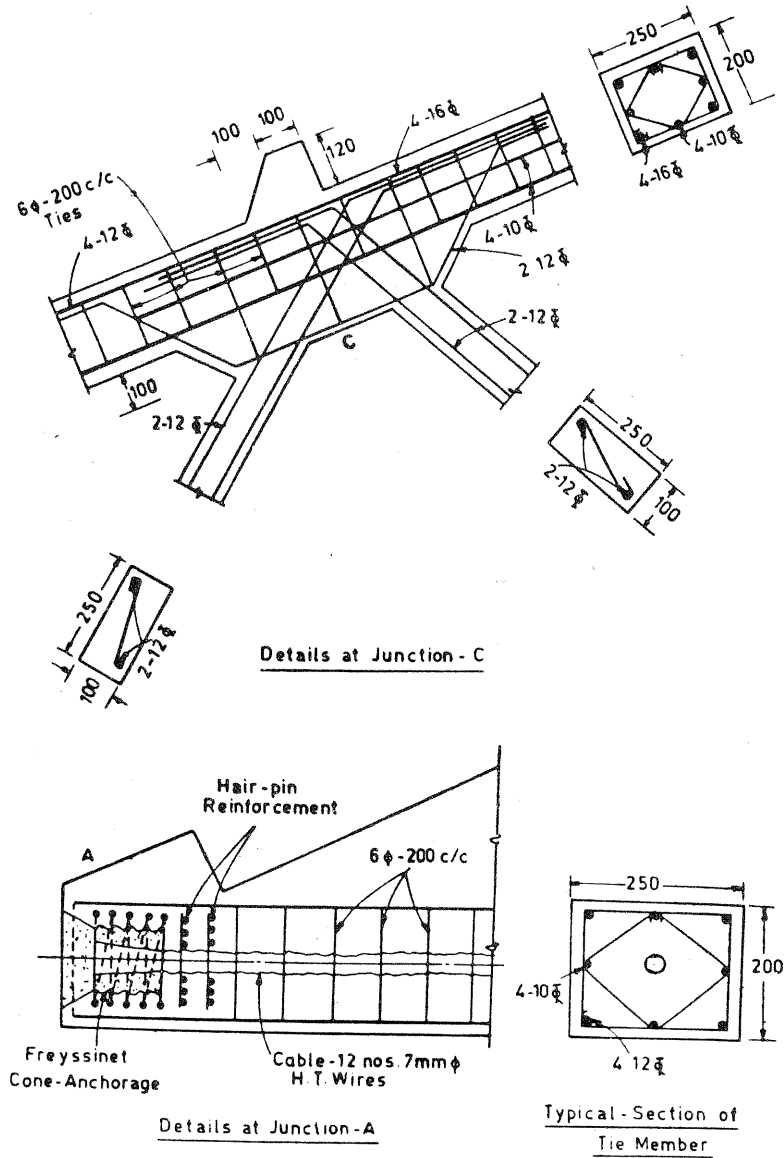


Fig. 16.3 Reinforcement details in truss

16.7 EXAMPLES FOR PRACTICE

1. A reinforced concrete truss of polygonal parallel chord type is proposed for the roof system of an industrial structure for a span of 24 m, comprising of 8 bays of 3 m each. The truss height is 2.5 m. Spacings of truss = 6 m. The roof is covered by precast ribbed slabs of size 3 m by 6 m each weighing 24 kN. Liveload on roof is 1.5 kN/m². Roof finishes = 1 kN/m². The top and bottom boom members are 250 mm wide by 300 mm deep, while the diagonal members and vertical members of cross section 250 mm by 150 mm. Using M-35 grade concrete and Fe-415 grade tor steel, design suitable reinforcements for the members of the truss. Design the bottom tie as a prestressed member.
2. The compression and tension members of a 18 m span reinforced concrete truss are subjected to the forces shown in table below:

Member	Cross sectional Dimensions (mm × mm)	Length of member (m)	Forces (kN)	Moment (kN·m)
AB	200 × 240	1.8 m	300 Compression	3.5
DE	200 × 240	3.6 m	240 Compression	12.0
AH	200 × 240	3.0 m	320 Tension	—

Using M-35 grade concrete and Fe-415 grade tor steel, design suitable reinforcements in the members. Design the suitable prestressing force for the tie member using 7 mm diameter high tensile wires initially stressed to 1000 N/mm². Sketch the details reinforcements in the cross section of the members.

3. A polygonal truss with an upward support diagonal having a sloping top chord is proposed for the roof system of an Industrial ware house 30 m by 120 m in plan. The spacings of trusses = 6 m. R.C.C. purlins are proposed between the trusses to support the asbestos roof cladding. M-35 grade concrete and Fe-415 grade tor steel are specified for precasting the trusses at site. Design the R.C.C. purlins and the typical compression and tension members of the truss. Also design the bottom chord tie as a prestressed member. Sketch the typical reinforcement details in the members of the truss.

17

Poles

17.1 GENERAL FEATURES

During the last two decades, reinforced concrete poles are being widely used and they have gradually replaced the traditional poles made of wood and steel. Reinforced concrete poles are mass produced and are extensively used in most countries for railway, power and signal lines, lighting poles, antenna masts, telephone transmission, low and high voltage electric power transmission and substation towers. The main advantages of reinforced concrete poles are:

1. Fire resistant, particularly to grass and bush fires near the ground level.
2. Freeze thaw resistance in cold regions.
3. Durability from corrosion in humid and temperate climates and from erosion in desert areas.
4. Economical, when mass produced using precast techniques.
5. Resistance to termite attack when embeded in earth.
6. R.C. poles can be moulded to any size and shape.
7. Easy installation in ground.
8. Clean and neat appearance and requiring negligible maintenance for a number of years, thus ideally suited for urban installations.

The various advantages have resulted in the rapid development and use of different types of reinforced concrete poles in the developed and developing countries. In view of the crash programme of rural electrification in India, it is estimated that there will be a demand for two million poles annually which will further increase due to the electrification of new lines during the next decade.

17.2 CLASSIFICATION OF POLES

According to the Indian Standard Code IS: 785-1964, the reinforced concrete poles are classified into 11 categories depending upon their transverse load carrying capacity as detailed in Table 17.1. The depth of embedment of pole below ground level depends upon the length of the pole. The embedded depth varies from 1.2 m for 6 m long poles to 2.40 m for 17 m long poles.

TABLE 17.1 Classification of R.C.C. Poles

Class of poles	Maximum over all length (m)	Minimum ultimate transverse load applied at 600 mm from top (kN)
1	17.0	30
2	17.0	23
3	17.0	20
4	17.0	14
5	16.0	11
6	12.5	10
7	12.0	8
8	12.0	7
9	11.0	5
10	9.0	3
11	7.5	2

Minimum over all length of all classes of poles shall be 6 m.

17.3 DESIGN FEATURES

Reinforced concrete poles are subjected to bending moments of equal magnitude in opposite directions. The poles are generally designed for the following critical conditions.

1. Bending due to wind load on cable and on exposed face.
2. Combined bending and torsion due to eccentric snapping of wires.
3. Maximum torsion due to skew snapping of wires.
4. Bending due to failure of all the wires on one side of the pole and
5. Handling and erection stresses.

The pole shall be designed such that when it is vertical, its strength in the transverse direction shall be sufficient to take an ultimate load equal to the horizontal wind load on wires and pole multiplied by the load factor. The design should ensure that at 40% of the ultimate load, the stresses and strains are such as not to cause any harmful cracking in pole. The minimum load factor for transverse strength of all classes of poles is prescribed as 2.5.

In the case of poles used for power transmission lines, the strength of the pole in the direction of the line should be not less than 1/4 of the strength

required in the transverse direction. The torsional strength of the pole should be not less than $0.75 \text{ kN}\cdot\text{m}$ for poles of class 1 to 9 and not less than $0.50 \text{ kN}\cdot\text{m}$ for poles of other classes.

The design of a typical reinforced concrete pole for power transmission lines is illustrated by the following example.

17.4 DESIGN EXAMPLE

A reinforced concrete pole is to be designed to suit the following data:

Line voltage—3 phase, 400 V—

No of circuits—one

Number of conductors (including neutral and street lighting wire)—five

Size and material of conductor—7.1 mm diameter copper

Size and material of neutral Wire—5 mm diameter copper

Size and material of street lighting wire—4 mm diameter copper

Normal span—50 m

Wind pressure— 1 kN/m^2

Load factor—2.5

Over all length of pole—7 m

Material M-20 grade concrete Fe-415 grade tor steel.

Arrangement of conductors—As shown in Fig. 17.1

Tension in conductors—3 kN

Design and detail the reinforcements in the pole.

1. Wind Load Calculations

Wind load on 3 conductors/span

$$= \left(1.00 \times \frac{2}{3} \times \frac{7.1}{1000} \times 3 \times 50 \right) = 0.71 \text{ kN}$$

Wind load on Neutral wire/span

$$= \left(1.00 \times \frac{2}{3} \times \frac{5}{1000} \times 50 \right) = 0.167 \text{ kN}$$

Wind load on street lighting wire

$$= \left(1.00 \times \frac{2}{3} \times \frac{4}{1000} \times 50 \right) = 0.133 \text{ kN}$$

2. Bending Moments

Total bending moment at ground level due to windage on all the wires = $(0.71 \times 5.6) + (0.167 \times 5.6) + (0.133 \times 5.95) = 5.70 \text{ kN}\cdot\text{m}$

Equivalent load at 0.6 m from top corresponding to windage on all the wires

$$= \frac{1}{5.20} [(0.71 \times 5.6) + (0.167 \times 5.6) + (0.133 \times 5.95)] = 1.097 \text{ kN}$$

Assuming the pole to have uniformly tapering square section having dimensions as shown in Fig. 17.1.

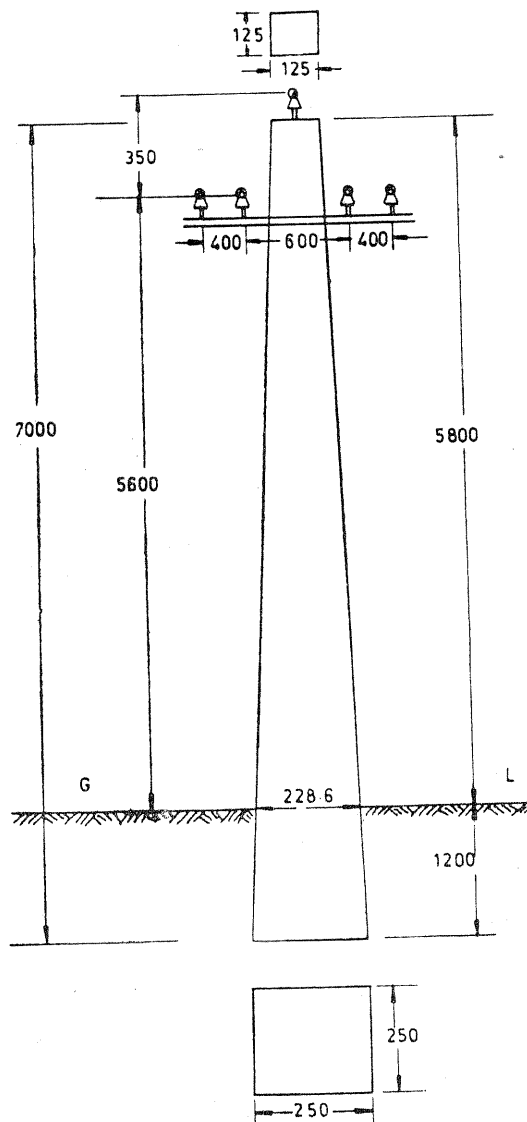


Fig. 17.1 R.C. transmission pole

Wind load on pole above ground level

$$= 1.00 \times 5.80 \left(\frac{0.125 + 0.2286}{2} \right) = 1.025 \text{ kN}$$

Bending moment at G.L. assuming the load to be acting at centre of gravity of pole

$$= 1.025 \times \frac{1}{3} \times 5.80 \left[\frac{0.2286 + 2 \times 0.125}{0.2286 + 0.125} \right] = 2.682 \text{ kN} \cdot \text{m}$$

Equivalent load at 0.6 m from top corresponding to windage on pole

$$= \left(\frac{2.682}{5.2} \right) = 0.515 \text{ kN}$$

Total transverse load at 0.6 m from top corresponding to windage on the conductors and pole $= (1.097 + 0.515) = 1.612 \text{ kg}$

Ultimate load with load factor $= (1.612 \times 2.5) = 4.03 \text{ kN}$

From Table 17.1, a reinforced concrete pole belonging to class 9 can be chosen.

3. Design of Reinforcements

Using M-20 grade concrete and Fe-415 grade tor steel, the ultimate moment capacity of the pole section at ground level is given by,

$$M_{u(\text{lim})} = 0.138 f_{ck} \cdot b \cdot d^2$$

Assuming an effective cover of 30 mm,

Effective depth at ground level $= (228.6 - 30.0) = 198.6 \text{ mm}$

$$\therefore M_{u(\text{lim})} = (0.138 \times 20 \times 50 \times 198.6^2) = 272 \times 10^6 \text{ N} \cdot \text{mm} \\ = 27.2 \text{ kN} \cdot \text{m}$$

Total service load bending moment at G.L. due to windage on wires and pole $= (5.70 + 2.682) = 8.382 \text{ kN} \cdot \text{m}$

Ultimate moment with a load factor of 2.5

$$M_u = (2.5 \times 8.382) = 20.955 \text{ kN} \cdot \text{m}$$

Since $M_u < M_{u(\text{lim})}$, the section is under reinforced

$$\therefore M_u = 0.87 f_y A_{st} \cdot d \left[1 - \frac{A_{st} f_y}{b d f_{ck}} \right]$$

$$20.955 \times 10^6 = 0.87 \times 415 \times A_{st} \times 198.6 \left[1 - \frac{A_{st} \cdot 415}{250 \times 198.6 \times 20} \right]$$

$$\text{Solving } A_{st} = 342 \text{ mm}^2$$

4. Torsional Moments

Due to skew snapping of the wires torsional moments develop in the pole.

Torsional moment $T = 3(1.4 + 0.6) = 6 \text{ kN} \cdot \text{m}$

$$T_u = (2.5 \times 6) = 15 \text{ kN} \cdot \text{m}$$

Bending moment at base $M_u = 20.955 \text{ kN} \cdot \text{m}$

$$V_u = 4.03 \text{ kN} \cdot \text{m}$$

Equivalent bending moment $M_{e1} = (M_u + M_t)$ where,

$$M_t = T_u \left[\frac{1 + (D/b)}{1.7} \right] = 15 \left[\frac{1 + (250/250)}{1.7} \right] = 17.6 \text{ kN} \cdot \text{m}$$

$$\therefore M_{e1} = (20.955 + 17.6) = 38.6 \text{ kN} \cdot \text{m}$$

The longitudinal steel should be designed for this bending moment.

For the assumed section at ground level $M_{u(lim)} = 27.2 \text{ kN} \cdot \text{m}$

$$[M_{e1} - M_{u(lim)}] = (38.6 - 27.2) = 11.4 \text{ kN} \cdot \text{m}$$

$$(11.4 - 10^6) = f_{sc} \cdot A_{sc} \cdot (d - d')$$

Assuming cover of 30 mm $d = 198.6 \text{ mm}$ $d' = 30 \text{ mm}$

$$f_{sc} > 0.87f_y > (0.87 \times 415) > 361 \text{ N/mm}^2$$

$$\therefore A_{sc} = \left(\frac{11.4 \times 10^6}{361 \times 168.6} \right) = 187.3 \text{ mm}^2$$

$$A_{st1} = \left[\frac{0.36f_{ck} \cdot b(0.48d)}{0.87f_y} \right] = \left[\frac{0.36 \times 20 \times 250(0.48 \times 198.6)}{0.87 \times 415} \right]$$

$$= 475 \text{ mm}^2$$

$$A_{st2} = \left(\frac{A_{sc} \cdot f_{sc}}{0.87f_y} \right) = \left(\frac{187.3 \times 361}{0.87 \times 415} \right) = 187.3 \text{ mm}^2$$

$$\therefore A_{st} = (A_{st1} + A_{st2}) = (475 + 187.3) = 662.3 \text{ mm}^2$$

Provide 4 bars of 22 Φ , one each at the corners ($A_{st} = 720 \text{ mm}^2$).

5. Transverse Reinforcements

$$V_e = V_u + 1.6 \left(\frac{T_u}{b} \right) = 4.03 + 1.6 \left(\frac{15}{0.25} \right) = 100 \text{ kN}$$

$$\tau_{ve} = \left(\frac{V_e}{bd} \right) = \left(\frac{100 \times 10^3}{250 \times 198.6} \right) = 2 \text{ N/mm}^2$$

$$\left(\frac{100A_{st}}{bd} \right) = \left(\frac{100 \times 720}{250 \times 198.6} \right) = 1.45$$

$$\therefore \tau_c = 0.70 \text{ N/mm}^2$$

$$\text{Balance shear} = V_s = \left[100 - \frac{0.70 \times 250 \times 198.6}{1000} \right] = 65.3 \text{ kN}$$

Using 10 mm Φ - 2 legged stirrups, spacing

$$s_v = \left(\frac{0.87 \times 415 - 2 - 79 - 198.6}{65.3 - 103} \right) = 173 \text{ mm}$$

Adopt 10 mm Φ - 2 legged stirrups at 170 mm centres spacing throughout the length of the pole.

The reinforcement details are as shown in Fig. 17.2.

17.5 EXAMPLES FOR PRACTICE

1. A reinforced concrete pole 10 m long is required to carry 4 conductors of 7 mm diameter each spaced at 500 mm intervals in a cross arm fixed at 600 mm from the top. The depth of embedment is 1.8 m, below ground level. Spacings of poles = 50 m
Wind pressure = 1.5 kN/m^2

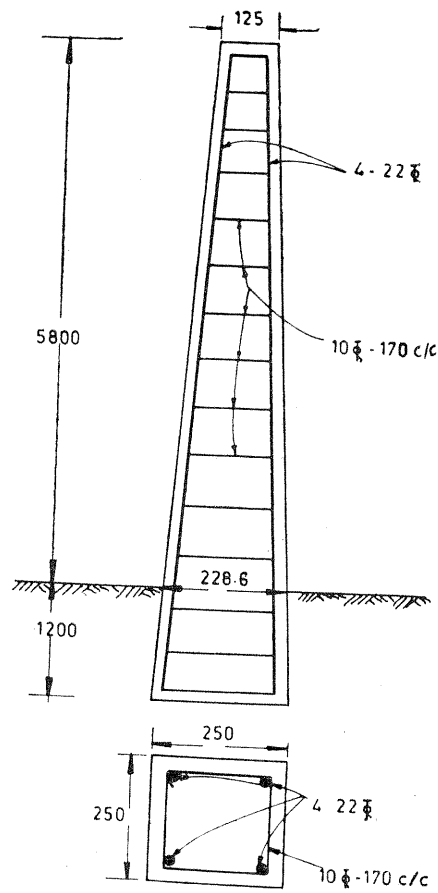


Fig. 17.2 Reinforcement details in R.C. pole

Load factor = 2.5

Tension in conductors = 3 kN

Adopting M-20 grade concrete and Fe-415 grade tor steel, design a suitable pole for the transmission line.

2. A reinforced concrete pole 12 m long is required to carry 4 conductors of 7 mm diameter each spaced at 600 mm intervals in a cross arm fixed at 500 mm from the top. The pole is embedded to a depth of 2 m below ground level. Spacings of poles = 60 m

Wind pressure = 1 kN/m²

Load factor = 2.5

Tension in conductors = 4 kN

Adopting M-20 grade concrete and Fe-415 grade tor steel, design a suitable pole for the transmission line.

18

Deep Beams

18.1 GENERAL FEATURES

A beam is considered as deep, if the depth of the beam is large in relation to the span of the beam. According to the Indian Standard Code IS : 456 – 1978, a beam is considered as a deep beam when the ratio of effective span to over all depth (L/D) is less than 2.0 for simply supported members and 2.5 for continuous members.

In deep beams, the bending stress distribution across any transverse section deviates appreciably from the straight line distribution assumed in the elementary beam theory. Consequently a transverse section which is plane before bending does not remain approximately plane after bending and the neutral axis does not usually lie at the mid depth. In the case of deep beams, shear flexure and shear modes dominated by tensile cleavage failure are common. The ultimate failure due to shear is generally brittle in nature in contrast to the ductile behaviour and progressive flexural failure with large number of cracks observed in normal beams.

18.2 PARAMETERS INFLUENCING DESIGN

The prominent parameters influencing the design of deep beams are:

- (a) C = Length of support
- (b) D = over all depth of beam
- (c) L = Effective span
- (d) b = Width of beam
- (e) w = uniform load on the beam

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The characteristic ratios used as parameters in design calculations and charts are,

$$\epsilon = \left(\frac{C}{L} \right) = \text{support to span ratio}$$

$$\beta = \left(\frac{D}{L} \right) = \text{Depth to span ratio}$$

The design charts and the procedure of design outlined here are based on the reports published by the Portland Cement Association, U.S.A. and the concrete Association of India. These reports are based on the original work reported by Dischinger in 1932. The parameters used in design are illustrated in Fig. 18.1.

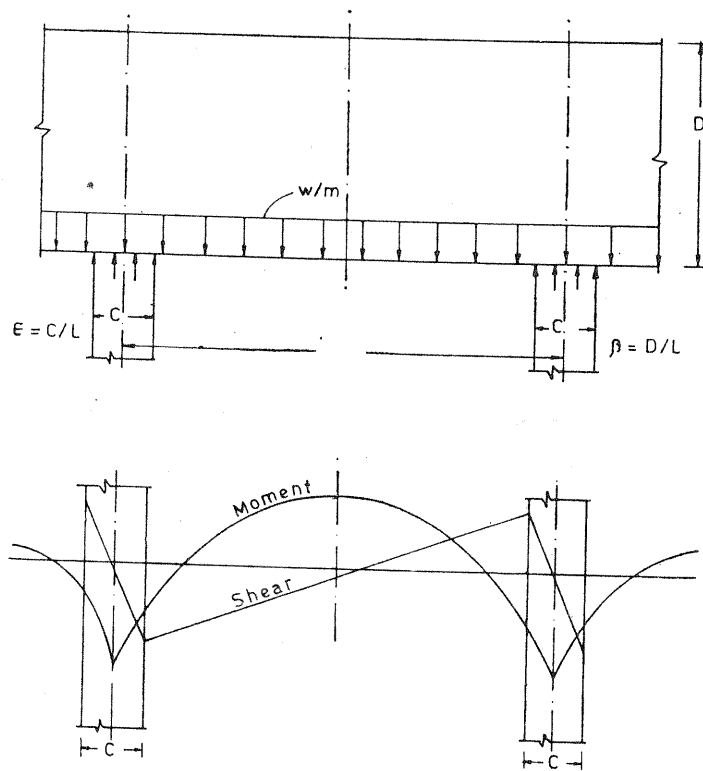


Fig. 18.1 Typical deep beam under uniform loading

18.3 FLEXURAL BENDING STRESSES

Figure 18.2 shows the flexural bending stress at mid span of a continuous deep beam for different ratios of β having values of 0.5, 0.67 and 1.0 and $\epsilon = 0.1$. As β increases from 0.5 to 1.0, the compressive stress decreases

rapidly at the top and the neutral axis moves towards the soffit of the beam. The tensile stress decreases gradually from $1.31 (w/b)$ to (w/b) .

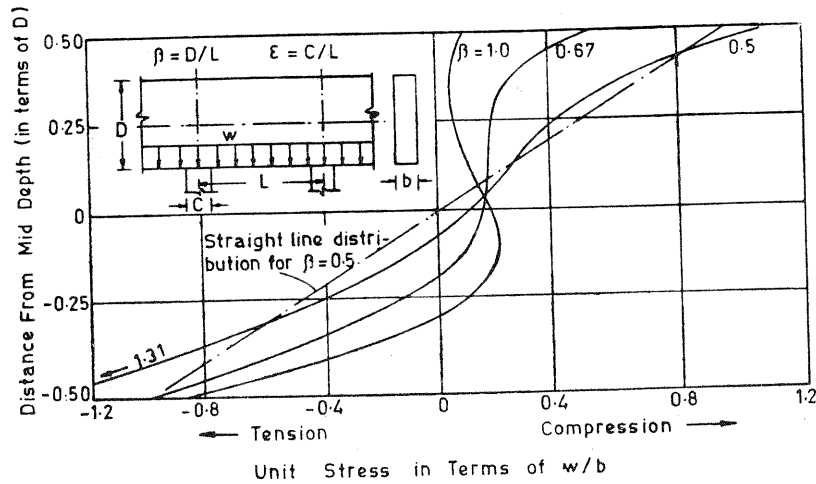


Fig. 18.2 Flexural stresses at mid-span of deep beams with uniform loading

Increasing the value of the depth D , beyond the value of the span L ($D/L > 1$), does not significantly influence the magnitude of the stresses within the depth ' L '. The curves are nearly the same for values of $\epsilon = 0.05$ to 0.2 .

The variation of stresses at support sections of a continuous deep beam for different values of β varying from 0.5 to 1.0 and $\epsilon = 0.1, 0.2$ and 0.05 are shown in Figs. 18.3, 18.4 and 18.5.

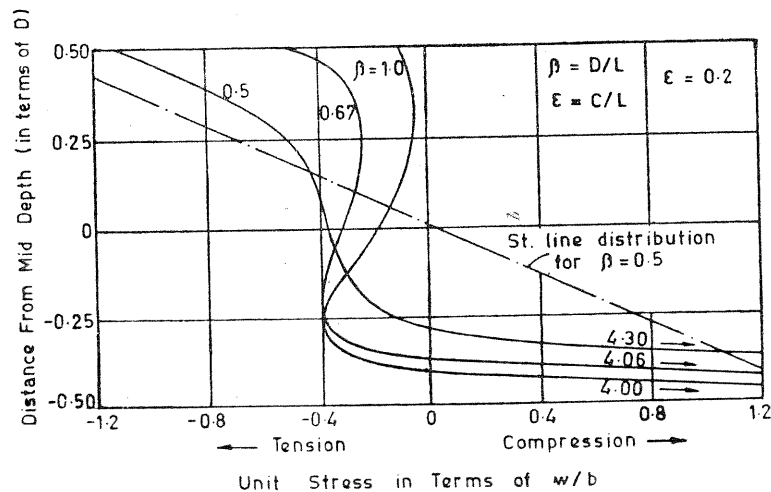


Fig. 18.3 Flexural stresses at support of deep beams with uniform loading

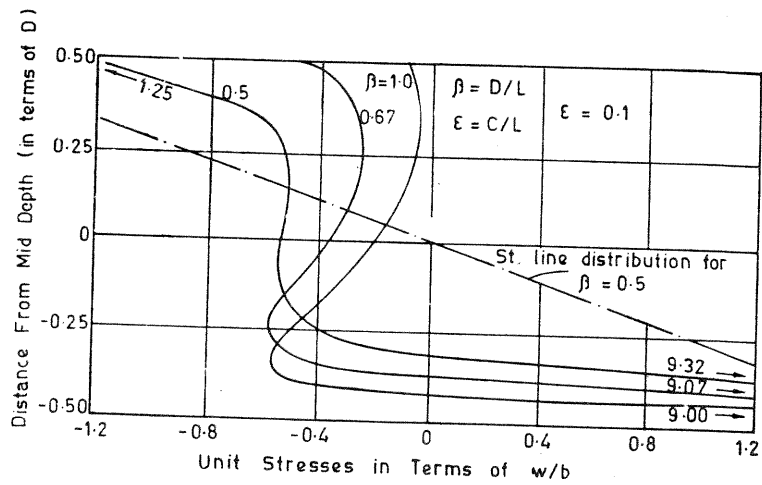


Fig. 18.4 Flexural stresses at support of deep beams with uniform loading

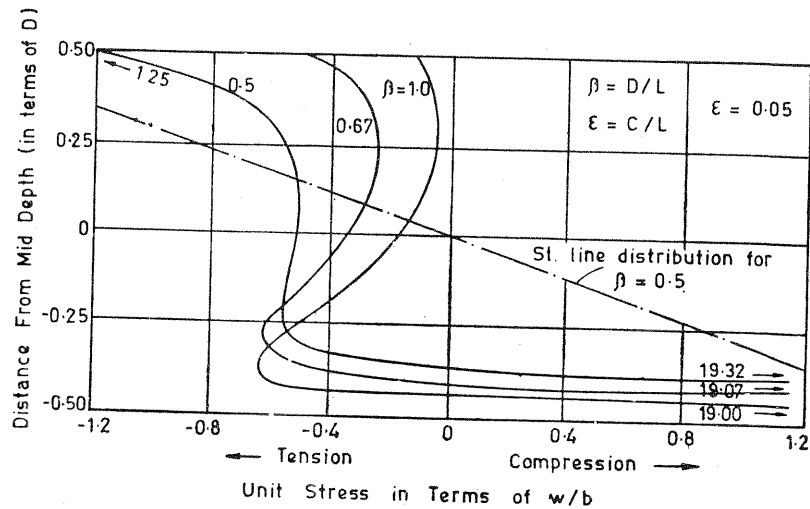


Fig. 18.5 Flexural stresses at support of deep beams with uniform loading

Assuming the uniformly distributed load shown in Figs. 18.2 to 18.5, the mid span and support moments in a typical interior span is given by

$$M_{(\text{span})} = \frac{wL^2}{24} (1 - \epsilon^2)$$

$$M_{(\text{support})} = \frac{wL^2}{24} (1 - \epsilon) (2 - \epsilon)$$

The reinforcements are designed for tension T , expressed as the product of a coefficient and wL for mid span and support sections. The coefficient for tension computation is shown in Fig. 18.6. The flexural stresses developed in simply supported single span girders is shown in Fig. 18.7.

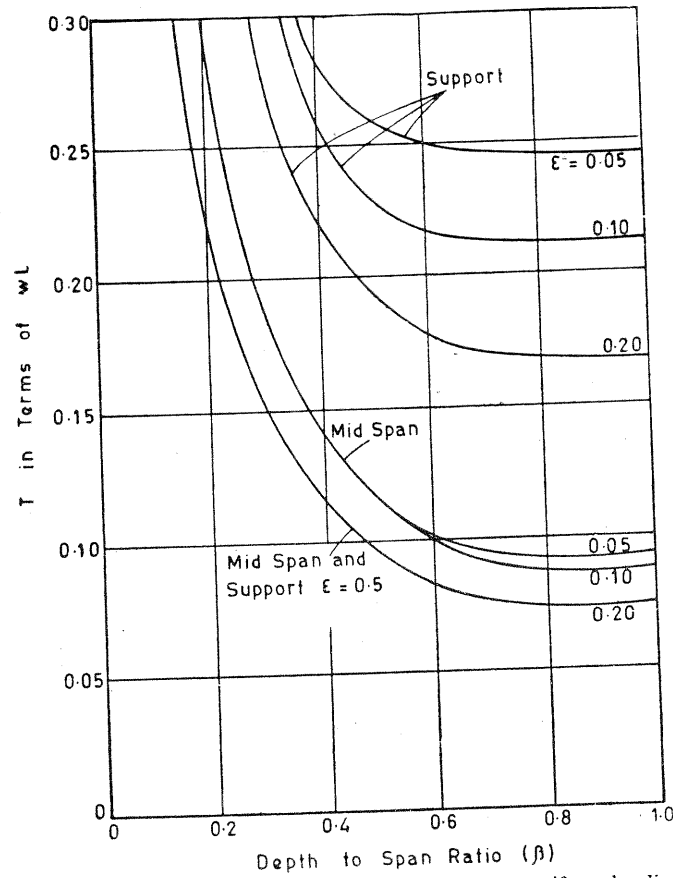


Fig 18.6 Resultant tension in deep beams having uniform loading

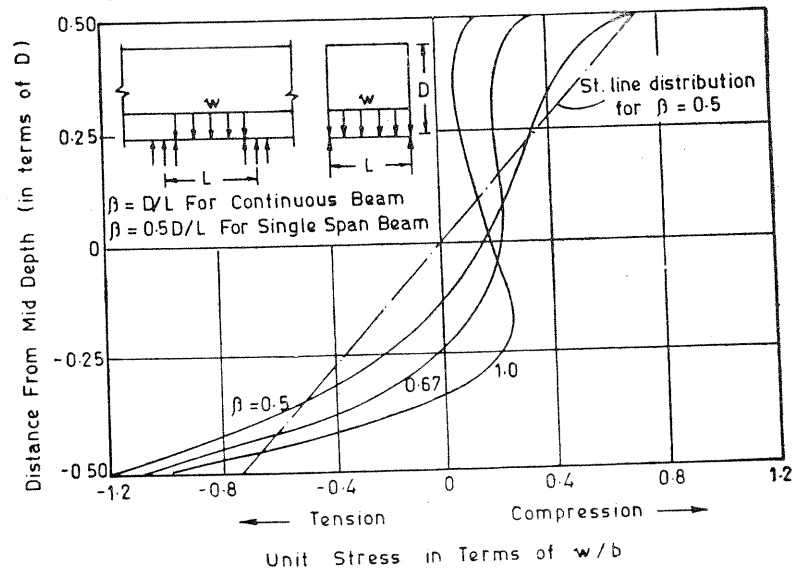


Fig. 18.7 Flexural stresses in simply supported single span deep girders

It is assumed that the stress distribution curves are similar at mid span for continuous girder with a length equal to $0.5L$ between the points of inflexion and that of a simply supported girder of span L . Hence the curves for the continuous girder shown in Fig. 18.7 may be adopted for single span girders when $\beta = 0.5D/L$. The design data is therefore generally selected for single span girders for $\beta = 0.5 D/L$ and $\epsilon = 0.5$ by using the Figs. 18.6 and 18.7.

The flexural stresses developed in continuous deep girder having concentrated loading at bottom and top are shown in Figs. 18.8 and 18.9 respectively.

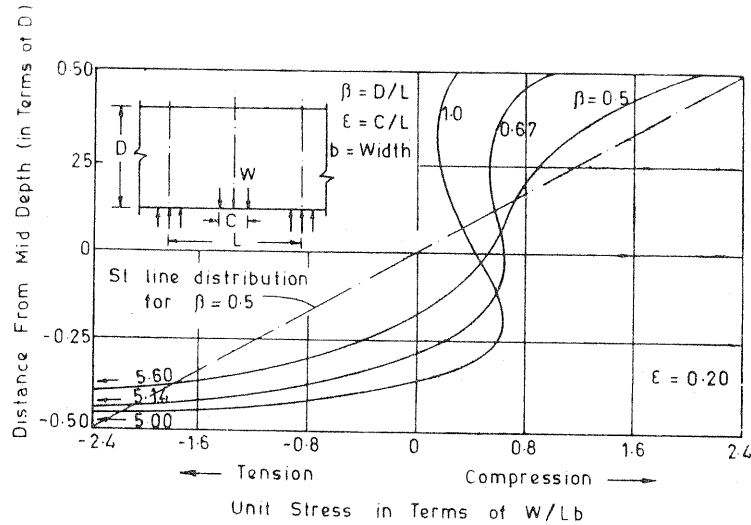


Fig. 18.8 Flexural stresses at mid span of deep beams with concentrated load at bottom edge

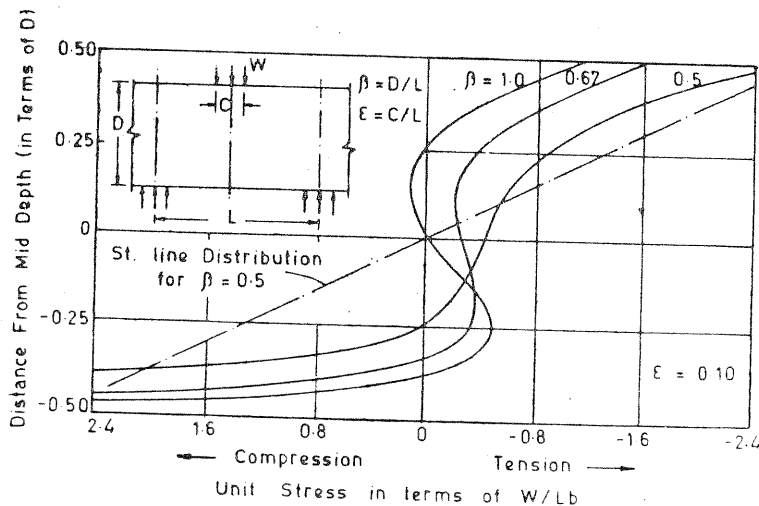


Fig. 18.9 Flexural stresses at centre line of support of deep beams with concentrated loading at top edge

A comparison of the curves indicate that they are nearly alike for $\beta = 0.5$. But as β increases there is increasing divergence of stresses for the two cases of loading. For $\beta = 1$, when the girder is loaded at top, the stress changes direction thrice and there are three neutral axis positions across the depth of the girder.

The resultant tension values as a function of depth to span ratio β and the parameter ϵ , are shown in Fig. 18.10. According to dischinger, selection of T -values based on loads applied at the bottom gives conservative results for continuous beams with loads applied on the top edge. For single span girder of span L , and depth ' D ' with a concentrated load W . The value of parameter β is taken as equal to $0.5 D/L$ and $\epsilon = 0.5 C/L$. The tension is computed as the product of the coefficient from Fig. 18.11 and the concentrated force W . The reinforcements are provided as near the soffit of the girder as possible.

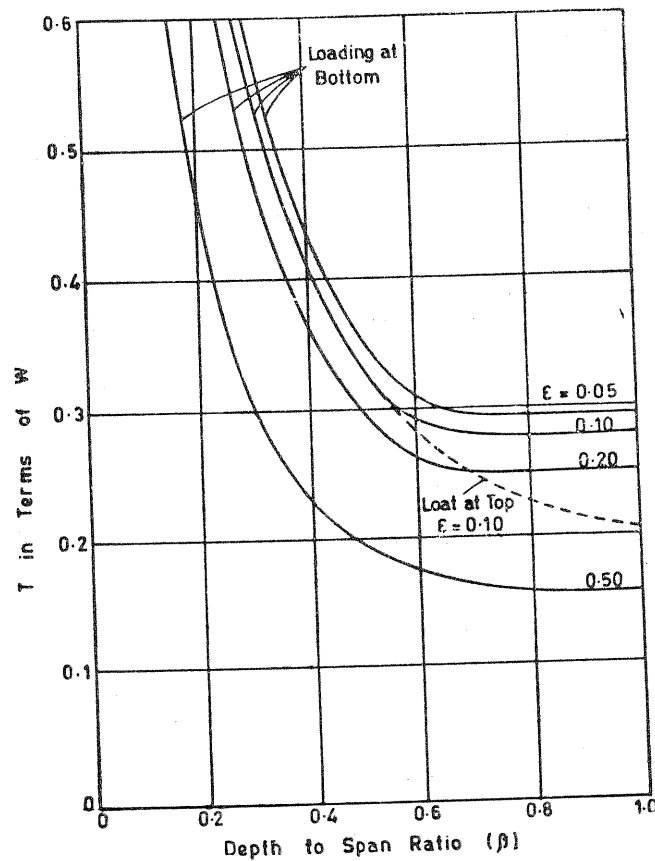


Fig. 18.10 Resultant tension in deep beams with concentrated load at mid span

18.4 SHEAR STRESSES IN DEEP BEAMS

In the case of deep beams with relatively small percentage of reinforcement, the cracks develop vertically from the soffit and remains practically vertical in comparison with the diagonal tension cracks observed in conventional shallow beams as shown in Fig. 18.11.

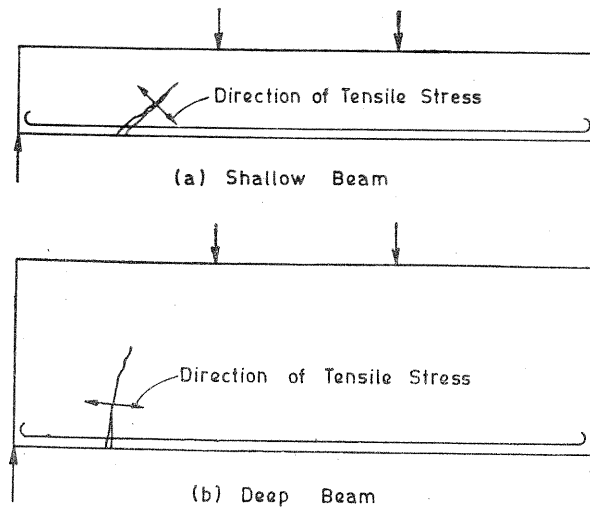


Fig. 18.11 Diagonal tension cracks in shallow and deep beams

It is clear from the figures that the diagonal tension which is characteristic of a shallow beam changes gradually into plain horizontal tension as the beam becomes a deep girder. Hence the conventional shear investigations are not strictly applicable to deep beams. Experimental investigations by V. Ramakrishnan and Y. Ananthanarayan have revealed that deep beams and shallow beams exhibit practically similar modes of shear failure for low shear span to depth ratios of less than 2. In deep beams the shear failure is always initiated by splitting action similar to that in a cylinder under diametrical compression as in the Brazzilian splitting test.

18.5 I.S. CODE PROVISIONS

According to the Indian Standard Code IS : 456-1978 provisions, a deep beam is a beam having a ratio of effective span to over all depth (L/D) less than 2 for simply supported and 2.5 for continuous members.

The reinforcements are provided for positive bending moment at mid span and negative bending moment at supports of continuous members computed on the basis of the lever arm, which is determined using the following relations:

(a) For simply supported beams:

$$\begin{aligned} Z &= 0.2 (L + 2D) \\ \text{when } 1 \leq L/D \leq 2 \\ \text{or } Z &= 0.6 L \\ \text{when } L/D < 1 \end{aligned}$$

(b) For continuous beams:

$$\begin{aligned} Z &= 0.2 (L + 1.5 D) \\ \text{when } 1 \leq L/D \leq 2.5 \\ \text{or } Z &= 0.5 L \\ \text{when } L/D < 1 \end{aligned}$$

where Z = Lever arm

L = Effective span taken as centre to centre distance between supports or 1.15 times the clear span, whichever is smaller and

D = over all depth

The positive reinforcements extending over the full span length without curtailment, should be placed within a zone of depth equal to $(0.25D - 0.05L)$ adjacent to the tension face of the beam.

The negative reinforcement over supports of a deep beam with a L/D ratio in the range of 1 to 2.5 should be distributed in two zones as detailed below:

(a) A zone of depth $0.2D$ adjacent to the tension face should contain a proportion of the tension steel given by

$$0.5 [L/D - 0.5]$$

(b) A zone measuring $0.3D$ on either side of the mid depth of the beam, containing the remainder of the tension steel evenly distributed.

For beams having a span to depth ratio less than 1, the steel is evenly distributed over a depth of $0.8D$ measured from the tension face.

In addition, side face reinforcement to the extent of 0.12% of gross concrete area, when deformed bars are used and 0.15% when mild steel bars are used should be provided as vertical reinforcements. Also the minimum percentage of longitudinal reinforcement is fixed at 0.2% for deformed bars and 0.25% for other types of bars. The spacing of the horizontal reinforcement should not be greater than three times the wall thickness or 450 mm. A deep beam complying with these reinforcement details is deemed to satisfy the provisions for shear according to the I.S. Code. The application of these design principles are illustrated in the following design examples.

18.6 DESIGN EXAMPLE

Design a typical interior span of a continuous deep beam using the following data:

Span of beam (L) = 9 m
 Over all depth (D) = 4.5 m
 Width of supports (C) = 0.9 m
 Width of beam (b) = 0.4 m
 Uniformly distributed load = $w = 200$ kN/m
 (including self weight)
 Concrete = M-20 grade
 Reinforcements = Fe-415 Tor steel.

Sketch the details of reinforcements at centre of span and support sections.

1. Design Parameters

$$\epsilon = \left(\frac{C}{L}\right) = \left(\frac{0.9}{9}\right) = 0.1$$

$$\beta = \left(\frac{D}{L}\right) = \left(\frac{4.5}{9}\right) = 0.5$$

$$L = 9 \text{ m} \quad D = 4.5 \quad w = 200 \text{ kN/m}$$

2. Moments

$$\begin{aligned} \text{Moment at mid span} &= \frac{wL^2}{24} (1 - \epsilon^2) \\ &= \left(\frac{200 \times 9^2}{24}\right) (1 - 0.01) = 675 \text{ kN} \cdot \text{m} \end{aligned}$$

$$\begin{aligned} \text{Moment at support} &= \left(\frac{wL^2}{24}\right) (1 - \epsilon)(2 - \epsilon) \\ &= \frac{wL^2}{24} (1 - 0.1)(2 - 0.1) \\ &= \left(\frac{wL^2}{14}\right) = \left(\frac{200 \times 9^2}{14}\right) = 1150 \text{ kN} \cdot \text{m} \end{aligned}$$

3. Tensile reinforcements (using design graphs)

The resultant T of all tensile stress in concrete is obtained by interpolating coefficients from Fig. 18.6.

For the given parameters $\epsilon = 0.1$ and $\beta = 0.5$

Coefficient for mid span = 0.12

Coefficient for interior support = 0.23

$\therefore$ Tension = (Coefficient) $\times wL$

Tension at mid span = $(0.12 \times 200 \times 9) = 216$ kN

Tension at support = $(0.23 \times 200 \times 9) = 414$ kN

$$\text{Mid span: } A_s = \left(\frac{216 \times 10^3}{230} \right) = 939 \text{ mm}^2$$

$$\text{Support: } A_s = \left(\frac{414 \times 10^3}{230} \right) = 1800 \text{ mm}^2$$

4. Steel Reinforcements according to I.S. 456 Code Procedure

Lever arm for continuous beam

$$Z = 0.2 (L + 1.5 D)$$

$$\text{when } 1 \leq L/D \leq 2.5$$

$$\text{In the present case } (L/D) = \left(\frac{9}{4.5} \right) = 2.0$$

$$\therefore Z = 0.2 (9 + 1.5 \times 4.5) = 3.15 \text{ m}$$

$$\therefore \text{ Tension at mid span} = \left(\frac{M}{Z} \right) = \left(\frac{675}{3.15} \right) = 214 \text{ kN}$$

$$\text{ Tension at support} = \left(\frac{M}{Z} \right) = \left(\frac{1150}{3.15} \right) = 365 \text{ kN}$$

$$\text{Mid span } A_s = \left(\frac{214 \times 10^3}{230} \right) = 930 \text{ mm}^2$$

$$\text{Support } A_s = \left(\frac{365 \times 10^3}{230} \right) = 1586 \text{ mm}^2$$

5. Minimum Reinforcements

According to IS : 456 minimum percentage of horizontal reinforcements are given by.

$$A_s = (0.002 \times 400 \times 4500) = 3600 \text{ mm}^2$$

Vertical reinforcements are given by

$$A_{sv} = (0.0012 \times 400 \times 1000) = 480 \text{ mm}^2/\text{m}$$

6. Arrangement of Reinforcements

According to the IS : 456 Code the reinforcements for positive and negative bending moments should be arranged as detailed below:

For positive B.M (centre of span)

$$\text{Zone of depth} = (0.25D \times 0.05L) = (0.25 \times 4.5 \times 0.05 \times 1) = 0.675 \text{ m}$$

$A_s = 3600 \text{ mm}^2$, to be distributed over a depth of 0.675 m from the tension face.

Use 20 bars of 16Φ ($A_s = 4020 \text{ mm}^2$) arranged in 5 rows of 4 bars each.

For negative B.M (support section)

$$\text{Zone of depth} = 0.2D = (0.2 \times 4.5) = 0.9 \text{ m}$$

$$\text{Proportion of steel} = 0.5 \left(\frac{L}{D} - 0.5 \right) = 0.5 \left(\frac{9}{4.5} - 0.5 \right) = 0.75$$

$$\therefore A_s = (0.75 \times 3600) = 2700 \text{ mm}^2$$

Use 16 bars of 16ϕ over a depth of 900 mm from tension face (4 rows of 4 bars each). Remaining steel = $(3600 - 2700) = 900 \text{ mm}^2$, Zone = $0.3 D = (0.3 \times 4.5) = 1.35 \text{ m}$ on either side of the mid depth of beam. Use 20 bars of 8 mm (5 rows of 4 bars). For vertical reinforcements at support and span sections, adopt 4 legged stirrups of 6 mm ϕ at 230 centres. The details of reinforcements are shown in Fig. 18.12.

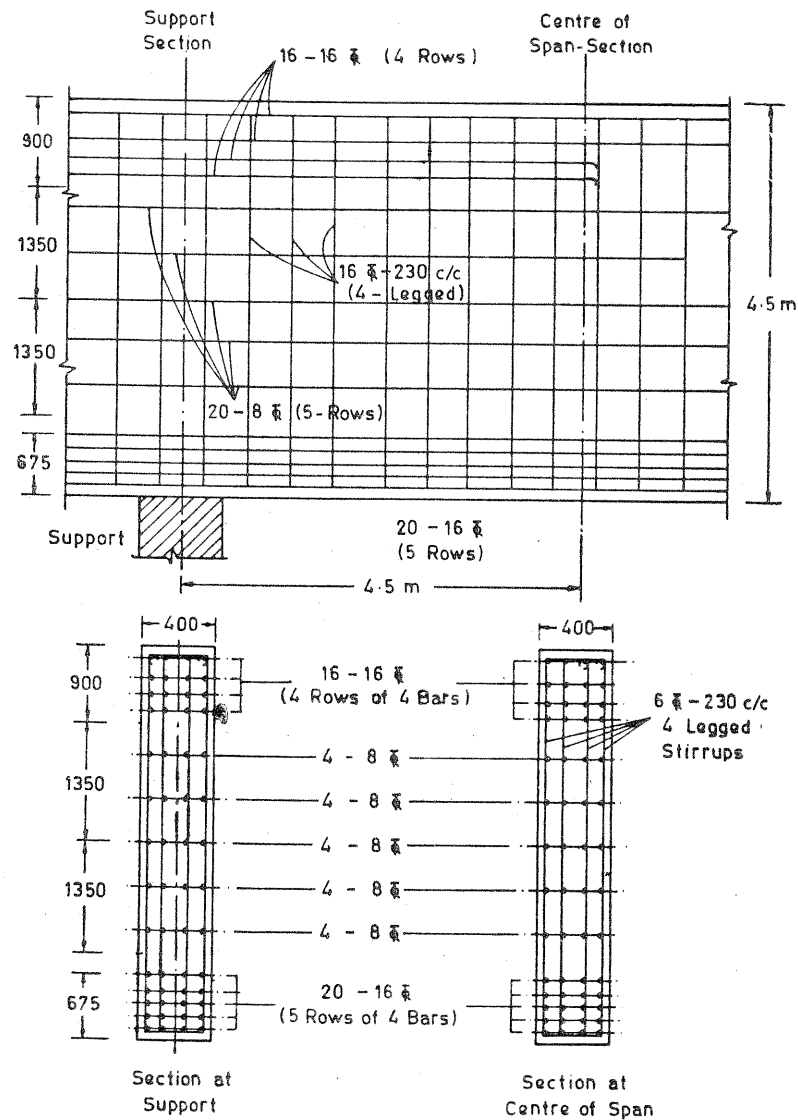


Fig. 18.12 Reinforcements in deep-beam

18.7 DESIGN EXAMPLE

Design a single span deep beam to suit the following data:

Effective span = 6 m

Over all depth = 6 m

Width of support = 0.6 m

Width of beam = 0.4 m

Total load on beam including self weight = 400 kN/m

Concrete = M-20 grade

Steel = Fe-415 grade Tor steel

1. Design Parameters

$$L = 6 \text{ m}$$

$$C = 0.6 \text{ m}$$

$$D = 6 \text{ m}$$

$$w = 400 \text{ kN/m}$$

For single span deep beams, the length of the support is disregarded and reaction assumed to act at the centre line of the support. Using the parameters of the continuous girder,

$$\epsilon = 0.5 \text{ and } \beta = \left(\frac{D}{2L} \right) = \left(\frac{6}{2 \times 6} \right) = 0.5$$

The total tension is obtained from Fig. 18.6 as

$$T = 0.095w(2L) = (0.095 \times 400 \times 2 \times 6) = 456 \text{ kN}$$

$$\therefore A_s = \left(\frac{456 \times 10^3}{230} \right) = 1983 \text{ mm}^2$$

Using the IS : Code procedure,

$$\text{Maximum moment} = \left(\frac{wL^2}{8} \right) = \left(\frac{400 \times 6^2}{8} \right) = 1800 \text{ kN}\cdot\text{m}$$

For simply supported beams,

$$Z = 0.2(L + 2D) \text{ for } 1 \leq L/D \leq 2 \text{ In this case } (L/D) = 1$$

$$\therefore Z = 0.2(6 + 2 \times 6) = 3.6 \text{ m}$$

2. Reinforcements

$$\text{Tension} = \left(\frac{M}{Z} \right) = \left(\frac{1800}{3.6} \right) = 500 \text{ kN}$$

$$\therefore A_s = \left(\frac{500 \times 10^3}{230} \right) = 2174 \text{ mm}^2$$

$$\text{Minimum horizontal reinforcement} = (0.002 \times 400 \times 6) = 4800 \text{ mm}^2$$

$$\text{Zone of depth} = (0.25D - 0.05L) = 0.25 \times 6 - 0.05 \times 6 = 1.2 \text{ m}$$

The tension reinforcement of 4800 mm², is arranged within a depth of 1.2 m from the tension face.

Adopt 24 bars of 16 mm Φ in 6 rows of 4 bars each.

$$\text{Vertical reinforcement area} = (0.0012 \times 400 \times 1000) = 480 \text{ mm}^2$$

Use 4 legged stirrups of 6 mm Φ at 230 mm centres.

The reinforcements details are as shown in Fig. 18.13.

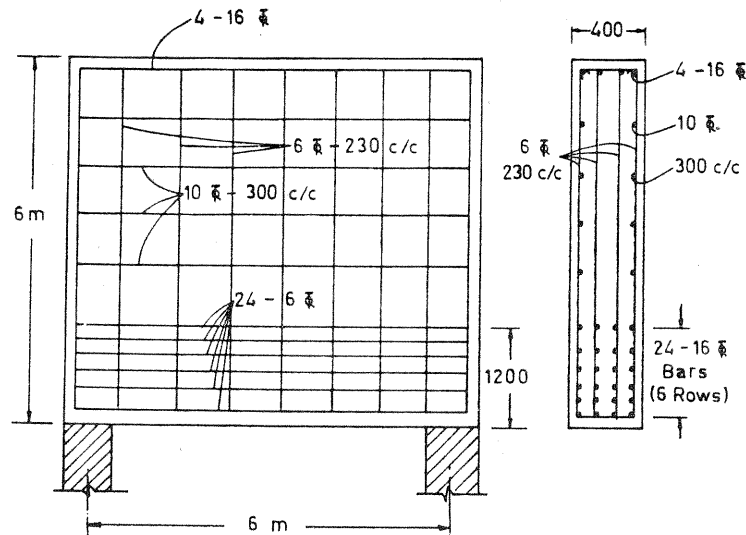


Fig. 18.13 Reinforcement details in single span deep beam

18.8 DESIGN EXAMPLE

A coal bunker 3 m by 3 m square with a height of 3 m stores 300 kN of coal. The side walls are 180 mm thick. Depth of hopper bottom = 1.2 m with a central opening of 0.5 m by 0.5 m. The bunker is supported on 4 R.C. columns at the corners. If the total weight of hopper bottom is 60 kN, design the side walls of the bunker as a deep beam using M-20 grade concrete and Fe-415 grade tor steel.

1. Data

Weight of coal = 300 kN

Weight of hopper bottom = 60 kN

Size of bunker = 3 m × 3 m × 3 m

Thickness of side walls = 180 mm

2. Loads

Weight of one side wall = $(3 \times 3 \times 0.18 \times 24) = 40$ kN

Total load on one wall acting as deep beam = $\left(\frac{360}{4} + 40\right) = 130$ kN

3. Moments

Span of deep beam = 3 m

$$M = \left(\frac{WL}{8}\right) = \left(\frac{130 \times 3}{8}\right) = 48.75 \text{ kN} \cdot \text{m}$$

4. Reinforcements

$$(L/D) = (3/3) = 1$$

$$Z = 0.2(L + 2D) = 0.2(3 + 2 \times 3) = 1.8 \text{ m}$$

$$\text{Tension } T = \left(\frac{M}{Z} \right) = \left(\frac{48.75}{1.8} \right) = 27 \text{ kN}$$

$$\therefore A_s = \left(\frac{27 \times 10^3}{230} \right) = 118 \text{ mm}^2$$

Minimum horizontal reinforcement

$$= (0.002 \times 180 \times 3000) = 1080 \text{ mm}^2$$

Use 10 bars of 12 mm Φ in a zone of depth

$$= (0.25D \times 0.05L) = (0.25 \times 3000 \times 0.05 \times 3000) = 900 \text{ mm}$$

$$\text{Vertical reinforcement} = (0.0012 \times 180 \times 1000) = 216 \text{ mm}^2$$

Use 6 mm Φ bars at 240 mm centres in vertical direction in both faces.

18.9 EXAMPLES FOR PRACTICE

1. A single span deep beam has an over all depth of 4 m and an effective span of 6 m. The width of the beam is 400 mm. The beam supports a uniformly distributed live load of 300 kN/m, over the entire span. Using M-20 grade concrete and Fe-415 grade tor steel, design suitable reinforcements for the beam and sketch the details.
2. A continuous deep beam spanning over three equal spans of 8 m each has an over all depth of 4 m. The width of support is 0.8 m and the width of beam = 0.4 m. The beam supports a uniformly distributed live load of 160 kN/m. Using M-15 grade concrete and Fe-250 grade mild steel, design suitable reinforcements for the central span of the continuous deep beam. Sketch the details of reinforcements.
3. A continuous deep beam with an interior span of 9 m, has an over all depth of 4.5 m. The width of support = 0.9 m. Width of beam = 400 mm. The beam supports a uniformly distributed live load of 100 kN/m together with concentrated loads of 200 kN at centre of span points. Using M-20 grade concrete and Fe-415 grade tor steel, design suitable reinforcements for the beam.
4. The total weight of contents and the hopper bottom of a bunker having 4 walls of size 4 m height by 4.7 m length is 840 kN. The Thickness of walls is 150 mm. The bunker is supported on 4 columns located at the corners. Adopting M-200 grade concrete and Fe-415 grade tor steel, design the walls of the bunker assuming them as deep beams.

19

Pipes

19.1 GENERAL ASPECTS

Reinforced cement concrete pipes are widely used for carrying drainage and irrigation water and for culverts over national highways carrying very heavy traffic. During the last two decades, R.C.C pipes have replaced the steel pipes for many works due to their negligible maintenance costs. R.C.C pipes are also used in gravity mains for carrying water under hydrostatic pressure. Developments of pipe casting by spinning process has resulted in better quality pipes. For conveying liquids under high pressure, prestressed concrete pipes are widely used.

19.2 CLASSIFICATION OF R.C.C. PIPES

According to the Indian Standard Code IS: 458-1971, Reinforced cement concrete pipes are classified as non pressure and pressure pipes with their applications as detailed below in Table 19.1.

19.3 DESIGN PRINCIPLES

Reinforced concrete pipes either spun or cast are designed to with stand the internal hydrostatic pressure without exceeding the permissible stresses of 126.5 N/mm^2 , for mild steel and 140 N/mm^2 , in the case of cold drawn steel wires. The thickness of the concrete pipe is designed in such a way that under specified test pressure, the maximum tensile stress in concrete when considered as effective to take stress along with the tensile reinforcement, should not exceed 2 N/mm^2 . The minimum thickness of pipe varies with the

TABLE 19.1 Classification of Pipes

Pipe designation		Conditions where used
NP-1	Unreinforced concrete nonpressure pipe	For drainage and irrigation use above ground or in shallow trenches
NP-2	Reinforced concrete light duty non pressure pipe	For drainage and irrigation use for culverts carrying light traffic
NP-3	R.C. heavy duty non pressur pipe	For drainage and irrigation use and for culverts carrying heavy traffic
NP-4	R.C. heavy duty non pressure pipe	For drainage and irrigation use and for culverts carrying very heavy traffic such as Railway loadings
P-1	R.C. pressure pipes tested to a hydrostatic pressure of 0.2 N/mm ² (20 m head)	For use in gravity mains the design pressure not exceeding 2/3 of test pressure
P-2	R.C. pressure pipes tested to a hydrostatic pressure of 0.4 N/mm ² (40 m head)	For use in pumping mains, the design pressure not exceeding half the test pressure
P-3	R.C. pressure pipes tested to a hydrostatic pressure of 0.6 N/mm ² (60 m head)	For use in pumping mains, the design pressure not exceeding half the test pressure

internal diameter and classification of pipes. For pressure pipes the thickness varies from 25 mm for diameter of 80 mm to 65 mm for a diameter of 1200 mm. The spigot dimensions of NP-1 class pipes are designated as shown in Fig. 19.1. The structural design of pipes should be in accordance with the Indian Standard Code IS: 783-1959.

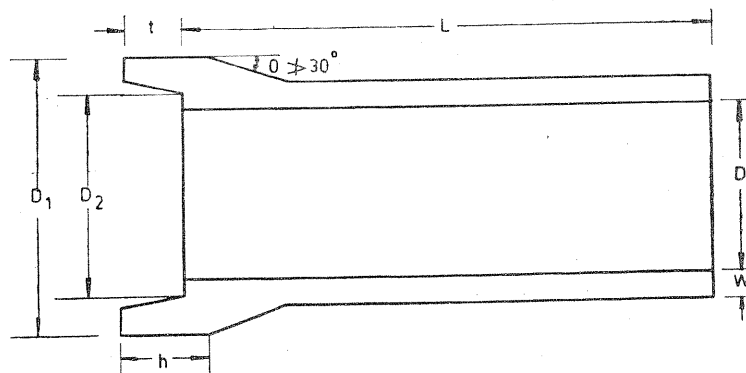


Fig. 19.1 Spigot dimension of NP-1 class R.C.C. pipes

The longitudinal reinforcement is designed to support the R.C.C pipe as a circular beam loaded with twice the self weight of the pipe and twice the weight of water to fill the pipe across a span equal to the length of the pipe. Under these loading conditions, the stresses in the reinforcement should not exceed the permissible stresses.

19.4 REINFORCEMENTS IN PIPES

The circumferential and longitudinal reinforcements are designed for the loads but minimum quantity of steel reinforcement are specified for different classes of pipes in IS: 458-1971. The typical reinforcement requirements for pipes of class-P-1 are shown in Table 19.2.

TABLE 19.2 Reinforcement Requirements in Pipes of Class P-1

Internal Diameter (mm)	Reinforcements	
	Longitudinal mild steel at permissible stress of 126.5 N/mm ² (kg/m)	Spiral hard drawn steel wire at permissible stress of 140 N/mm ² (kg/m)
100	0.863	0.327
200	0.863	0.575
400	1.00	3.800
600	1.25	8.150
800	1.78	14.500
1000	2.50	22.50
1200	3.36	32.50

The pitch of spirals should neither be more than 100 mm or four times the thickness of the barrel, whichever is less, nor less than the maximum size of aggregate plus the diameter of the bar used.

The minimum clear cover for concrete pipes specified in the I.S. Code for different types of pipes are as shown in Table 19.3.

TABLE 19.3 Cover Requirements

Barrel thickness	For spun pipes (mm)	For pipes other than spun pipes (mm)
25 mm and below	8.5	12.0
Over 25 and including 30 mm	9.0	12.0
Over 30 and below 75	12.0	16.0
75 mm and above	18.0	18.0

19.5 TESTS ON PIPES

The R.C.C. pipes should conform to the following tests specified in Indian Standard Code IS : 3597-1966:

- Hydrostatic test
- Three edge bearing test or sand bearing test
- Absorption test
- Bursting test

The requirements of load to produce a crack of size 0.25 mm and the ultimate load to be sustained before failure for different classes of pipes are specified in the relevant IS. Code.

19.6 DESIGN EXAMPLE

A reinforced concrete pressure pipe is to be designed to withstand a working pressure of 0.2 N/mm². The internal diameter of the pipe is 1000 mm and the length of the pipe is 3 m. Design the pipe and sketch the details of reinforcements. Adopt M-20 grade concrete and hard drawn steel wire conforming to I.S. 432.

1. Data

Intensity of pressure = 0.2 N/mm² = 200 kN/m²

Internal diameter = 1000 mm = 1 m

Length of pipe = 3 m, $m = 13$ m

2. Permissible Stresses

According to IS : 458-1971

$\sigma_{st} = 140$ N/mm²

$\sigma_{ct} = 2$ N/mm²

3. Hoop Tension and Reinforcement

$$\text{Maximum hoop tension} = \left(\frac{pD}{2} \right) = \left(\frac{200 \times 1}{2} \right) = 100 \text{ kN/m}$$

$$\therefore A_{st} = \left(\frac{100 \times 10^3}{140} \right) = 714 \text{ mm}^2/\text{m Length}$$

Provide 10 mm Φ bars at 70 mm c/c as spiral reinforcement

($A_{st} = 1122$ mm²)

Minimum quantity of steel as per IS : 458 is 22.5 kg/m length of pipe

Assuming 80 mm as thickness of pipe

Weight of one spiral of 10 mm $\Phi = (\pi \times 1.06 \times 0.612) = 2.03$ kg

$$\text{Number of spirals in 1 m} = \left(\frac{1000}{70} \right) = 14.3$$

$\therefore$ Weight of spiral reinforcement/metre length of pipe

$$= (2.03 \times 14.3) = 29.14 \text{ kg/m}$$

The quantity of steel reinforcement provided is greater than the minimum of 22.5 kg/m specified in the code.

4. Thickness of Pipe

Let t = Thickness of pipe from cracking considerations

$$\left[\frac{(pD/2)}{1000t + (m-1)A_{st}} \right] = \sigma_{ct}$$

$$\left[\frac{100 \times 10^3}{1000t + (13-1)1122} \right] = 2$$

Solving $t = 36.53 \text{ mm}$

But minimum thickness is not less than 55 mm. Adopt $t = 60 \text{ mm}$.

5. Longitudinal Reinforcement

Assuming the pipe to span over a length of 3 m

Self weight of pipe $= (\pi \times 1.06 \times 0.06 \times 24) = 4.79 \text{ kN/m}$

Weight of water $= \left(\frac{\pi \times 1^2}{4} \times 10 \right) = 7.85 \text{ kN/m}$

$\therefore$ Total design load $= 2(4.79 + 7.85) = 25.28 \text{ kN/m}$

$$I = \frac{\pi}{64} [D^4 - d^4] = \frac{\pi}{64} [1.12^4 - 1^4] = 0.028 \text{ m}^4$$

$$M_{\max} = \left(\frac{25.28 \times 3^2}{8} \right) = 28.44 \text{ kN} \cdot \text{m}$$

$$\text{Stress } \sigma = \left(\frac{28.44 \times 10^6 \times 560}{0.028 \times 10^{12}} \right) = 0.568 \text{ N/mm}^2$$

Stresses are negligibly small. Provide minimum longitudinal reinforcement of 2.5 kg/m using 5 mm ϕ bars,

$$\text{Weight of each bar} = \left(\frac{\pi \times 0.005^2}{4} \times 7800 \right) = 0.153 \text{ kg/m}$$

$$\therefore \text{Number of bars required} = \left(\frac{2.5}{0.153} \right) = 16.33$$

$\therefore$ Provide 20 bars of 5 mm ϕ spaced 166 mm along the circumference as longitudinal reinforcement.

19.7 DESIGN OF NON PRESSURE R.C.C. PIPES FOR CULVERTS

R.C.C. pipes are commonly used as a cross drainage work for a road or railway embankment. The hydraulic design consists of computing the area of the pipe required to pass a given discharge. If Q is the discharge, A = cross sectional area of the pipe and v = velocity of flow, and d = diameter of the pipe, then

$$A = \left(\frac{\pi d^2}{4} \right) = \left(\frac{Q}{v} \right) \quad \therefore \quad d = \sqrt{\frac{4Q}{\pi v}}$$

The structural design of the pipe involves the computations of the three edge bearing strength of pipe, the weight of earth fill over the fill and the load on pipe due to a surface concentrated live load, each associated with a strength factor generally taken as 1.5. The type of non-pressure pipe and bedding are so chosen that under the worst combination of field loading, a factor of safety of 1.5 is available as given by the equation,

$$\left[\frac{\text{Three edge bearing strength (kN/m)}}{\text{Factor of safety (1.5)}} \right] = \left[\frac{W \text{ due to filling material (kN/m)}}{\text{Corresponding strength factor}} \right] + \left[\frac{W \text{ due to surface load (kN/m)}}{\text{strength factor (1.5)}} \right]$$

The loading acting on the pipe due to the soil in embankment is computed from the equation,

$$W = C_e w D^2$$

where, W = Vertical external load in kN/m of pipe due to embankment material as shown in Table 19.4.

C_e = Coefficient depending on the ratio of height of embankment H to the external diameter of the pipe and condition of laying as specified in IS : 783-1959,

w = Density of the embankment material in kN/m³

D = External diameter of pipe (m)

The load on the pipe due to a concentrated high way wheel load P is obtained from the equation,

$$W = 4C_s \cdot I \cdot P$$

where, W = Vertical external load in kN/m due to concentrated surface load

C_s = Influence coefficient depending upon D and H as compiled in Table 19.5

H = Vertical depth of top of pipe below the surface (m)

D = External diameter of pipe (m)

P = Concentrated wheel load (kN)

I = Impact factor (1.5 for high ways)

In the case of Railway loading, the load is uniformly distributed because of the sleepers and ballast. The load on a buried pipe in a railway embankment is given by the equation,

$$W = 4C_s \cdot U \cdot D$$

where, W = Load on pipe in kN/m

C_s = Influence coefficient depending on the length of the sleeper, distance between two axles and depth of the top of pipe below surface

U = Uniformly distributed load in kN/m², on the surface directly above the pipe = $\left[\frac{PI}{4AB} + 2W_t \cdot B \right]$

P = Axle load in kN (229 kN for B.G.)

A = Half the length of the sleeper in m (1.35 m for B.G.)

B = Half the distance between the two driving axles in m [0.92 m for B.G.]

W_t = Weight of track structure in kN/m (generally 3 kN/m)

D = Outside diameter of the pipe in m.

TABLE 19.4 Load on Pipe Due to Earth Fill

Pipe size d (mm)	Outer diameter D (mm)	Embankment loading on pipe in kN/m for various depths H in m									
		1	2	3	4	5	6	7	8	9	10
NP-3											
500	650	16.8	34.9	54.0	65.3	99.0	118.5	150.0	171.0	191.0	212.0
600	760	18.7	42.6	62.4	75.5	103.0	133.0	160.5	186.0	197.0	230.0
700	860	20.0	48.0	68.0	86.5	118.5	154.5	174.0	209.0	228.0	246.0
800	980	24.2	55.3	79.5	107.0	136.5	163.0	202.0	230.0	260.0	288.0
900	1100	28.3	58.7	91.5	122.0	150.0	185.0	209.0	257.0	297.0	338.0
1100	1200	28.6	59.5	101.0	132.0	163.0	202.0	228.0	259.0	324.0	357.0
1200	1430	33.4	74.0	115.0	159.0	200.0	226.0	274.0	315.0	352.0	427.5

TABLE 19.5 Influence Coefficient C_s for Concentrated Surface Load for High Ways

Pipe size d (mm)	Outer diameter D (mm)	C_s for various depths H in m									
		0.1	0.2	0.3	0.4	0.6	0.8	1	2	3	4
NP-3											
500	650	0.246	0.228	0.198	0.169	0.117	0.083	0.060	0.017	0.008	0.005
600	760	0.247	0.234	0.210	0.182	0.131	0.094	0.068	0.022	0.010	0.006
700	860	0.247	0.236	0.215	0.186	0.140	0.102	0.075	0.024	0.010	0.006
800	980	0.249	0.240	0.220	0.196	0.149	0.110	0.083	0.027	0.013	0.007
900	1100	0.249	0.241	0.255	0.202	0.156	0.117	0.089	0.029	0.014	0.008
1100	1200	0.249	0.242	0.228	0.205	0.162	0.123	0.095	0.032	0.015	0.010
1200	1430	0.249	0.242	0.230	0.209	0.171	0.131	0.104	0.036	0.020	0.011

For broad gauge loading, the equation reduces to $W = 339 \cdot C_s \cdot D$
The values of the coefficient C_s is compiled in Table 19.6.

TABLE 19.6 Influence Coefficient C_s for Broad Gauge Railway Loading

H (m)	C_s	H (m)	C_s
0.1	0.250	1.0	0.183
0.2	0.249	2.0	0.094
0.3	0.245	3.0	0.052
0.4	0.240	4.0	0.032
0.5	0.233	5.0	0.021
0.6	0.224	6.0	0.015
0.7	0.218	7.0	0.011
0.8	0.205	8.0	0.009
0.9	0.193	9.0	0.007
		10.0	0.005

19.8 DESIGN EXAMPLE

Design a suitable pipe culvert to suit the following data:

Discharge through pipe culvert = $1.57 \text{ m}^3/\text{s}$

Velocity of flow through pipe = 2 m/s

Width of road (two lane) = 7.5 m

Top width of embankment = 15 m

Side slopes of embankment = $1.5 : 1$

Bed level = 100.00

Top of embankment = 103.00

Loading – I.R.C. Class AA.

Draw the longitudinal section, and end view of the pipe culvert.

1. Diameter of Pipe Culvert

Discharge $Q = Av$

$$\therefore A = \left(\frac{Q}{v} \right) = \left(\frac{1.57}{2} \right) = 0.785 \text{ m}^2$$

$$\left(\frac{\pi d^2}{4} \right) = 0.785 \quad \therefore d = 1.00 \text{ m}$$

Adopt NP-3, R.C.C. pipe heavy duty non-pressure pipe for carrying heavy road traffic.

From IS : 458-1971, for a pipe of internal diameter 1 m , the external diameter $D = 1.2 \text{ m}$.

2. Load Due to Earth Fill

Height of embankment over pipe = 2 m

From Table 19.4, for $d = 1000$ mm and $H = 2$ m,

Load due to earth fill = 59.5 kN/m

3. Load on Pipe Due to I.R.C. Class AA Wheel Loads

Assuming I.R.C. Class AA tracked vehicle,

$$\text{Load/m length} = \left(\frac{350}{3.6} \right) = 97.22 \text{ kN/m}$$

$$\text{Loading on pipe} = 4 \cdot C_s \cdot I \cdot P$$

From Table 19.5, for $d = 1000$ mm and

$$H = 2 \text{ m}, \quad C_s = 0.032, \quad I = 1.5, \quad P = 97.22$$

$$\text{Loading on pipe} = (4 \times 0.032 \times 1.5 \times 97.22) = 18.66 \text{ kN/m}$$

4. Check for Safety Factor

Referring to IS : 458-1971

Three edge bearing strength for NP-3 class pipe of 1000 mm internal diameter is 111 kN/m.

$\therefore$ The strength factor (S.F.) required for bending is obtained from the equation,

$$\left(\frac{111}{1.5} \right) = \left(\frac{59.5}{\text{S.F.}} \right) + \left(\frac{18.66}{1.5} \right) \quad \therefore \text{S.F.} = 0.96.$$

The strength factor for first class bending is 2.3 and for concrete grade bedding shown in Fig. 19.2 is 3.7. Any of these two bedding can be provided for the pipe culvert.

5. Reinforcements

The minimum reinforcements in the pipe according to IS : 458-1971 are as follows.

Spiral reinforcement of hard drawn steel wire with a permissible stress of $140 \text{ N/mm}^2 = 44 \text{ kg/m}$

Longitudinal reinforcement of mild steel with a permissible stress of $126.5 \text{ N/mm}^2 = 5.80 \text{ kg/m}$

Provide 12 mm ϕ bars at 60 mm c/c as spiral reinforcement

Average diameter of spiral = 1.1 m

Weight of one spiral of 12 mm $\phi = (\pi \times 1.1 \times 0.88) = 3.045 \text{ kg}$

$$\text{Number of spirals in 1 m} = \left(\frac{1000}{60} \right) = 16.66$$

$\therefore$ Weight of spiral reinforcement/metre length of pipe

$$= (3.045 \times 16.66) = 50.7 \text{ kg/m}$$

The quantity of spiral steel provided is greater than the minimum of 44 kg/m specified in the code. Providing 6 mm ϕ mild steel bars as longitudinal reinforcement.

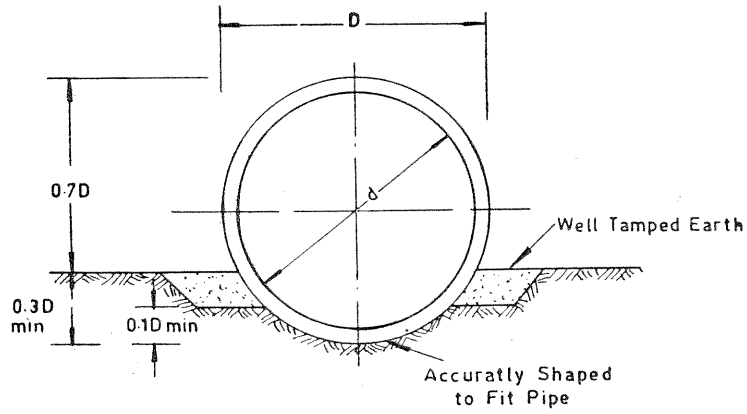
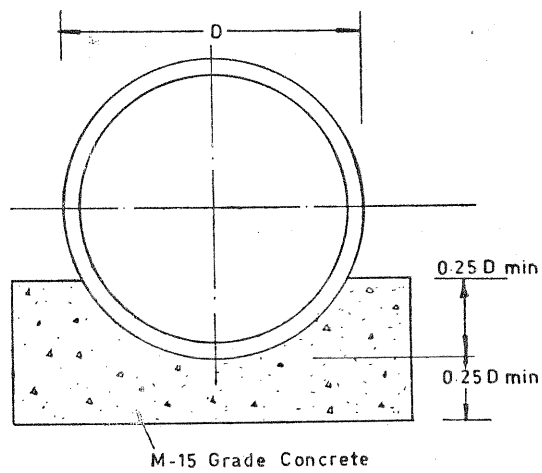

 a) First-Class Bedding

 b). Concrete Cradle Bedding

Fig. 19.2 Beddings for concrete pipes in pipe culverts

$$\text{Weight of each bar} = \left(\frac{\pi \times 0.006^2}{4} \times 7800 \right) = 0.22 \text{ kg/m}$$

$$\therefore \text{Number of bars required} = \left(\frac{5.80}{0.22} \right) = 26.36$$

$$\text{Spacing} = \left(\frac{\pi \times 1100}{26.36} \right) = 131 \text{ mm}$$

Adopt 130 mm spacing for the longitudinal reinforcement.
The details of the pipe culvert are shown in Fig. 19.3.

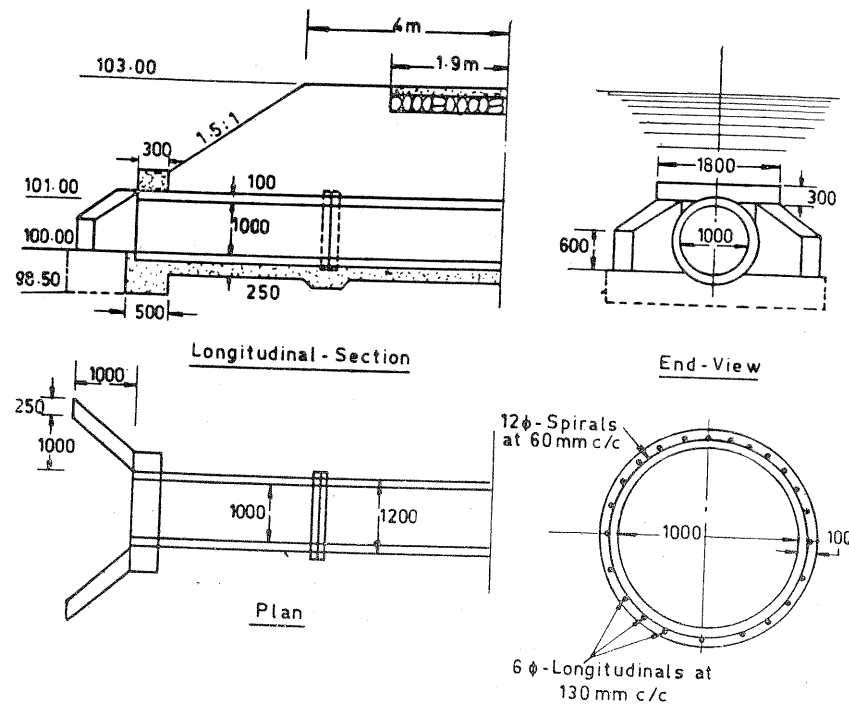


Fig. 19.3 R.C.C. pipe-culvert

19.9 EXAMPLES FOR PRACTICE

1. A reinforced concrete pressure pipe is to be designed to with stand a working pressure of 0.1 N/mm^2 . The internal diameter of the pipe is 500 mm and the length of the pipe is 3 m. Design the pipe according to the I.S. Code requirements and sketch the details of reinforcements. Adopt M-20 grade concrete and hard drawn steel wire conforming to IS : 432.
2. A R.C.C pipe is required to with stand 15 m head of water Using M-20 grade concrete and hard drawn steel wire, design the pipe and sketch the details of reinforcements.
3. Design a suitable pipe culvert to carry a discharge of $1 \text{ m}^3/\text{s}$ with a velocity of 2 m/s. The depth of earth filling over the pipe is 2 m. Adopt I.R.C. class AA loading with M-20 grade concrete and steel conforming to I.S. 432. Sketch the details of reinforcements and bedding for the pipe.

20

Bridge Deck Systems

20.1 GENERAL TYPES

The most common types of reinforced concrete bridge decks are (a) the slab type, (b) the tee beam and slab type with cross girders. The reinforced concrete slab type deck is generally used for small spans. This type of super structure is economical up to spans of about 8 m. Slab decks are simpler for construction due to the easier fabrication of form work and reinforcements and placement of concrete. The slab decks are supported on the two opposite sides on piers or abutments.

Tee beam and slab types decks are generally adopted in the span range of 10 to 25 m. For longer spans, the dead weight of girders becomes too heavy and to reduce the dead weight moments, prestressed concrete bridge decks are most commonly used. The tee beam deck comprises of the continuous slab cast monolithic with the longitudinal tee girders and cross girders spaced at regular intervals as shown in Fig. 20.1.

20.2 ANALYSIS AND DESIGN OF SLAB DECKS

(a) *Slabs Spanning in one Direction*

For slabs spanning in one direction, the dead load moments can directly be computed assuming the slab to be simply supported between the supports. Bridge deck slabs have to be designed for I.R.C. loads, specified as Class AA or Class A depending on the importance of the bridge. The different classes of I.R.C. loads specified in I.R.C. bridge code are shown in Figs. 20.2 and 20.3. For slabs supported on two opposite sides, the maximum bending moment caused by a wheel load may be assumed to be resisted by an

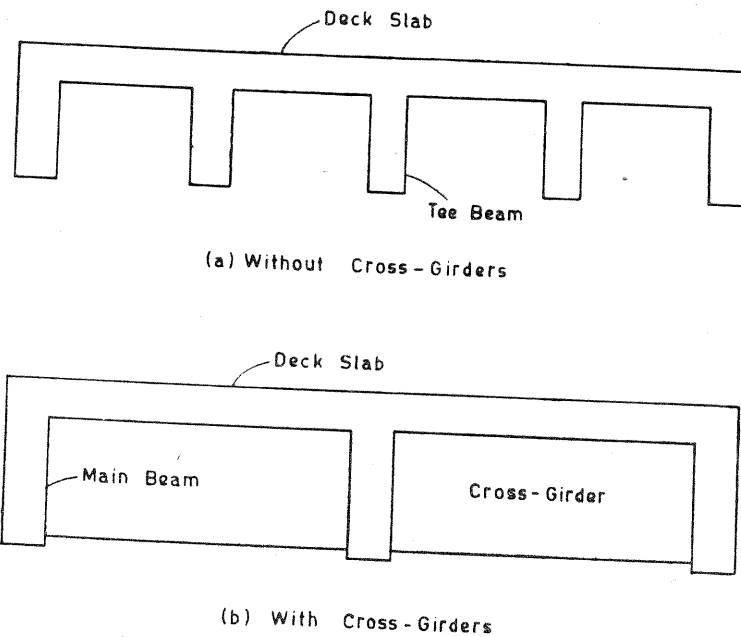


Fig. 20.1 Tee beam and slab bridge decks

effective width of slab measured parallel to the supporting edges. For a single concentrated load the effective width may be calculated by the equation:

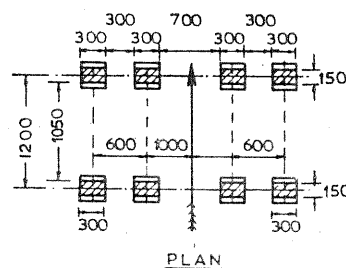
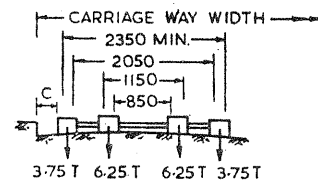
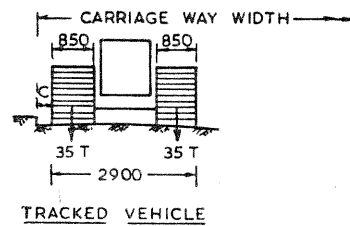
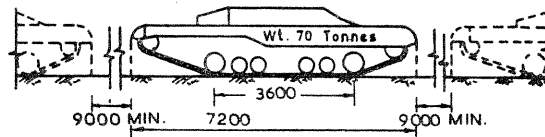
$$b_e = k \cdot x \left(1 - \frac{x}{L} \right) + b_w$$

- where
- b_e = The effective width of slab on which the load acts
 - L = Effective span
 - x = Distance of centre of gravity of load from nearer support
 - b_w = breadth of concentration area of load
 - k = A constant depending on the ratio (B/L) where B is the width of the slab.

The values of the constant 'K' for different values of the ratio (B/L) is compiled in Table 20.1.

Notes for Fig. 20.2

1. The nose to tail spacing between two successive vehicles shall not be less than 90 mm.
2. For multi lane bridges and culverts, one train of Class AA tracked or wheeled vehicles whichever creates severer conditions shall be considered for every two traffic lane width.
3. No other live load shall be considered on any part of the said 2-lane width carriageway of the bridge when the above mentioned train of vehicles is crossing the bridge.



WHEELED VEHICLE

Fig. 20.2 I.R.C. Class AA loading

4. The maximum loads for the wheeled vehicle shall be 20 tonnes for a single axle or 40 tonnes for a bogie of two axles spaced not more than 1.2 m centres.
5. The minimum clearance between the road face of the kerb and the outer edge of the wheel or track, C , shall be as under:

Carriage width		Minimum value of C
3.8 m and above	single lane bridges	0.3 m
less than 5.5 m	multi-lane bridges	0.6 m
5.5 m or above		1.2 m

TABLE 20.1 Values of k

$\frac{B}{L}$	k For simply supported slab	k For continuous slab	$\frac{B}{L}$	k For simply supported slab	k For continuous slab
0.1	0.40	0.40	1.1	2.60	2.28
0.2	0.80	0.80	1.2	2.64	2.36
0.3	1.16	1.16	1.3	2.72	2.40
0.4	1.48	1.44	1.4	2.80	2.48
0.5	1.72	1.68	1.5	2.84	2.48
0.6	1.96	1.84	1.6	2.88	2.52
0.7	2.12	1.96	1.7	2.92	2.56
0.8	2.24	2.08	1.8	2.96	2.60
0.9	2.36	2.16	1.9	3.00	2.60
1.0	2.48	2.24	2 and above	3.00	2.60

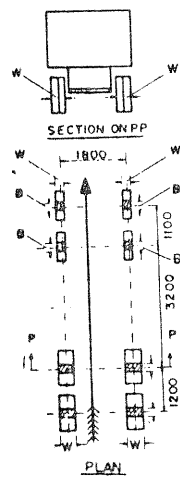
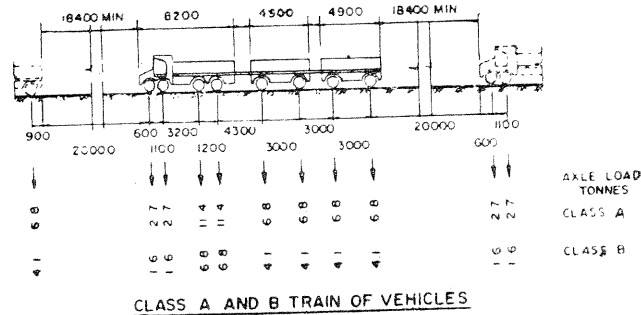
Notes for Fig. 20.3

1. The nose to tail distance between successive trains shall not be less than 18.4 m.
2. No other live load shall cover any part of the carriage way when a train of vehicles (or trains of vehicles in multi-lane bridge) is crossing the bridge.
3. The ground contact area of the wheels shall be as under:

Axle load tonnes	Ground contact area	
	B mm	W mm
11.4	250	500
6.8	200	380
4.1	150	300
2.7	150	200
1.6	125	175

4. The minimum clearance, f , between outer edges of the wheel and the roadway face of the kerb, and the minimum clearance, g , between the outer edges of passing or crossing vehicles on multi lane bridges shall be as given below:

Clear carriage way width	g	f
5.5 to 7.5 m	Uniformly increasing from 0.4 to 1.2 m	150 mm for all carriage way widths
above 7.5 m	1.2 m	



DRIVING VEHICLE

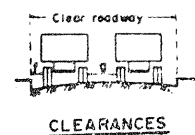


Fig. 20.3 I.R.C. class A and B loadings

The maximum bending moment in the slab develops for Class AA wheeled vehicle for spans up to 4 m and for Class AA tracked vehicle for spans exceeding 4 m. For two lane bridges, the shear due to Class AA tracked vehicle controls the design for all spans from 1 to 8 m. The distribution reinforcement is designed for 0.3 times the live load moment and 0.2 times the dead load moment.

(b) Slabs Spanning in two Directions

In the case of bridge decks with tee beam and cross girders, the deck slab is supported on all the four sides and is spanning in two directions. The moments in the two directions can be computed by using the design curves

developed by M. Pigeaud. The method developed by Pigeaud is applicable to rectangular slabs supported freely on all the four sides and subjected to a symmetrically placed concentrated load as shown in Fig. 20.4.

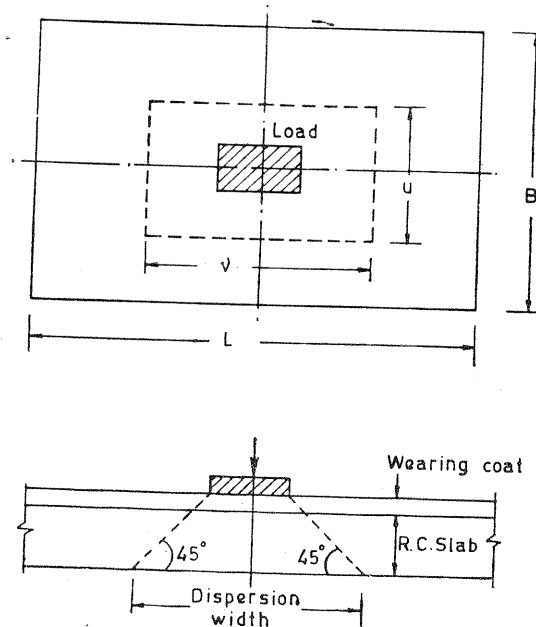


Fig. 20.4 Dispersion of live load through deck slab

The notations used are as follows:

- L = Long span length
- B = Short span length
- u, v = Dimensions of the load spread after allowing for dispersion through the deck slab
- k = Ratio of short to long span $= (B/L)$
- M_1 = Moment in the short span direction
- M_2 = Moment in the long span direction
- m_1 and m_2 = Coefficients for moments along the short and long spans
- μ = Poisson's ratio for concrete generally assumed as 0.15
- W = Load from the wheel under consideration

The dispersion of the load may be assumed to be at 45 degrees through the wearing coat and deck slab.

The bending moments are computed as

$$M_1 = (m_1 + \mu m_2)W$$

$$M_2 = (m_2 + \mu m_1)W$$

The values of the moment coefficients m_1 and m_2 , depend upon the parameters, (u/B) , (v/L) and K .

Figures 20.5 to 20.11 are the Pigeaud's curves used for the estimation of moment coefficients m_1 and m_2 , for values of K ranging from 0.4 to 1.0. Moment coefficients for slabs completely loaded with uniformly distributed load for different values of K and $1/K$ are obtained from Fig. 20.12.

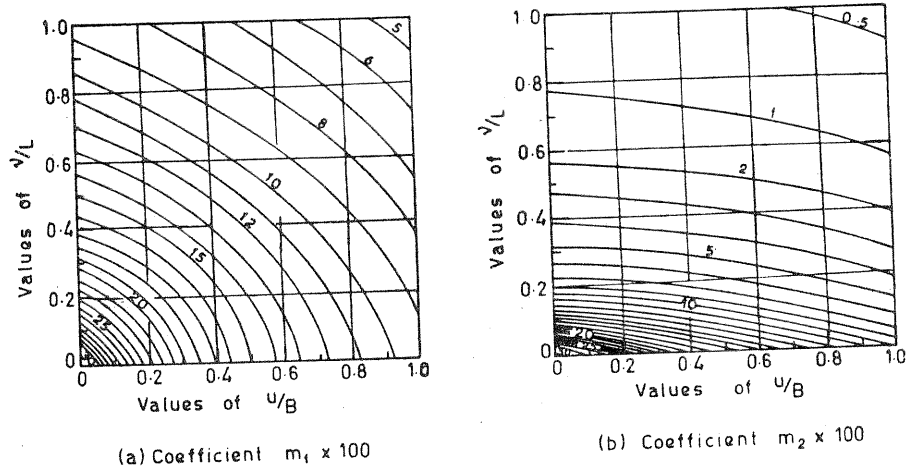


Fig. 20.5 Moment coefficients for m_1 and m_2 for $K=0.4$

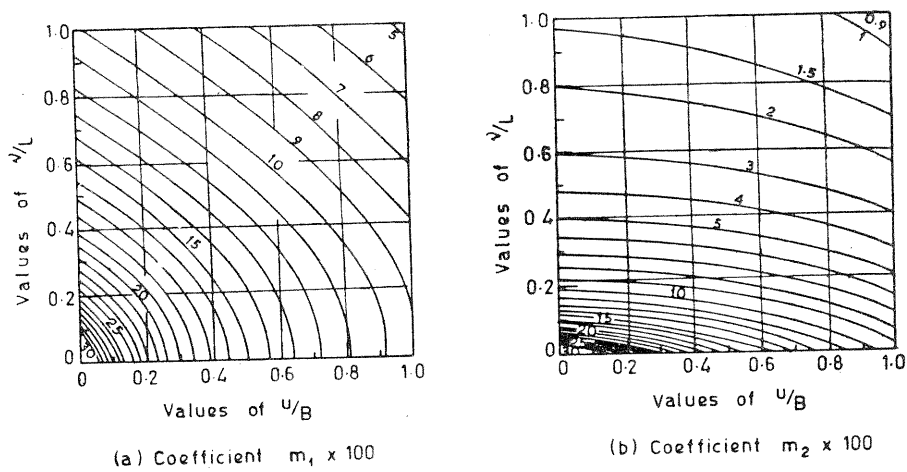
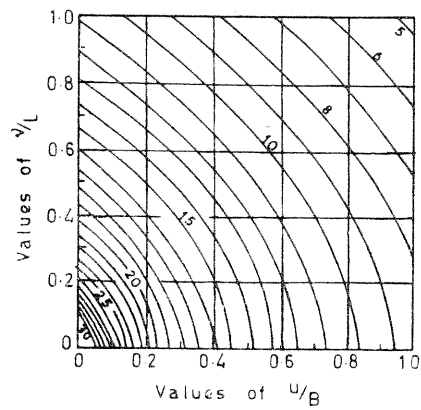
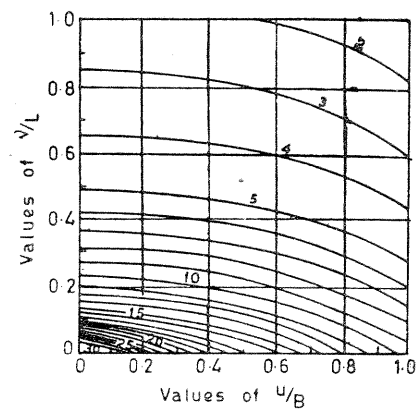


Fig. 20.6 Moment coefficients m_1 and m_2 for $K=0.5$

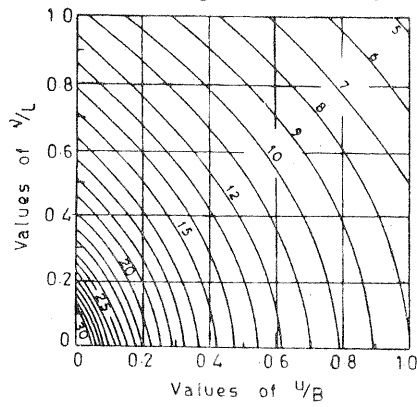


(a) Coefficient $m_1 \times 100$

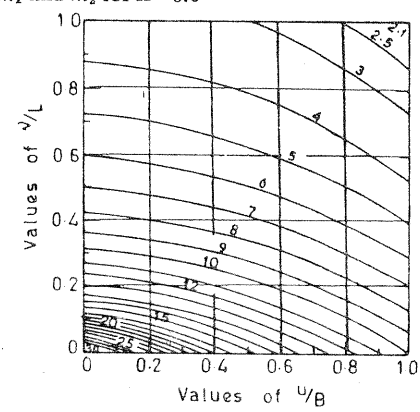


(b) Coefficient $m_2 \times 100$

Fig. 20.7 Moment coefficients m_1 and m_2 for $K=0.6$

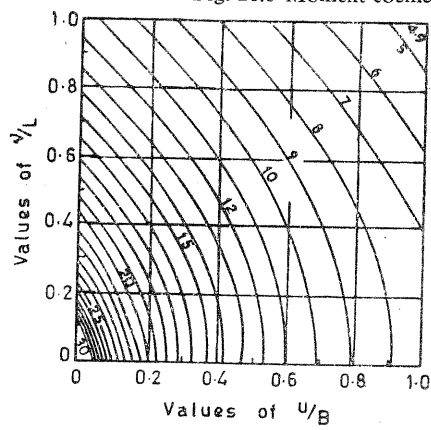


(a) Coefficient $m_1 \times 100$

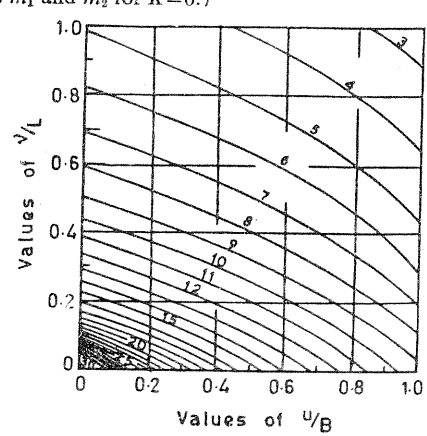


(b) Coefficient $m_2 \times 100$

Fig. 20.8 Moment coefficients m_1 and m_2 for $K=0.7$

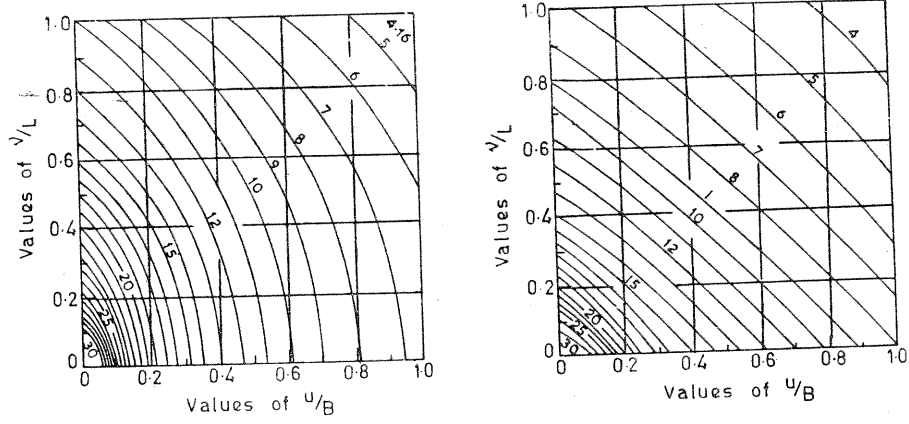


(a) Coefficient $m_1 \times 100$



(b) Coefficient $m_2 \times 100$

Fig. 20.9 Moment coefficients m_1 and m_2 for $K=0.8$



(a) Coefficient $m_1 \times 100$

(b) Coefficient $m_2 \times 100$

Fig. 20.10 Moment coefficients for m_1 and m_2 for $K=0.9$

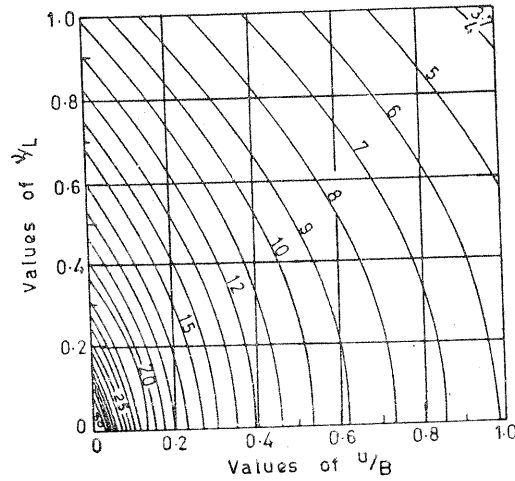


Fig. 20.11 Moment coefficients for m_1 (or m_2) $\times 100$ for $K=1.0$

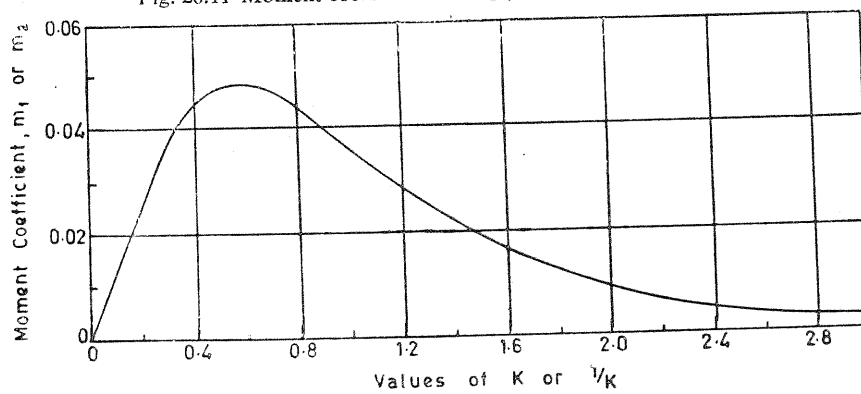


Fig. 20.12 Moment coefficients for slabs completely loaded with uniformly distributed load, coefficient is m_1 for K and m_2 for $1/K$

20.3 ANALYSIS AND DESIGN OF TEE-BEAM AND SLAB DECKS

A typical tee beam deck slab generally comprises of the longitudinal girders, continuous deck slab between the tee beams and cross girders to provide lateral rigidity to the bridge deck. The distribution of live loads among the longitudinal girders can be estimated by any of the following rational methods.

1. Courbon's method
2. Hendry-Jaegar method
3. Guyon-Massonnet method

Among those methods, Courbon's method is the simplest and is applicable when the following conditions are satisfied:

- (a) The ratio of span to width of deck is greater than 2 but less than 4
- (b) The longitudinal girders are interconnected by at least five symmetrically spaced cross girders
- (c) The cross girders extend to a depth of at least 0.75 of the depth of the longitudinal girder

Courbon's method is popular due to the simplicity of computations as detailed below:

When the live loads are positioned nearer to the Kerb, the centre of gravity of live load acts eccentrically with the centre of gravity of the girder system. Due to this eccentricity, the loads are increased and decreased on each girder, depending upon the position of the girders. This is calculated by Courbon's theory by a reaction factor given by,

$$R_x = \frac{\sum W}{n} \left[1 + \frac{\sum I}{\sum d_x^2 \cdot I} \cdot d_x e \right]$$

Where R_x = Reaction factor for girder under consideration

I = Moment of inertia of each longitudinal girder

d_x = Distance of the girder under consideration from the central axis of the bridge

W = Total concentrated live load

n = Number of longitudinal girders

e = Eccentricity of live load with respect to the axis of the bridge.

The live load bending moments and shear forces are computed for each of the girders. The maximum design moments and shear forces are obtained by adding the live load and dead load bending moments. The reinforcements in the main longitudinal girders are designed for the maximum moments and shears developed in the girders.

An approximate method may be used for the computation of the bending moments and shear forces in cross girders. The cross girders are assumed to be rigid so that the reactions due to dead and live loads are assumed to

be equally shared by the cross girders. This assumption will simplify the computation of bending moments and shear forces in the cross girders.

The complete design of slab deck and tee beam decks are illustrated by the following examples.

20.4 DESIGN EXAMPLE

Design a reinforced concrete slab culvert for a state high way to suit the following data:

Carriage way—Two lane (7.5 m wide)

Materials—M-20 grade concrete Fe-415 grade tor steel

Kerbs—600 mm wide

Clear span = 6 m, Wearing coat = 80 mm

Width of bearing = 0.4 m

Loading—I.R.C. Class A or AA

Whichever gives the worst effect. Design the R.C. deck slab and sketch the details of reinforcements in the longitudinal and cross sections of the slab.

1. Data

Clear span = 6 m

Width of bearing = 0.4 m

Kerbs = 600 mm

M-20 grade concrete and Fe-415 grade tor steel

2. Allowable Stresses

$$\sigma_{cb} = 7 \text{ N/mm}^2$$

$$m = 13$$

$$\sigma_{st} = 190 \text{ N/mm}^2$$

$$j = 0.89$$

$$Q = 1.008$$

3. Depth of Slab and Effective Span

Assume thickness of slab at 80 mm per metre of span for high way bridge decks.

$$\therefore \text{Over all slab thickness} = (80 \times 6) = 500 \text{ mm}$$

Using 25 mm Φ bars with clear cover of 25 mm

$$\text{Effective depth} = [500 - (25 + 12.5)] = 462.5 \text{ mm}$$

Width of bearing = 0.4 m

$\therefore$ Effective span is the least of

$$(1) \text{ Clear span} + \text{Effective depth} = (6 \text{ m} + 0.4625 \text{ m}) = 6.4625 \text{ m}$$

$$(2) \text{ Centre to centre of bearings} = (6 \text{ m} + 0.4 \text{ m}) = 6.4 \text{ m}$$

$$\therefore \text{Effective span} = L = 6.4 \text{ m}$$

The cross section of the deck slab is shown in Fig. 20.13.

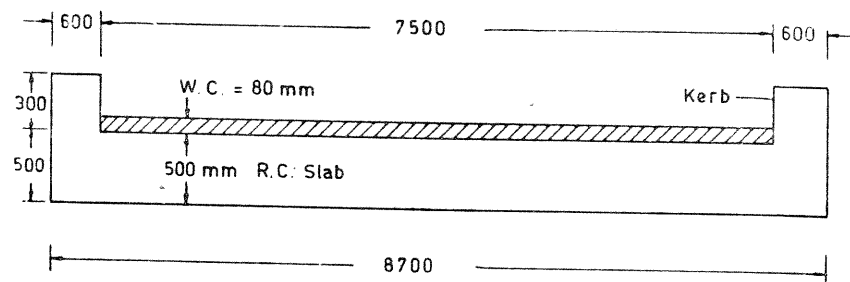


Fig. 20.13 Cross section of deck slab

4. Dead Load Bending Moments

$$\text{Dead weight of slab} = (0.5 \times 24) = 12 \text{ kN/m}^2$$

$$\text{Wearing coat} = (0.08 \times 22) = 1.76$$

$$\therefore \text{Total load} = 13.76 \text{ kN/m}^2$$

$$\therefore \text{Dead load bending moment} = \left(\frac{13.76 \times 6.4^2}{8} \right) = 70.4 \text{ kN}\cdot\text{m}$$

5. Live Load Bending Moment

The bending moment will be maximum for I.R.C. Class AA-tracked vehicle and hence computations will be made for this class of loading. Impact factor of class AA-tracked vehicle is 25% for 5 m span decreasing linearly to 10% for 9 m span.

$\therefore$ For 6.4 m span,

$$\text{Impact factor} = \left[25 - \frac{15}{4}(6.4 - 5) \right] = 19.7\%$$

The tracked vehicle is placed symmetrically on the span.

$$\text{Effective length of load} = [3.6 + 2(0.5 + 0.08)] = 4.76 \text{ m}$$

Effective width of slab, perpendicular to span is expressed as,

$$b_e = k \cdot x(1 - x/L) + b_w$$

Referring to Fig. 20.14

$$x = 3.2 \text{ m}, \quad L = 6.4 \text{ m}, \quad B = 8.7 \text{ m}, \quad \left(\frac{B}{L} \right) = 1.36$$

$$b_w = (0.85 + 2 \times 0.08) = 1.01 \text{ m}$$

From Table 20.1, For $\left(\frac{B}{L} \right) = 1.36$, simply supported slabs, $k = 2.77$

$$\therefore b_e = 2.77 \times 3.2 \left(1 - \frac{3.2}{6.4} \right) + 1.01 = 5.442 \text{ m}$$

The tracked vehicle is placed close to the kerb with the required minimum clearance as shown in Fig. 20.15.

Net effective width of dispersion = 6.996 m

$$\text{Total load of two tracks with impact} = (700 \times 1.197) = 838 \text{ kN}$$

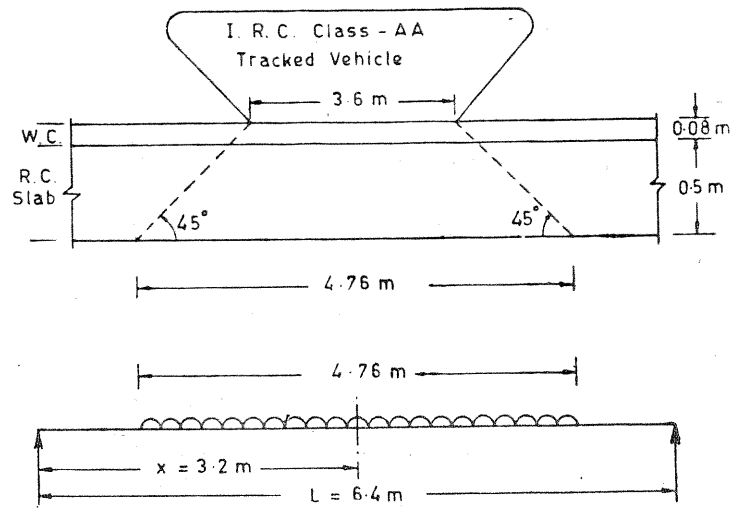
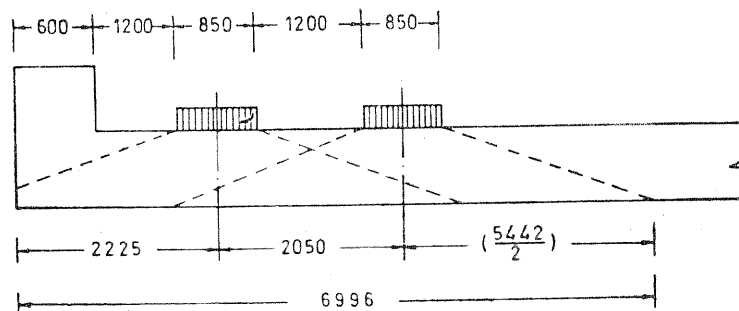
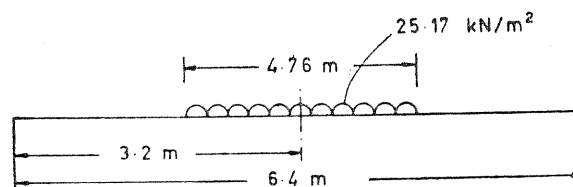


Fig. 20.14 Dispersion of loads due to I.R.C. Class-AA vehicle



(a) Effective Width of Dispersion



(b) Position of Load For Maximum B.M.

Fig. 20.15 Effective width of dispersion of live loads

$$\text{Average intensity of load} = \left(\frac{838}{4.76 \times 6.996} \right) = 25.17 \text{ kN/m}^2$$

Maximum bending moment due to live load is given by

$$M_{\max} = \left(\frac{25.17 \times 4.76}{2} \times 3.2 \right) - \left(\frac{25.17 \times 4.76}{2} \times \frac{4.76}{4} \right) \\ = 120.36 \text{ kN} \cdot \text{m}$$

$$\therefore \text{Total design bending moment} = (120.36 + 70.4) = 191 \text{ kN} \cdot \text{m}$$

6. *Shear Due to Class AA-Tracked Vehicle*

For maximum shear at support, the I.R.C. Class AA-tracked vehicle is arranged as shown in the Fig. 20.16.

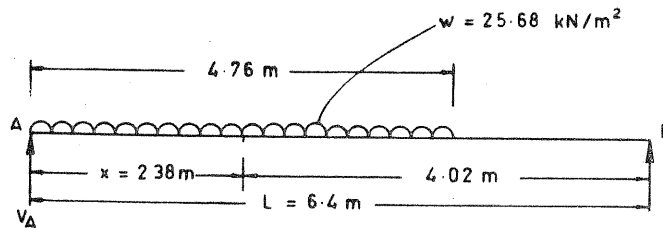


Fig. 20.16 Load position for maximum shear

Effective width of dispersion is given by,

$$b_e = k \cdot x(1 - x/L) + b_w$$

$$\text{where } x = 2.38 \text{ m} \quad \left(\frac{B}{L} \right) = 1.36$$

$$B = 8.7 \text{ m}$$

$$L = 6.4 \text{ m}$$

$$k = 2.77$$

$$b_w = 1.01 \text{ m}$$

$$\therefore b_e = 2.77 \times 2.38 \left(1 - \frac{2.38}{6.4} \right) + 1.01 = 5.16 \text{ m}$$

$$\therefore \text{Width of dispersion} = \left(2225 + 2050 + \frac{5160}{2} \right) = 6.855 \text{ m}$$

(Refer Fig. 20.15)

$$\therefore w = \left(\frac{838}{4.76 \times 6.855} \right) = 25.68 \text{ kN/m}^2$$

$$\therefore \text{Shear force } V_A = \left(\frac{25.68 \times 4.76 \times 4.02}{6.4} \right) = 76.80 \text{ kN}$$

$$\text{Dead load shear} = \left(\frac{13.76 \times 6.4}{2} \right) = 43.75 \text{ kN}$$

$$\therefore \text{Total design shear} = (76.80 + 43.75) = 121 \text{ kN}$$

7. Design of Deck Slab

Effective depth required is

$$d = \left(\sqrt{\frac{M}{Q \cdot b}} \right) = \left(\sqrt{\frac{191 \times 10^6}{1.008 \times 1000}} \right) = 435.3 \text{ mm}$$

Effective depth provided = 462.5 mm

$$\therefore A_{st} = \left(\frac{M}{\sigma_{st} \cdot j \cdot d} \right) = \left(\frac{191 \times 10^6}{190 \times 0.89 \times 462.5} \right) = 2442 \text{ mm}^2$$

$$\text{Spacing of 25 mm diameter bars} = \left(\frac{1000 \times 491}{2442} \right) = 201 \text{ mm}$$

Provide 25 mm Φ at 200 mm centres.

Bending moment for distribution steel is

$$= (0.3M_L + 0.2M_d) = (0.3 \times 120.36) + (0.2 \times 70.4) = 50.2 \text{ kN} \cdot \text{m}$$

Using 12 mm Φ bars

$$\text{Effective depth} = [462.5 - (12.5 + 6)] = 444 \text{ mm}$$

$$\therefore A_{st} = \left(\frac{50.2 \times 10^6}{1900 \times 0.89 \times 444} \right) = 669 \text{ mm}^2$$

$$\text{Spacing of 12 mm } \Phi \text{ bars} = \left(\frac{1000 \times 113}{669} \right) = 169 \text{ mm}$$

Provide 12 mm Φ bars at 150 mm centres.

8. Check for Shear Stresses

$$\tau_v = \left(\frac{V}{bd} \right) = \left(\frac{121 \times 10^3}{1000 \times 462.5} \right) = 0.26 \text{ N/mm}^2$$

$$\left(\frac{100A_{st}}{bd} \right) = \left(\frac{100 \times 2442}{1000 \times 462.5} \right) = 0.528$$

From Table 17 of IS: 456-1978 code

Permissible shear stress $\tau_c = 0.30 \text{ N/mm}^2$

since $\tau_v < \tau_c$, shear reinforcements are not required.

The details of reinforcements in the slab is shown in Fig.20.17.

20.5 DESIGN EXAMPLE

Design a R.C.C. Tee beam girder bridge to suit the following data:

Clear width of road way = 7.5 m

Span (centre to centre of bearings) = 16 m

Live load—I.R.C. Class AA or A which ever gives the worst effect

Average thickness of wearing coat = 80 mm

Concrete mix = M-20 grade

Steel = Fe-415 grade tor steel

Design the deck slab, main girder and cross girder and sketch the typical details of reinforcements.

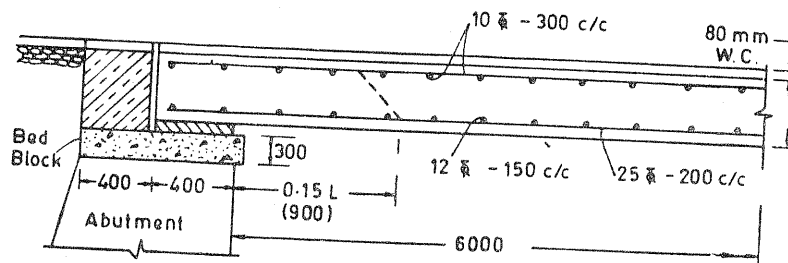
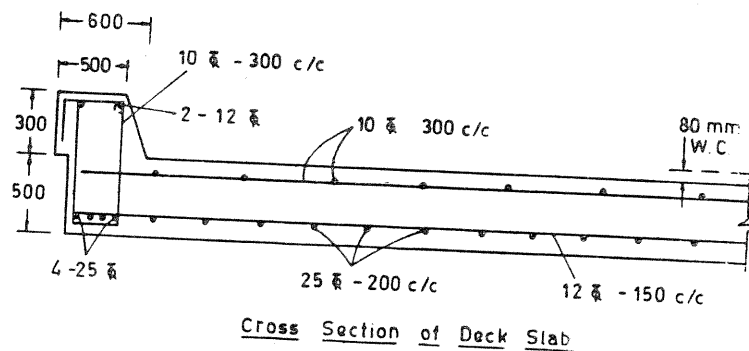


Fig. 20.17 Reinforcement details in deck slab

1. Data

Effective span of the beams = 16 m
 Road width = 7.5 m
 Thickness of the wearing coat = 80 mm
 M-20 grade concrete and Fe-415 grade tor steel.

2. Allowable Stresses

$$\begin{aligned}\sigma_{cb} &= 7 \text{ N/mm}^2 & m &= 13 \\ \sigma_{st} &= 190 \text{ N/mm}^2 & j &= 0.89 \\ & & Q &= 1.008\end{aligned}$$

3. Cross Section of Deck

3-main girders are provided at 2.5 m centres. Thickness of deck slab = 80 mm
 Wearing coat = 80 mm
 Width of main girders = 300 mm
 Kerbs 600 mm wide and 300 mm deep. Cross girders are provided at every 4 m intervals. Breadth of cross girder = 300 mm Depth of main girder = 160 cm at the rate of 10 cm per metre of span.

The depth of cross girder is taken as equal to the depth of the main girder to simplify the computations. The cross section of the deck and the plan showing the spacing of cross girders are shown in Fig. 20.18.

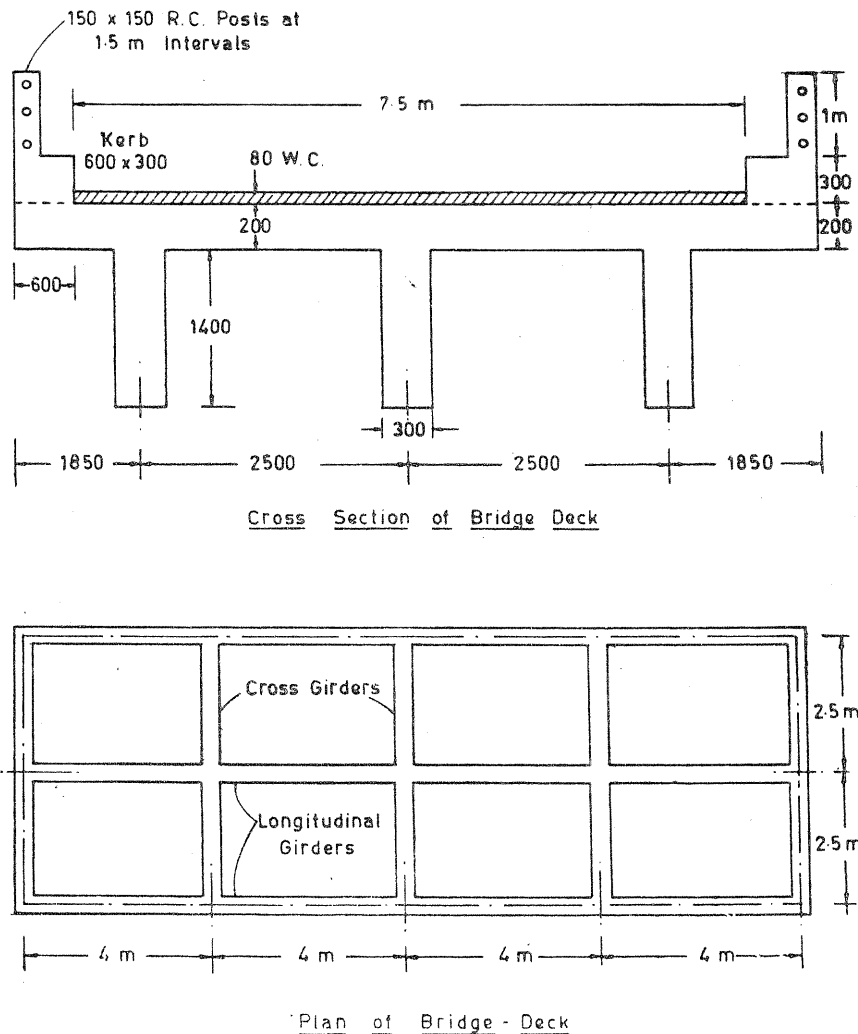


Fig. 20.18 Tee-beam and slab bridge deck

4. Design of Interior Slab Panels

(a) Bending Moments

$$\text{Dead weight of slab} = (1 \times 1 \times 0.2 \times 24) = 4.80 \text{ kN/m}^2$$

$$\text{Dead weight of W.C} = (0.08 \times 2.2) = 1.76$$

$$\text{Total dead load} = 6.56 \text{ kN/m}^2$$

Live load: Class AA-tracked vehicle one wheel is placed at the centre of panel as shown in Fig. 20.19.

$$u = (0.85 + 2 \times 0.08) = 1.01 \text{ m}$$

$$v = (3.60 + 2 \times 0.08) = 3.76 \text{ m}$$

$$\left(\frac{u}{B}\right) = \left(\frac{1.01}{2.5}\right) = 0.404 \quad \left(\frac{v}{L}\right) = \left(\frac{3.76}{4.0}\right) = 0.94$$

$$k = \left(\frac{B}{L}\right) = \left(\frac{2.5}{4.5}\right) = 0.625$$

Referring to Pigeaud's curve (Fig. 20.7)

$$M_1 = 8.5 \times 10^{-2} \quad \text{and} \quad M_2 = 2.4 \times 10^{-2}$$

$$\therefore M_B = W(M_1 + 0.15M_2) = 350(8.5 \times 10^{-2} + 0.15 \times 2.4 \times 10^{-2}) = 31.01 \text{ kN}\cdot\text{m}$$

As the slab is continuous

Design B.M = $0.8M_B$

M_B (including impact and continuity factor)

$$= (1.25 \times 0.8 \times 31.01) = 31.01 \text{ kN}\cdot\text{m}$$

$$M_L = 350(2.4 \times 10^{-2} + 0.15 \times 8.5 \times 10^{-2}) = 8.855 \text{ kN}\cdot\text{m}$$

$$\text{Design } M_L = (1.25 \times 0.8 \times 8.855) = 8.855 \text{ kN}\cdot\text{m}$$

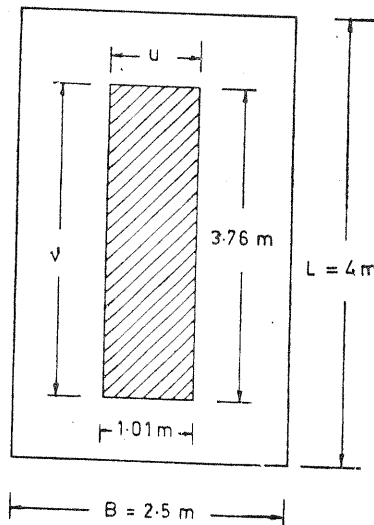


Fig. 20.19 Position of wheel load for maximum bending moment

(b) *Shear Forces*

Dispersion in the direction of span = $[0.85 + 2(0.08 + 0.2)] = 1.41 \text{ m}$. For maximum shear, load is kept such that the whole dispersion is in span, the load is kept at $\left(\frac{1.41}{2}\right) = 0.705 \text{ m}$ from the edge of beam as shown in Fig. 20.20.

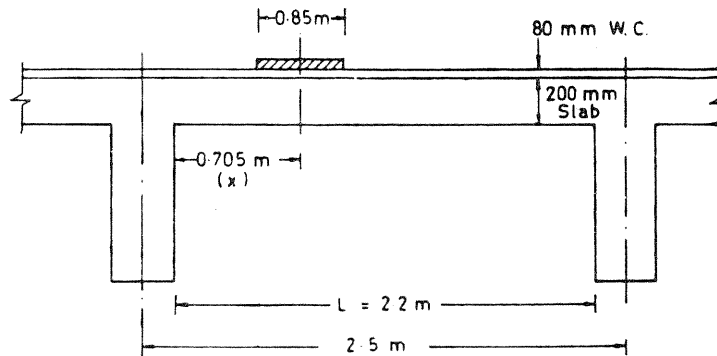


Fig. 20.20 Position of wheel load for maximum shear

$$\text{Effective width of slab} = k \cdot x(1 - x/L) + b_w$$

Breadth of cross girder = 30 cm

Clear length of panel = $L = 3.7$ m

$$\therefore \left(\frac{B}{L}\right) = \left(\frac{3.7}{2.2}\right) = 1.68$$

From Table 20.1 k for continuous slab is obtained as $k = 2.52$

Effective width of slab

$$= \left[2.52 \times 0.705 \left(1 - \frac{0.705}{2.2} \right) + 3.6 + (2 \times 0.08) \right] = 5 \text{ m}$$

$$\therefore \text{Load/m width} = \left(\frac{350}{5} \right) = 70 \text{ kN}$$

$$\therefore \text{Shear force} = 70 \left[\frac{2.2 - 0.705}{2.2} \right] = 47.60 \text{ kN}$$

$$\text{Shear force with impact} = (1.25 \times 47.60) = 59.50 \text{ kN}$$

(b) *Dead Load Bending Moments and Shear Forces*

Dead Load = 6.56 kN/m^2

$$\text{Total load on panel} = (4 \times 2.5 \times 6.56) = 65.6 \text{ kN}$$

$$\left(\frac{u}{B}\right) = 1, \left(\frac{v}{L}\right) = 1 \text{ as panel is loaded with uniformly distributed load.}$$

$$k = \left(\frac{B}{L}\right) = \left(\frac{2.5}{4}\right) = 0.625 \quad \text{and} \quad \frac{1}{k} = 1.6$$

From Pigeaud's curve (Fig. 20.11)

$$M_1 = 4.9 \times 10^{-2} \quad \text{and} \quad M_2 = 1.5 \times 10^{-2}$$

$$\therefore M_B = 65.6(4.9 \times 10^{-2} + 0.15 \times 1.5 \times 10^{-2}) = 3.36 \text{ kN} \cdot \text{m}$$

Taking continuity into effect

$$M_B = (0.2 \times 3.36) = 2.688 \text{ kN} \cdot \text{m}$$

$$M_L = 65.6(1.5 \times 10^{-2} + 0.15 \times 4.9 \times 10^{-2}) = 1.468 \text{ kN} \cdot \text{m}$$

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Taking continuity into effect

$$M_L = (0.8 \times 1.468) = 1.174 \text{ kN} \cdot \text{m}$$

$$\text{Dead load shear force} = \left(\frac{6.56 \times 2.2}{2} \right) = 7.216 \text{ kN}$$

(d) *Design Moments and Shears*

$$\text{Total } M_B = (31.01 + 2.688) = 33.698 \text{ kN} \cdot \text{m}$$

$$\text{Total } M_L = (8.855 + 1.174) = 10.029 \text{ kN} \cdot \text{m}$$

$$\text{Total shear force} = (59.5 + 7.216) = 66.716 \text{ kN}$$

(e) *Design of Section*

$$\text{Effective depth} = d = \left(\frac{33.698 \times 10^6}{1.008 \times 1000} \right) = 180 \text{ mm}$$

Adopt over all depth = 200 mm

Effective depth = 180 mm

$$A_{st} = \left(\frac{33.698 \times 10^6}{190 \times 0.89 \times 180} \right) = 1107 \text{ mm}^2$$

(For short span)

Use 12 mm Φ at 100 mm centres ($A_{st} = 1131 \text{ mm}^2$)

Effective depth for long span using 10 mm diameter bars
 $= (180 - 6 - 5) = 169 \text{ mm}$

$$\therefore A_{st} = \left(\frac{10.029 \times 10^6}{190 \times 0.89 \times 169} \right) = 351 \text{ mm}^2$$

(Long span)

Use 10 mm Φ at 200 mm centres ($A_{st} = 393 \text{ mm}^2$)

(f) *Check for Shear Stress*

Nominal shear stress

$$= \tau_v = \left(\frac{66.716 \times 10^3}{1000 \times 180} \right) = 0.87 = 0.37 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd} \right) = \left(\frac{100(1131 + 393)}{1000 \times 174.5} \right) = 0.87$$

From Table 17 of IS: 456-1978 code

Permissible shear stress $= k \cdot \tau_c = (1.2 \times 0.37)$

$$= 0.444 \text{ N/mm}^2 > 0.37 \text{ N/mm}^2$$

Hence the shear stresses in the slab are within permissible limits.

5. Design of Longitudinal Girders

(a) *Reaction Factors*

Using Courbon's theory, the I.R.C. Class AA loads are arranged for maximum eccentricity as shown in Fig. 20.21.

Reaction factor for outer girder is,

$$R_A = \frac{2W_1}{3} \left(1 + \frac{3I \times 2.5 \times 1.1}{2I \times 2.5^2} \right) = 1.107W_1$$

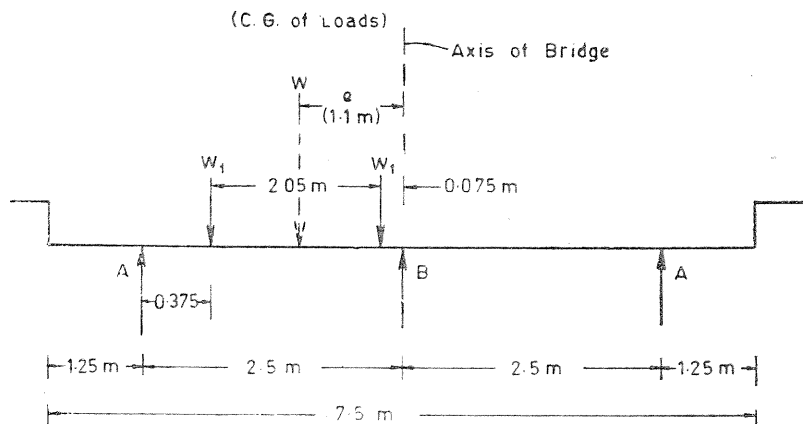


Fig. 20.21 Transverse disposition of I.R.C. Class AA-tracked vehicle

Reaction factor for inner girder is,

$$R_B = \frac{2W_1}{3}[1 + 0] = \left(\frac{2W_1}{3}\right)$$

If $W = \text{Axle load} = 700 \text{ kN}$

$$W_1 = 0.5W$$

$$\therefore R_A = (1.107 \times 0.5W) = 0.5536W$$

$$R_B = (0.667 \times 0.5W) = 0.3333W$$

(b) *Dead Load from Slab per Girder*

The dead load of deck slab is calculated with reference to Fig. 20.22.

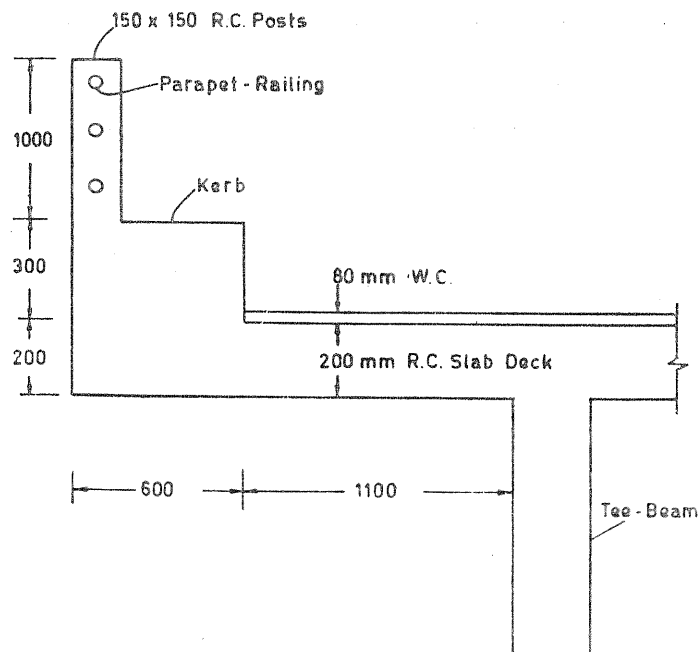


Fig. 20.22 Details of deck-slab, kerb and Parapet

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1. Parapet railing 0.700 kN/m
2. Wearing coat = $(0.08 \times 1.10 \times 22)$ 1.936
3. Deck slab = $(0.2 \times 1.10 \times 24)$ 5.280
4. Kerb = $(0.5 \times 0.6 \times 1 \times 24)$ 7.200

15.116 kN/m

Total dead load of deck = $(2 \times 15.116) + (6.56 \times 5.3) = 65 \text{ kN/m}$

It is assumed that the dead load is shared equally by all the girders

$$\therefore \text{Dead load/girder} = \left(\frac{65}{3}\right) = 21.66 \text{ kN/m}$$

(c) Live Load Bending Moments in Girder

Span of girder = 16 m

Impact factor (for Class AA) = 10%

The live load is placed centrally on the span as shown in Fig. 20.23.

$$\text{Bending moment} = \left(\frac{4 + 3 \cdot 1}{2}\right) 700 = 2485 \text{ kN}\cdot\text{m}$$

$\therefore$ Bending moment including impact and reaction factor for outer girder is

$$= (2485 \times 1.1 \times 0.5536) = 1513 \text{ kN}\cdot\text{m}$$

Bending moment including impact and reaction factor for inner girder is

$$= (2485 \times 1.1 \times 0.3333) = 912 \text{ kN}\cdot\text{m}$$

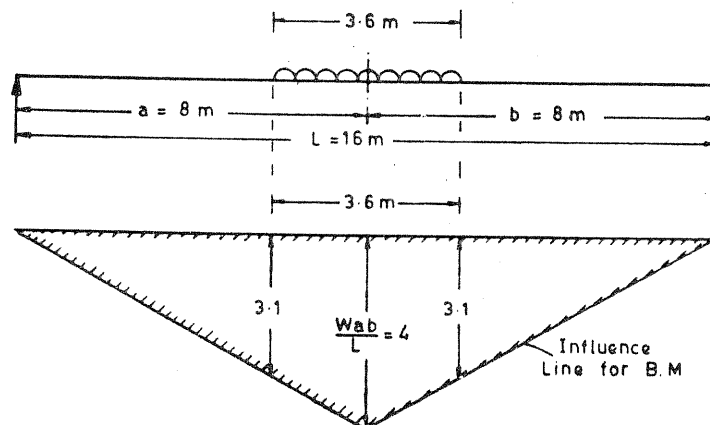


Fig. 20.23 Influence line for bending moment in Girder

(d) Live Load shear

For estimating the maximum live load shear in the girders, the I.R.C Class AA loads are placed as shown in the Fig. 20.24.

$$\text{Reaction of } W_2 \text{ on girder } B = \left(\frac{350 \times 0.45}{2.5}\right) = 63 \text{ kN}$$

$$\text{Reaction of } W_2 \text{ on girder } A = \left(\frac{350 \times 2.05}{2.5}\right) = 287 \text{ kN}$$

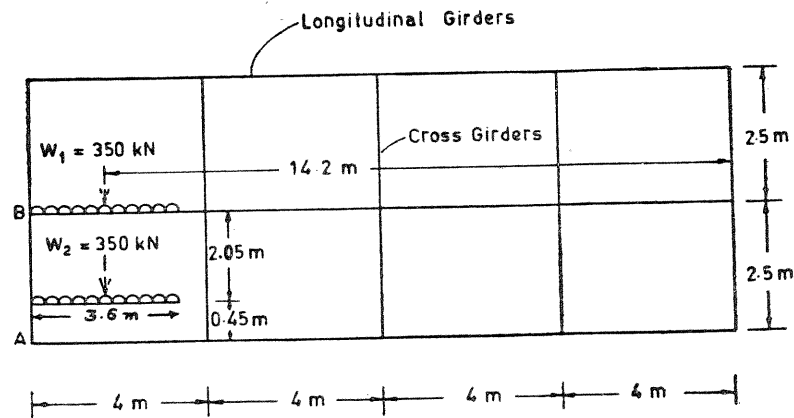


Fig. 20.24 Position of I.R.C. Class AA loads for maximum shear

Total load on girder $B = (350 + 63) = 413 \text{ kN}$

Maximum reaction in girder $B = \left(\frac{413 \times 14.2}{16} \right) = 366 \text{ kN}$

Maximum reaction in girder $A = \left(\frac{287 \times 14.2}{16} \right) = 255 \text{ kN}$

Maximum live load shears with impact factors in inner girder

Inner girder $= (366 \times 1.1) = 402.6 \text{ kN}$

Outer girder $= (255 \times 1.1) = 280.5 \text{ kN}$

(e) *Dead Load Moments and Shear Force in Main Girder*

The depth of girder is assumed as 1600 mm (100 mm for every metre of span)

Depth of rib = 1.4 m Width = 0.3 m

Weight of rib/m $= (1 \times 0.3 \times 1.4 \times 24) = 10.08 \text{ kN/m}$

The cross girder is assumed to have the same cross sectional dimensions of the main girder.

Weight of cross girder = 10.08 kN/m.

Reaction on main girder $= (10.08 \times 2.5) = 25.2 \text{ kN}$

Reaction from deck slab on each girder = 21.66 kN/m

$\therefore$ Total dead load/m on girder $= (21.66 + 10.08) = 31.74 \text{ kN/m}$

Referring to Fig. 20.25, the maximum bending moment are computed.

$$M_{\max} = \left(\frac{31.74 \times 16^2}{8} \right) + \left(\frac{25.2 \times 16}{4} \right) + \left(\frac{25.2 \times 16}{4} \right)$$

(Centre of span)

$$= 1218 \text{ kN}\cdot\text{m}$$

Dead load shear at support

$$= \left(\frac{31.74 \times 16}{2} \right) + 25.2 + \left(\frac{25.2}{2} \right) = 292 \text{ kN.}$$

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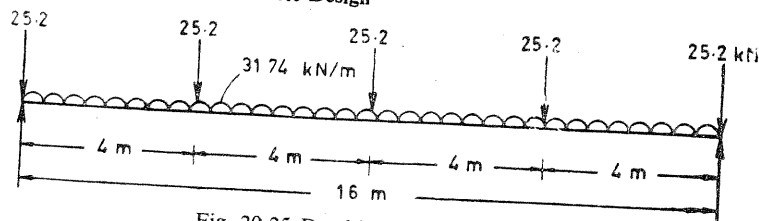


Fig. 20.25 Dead loads on main Girders

(f) Design Bending Moments and Shear Forces

The design moments and shears are compiled in Table 20.2.

TABLE 20.2 Design Moments and Shear Forces

B.M.	D.L. B.M	L.L. B.M	Total B.M	Units
Outer girder	1218	1513	2731	kN·m
Inner girder	1218	912	2130	kN·m
S.F.	D.L. S.F	L.L. S.F	Total S.F	Units
Outer girder	292	280.1	572.1	kN
Inner girder	292	402.6	694.6	kN

(g) Design of Sections for Maximum B.M. and S.F.

$$M_{\max} = 2731 \text{ kN·m}$$

$$V_{\max} = 694.6 \text{ kN}$$

The beam is designed as a Tee-section,
Assuming an effective depth $d = 1450 \text{ mm}$

$$\text{Approximate lever arm} = \left(1450 - \frac{200}{2}\right) = 1350 \text{ mm}$$

$$A_{st} = \left(\frac{2731 \times 10^6}{290 \times 1350}\right) = 10647 \text{ mm}^2$$

Provide 12 bars of 36ϕ in three rows ($A_{st} = 12,216 \text{ mm}^2$)

Shear reinforcements are designed to resist the maximum shear at supports.

Nominal shear stress

$$\tau_v = \left(\frac{V}{bd}\right) = \left(\frac{694.6 \times 10^3}{300 \times 1450}\right) = 1.596 \text{ N/mm}^2$$

If 4 bars of 36ϕ are bent up near supports to resist shear,

$$\left(\frac{100A_s}{bd}\right) = \left(\frac{100 \times 8 \times 1018}{300 \times 1450}\right) = 1.8$$

From Table 17 of IS: 456, for M-20 grade concrete $\tau_c = 0.47 \text{ N/mm}^2$

$\therefore$ Shear taken by concrete $= (\tau_c \cdot b \cdot d)$

$$= \left(\frac{0.47 \times 300 \times 1450}{1000}\right) = 204.45 \text{ kN}$$

Assuming 2 bars of 36ϕ to be bent up at support section,

$$\begin{aligned}\text{Shear resisted by bent up bars} &= V_s = (\sigma_{sv} \cdot A_{sv} \cdot \sin \alpha) \\ &= \left(\frac{150 \times 2 \times 1018 \times 1}{1000 \times \sqrt{2}} \right) = 215.9 \text{ kN}\end{aligned}$$

$\therefore$ Shear to be resisted by vertical stirrups.

$$V = (694.6 - 204.45 - 215.90) = 274.25 \text{ kN}$$

Using 10 mm Φ - 4 legged stirrups,

$$\text{Spacing } s_v = \left(\frac{\sigma_{sv} \cdot A_{sv} \cdot d}{V} \right) = \left(\frac{150 \times 4 \times 79 \times 1450}{274.25 \times 10^3} \right) = 250.6 \text{ mm}$$

Provide 10 mm Φ - 4 legged stirrups at 250 mm centres near supports and 450 mm centres towards the centre of span.

6. Design of Cross Girders

Self weight of cross girder = 10.08 kN/m

Referring to Fig. 20.26

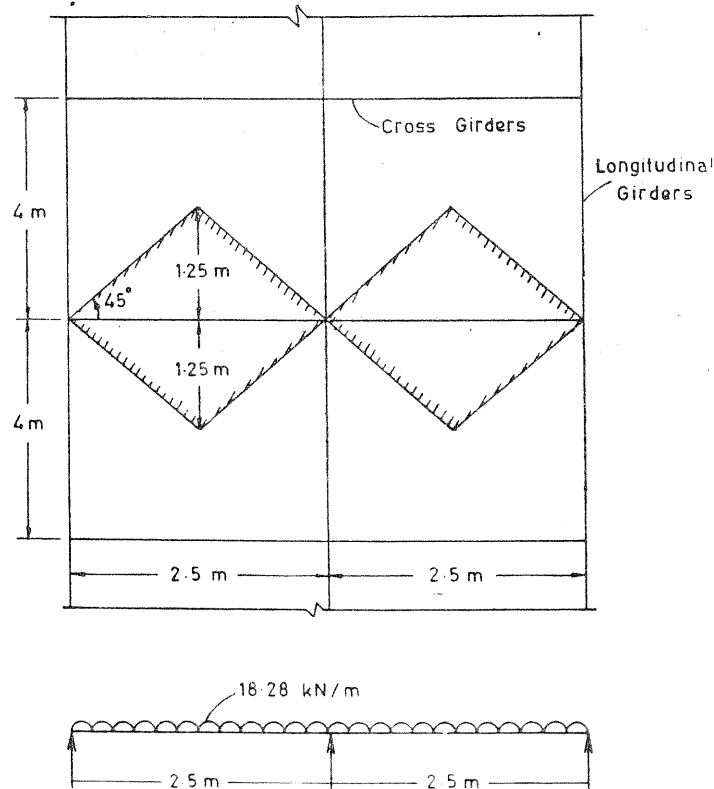


Fig. 20.26 Loads on cross-girder

$$\text{Dead load from slab} = \left(2 \times \frac{1}{2} \times 2.5 \times 1.25 \times 6.56 \right) = 20.5 \text{ kN}$$

$$\text{Uniformly distributed load} = \left(\frac{20.5}{2.5} \right) = 8.2 \text{ kN/m}$$

Total load on cross girder = $(10.08 + 8.2) = 18.28 \text{ kN/m}$

Assuming the cross girder to be rigid,

$$\text{Reaction on each cross girder} = \left(\frac{18.28 \times 5}{3} \right) = 30.47 \text{ kN}$$

For maximum bending moment in the cross girder, the loads of I.R.C. Class AA should be placed as shown in Fig. 20.27.

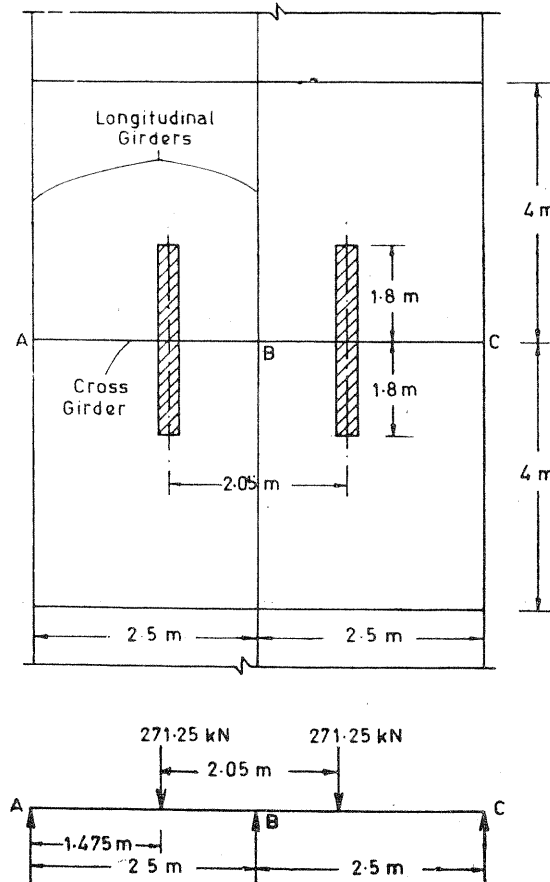


Fig. 20.27 Position of live loads for maximum B.M. in cross girder

$$\text{Load coming on cross girder} = \frac{350 \left(4 - \frac{1.8}{2} \right)}{4} = 271.25 \text{ kN}$$

Assuming the cross girder as rigid, reaction on each longitudinal girder is

$$= \left(\frac{2 \times 271.25}{3'} \right) = 180.83 \text{ kN}$$

$$\begin{aligned}\text{Maximum B.M. in cross girder under the load} &= (180.83 \times 1.475) \\ &= 266.7 \text{ kN}\cdot\text{m}\end{aligned}$$

L.L. B.M. including impact $= (1.1 \times 266.7) = 293.37 \text{ kN}\cdot\text{m}$

Dead load B.M. at 1.475 m from support

$$= \left(30.47 \times 1.475 - 18.28 \times \frac{1.475}{2} \right) = 25.10 \text{ kN}\cdot\text{m}$$

$\therefore$ Total design B.M. $= (293.37 + 25.10) = 318.47 \text{ kN}\cdot\text{m}$

$$\begin{aligned} \text{Live load shear including impact} &= \left(\frac{(2 \times 271.25)}{3} \times 1.1 \right) \\ &= 198.917 \text{ kN} \end{aligned}$$

$\therefore$ Dead load shear $= 30.47 \text{ kN}$

$\therefore$ Total design shear $= (198.917 + 30.47) = 229.39 \text{ kN}$

Assuming an effective depth for cross girder as 1540 mm

$$A_{st} = \left(\frac{318.47 \times 10}{190 \times 0.9 \times 1540} \right) = 1209 \text{ mm}^2$$

Provide 4 bars of 20 mm Φ ($A_{st} = 1256 \text{ mm}^2$)

$$\text{Shear stress} = \tau_v = \left(\frac{229.39 \times 10^3}{300 \times 1540} \right) = 0.496 \text{ N/mm}^2$$

$$\left(\frac{100 A_s}{bd} \right) = \left(\frac{100 \times 1256}{300 \times 1540} \right) = 0.271$$

$\therefore$ From Table 17 of IS : 456, $\tau_c = 0.22 \text{ N/mm}^2$

$$(\tau_c \cdot b \cdot d) = \left(\frac{0.22 \times 300 \times 1540}{1000} \right) = 101.6 \text{ kN}$$

Balance shear $= (229.39 - 101.6) = 127.79 \text{ kN}$

Using 10 mm Φ - 2 legged stirrups,

$$\text{Spacing } s_v = \left(\frac{150 \times 2 \times 79 \times 1540}{127.79 \times 10^3} \right) = 285 \text{ mm}$$

Adopt 10 mm Φ - 2 legged stirrups at 250 mm centres, throughout the length of the cross girder.

The details of reinforcements are shown in the cross section of the deck slab, and longitudinal section of main and cross girders in Figs. 20.28 and 20.29.

20.6 EXAMPLES FOR PRACTICE

1. A road bridge deck consists of a reinforced concrete slab continuous over the beams spaced at 2 m centres and cross girders spaced at 5 m centres. Thickness of the wearing coat is 100 mm. Type of loading-I.R.C. Class AA or A whichever gives the worst effect. Adopt M-20 grade concrete and Fe-415 grade tor steel. Design the deck slab and sketch the details of reinforcements.
2. A reinforced concrete simply supported slab is required for the deck of a road bridge having the data given below:

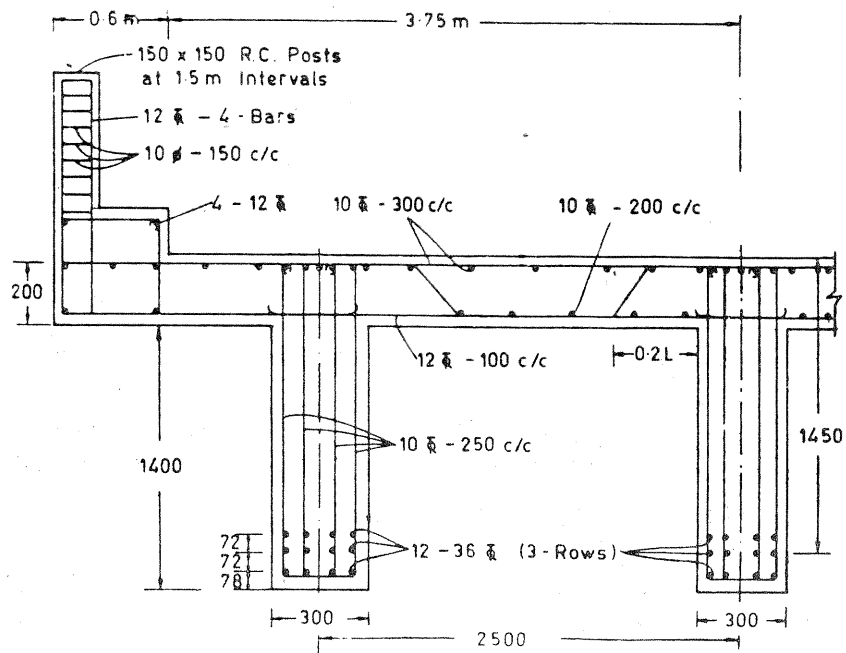


Fig. 20.28 Cross-section of tee-beam and slab-deck

Width of carriage way = 7.5 m

Kerbs = 600 mm wide

Clear span = 5 m

Type of loading = I.R.C. Class AA or A whichever gives the worst effect

Materials = M-20 grade concrete Fe-415 grade tor steel.

Design the deck slab and sketch the details of reinforcements in the longitudinal and cross sections of the slab.

3. Design the centre girder of a 3-girder R.C. tee beam and slab deck system using the following data:

Clear width of read way = 7.5 m

Span (centre to centre of bearings) = 15 m

Loading-I.R.C. Class AA and A

Wearing coat = 100 mm thick

Thickness of slab = 200 mm

Spacing of cross girders = 3.75 m

Materials— concrete M-20 grade, tor steel of Fe-415 grade.

Sketch the details of reinforcements in the girder.

4. An R.C.C. tee beam and slab deck is to be designed for a major river crossing in a National Highway. The following data is available.

Clear width of road way = 7.5 m

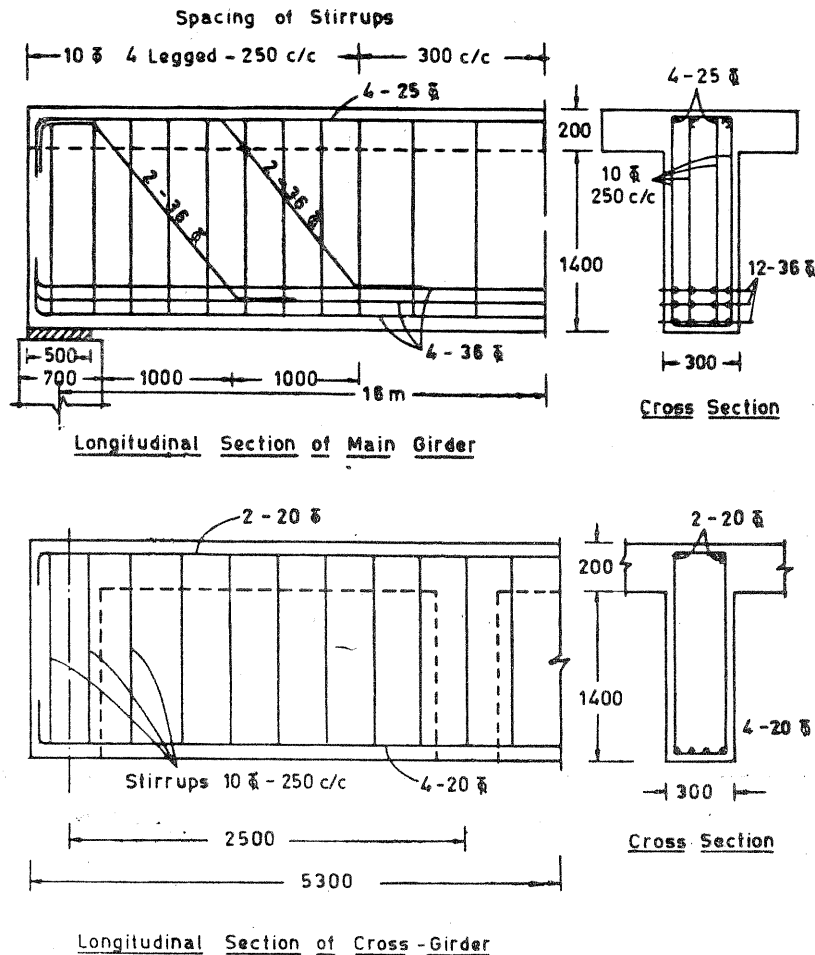


Fig. 20.29 Reinforcement details in longitudinal and cross girders

Foot paths = 1.5 m on either side

Wearing coat = 100 mm thick

Number of main girders = 4

Span (centre to centre of bearings) = 20 m

Spacings of cross girders = 5 m c/c

Loading = I.R.C. Class AA or A which ever gives the worst effect.

Materials = concrete M-20 grade or steel of Fe-415 grade

Design the deck slab and the main girders of the deck and sketch the details of reinforcements.

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5. The beam and slab deck of a high way bridge consists of two main longitudinal girders spaced at 4 m centres with cross girders at every 4 m intervals. Design the deck slab using the following data:
Thickness of wearing coat = 80 mm
Loading = I.R.C. Class AA or A whichever gives the worst effect
Materials = M-20 grade concrete and Fe-415 grade tor steel.
Design the deck slab and sketch the details of reinforcements in the slab.

APPENDIX - 1

Bending Moment and Shear Force Coefficients For Continuous Beams.		
Max. Bending Moment Coefficients	All Beams Freely Supported at End Supports	
	All Spans Loaded	Incidental Load
	Uniformly Distributed Load 	Uniformly Distributed Load
	Central Point Load 	Central Point Load
	Loads at 1/3 Points 	Loads at 1/3 Points
	Max. Forces (Shear) Uniformly Distributed Load 	Max. Forces (Shear) Uniformly Distributed Load
B.M. Coefficients :- Multiply by (Span x Total Load on Span) Coefficients Above The Line are For Negative B.M. at Supports ; Those Under The Line are For Positive Mid-Span B.M. Shear Coefficients :- Multiply by (Total Load on Span)		

APPENDIX - 2
Values of Exponential Functions

x	e^{-x}	x	e^{-x}
0.1	0.9048	2.6	0.0743
0.2	0.8187	2.7	0.0672
0.3	0.7408	2.8	0.0608
0.4	0.6703	2.9	0.0550
0.5	0.6065	3.0	0.0498
0.6	0.5488	3.1	0.0450
0.7	0.4966	3.2	0.0408
0.8	0.4493	3.3	0.0369
0.9	0.4066	3.4	0.0334
1.0	0.3679	3.5	0.0302
1.1	0.3329	3.6	0.0273
1.2	0.3012	3.7	0.0247
1.3	0.2725	3.8	0.0224
1.4	0.2466	3.9	0.0202
1.5	0.2231	4.0	0.0183
1.6	0.2019	4.1	0.0166
1.7	0.1827	4.2	0.0150
1.8	0.1653	4.3	0.0136
1.9	0.1496	4.4	0.0123
2.0	0.1353	4.5	0.0111
2.1	0.1225	4.6	0.0101
2.2	0.1108	4.7	0.0091
2.3	0.1003	4.8	0.0082
2.4	0.0907	4.9	0.0074
2.5	0.0821	5.0	0.0067

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