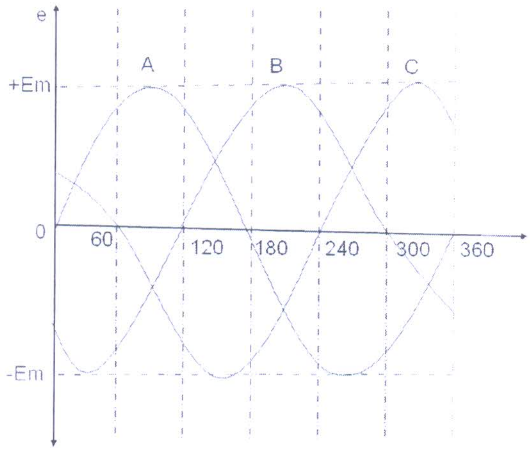
Three Phase AC Circuit

A three phase supply is a set of three alternating quantities displaced from each other by an angle of 120°. A three phase voltage is shown in the figure. It consists of three phases- phase A, phase B and phase C. Phase A waveform starts at 0°. Phase B waveform stars at 120° and phase C waveform at 240°.



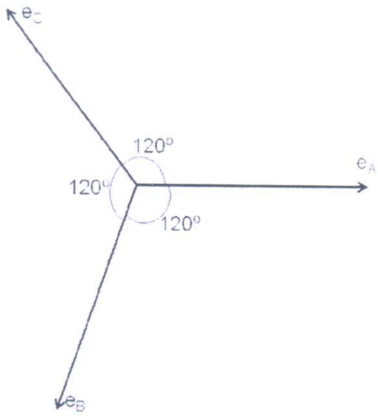
The three phase voltage can be represented by a set of three equations as shown below.

$$e_A = E_m \sin \omega t$$
$$e_B = E_m \sin(\omega t - 120^\circ)$$
$$e_C = E_m \sin(\omega t - 240^\circ) = E_m \sin(\omega t + 120^\circ)$$

The sum of the three phase voltages at any instant is equal to zero.

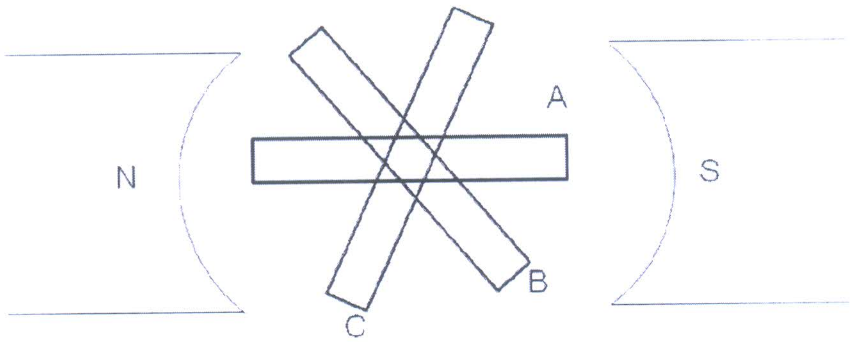
$$e_A + e_B + e_C = 0$$

The phasor representation of three phase voltages is as shown.



The phase A voltage is taken as the reference and is drawn along the x-axis. The phase B voltage lags behind the phase A voltage by 120°. The phase C voltage lags behind the phase A voltage by 240° and phase B voltage by 120°.

Generation of Three Phase Voltage



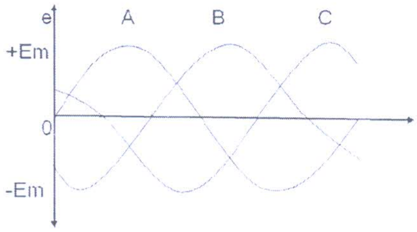
Three Phase voltage can be generated by placing three rectangular coils displaced in space by 120° in a uniform magnetic field. When these coils rotate with a uniform angular velocity of ω rad/sec, a sinusoidal emf displaced by 120° is induced in these coils.

Necessity and advantages of three phase systems

- 3Φ power has a constant magnitude whereas 1Φ power pulsates from zero to peak value at twice the supply frequency
- A 3Φ system can set up a rotating magnetic field in stationary windings. This is not possible with a 1Φ supply.
- For the same rating 3Φ machines are smaller, simpler in construction and have better operating characteristics than 1Φ machines
- To transmit the same amount of power over a fixed distance at a given voltage, the 3Φ system requires only 3/4th the weight of copper that is required by the 1Φ system
- The voltage regulation of a 3Φ transmission line is better than that of 1Φ line

Phase Sequence

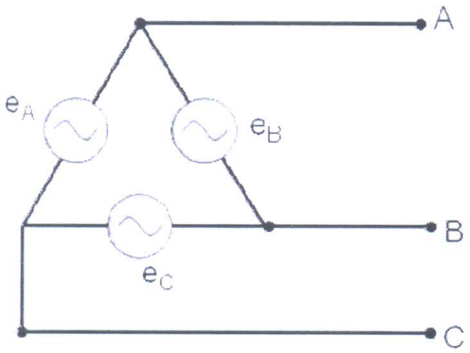
The order in which the voltages in the three phases reach their maximum value



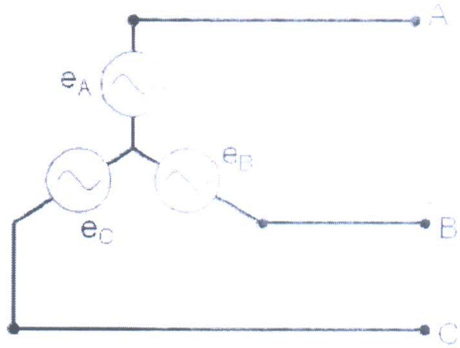
For the waveform shown in figure, phase A reaches the maximum value first, followed by phase B and then by phase C. hence the phase sequence is A-B-C.

Balanced Supply

A supply is said to be balanced if all three voltages are equal in magnitude and displaced by 120^0
A three phase supply can be connected in two ways - Either in Delta connection or in Star connection as shown in the figure.



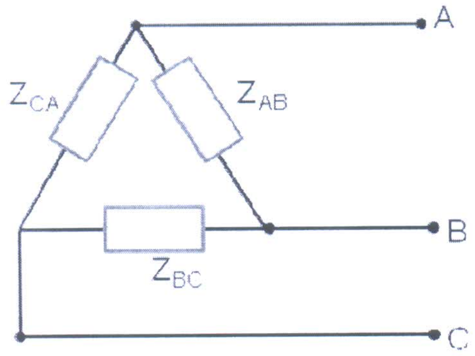
Delta Connection



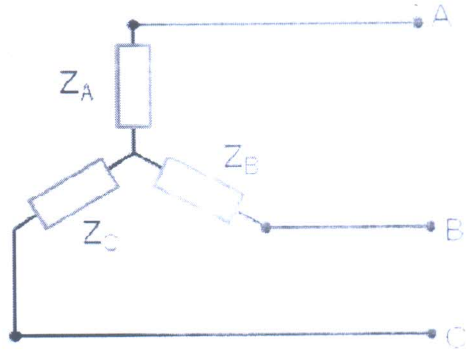
Star Connection

Balanced Load

A load is said to be balanced if the impedances in all three phases are equal in magnitude and phase
A three phase load can be connected in two ways - Either in Delta connection or in Star connection as shown in the figure.



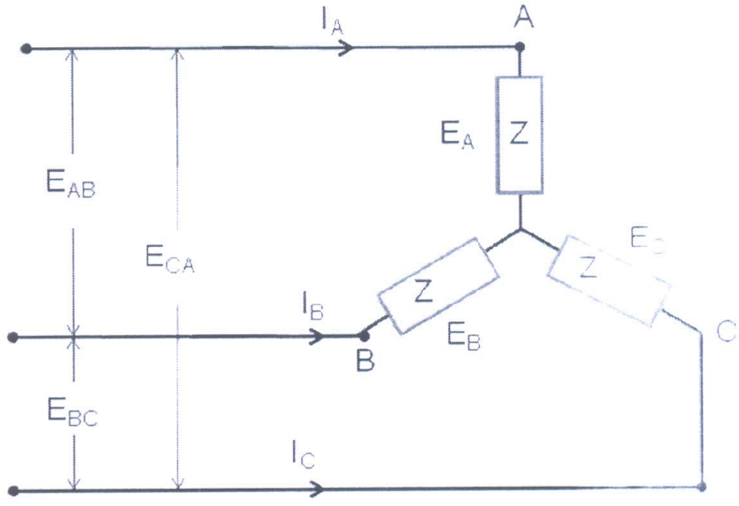
Delta Connection



Star Connection

Balanced Star Connected Load

A balanced star connected load is shown in the figure. A phase voltage is defined as voltage across any phase of the three phase load. The phase voltages shown in figure are E_A, E_B and E_C. A line voltage is defined as the voltage between any two lines. The line voltages shown in the figure are E_{AB}, E_{BC} and E_{CA}. The line currents are I_A, I_B and I_C. For a star connected load, the phase currents are same as the line currents.



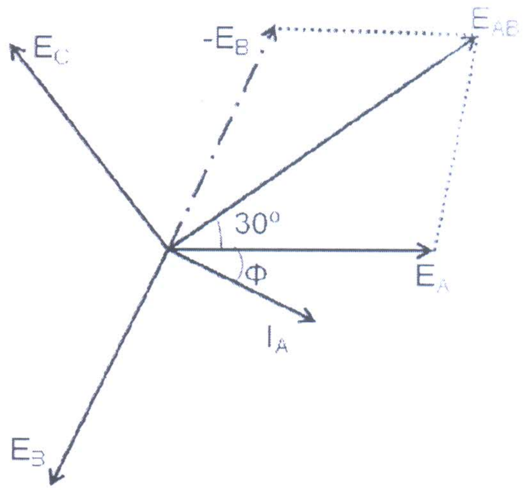
Using Kirchoff's voltage law, the line voltages can be written in terms of the phase voltages as shown below.

$$E_{AB} = E_A - E_B$$

$$E_{BC} = E_B - E_C$$

$$E_{CA} = E_C - E_A$$

The phasor diagram shows the three phase voltages and the line voltage E_{AB} drawn from E_A and $-E_B$ phasors. The phasor for current I_A is also shown. It is assumed that the load is inductive.



From the phasor diagram we see that the line voltage E_{AB} leads the phase voltage E_A by 30°. The magnitude of the two voltages can be related as follows.

$$E_{AB} = 2E_A \cos 30^\circ = \sqrt{3}E_A$$

Hence for a balanced star connected load we can make the following conclusions.

$$E_l = \sqrt{3}E_{ph}$$

$$I_l = I_{ph}$$

Line voltage leads phase voltage by 30^0

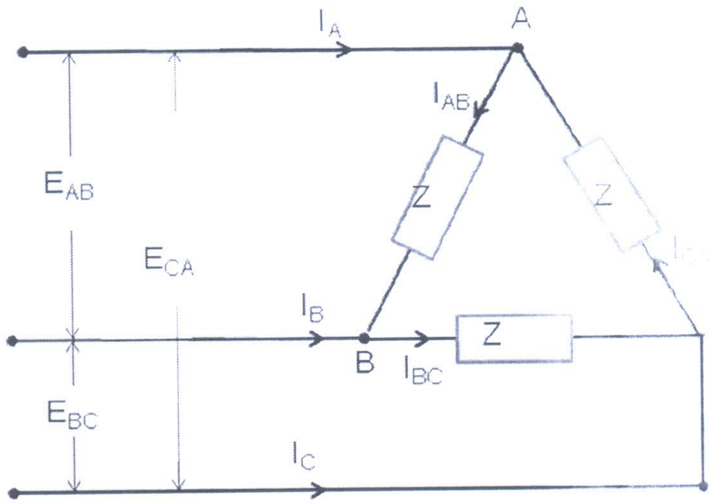
Three phase Power

In a single phase circuit, the power is given by VIcosΦ. It can also be written as $V_{ph}I_{ph}\cos\Phi$. The power in a three circuit will be three times the power in a single phase circuit.

$$P = 3E_{ph}I_{ph}\cos\Phi$$

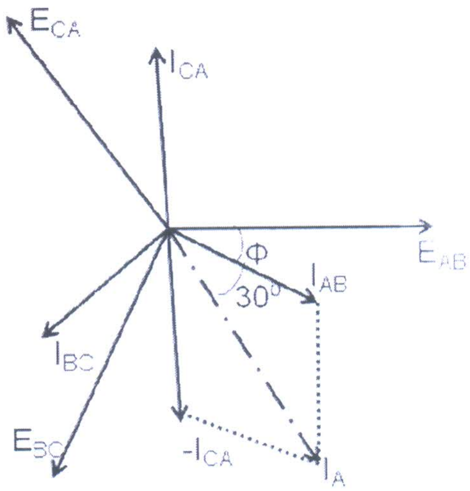
$$P = \sqrt{3}E_l I_l \cos\Phi$$

A balanced delta connected load is shown in the figure. The phase currents are I_{AB}, I_{BC} and I_{CA}. The line currents are I_A, I_B and I_C. For a delta connected load, the phase voltages are same as the line voltages given by E_{AB}, E_{BC} and E_{CA}.



Using Kirchoff's current law, the line currents can be written in terms of the phase currents as shown below.

$$I_A = I_{AB} - I_{CA}$$
$$I_B = I_{BC} - I_{AB}$$
$$I_C = I_{CA} - I_{BC}$$



The phasor diagram shows the three voltages E_{AB}, E_{BC} and E_{CA} and the three phase currents I_{AB}, I_{BC} and I_{CA} lagging behind the respective phase voltages by an angle Φ. This is drawn by assuming that the load is inductive. From the phase currents I_{AB} and $-I_{CA}$, the line current I_A is drawn as shown in the figure.

From the phasor diagram we see that the line current I_A lags behind the phase phase current I_{AB} by 30^0. The magnitude of the two currents can be related as follows.

$$I_A = 2I_{AB}\cos 30^\circ = \sqrt{3}I_{AB}$$

Hence for a balanced delta connected load we can make the following conclusions.

$$I_l = \sqrt{3}I_{ph}$$

$$E_l = E_{ph}$$

Line current lags behind phase current by 30^0

Three phase Power

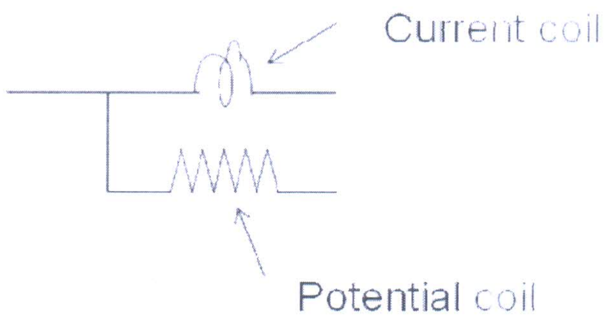
The three phase power for a delta connected load can be derived in the same way as that for a star connected load.

$$P = 3E_{ph}I_{ph}\cos\Phi$$

$$P = \sqrt{3}E_l I_l \cos\Phi$$

Measurement of power and power factor by two wattmeter method

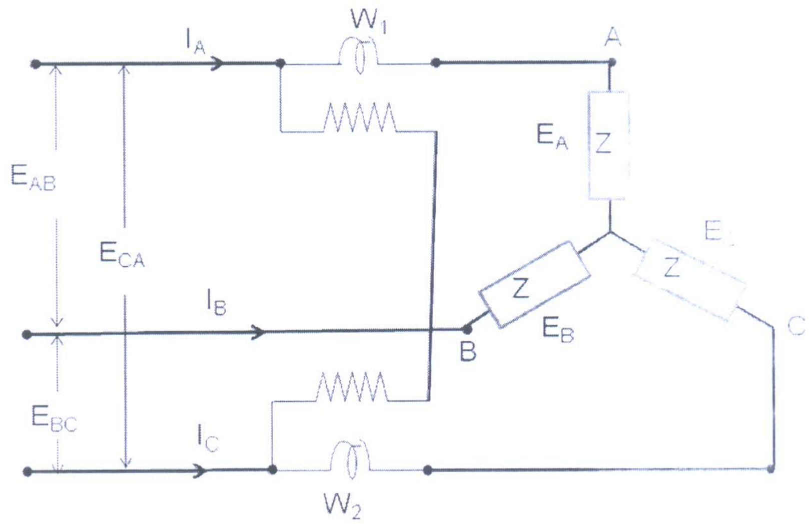
The power in a three phase circuit can be measured by connecting two wattmeters in any of the two phases of the three phase circuit. A wattmeter consists of a current coil and a potential coil as shown in the figure.



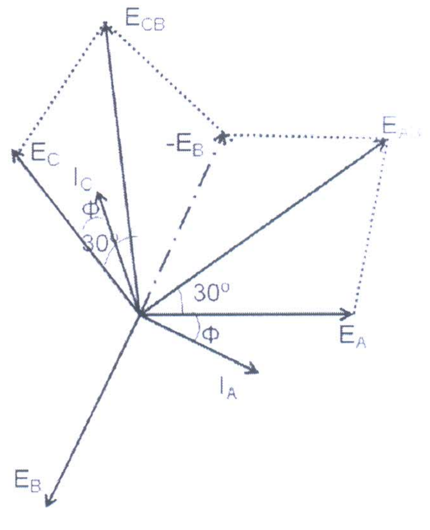
The wattmeter is connected in the circuit in such a way that the current coil is in series and carries the load current and the potential coil is connected in parallel across the load voltage. The wattmeter reading will then be equal to the product of the current carried by the current coil, the voltage across the potential coil and the cosine of the angle between the voltage and current.

The measurement of power is first given for a balanced star connected load and then for a balanced delta connected load.

(i) Balanced star connected load



The circuit shows a balanced star connected load for which the power is to be measured. Two wattmeter W_1 and W_2 are connected in phase A and phase C as shown in the figure.



The current coil of wattmeter W_1 carries the current I_A and its potential coil is connected across the voltage E_{AB}. A phasor diagram is drawn to determine the angle between I_A and E_{AB} as shown.
From the phasor diagram we determine that the angle between the phasors I_A and E_{AB} is $(30+\Phi)$. Hence the wattmeter reading W_1 is given by

$W_1=E_{AB}I_A\cos(30+\Phi)$

The current coil of wattmeter W_2 carries the current I_C and its potential coil is connected across the voltage E_{CB}. From the phasor diagram we determine that the angle between the phasors I_C and E_{CB} is $(30-\Phi)$. Hence the wattmeter reading W_2 is given by

$W_2=E_{CB}I_C\cos(30-\Phi)$

Line voltages $E_{AB}=E_{CB}=E_L$
And line currents $I_A=I_C=I_L$

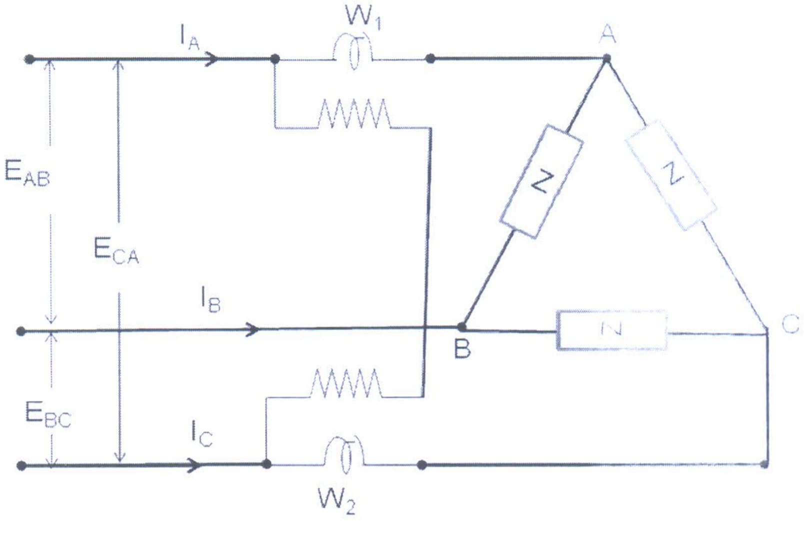
Hence

$$
\begin{aligned}
W_1 &= E_L I_L \cos(30+\Phi) \\
W_2 &= E_L I_L \cos(30-\Phi) \\
W_1 + W_2 &= E_L I_L \cos(30+\Phi) + E_L I_L \cos(30-\Phi) \\
W_1 + W_2 &= E_L I_L (2\cos 30^\circ \cos\Phi) \\
W_1 + W_2 &= \sqrt{3} E_L I_L \cos\Phi
\end{aligned}
$$

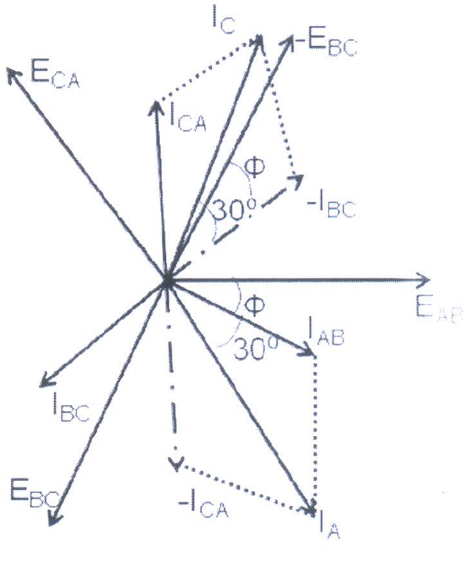
From the above equations we observe that the sum of the two wattmeter reading gives the three phase power.

(ii) Balanced delta connected load

The circuit shows a balanced delta connected load for which the power is to be measured. Two wattmeter W_1 and W_2 are connected in phase A and phase C as shown in the figure.



The current coil of wattmeter W_1 carries the current I_A and its potential coil is connected across the voltage E_{AB}. A phasor diagram is drawn to determine the angle between I_A and E_{AB} as shown.



From the phasor diagram we determine that the angle between the phasors I_A and E_{AB} is (30+Φ). Hence the wattmeter reading W_1 is given by

$W_1 = E_{AB} I_A \cos(30+\Phi)$

The current coil of wattmeter W_2 carries the current I_C and its potential coil is connected across the voltage E_{CB}. From the phasor diagram we determine that the angle between the phasors I_C and E_{CB} is (30-Φ). Hence the wattmeter reading W_2 is given by

$W_2=E_{CB}I_C\cos(30-\Phi)$

Line voltages $E_{AB}=E_{CB}=E_L$

And line currents $I_A=I_C=I_L$

Hence

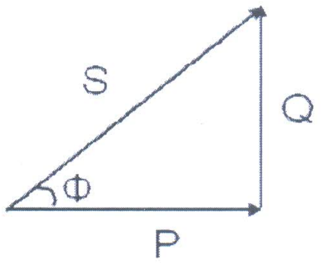
$$W_1 = E_L I_L \cos(30+\Phi)$$
$$W_2 = E_L I_L \cos(30-\Phi)$$
$$W_1 + W_2 = E_L I_L \cos(30+\Phi) + E_L I_L \cos(30-\Phi)$$
$$W_1 + W_2 = E_L I_L (2\cos 30^\circ \cos\Phi)$$
$$W_1 + W_2 = \sqrt{3} E_L I_L \cos\Phi$$

From the above equations we observe that the sum of the two wattmeter reading gives the three phase power.

Determination of Real power, Reactive power and Power factor

$$W_1 = E_L I_L \cos(30+\Phi)$$
$$W_2 = E_L I_L \cos(30-\Phi)$$
$$W_1 + W_2 = \sqrt{3} E_L I_L \cos\Phi$$
$$W_2 - W_1 = E_L I_L \sin\Phi$$
$$\tan\Phi = \sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)$$
$$\Phi = \tan^{-1}\left[\sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)\right]$$
$$P = W_1 + W_2$$
$$Q = \sqrt{3}(W_2 - W_1)$$
$$pf = \cos\Phi = \cos\left[\tan^{-1}\left[\sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)\right]\right]$$

The power factor can also be determined from the power triangle



From the power triangle,

$$P = W_1 + W_2$$
$$Q = \sqrt{3}(W_2 - W_1)$$
$$S = \sqrt{(W_1 + W_2)^2 + 3(W_2 - W_1)^2}$$
$$pf = \cos\Phi = \frac{P}{S} = \frac{W_1 + W_2}{\sqrt{(W_1 + W_2)^2 + 3(W_2 - W_1)^2}}$$

Wattmeter readings at different Power Factors

$(i) upf$

$\Phi = 0^\circ$

$$W_1 = E_L I_L \cos(30 + \Phi) = E_L I_L \cos(30) = \frac{\sqrt{3}}{2} E_L I_L$$
$$W_2 = E_L I_L \cos(30 - \Phi) = E_L I_L \cos(30) = \frac{\sqrt{3}}{2} E_L I_L$$
$$W_1 = W_2$$

$(ii) pf = 0.866$

$\Phi = 30^\circ$

$$W_1 = E_L I_L \cos(30 + \Phi) = E_L I_L \cos(30 + 30) = \frac{E_L I_L}{2}$$
$$W_2 = E_L I_L \cos(30 - \Phi) = E_L I_L \cos(30 - 30) = E_L I_L$$
$$W_2 = 2W_1$$

$$(iii)\, pf = 0.5$$
$$\Phi = 60^\circ$$
$$W_1 = E_L I_L \cos(30+\Phi) = E_L I_L \cos(30+60) = 0$$
$$W_2 = E_L I_L \cos(30-\Phi) = E_L I_L \cos(30-60) = \frac{\sqrt{3}}{2} E_L I_L$$

$$(iv)\, pf < 0.5$$
$$\Phi > 60^\circ$$
$$W_1 = E_L I_L \cos(30+\Phi) < 0$$
$$W_2 = E_L I_L \cos(30-\Phi) > 0$$

$$(v)\, pf = 0$$
$$\Phi = 90^\circ$$
$$W_1 = E_L I_L \cos(30+\Phi) = E_L I_L \cos(30+90) = \frac{E_L I_L}{2}$$
$$W_2 = E_L I_L \cos(30-\Phi) = E_L I_L \cos(30-90) = -\frac{E_L I_L}{2}$$
$$W_1 = -W_2$$

Problem 1

A balanced 3Φ delta connected load has per phase impedance of (25+j40)Ω. If 400V, 3Φ supply is connected to this load, find (i) phase current (ii) line current (iii) power supplied to the load.

$$Z_{ph} = \sqrt{25^2 + 40^2} = 47.17\Omega$$
$$\Phi = \tan^{-1}\left(\frac{40}{25}\right) = 60^\circ$$
$$Z_{ph} = 47.17\angle 60^\circ\, \Omega$$
$$E_L = 400V = E_{ph}$$
$$(i)\, I_{ph} = \frac{E_{ph}}{Z_{ph}} = \frac{400}{47.17\angle 60^\circ} = 8.48\angle -60^\circ\, A$$
$$(ii)\, I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 8.48 = 14.7\angle -90^\circ\, A$$
$$(iii)\, P = \sqrt{3} E_L I_L \cos\Phi = \sqrt{3} \times 400 \times 14.7 \times \cos 60^\circ$$
$$P = 5397.76W$$

Problem 2

Two wattmeter method is used to measure the power absorbed by a 3Φ induction motor. The wattmeter readings are 12.5kW and -4.8kW. Find (i) the power absorbed by the machine (ii) load power factor (iii) reactive power taken by the load.

$$W_1 = 12.5kW$$
$$W_2 = -4.8kW$$
$$(i) P = W_1 + W_2 = 12.5 - 4.8 = 7.7kW$$
$$(ii) \tan\Phi = \sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right) = \sqrt{3}\left(\frac{-4.8 - 12.5}{12.5 - 4.8}\right) = -3.89$$
$$\Phi = \tan^{-1}[-3.89] = -75.6^\circ$$
$$pf = \cos\Phi = \cos(-75.6^\circ) = 0.2487$$
$$(iii) Q = \sqrt{3}(W_2 - W_1) = \sqrt{3}(-4.8 - 12.5) = 29.96kVAR$$

Problem 3

Calculate the active and reactive components of each phase of a star connected 10kV, 3Φ alternator supplying 5MW at 0.8 pf.

$$E_L = 10kV$$
$$P = 5MW$$
$$pf = \cos\Phi = 0.8$$
$$\Phi = 36.87^\circ$$
$$P = \sqrt{3}E_L I_L \cos\Phi$$
$$I_L = \frac{P}{\sqrt{3}E_L \cos\Phi} = \frac{5\times10^6}{\sqrt{3}\times10\times10^3\times0.8} = 360.84A$$
$$P_{ph} = \frac{5\times10^6}{3} = 166.7MW$$
$$Q_{ph} = E_{ph} I_{ph} \sin\Phi = \frac{10\times10}{\sqrt{3}}\times360.8\times\sin 36.87^\circ = 1.25MVAR$$

Problem 4

A 3Φ load of three equal impedances connected in delta across a balanced 400V supply takes a line current of 10A at a power factor of 0.7 lagging. calculate (i) the phase current (ii) the total power (iii) the total reactive kVAR

$$E_L = 400V = E_{ph}$$

$$I_L = 10A$$

$$pf = \cos\Phi = 0.7 lag$$

$$\Phi = 45.57^\circ$$

$$(i) I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.8A$$

$$(ii) P = \sqrt{3} E_L I_L \cos\Phi = \sqrt{3} \times 400 \times 10 \times 0.7 = 4.84kW$$

$$(iii) Q = \sqrt{3} E_L I_L \sin\Phi = \sqrt{3} \times 400 \times 10 \times \sin 45.57^\circ = 4.94kVAR$$

Problem 5

The power flowing in a 3Φ, 3 wire balanced load system is measured by two wattmeter method. The reading in wattmeter A is 750W and wattmeter B is 1500W. What is the power factor of the system?

$$W_1 = 750W$$

$$W_2 = 1500W$$

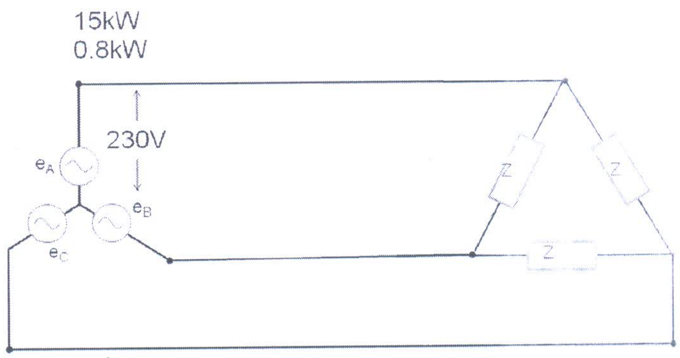
$$\Phi = \tan^{-1}\left[\sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)\right] = \tan^{-1}\left[\sqrt{3}\left(\frac{1500 - 750}{750 + 1500}\right)\right]$$

$$\Phi = 30^\circ$$

$$pf = \cos\Phi = \cos 30^\circ = 0.866$$

Problem 6

A 3Φ star connected supply with a phase voltage of 230V is supplying a balanced delta connected load. The load draws 15kW at 0.8pf lagging. Find the line currents and the current in each phase of the load. What is the load impedance per phase.



Alternator

$$E_{ph} = 230V$$

$$E_L = \sqrt{3} \times 230V = 398.37V$$

$$P = 15kW$$

$$pf = \cos\Phi = 0.8 lagging$$

$$I_L = \frac{P}{\sqrt{3} E_L \cos\Phi} = 27.17A$$

Load

$$E_{ph} = E_L = 398.37V$$

$$I_L = 27.17A$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = 15.68A$$

$$Z_{ph} = \frac{E_{ph}}{I_{ph}} = 25.4\Omega$$