

Subject

Basic Electrical Engineering

Chapters

Electrical Fundamentals

DC Circuits

Network Theorem

Magnetic Circuits

Single Phase Transformer

PK Nayak

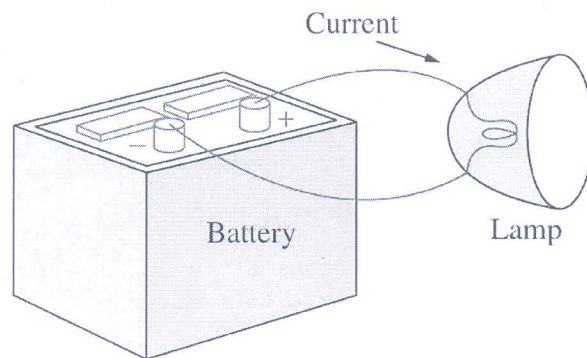
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CHAPTER 1

Basic Concepts

In electrical engineering, we are often interested in communicating or transferring energy from one point to another. To do this requires an interconnection of electrical devices. Such interconnection is referred to as an **electric circuit**, and each component of the circuit is known as an **element**.

DEFINITION 1.0.1. An **electric circuit** is an interconnection of electrical elements.



1.1. Systems of Units

1.1.1. As engineers, we deal with measurable quantities. Our measurement must be communicated in standard language that virtually all professionals can understand irrespective of the country. Such an international measurement language is the **International System of Units (SI)**.

- In this system, there are six principal units from which the units of all other physical quantities can be derived.

| Quantity | Basic Unit | Symbol |
|--------------------|------------|--------|
| Length | meter | m |
| Mass | kilogram | Kg |
| Time | second | s |
| Electric Current | ampere | A |
| Temperature | kelvin | K |
| Luminous Intensity | candela | cd |
| Charge | coulomb | C |

- One great advantage of SI unit is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit.

| Multiplier | Prefix | Symbol |
|------------|--------|--------|
| 10^{12} | tera | T |
| 10^9 | giga | G |
| 10^6 | mega | M |
| 10^3 | kilo | k |
| 10^{-2} | centi | c |
| 10^{-3} | milli | m |
| 10^{-6} | micro | μ |
| 10^{-9} | nano | n |
| 10^{-12} | pico | p |

EXAMPLE 1.1.2. Change of units:

$$600,000,000 \text{ mm} =$$

1.2. Circuit Variables

1.2.1. Charge: The concept of electric charge is the underlying principle for all electrical phenomena. Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C). The charge of an electron is $-1.602 \times 10^{-19} \text{ C}$.

- The coulomb is a large unit for charges. In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons. Thus realistic or laboratory values of charges are on the order of pC, nC, or μC .
- A large power supply capacitor can store up to 0.5 C of charge.

1.2.2. Law of Conservation of Charge: Charge can neither be created nor destroyed, only transferred.

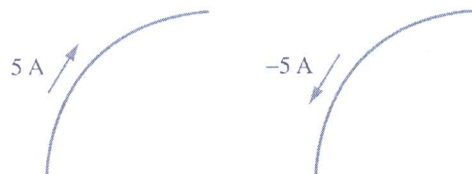
DEFINITION 1.2.3. Current: The time rate of change of charge, measured in amperes (A). Mathematically,

$$i(t) = \frac{d}{dt}q(t)$$

Note:

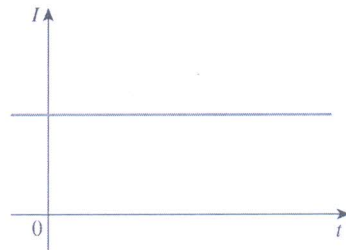
- 1 ampere (A) = 1 coulomb/second (C/s).
- The charge transferred between time t_1 and t_2 is obtained by

$$q = \int_{t_1}^{t_2} i dt$$

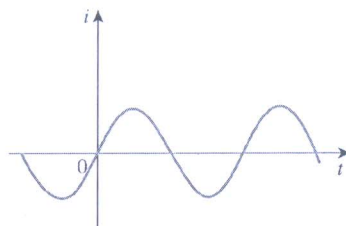


1.2.4. Two types of currents:

- (a) A **direct current** (DC) is a current that remains constant with time.



- (b) An **alternating current** (AC) is a current that varies with time.



- Such AC current is used in your household, to run the air conditioner, refrigerator, washing machine, and other electric appliances.

1.2.5. By convention the symbol I is used to represent such a constant current. A time-varying current is represented by the symbol i .

DEFINITION 1.2.6. Voltage (or potential difference): the energy required to move a unit charge through an element, measured in volts (V). The voltage between two points a and b in a circuit is denoted by v_{ab} and can be interpreted in two ways:

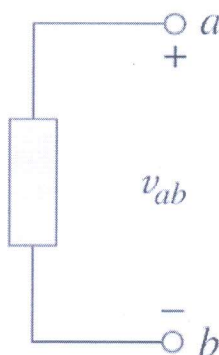
- (a) point a is at a potential of v_{ab} volts higher than point b , or
- (b) the potential at point a with respect to point b is v_{ab} .

Note:

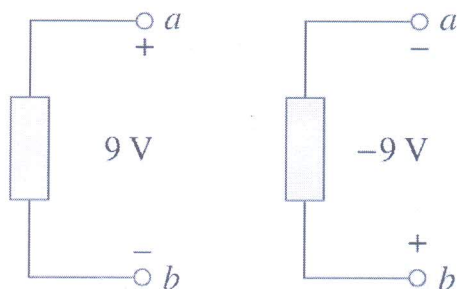
- 1 volt (V) = 1 joule/coulomb = 1 newton-meter/coulomb
- $v_{ab} = -v_{ba}$
- Mathematically,

$$v_{ab} = \frac{dw}{dq}$$

where w is the energy in joules (J) and q is charge in coulombs (C).



The plus (+) and minus (-) signs at the points a and b are used to define reference direction or voltage polarity.



1.2.7. Like electric current, a constant voltage is called a **DC voltage** and is represented by V , whereas a sinusoidally time-varying voltage is called an **AC voltage** and is represented by v . A dc voltage is commonly produced by a battery; ac voltage is produced by an electric generator.

1.2.8. Current and voltage are the two basic variables in electric circuits. The common term *signal* is used for an electric quantity such as a current or a voltage (or even electromagnetic wave) when it is used for conveying information. Engineers prefer to call such variables *signals* rather than mathematical functions of time because of their importance in communications and other disciplines.

For practical purposes, we need to be able to find/calculate/measure more than the current and voltage. We all know from experience that a 100-watt bulb gives more light than a 60-watt bulb. We also know that when we pay our bills to the electric utility companies, we are paying for the electric energy consumed over a certain period of time. Thus power and energy calculations are important in circuit analysis.

DEFINITION 1.2.9. Power: time rate of *expending* or *absorbing* energy, measured in watts (W). Mathematically, the instantaneous power

$$p = \frac{dw}{dt} = \frac{dw}{dq} \frac{dq}{dt} = vi$$

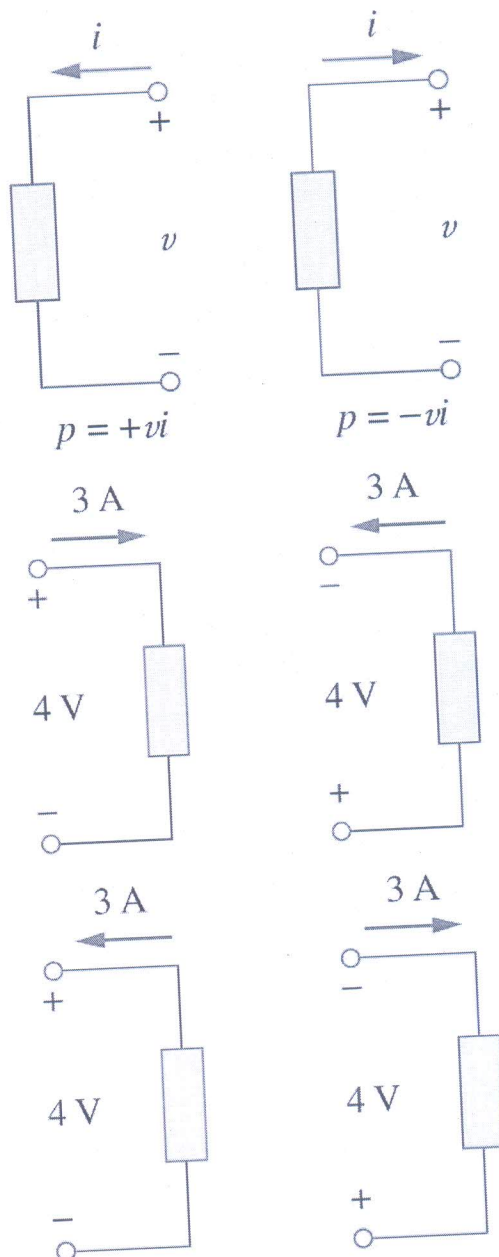
DEFINITION 1.2.10. Sign of power

- **Plus sign:** Power is absorbed by the element. (resistor, inductor)
- **Minus sign:** Power is supplied by the element. (battery, generator)

DEFINITION 1.2.11. Passive sign convention:

- If the current enters through the positive polarity of the voltage,
 $p = vi$.
- If the current enters through the negative polarity of the voltage,
 $p = -vi$.

1. BASIC CONCEPTS



Law of Conservation of Energy: Energy can neither be created nor destroyed, only transferred.

- For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero.
- The total power supplied to the circuit must balance the total power absorbed.

EXAMPLE 1.2.12.

Energy: the energy absorbed or supplied by an element from time 0 to t is

$$w = \int_0^t p \, dt = \int_0^t vi \, dt.$$

- Integration suggests finding area under the curve. Need to be careful with negative area.

The electric power utility companies measure energy in kilowatt-hours (kWh), where $1 \text{ kWh} = 3600 \text{ kJ}$.

1.3. Circuit Elements

DEFINITION 1.3.1. There are 2 types of elements found in electrical circuits.

1) **Active elements** (is capable of generating energy), e.g., generators, batteries, and operational amplifiers (Op-amp).

2) **Passive element**, e.g., resistors, capacitors and inductors.

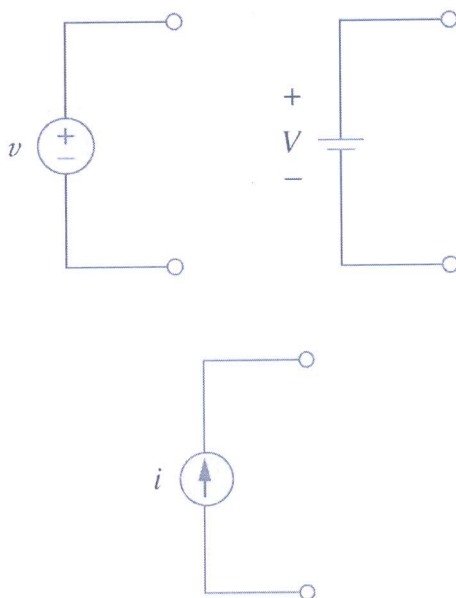
1.3.2. The most important active elements are voltage and current sources:

(a) **Voltage source** provides the circuit with a specified voltage (e.g. a 1.5V battery)

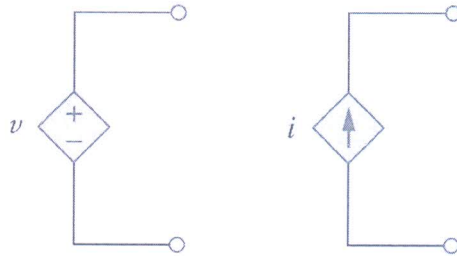
(b) **Current source** provides the circuit with a specified current (e.g. a 1A current source).

DEFINITION 1.3.3. In addition, we may characterize the voltage or current sources as:

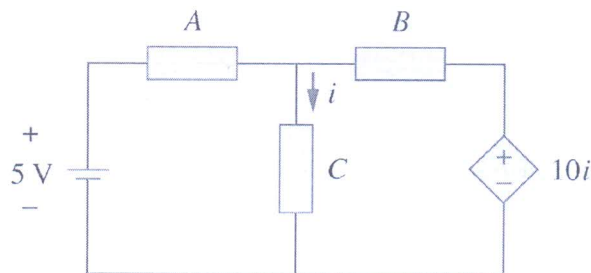
1) **Independent source**: An active element that provides a specified voltage or current that is completely independent of other circuit elements.



2) **Dependent source:** An active element in which the source quantity is controlled by another voltage or current.



1.3.4. The key idea to keep in mind is that a voltage source comes with polarities (+ -) in its symbol, while a current source comes with an arrow, irrespective of what it depends on.



1.3.5. Remarks:

- Dependent sources are useful in modeling elements such as transistors, operational amplifiers and integrated circuits.
- An ideal voltage source (dependent or independent) will produce any current required to ensure that the terminal voltage is as stated.
- An ideal current source will produce the necessary voltage to ensure the stated current flow.
- Thus an ideal source could in theory supply an infinite amount of energy.
- Not only do sources supply power to a circuit, they can absorb power from a circuit too.
- For a voltage source, we know the voltage but not the current supplied or drawn by it. By the same token, we know the current supplied by a current source but not the voltage across it.

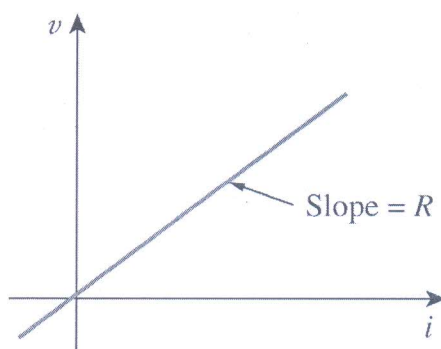
CHAPTER 2

Basic Laws

Here we explore two fundamental laws that govern electric circuits (Ohm's law and Kirchhoff's laws) and discuss some techniques commonly applied in circuit design and analysis.

2.1. Ohm's Law

Ohm's law shows a relationship between voltage and current of a resistive element such as conducting wire or light bulb.



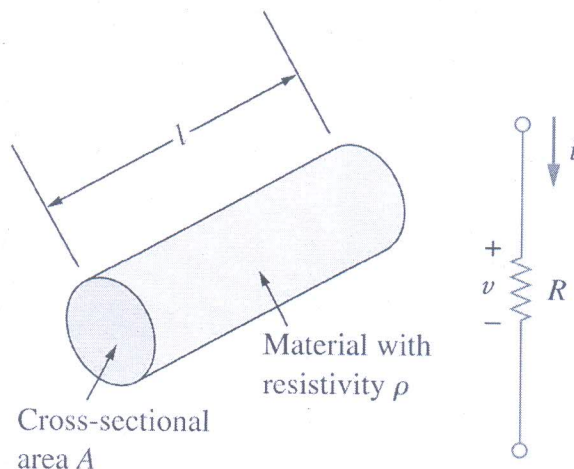
2.1.1. Ohm's Law: The voltage v across a resistor is directly proportional to the current i flowing through the resistor.

$$v = iR,$$

where R = resistance of the resistor, denoting its ability to resist the flow of electric current. The resistance is measured in ohms (Ω).

- To apply Ohm's law, the direction of current i and the polarity of voltage v must conform with the passive sign convention. This implies that current flows from a higher potential to a lower potential

in order for $v = iR$. If current flows from a lower potential to a higher potential, $v = -iR$.



2.1.2. The resistance R of a cylindrical conductor of cross-sectional area A , length L , and conductivity σ is given by

$$R = \frac{L}{\sigma A}.$$

Alternatively,

$$R = \rho \frac{L}{A}$$

where ρ is known as the resistivity of the material in ohm-meters. Good conductors, such as copper and aluminum, have low resistivities, while insulators, such as mica and paper, have high resistivities.

2.1.3. Remarks:

(a) $R = v/i$

(b) Conductance :

$$G = \frac{1}{R} = \frac{i}{v}$$

The unit of G is the mho¹ (\mathcal{U}) or siemens² (S)

¹Yes, this is NOT a typo! It was derived from spelling ohm backwards.

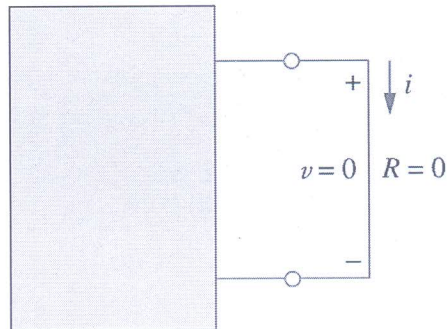
²In English, the term siemens is used both for the singular and plural.

(c) The two extreme possible values of R .

(i) When $R = 0$, we have a **short circuit** and

$$v = iR = 0$$

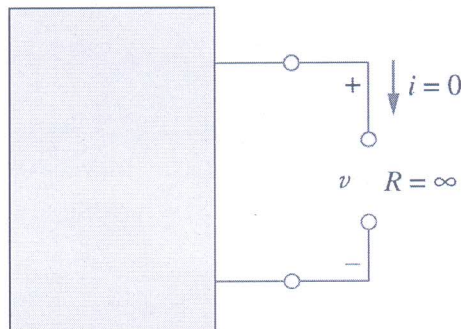
showing that $v = 0$ for any i .



(ii) When $R = \infty$, we have an **open circuit** and

$$i = \lim_{R \rightarrow \infty} \frac{v}{R} = 0$$

indicating that $i = 0$ for any v .



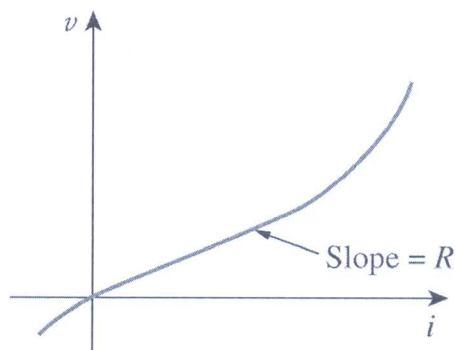
2.1.4. A resistor is either fixed or variable. Most resistors are of the fixed type, meaning their resistance remains constant.

A common variable resistor is known as a **potentiometer** or **pot** for short



2.1.5. Not all resistors obey Ohms law. A resistor that obeys Ohms law is known as a **linear** resistor.

- A **nonlinear** resistor does not obey Ohms law.

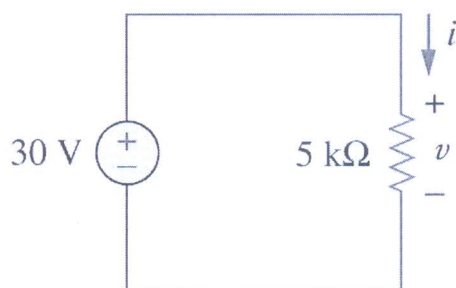


- Examples of devices with nonlinear resistance are the lightbulb and the diode.
- Although all practical resistors may exhibit nonlinear behavior under certain conditions, we will assume in this class that all elements actually designated as resistors are linear.

2.1.6. Using Ohm's law, the power p dissipated by a resistor R is

$$p = vi = i^2 R = \frac{v^2}{R}.$$

EXAMPLE 2.1.7. In the circuit below, calculate the current i , and the power p .



DEFINITION 2.1.8. The **power rating** is the maximum allowable power dissipation in the resistor. Exceeding this power rating leads to overheating and can cause the resistor to burn up.

EXAMPLE 2.1.9. Determine the minimum resistor size that can be connected to a 1.5V battery without exceeding the resistor's $\frac{1}{4}$ -W power rating.

2.2. Node, Branches and Loops

DEFINITION 2.2.1. Since the elements of an electric circuit can be interconnected in several ways, we need to understand some basic concept of network topology.

- Network = interconnection of elements or devices
- Circuit = a network with closed paths

DEFINITION 2.2.2. **Branch:** A branch represents a single element such as a voltage source or a resistor. A branch represents any two-terminal element.

DEFINITION 2.2.3. **Node:** A node is the point of connection between two or more branches. It is usually indicated by a dot in a circuit.

- If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node.

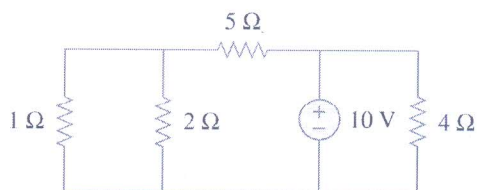
DEFINITION 2.2.4. **Loop:** A loop is any closed path in a circuit. A closed path is formed by starting at a node, passing through a set of nodes and returning to the starting node without passing through any node more than once.

DEFINITION 2.2.5. **Series:** Two or more elements are in **series** if they are cascaded or connected sequentially and consequently carry the *same current*.

DEFINITION 2.2.6. **Parallel:** Two or more elements are in **parallel** if they are connected to the same two nodes and consequently have the *same voltage* across them.

Elements may be connected in a way that they are neither in series nor in parallel.

EXAMPLE 2.2.7. How many branches and nodes does the circuit in the following figure have? Identify the elements that are in series and in parallel.



2.2.8. A loop is said to be **independent** if it contains a branch which is not in any other loop. Independent loops or paths result in independent sets of equations. A network with b branches, n nodes, and ℓ independent loops will satisfy the fundamental theorem of network topology:

$$b = \ell + n - 1.$$

2.3. Kirchhoff's Laws

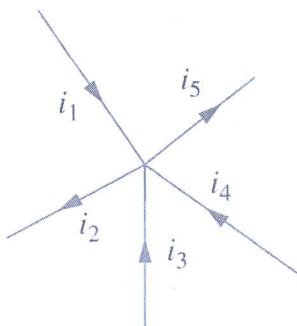
Ohm's law coupled with Kirchhoff's two laws gives a sufficient, powerful set of tools for analyzing a large variety of electric circuits.

2.3.1. **Kirchhoff's current law (KCL)**: the *algebraic sum* of current entering a node (or a closed boundary) is zero. Mathematically,

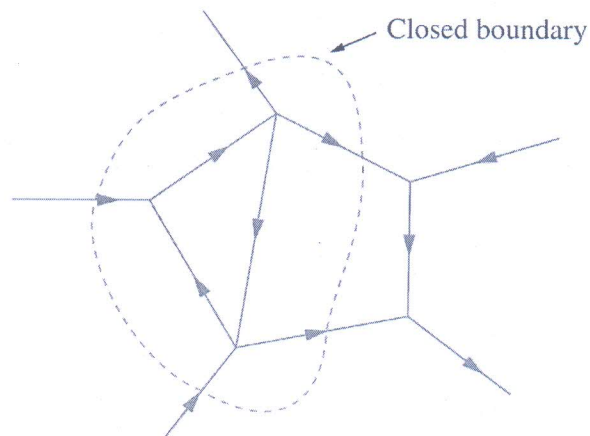
$$\sum_{n=1}^N i_n = 0$$

KCL is based on **the law of conservation of charge**. An alternative form of KCL is

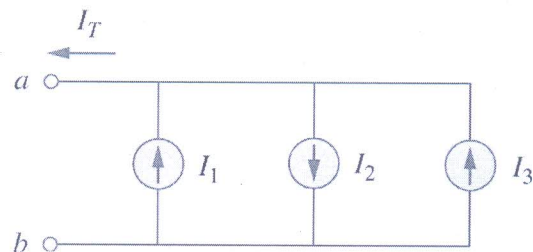
Sum of currents (or charges) entering a node
= Sum of the currents (charges) leaving the node.



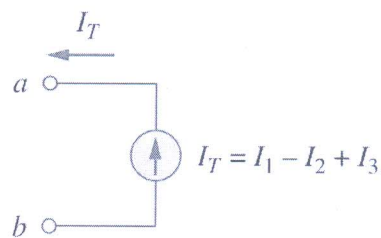
Note that KCL also applies to a closed boundary. This may be regarded as a generalized case, because a node may be regarded as a closed surface shrunk to a point. In two dimensions, a closed boundary is the same as a closed path. The total current entering the closed surface is equal to the total current leaving the surface.



EXAMPLE 2.3.2. A simple application of KCL is combining current sources in parallel.



(a)



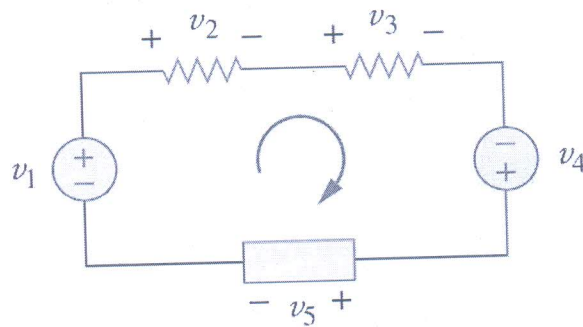
(b)

A Kirchhoff's voltage law (KVL): the algebraic sum of all voltages around a closed path (or loop) is zero. Mathematically,

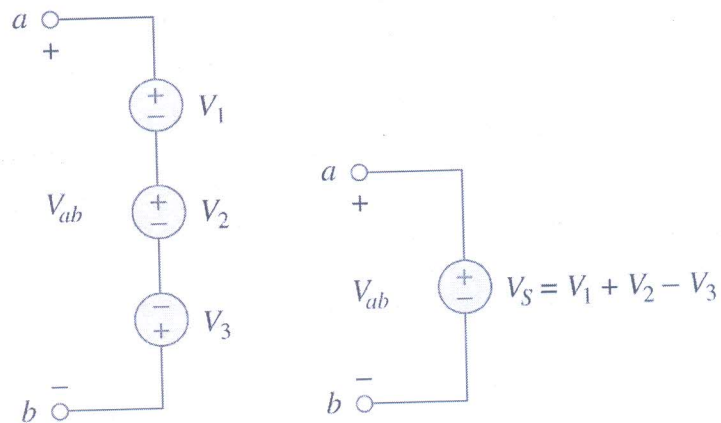
$$\sum_{m=1}^M v_m = 0$$

KVL is based on **the law of conservation of energy**. An alternative form of KVL is

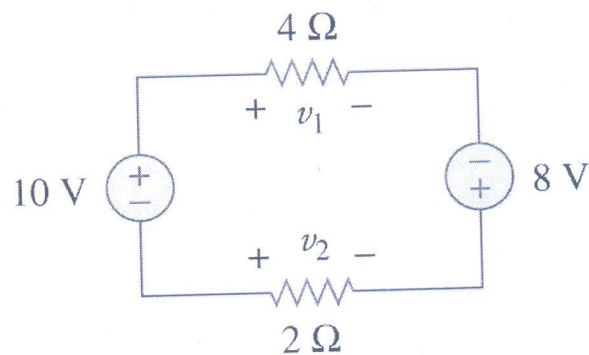
Sum of voltage drops = Sum of voltage rises.



EXAMPLE 2.3.3. When voltage sources are connected in series, KVL can be applied to obtain the total voltage.



EXAMPLE 2.3.4. Find v_1 and v_2 in the following circuit.

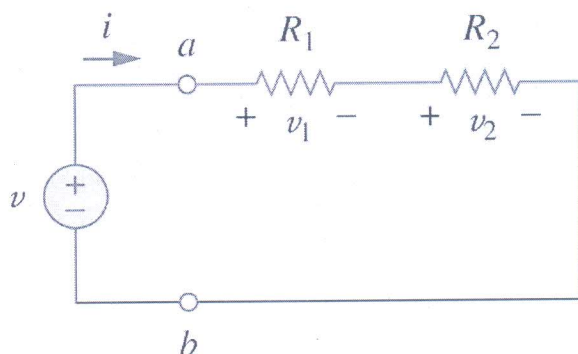


2.4. Series Resistors and Voltage Division

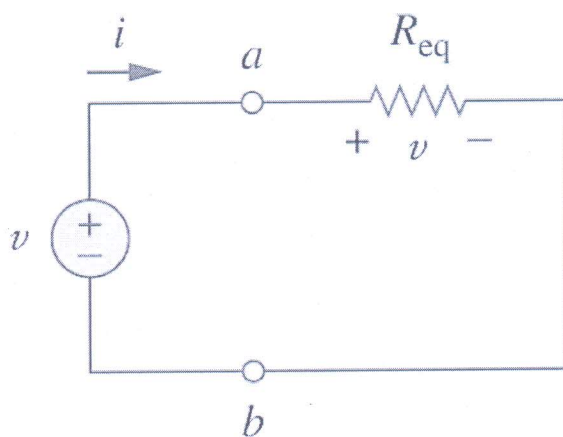
2.4.1. When two resistors R_1 and R_2 ohms are connected in series, they can be replaced by an *equivalent* resistor R_{eq} where

$$R_{eq} = R_1 + R_2$$

In particular, the two resistors in series shown in the following circuit



can be replaced by an equivalent resistor R_{eq} where $R_{eq} = R_1 + R_2$ as shown below.



The two circuits above are equivalent in the sense that they exhibit the same voltage-current relationships at the terminals a - b .

Voltage Divider: If R_1 and R_2 are connected in series with a voltage source v volts, the voltage drops across R_1 and R_2 are

$$v_1 = \frac{R_1}{R_1 + R_2} v \quad \text{and} \quad v_2 = \frac{R_2}{R_1 + R_2} v$$

Note: The source voltage v is *divided* among the resistors in direct proportion to their resistances.

2.4.2. In general, for N resistors whose values are R_1, R_2, \dots, R_N ohms connected in series, they can be replaced by an *equivalent* resistor R_{eq} where

$$R_{eq} = R_1 + R_2 + \dots + R_N = \sum_{j=1}^N R_j$$

If a circuit has N resistors in series with a voltage source v , the j th resistor R_j has a voltage drop of

$$v_j = \frac{R_j}{R_1 + R_2 + \dots + R_N} v$$

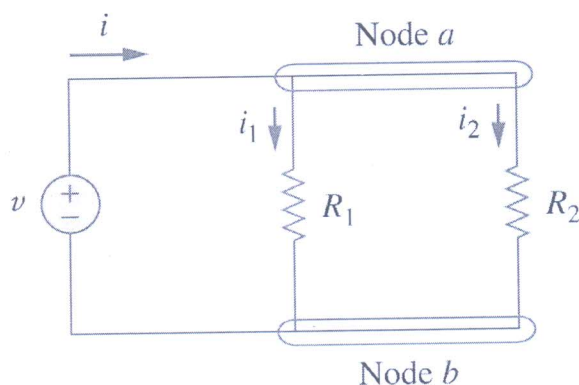
2.5. Parallel Resistors and Current Division

When two resistors R_1 and R_2 ohms are connected in parallel, they can be replaced by an *equivalent* resistor R_{eq} where

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$R_{eq} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$



Current Divider: If R_1 and R_2 are connected in parallel with a current source i , the current passing through R_1 and R_2 are

$$i_1 = \frac{R_2}{R_1 + R_2} i \quad \text{and} \quad i_2 = \frac{R_1}{R_1 + R_2} i$$

Note: The source current i is *divided* among the resistors in inverse proportion to their resistances.

EXAMPLE 2.5.1.

EXAMPLE 2.5.2. $6 \parallel 3 =$

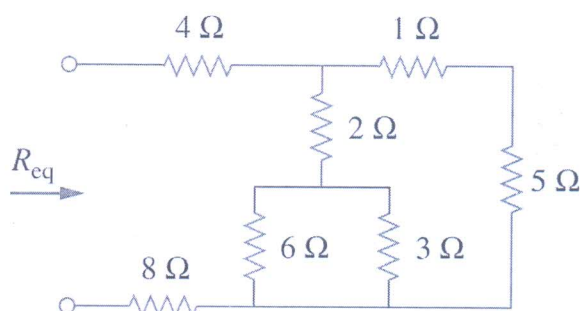
EXAMPLE 2.5.3. $(a) \parallel (na) =$

EXAMPLE 2.5.4. $(ma) \parallel (na) =$

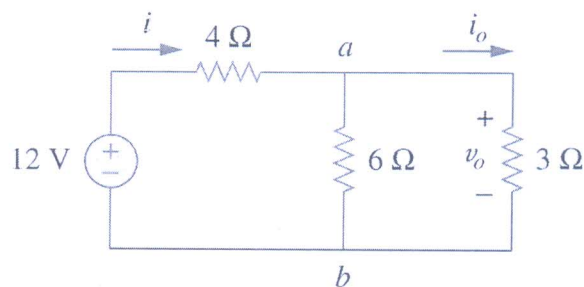
In general, for N resistors connected in parallel, the *equivalent* resistor $R_{eq} = R_1 \parallel R_2 \parallel \cdots \parallel R_N$ is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

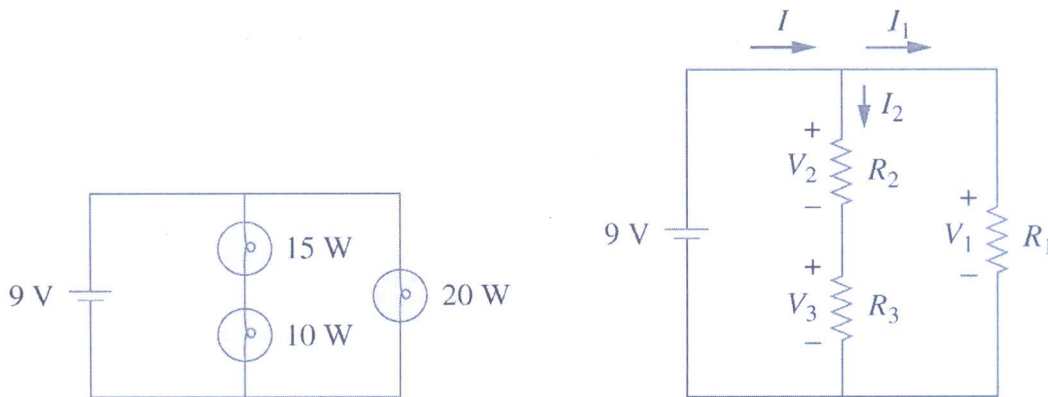
EXAMPLE 2.5.5. Find R_{eq} for the following circuit.



EXAMPLE 2.5.6. Find i_o , v_o , p_o (power dissipated in the 3Ω resistor).



EXAMPLE 2.5.7. Three light bulbs are connected to a 9V battery as shown below. Calculate: (a) the total current supplied by the battery, (b) the current through each bulb, (c) the resistance of each bulb.



2.6. Practical Voltage and Current Sources

An ideal voltage source is assumed to supply a constant voltage. This implies that it can supply very large current even when the load resistance is very small.

However, a practical voltage source can supply only a finite amount of current. To reflect this limitation, we model a practical voltage source as an ideal voltage source connected in series with an internal resistance r_s , as follows:

Similarly, a practical current source can be modeled as an ideal current source connected in parallel with an internal resistance r_s .

2.7. Measuring Devices

Ohmmeter: measures the resistance of the element.

Important rule: Measure the resistance only when the element is disconnected from circuits.

Ammeter: connected in **series** with an element to measure current flowing through that element. Since an ideal ammeter should not restrict the flow of current, (i.e., cause a voltage drop), *an ideal ammeter has zero internal resistance.*

Voltmeter: connected in **parallel** with an element to measure voltage across that element. Since an ideal voltmeter should not draw current away from the element, *an ideal voltmeter has infinite internal resistance.*

CHAPTER 3

Methods of Analysis

Here we apply the fundamental laws of circuit theory (Ohm's Law & Kirchhoff's Laws) to develop two powerful techniques for circuit analysis.

1. Nodal Analysis (based on KCL)
2. Mesh Analysis (based on KVL)

This is the *most important* chapter for our course.

3.1. Nodal Analysis

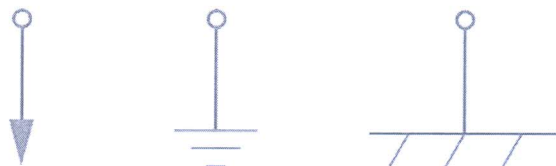
Analyzing circuit using **node voltages** as the circuit variables.

3.1.1. Steps to Determine Node Voltages:

Step 0: Determine the number of nodes n .

Step 1: Select a node as a reference node (ground node). Assign voltages v_1, v_2, \dots, v_{n-1} to the remaining $n - 1$ nodes. The voltage are referenced with respect to the reference node.

- The ground node is assumed to have 0 potential.



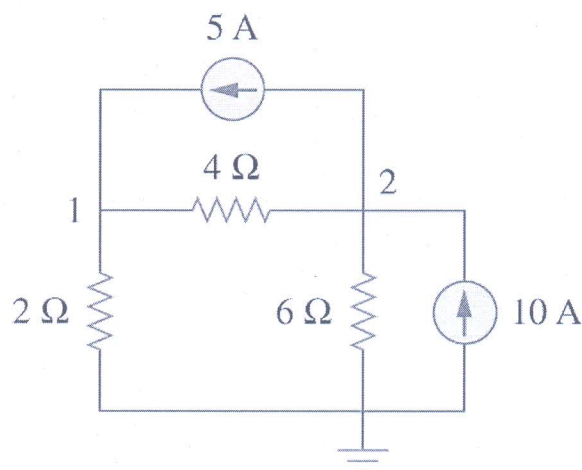
Step 2: Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.

Step 3: Solve the resulting *simultaneous equations* to obtain the unknown node voltages.

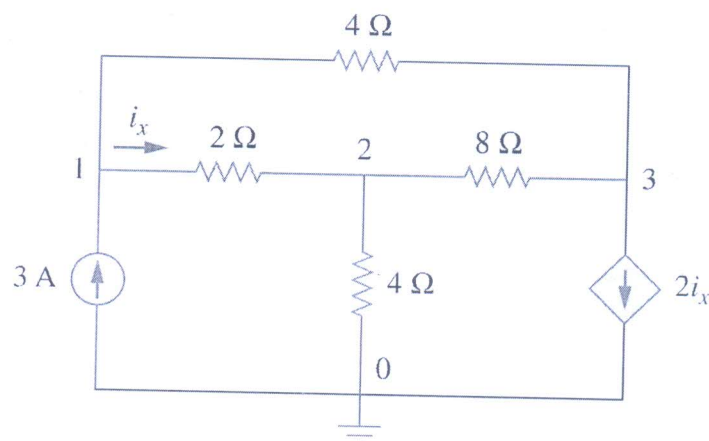
3.1.2. Remark:

- (a) Current flows from a higher potential to a lower potential in a resistor.
- (b) If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source.
- (c) Multiple methods to solve the simultaneous equations in Step 3.
 - Method 1: Elimination technique (good for a few variables)
 - Method 2: Write in term of matrix and vectors (write $Ax = b$), then use Cramer's rule.
 - Method 3: Use computer (e.g., Matlab) to find A^{-1} and $x = A^{-1}b$

EXAMPLE 3.1.3. Calculate the node voltages in the circuit below.

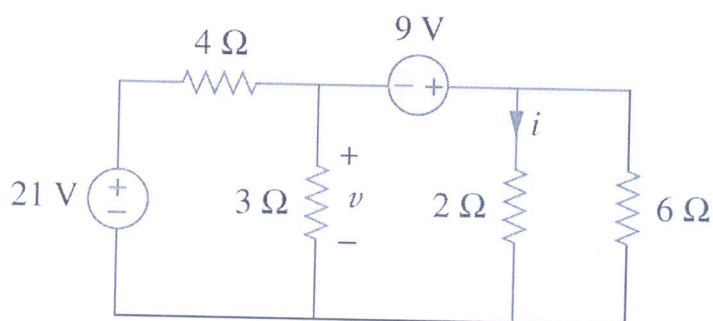


EXAMPLE 3.1.4. Calculate the node voltages in the circuit below.



3.1.5. **Special Case:** If there is a voltage source connected between two nonreference nodes, the two nonreference nodes form a **supernode**. We apply both KCL and KVL to determine the node voltages.

EXAMPLE 3.1.6. Find v and i in the circuit below.



3.1.7. Remarks: Note the following properties of a supernode:

- (a) The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.
- (b) A supernode has no voltage of its own.
- (c) We can have more than two nodes forming a single supernode.
- (d) The supernodes are treated differently because nodal analysis requires knowing the current through each element. However, there is no way of knowing the current through a voltage source in advance.

3.2. Mesh Analysis

Mesh analysis provides another general procedure for analyzing circuits, using **mesh currents** as the circuit variables.

DEFINITION 3.2.1. Mesh is a loop which does not contain any other loop within it.

3.2.2. Steps to Determine Mesh Currents:

Step 0: Determine the number of meshes n .

Step 1: Assign mesh current i_1, i_2, \dots, i_n , to the n meshes.

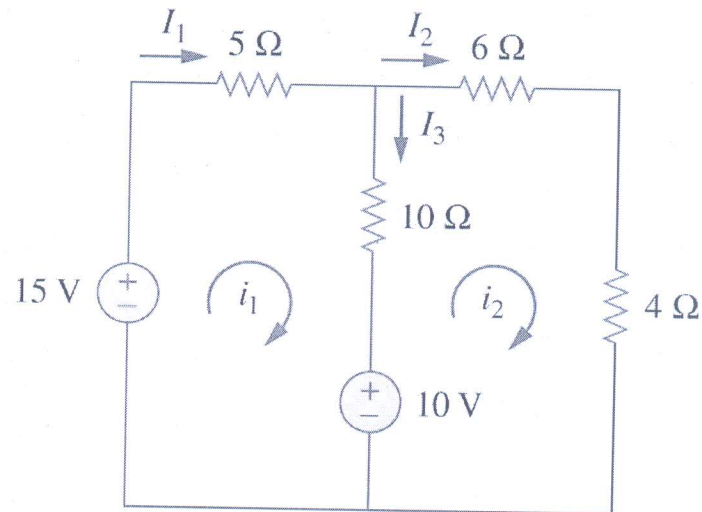
- The direction of the mesh current is arbitrary—(clockwise or counterclockwise)—and does not affect the validity of the solution.
- For convenience, we define currents flow in the clockwise (CW) direction.

Step 2: From the current direction in each mesh, define the voltage drop polarities.

Step 3: Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh current.

Step 4: Solve the resulting n simultaneous equations to get the mesh current.

EXAMPLE 3.2.3. Find the branch currents I_1 , I_2 , and I_3 using mesh analysis.



3.2.4. Remarks:

- (a) Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents.
- (b) Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously.
- (c) Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is *planar*.
 - A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is nonplanar.

3.3. Nodal Versus Mesh Analysis

You should be familiar with both methods. However, given a network to be analyzed, how do we know which method is better or more efficient?

Suggestion: Choose the method that results in smaller number of variables or equations.

- A circuit with fewer nodes than meshes is better analyzed using nodal analysis, while a circuit with fewer meshes than nodes is better analyzed using mesh analysis.

You can also use one method to check your results of the other method.

Network Theorem

NETWORK THEOREMS

Thevenin's Theorem

Norton's Theorem

Superposition Theorem

INTRODUCTION:

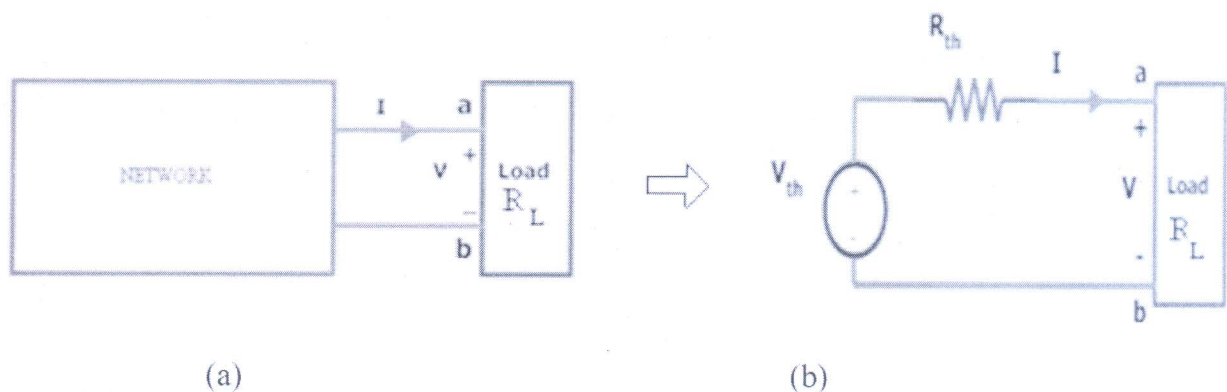
Any complicated network i.e. several sources, multiple resistors are present if the single element response is desired then use the network theorems. Network theorems are also can be termed as network reduction techniques. Each and every theorem got its importance of solving network. Let us see some important theorems with DC with detailed procedures.

Thevenin's Theorem and Norton's theorem (Introduction):

Thevenin's Theorem and Norton's theorem are two important theorems in solving Network problems having many active and passive elements. Using these theorems the network can be reduced to simple equivalent circuits with one active source and one element. In circuit analysis many a times the current through a branch is required to be found when its value is changed with all other element values remaining same. In such cases finding out every time the branch current using the conventional mesh and node analysis methods is quite awkward and time consuming. But with the simple equivalent circuits (with one active source and one element) obtained using these two theorems the calculations become very simple. Thevenin's and Norton's theorems are dual theorems.

Thevenin's Theorem Statement :

Any linear, bilateral two terminal network consisting of sources and resistors (Impedance), can be replaced by an equivalent circuit consisting of a voltage source in series with a resistance (Impedance). The equivalent voltage source V_{th} is the open circuit voltage looking into the terminals (with concerned branch element removed) and the equivalent resistance R_{th} while all sources are replaced by their internal resistors at ideal condition i.e. voltage source is short circuit and current source is open circuit.



Network Theorem

Figure (a) shows a simple block representation of a network with several active / passive elements with the load resistance R_L connected across the terminals 'a & b' and figure (b) shows the Thevenin equivalent circuit with V_{Th} connected across R_{Th} & R_L .

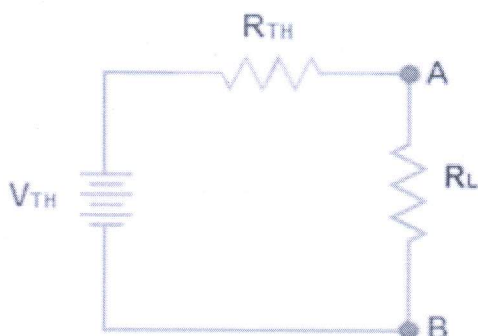
Main steps to find out V_{Th} and R_{Th} :

1. The terminals of the branch/element through which the current is to be found out are marked as say a & b after removing the concerned branch/element.
2. Open circuit voltage V_{OC} across these two terminals is found out using the conventional network mesh/node analysis methods and this would be V_{Th} .
3. Thevenin resistance R_{Th} is found out by the method depending upon whether the network contains dependent sources or not.

a. With dependent sources: $R_{Th} = V_{oc} / I_{sc}$

b. Without dependent sources : R_{Th} = Equivalent resistance looking into the concerned terminals with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)

4. Replace the network with V_{Th} in series with R_{Th} and the concerned branch resistance (or) load resistance across the load terminals(A&B) as shown in below fig.



Example: Find V_{Th} , R_{Th} and the load current and load voltage flowing through R_L resistor as shown in fig. by using Thevenin's Theorem?

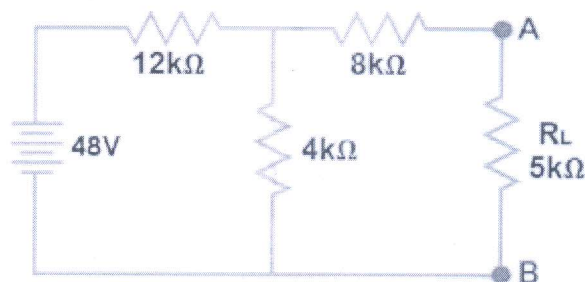


Fig.(a)

Network Theorem

Solution:

The resistance R_L is removed and the terminals of the resistance R_L are marked as A & B as shown in the fig. (1)

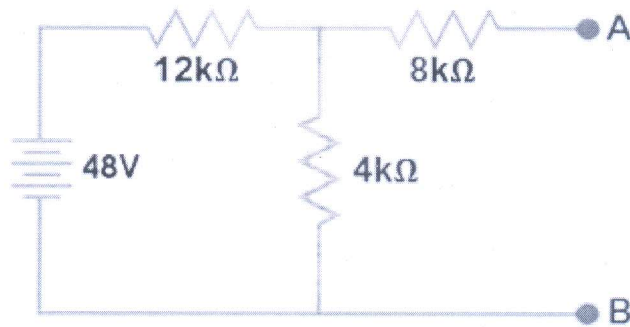
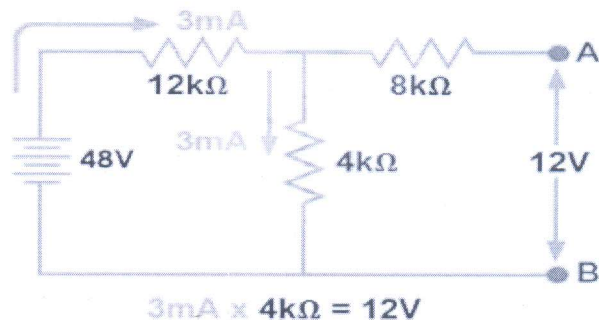


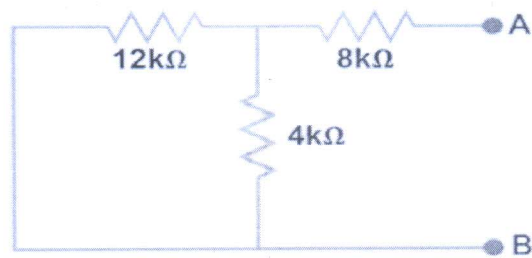
Fig.(1)

Calculate / measure the Open Circuit Voltage. This is the Thevenin Voltage (V_{TH}). We have already removed the load resistor from fig.(a), so the circuit became an open circuit as shown in fig (1). Now we have to calculate the Thevenin's Voltage. Since $3mA$ Current flows in both $12k\Omega$ and $4k\Omega$ resistors as this is a series circuit because current will not flow in the $8k\Omega$ resistor as it is open. So $12V$ ($3mA \times 4k\Omega$) will appear across the $4k\Omega$ resistor. We also know that current is not flowing through the $8k\Omega$ resistor as it is open circuit, but the $8k\Omega$ resistor is in parallel with $4k\Omega$ resistor. So the same voltage (i.e. $12V$) will appear across the $8k\Omega$ resistor as $4k\Omega$ resistor. Therefore $12V$ will appear across the AB terminals. So, $V_{TH} = 12V$



Fig(2)

All voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited) as shown in fig.(3)



Fig(3)

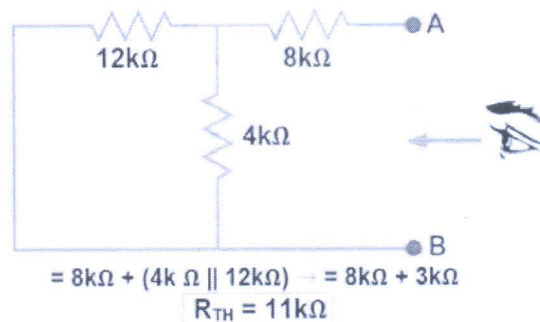
Calculate /measure the Open Circuit Resistance. This is the Thevenin Resistance (R_{TH}) We have Reduced the 48V DC source to zero is equivalent to replace it with a short circuit as shown in figure (3) We can see that 8kΩ resistor is in series with a parallel connection of 4kΩ resistor and 12k Ω resistor. i.e.:

$$8k\Omega + (4k\Omega \parallel 12k\Omega) \dots (\parallel = \text{in parallel with})$$

$$R_{TH} = 8k\Omega + [(4k\Omega \times 12k\Omega) / (4k\Omega + 12k\Omega)]$$

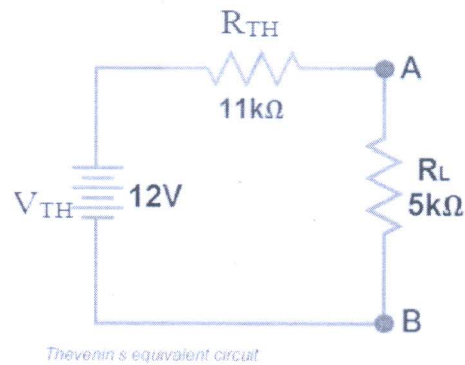
$$R_{TH} = 8k\Omega + 3k\Omega$$

$$R_{TH} = 11k\Omega$$



Fig(4)

Connect the R_{TH} in series with Voltage Source V_{TH} and re-connect the load resistor across the load terminals(A&B) as shown in fig (5) i.e. Thevenin circuit with load resistor. This is the Thevenin's equivalent circuit



Fig(5)

Now apply Ohm's law and calculate the total load current from fig 5.

$$I_L = V_{TH} / (R_{TH} + R_L) = 12V / (11k\Omega + 5k\Omega) = 12/16k\Omega$$

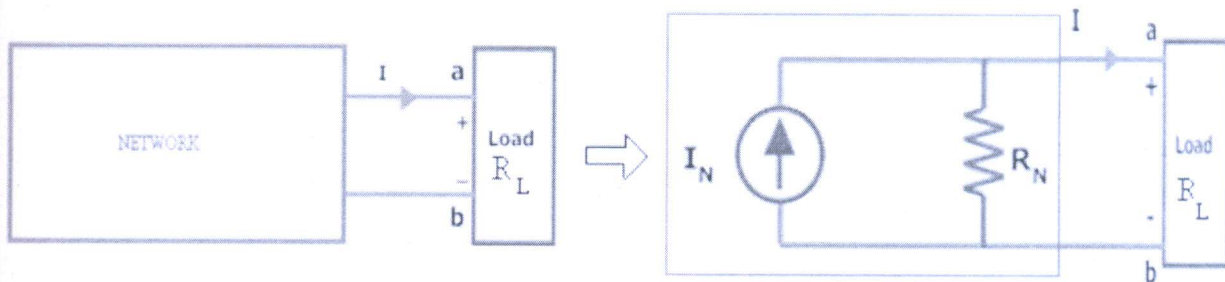
$$I_L = 0.75mA$$

$$\text{And } V_L = I_L \times R_L = 0.75mA \times 5k\Omega$$

$$V_L = 3.75V$$

Norton's Theorem Statement:

Any linear, bilateral two terminal network consisting of sources and resistors (Impedance), can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance (Impedance), the current source being the short circuited current across the load terminals and the resistance being the internal resistance of the source network looking through the open circuited load terminals.



(a) (b)

Figure (a) shows a simple block representation of a network with several active / passive elements with the load resistance R_L connected across the terminals 'a & b' and figure (b) shows the Norton equivalent circuit with I_N connected across R_N & R_L .

Main steps to find out I_N and R_N :

1. The terminals of the branch/element through which the current is to be found out are marked as say a & b after removing the concerned branch/element.

Network Theorem

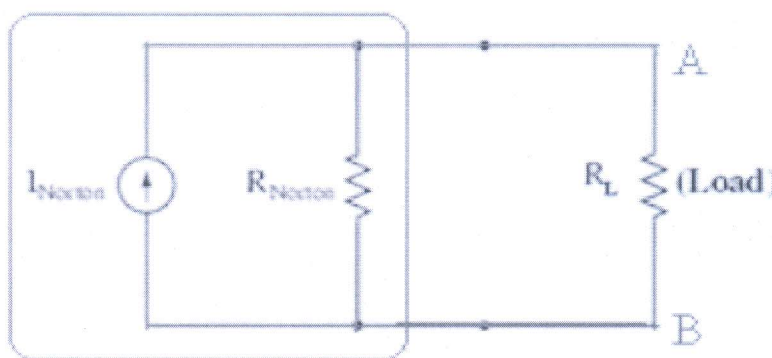
2. Open circuit voltage V_{OC} across these two terminals and I_{SC} through these two terminals are found out using the conventional network mesh/node analysis methods and they are same as what we obtained in Thevenin's equivalent circuit.

3. Next Norton resistance R_N is found out depending upon whether the network contains dependent sources or not.

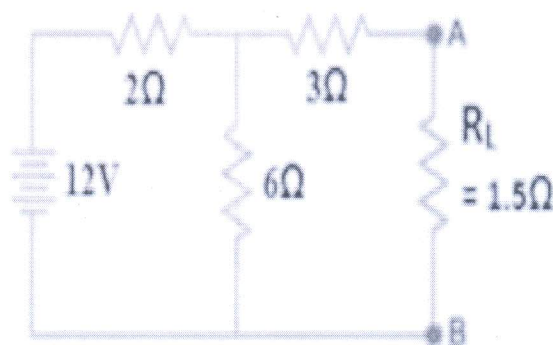
a) With dependent sources: $R_N = V_{oc} / I_{sc}$

b) Without dependent sources : $R_N =$ Equivalent resistance looking into the concerned terminals with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)

4. Replace the network with I_N in parallel with R_N and the concerned branch resistance across the load terminals (A & B) as shown in below fig

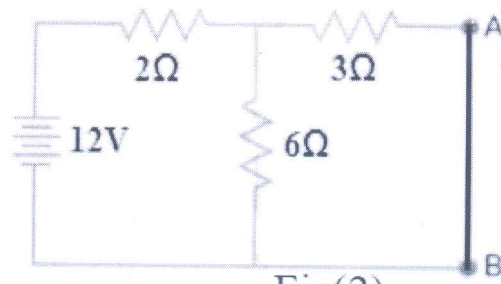


Example: Find the current through the resistance R_L (1.5Ω) of the circuit shown in the figure (a) below using Norton's equivalent circuit?



Fig(a)

Solution: To find out the Norton's equivalent ckt we have to find out $I_N = I_{sc}$, $R_N = V_{oc} / I_{sc}$. Short the 1.5Ω load resistor as shown in (Fig 2), and Calculate / measure the Short Circuit Current. This is the Norton Current (I_N).



Fig(2)

We have shorted the AB terminals to determine the Norton current, I_N . The 6Ω and 3Ω are then in parallel and this parallel combination of 6Ω and 3Ω are then in series with 2Ω . So the Total Resistance of the circuit to the Source is:-

$$2\Omega + (6\Omega \parallel 3\Omega) \dots (\parallel = \text{in parallel with})$$

$$R_T = 2\Omega + [(3\Omega \times 6\Omega) / (3\Omega + 6\Omega)]$$

$$R_T = 2\Omega + 2\Omega$$

$$R_T = 4\Omega$$

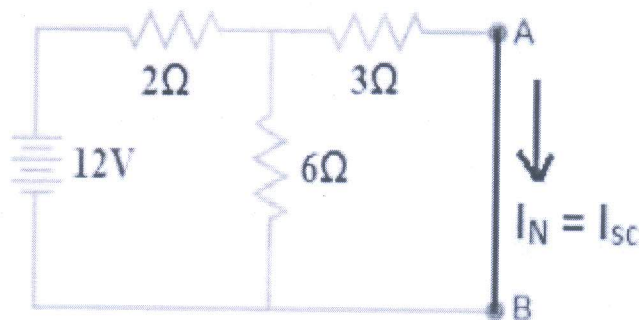
$$I_T = V / R_T$$

$$I_T = 12V / 4\Omega = 3A..$$

Now we have to find $I_{SC} = I_N$... Apply CDR... (Current Divider Rule) ...

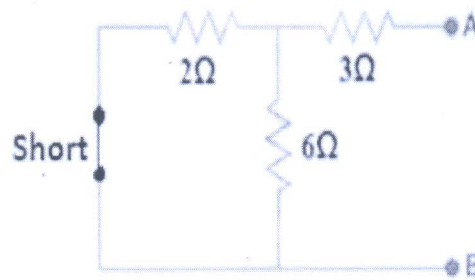
$$I_{SC} = I_N = 3A \times [(6\Omega / (3\Omega + 6\Omega))] = 2A.$$

$$I_{SC} = I_N = 2A.$$



Fig(3)

All voltage & current sources replaced by their internal impedances (i.e. ideal voltage source short circuited and ideal current sources open circuited) and Open Load Resistor. as shown in fig.(4)



Fig(4)

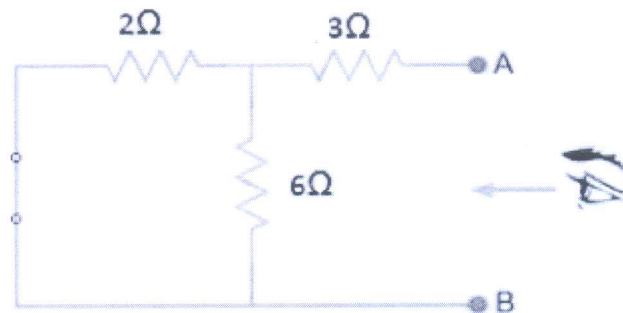
Calculate /measure the Open Circuit Resistance. This is the Norton Resistance (R_N) We have Reduced the 12V DC source to zero is equivalent to replace it with a short circuit as shown in fig(4), We can see that 3Ω resistor is in series with a parallel combination of 6Ω resistor and 2Ω resistor. i.e.:

$$3\Omega + (6\Omega \parallel 2\Omega) \dots \dots (|| = \text{in parallel with})$$

$$R_N = 3\Omega + [(6\Omega \times 2\Omega) / (6\Omega + 2\Omega)]$$

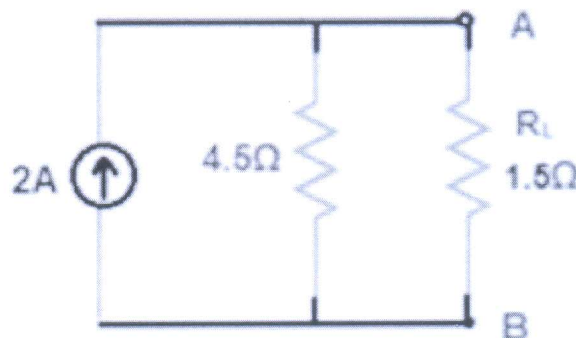
$$R_N = 3\Omega + 1.5\Omega$$

$$R_N = 4.5\Omega$$



Fig(5)

Connect the R_N in Parallel with Current Source I_N and re-connect the load resistor. This is shown in fig (6) i.e. Norton Equivalent circuit with load resistor.



Fig(6)

Network Theorem

Now apply the Ohm's Law and calculate the load current through Load resistance across the terminals A&B. Load Current through Load Resistor is

$$I_L = I_N \times [R_N / (R_N + R_L)]$$

$$I_L = 2A \times (4.5\Omega / 4.5\Omega + 1.5k\Omega)$$

$$I_L = 1.5A \quad I_L = 1.5A$$

Superposition Theorem:

The principle of superposition helps us to analyze a linear circuit with more than one current or voltage sources sometimes it is easier to find out the voltage across or current in a branch of the circuit by considering the effect of one source at a time by replacing the other sources with their ideal internal resistances.

Superposition Theorem Statement:

Any linear, bilateral two terminal network consisting of more than one sources, The total current or voltage in any part of a network is equal to the algebraic sum of the currents or voltages in the required branch with each source acting individually while other sources are replaced by their ideal internal resistances. (i.e. Voltage sources by a short circuit and current sources by open circuit) Steps to Apply Superposition Principle:

1. Replace all independent sources with their internal resistances except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Example: By Using the superposition theorem find I in the circuit shown in figure?

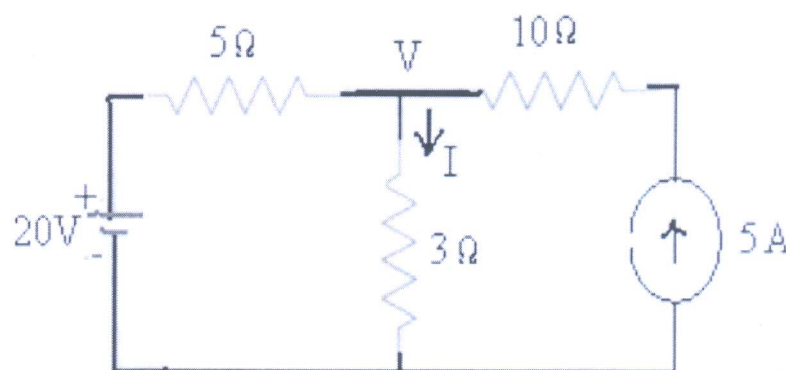


Fig. (a)

Network Theorem

Solution: Applying the superposition theorem, the current I_2 in the resistance of $3\ \Omega$ due to the voltage source of 20V alone, with current source of 5A open circuited [as shown in the figure.1 below] is given by :

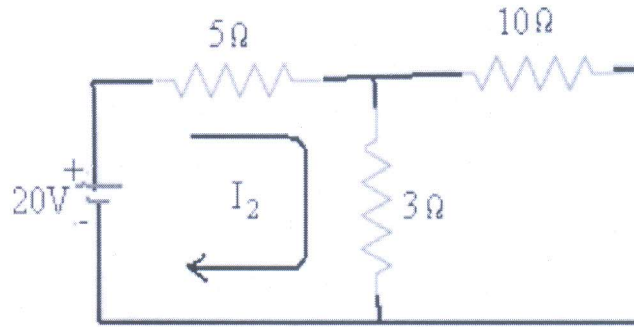


Fig1

$$I_2 = 20/(5+3) = 2.5\text{A}$$

Similarly the current I_5 in the resistance of $3\ \Omega$ due to the current source of 5A alone with voltage source of 20V short circuited [as shown in the figure.2 below] is given by :

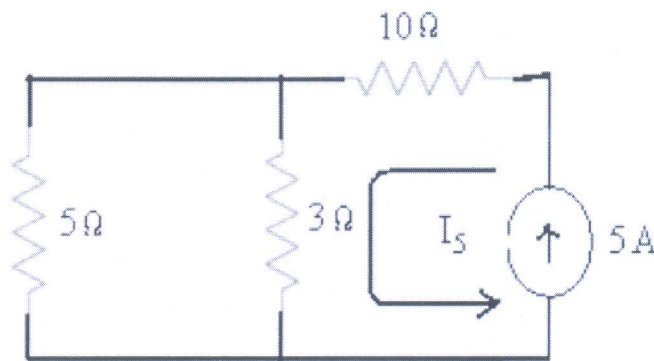


Fig.2

$$I_5 = 5 \times 5/(3+5) = 3.125\text{ A}$$

The total current passing through the resistance of $3\ \Omega$ is then $= I_2 + I_5 = 2.5 + 3.125 = 5.625\text{ A}$

MAGNETIC CIRCUITS

INTRODUCTION

Although the lines of magnetic flux have no physical existence, they do form a very convenient and useful basis for explaining various magnetic effects and to calculate the magnitudes of various magnetic quantities. The complete closed path followed by any group of magnetic flux lines is referred as magnetic circuit. The lines of magnetic flux never intersect, and each line forms a closed path. Whenever a current is flowing through the coil there will be magnetic flux produced and the path followed by the magnetic flux is known as magnetic circuit. The operation of all the electrical devices like generators, motors, transformers etc. depend upon the magnetism produced by this magnetic circuit. Therefore, to obtain the required characteristics of these devices, their magnetic circuits have to be designed carefully.

Magneto Motive Force (MMF)

The magnetic pressure which sets up or tends to set up magnetic flux in a magnetic circuit is known as MMF.

1. Magneto motive force is the measure of the ability of a coil to produce flux.
2. The magnetic flux is due to the existence of the MMF caused by a current flowing through a coil having no. of turns.
3. A coil with 'N' turns carrying a current of 'I' amperes represents a magnetic circuit producing an MMF of NI $MMF=NI$
4. Units of MMF = Ampere turns (AT)

Magnetic Flux:

1. The amount of magnetic lines of force set-up in a magnetic circuit is called magnetic flux.
2. The magnetic flux that is established in a magnetic circuit is proportional to the MMF and the proportional constant is the reluctance of the magnetic circuit.

Magnetic flux \propto MMF

$$\text{Magnetic flux} = MMF / RELUCTANCE = NI / S$$

Magnetic Circuits

3. The unit of magnetic flux is Weber.

Reluctance:

1. The opposition offered to the flow of magnetic flux in a magnetic circuit is called reluctance
2. Reluctance of a magnetic circuit is defined as the ratio of magneto motive force to the flux established.
3. Reluctance depends upon length (l), area of cross-section (A) and permeability of the material that makes up the magnetic circuit.

$$S = \frac{l}{\mu A}$$

$$\text{Reluctance} = \frac{\text{MMF}}{\text{Flux}}$$

4. The unit of reluctance is AT/ Wb

Magnetic field strength (H)

1. If the magnetic circuit is homogeneous, and of uniform cross-sectional area, the magnetic field strength is defined as the magneto motive force per unit length of magnetic circuit.

$$H = \text{MMF} / \text{LENGTH} = NI / l$$

2. The unit of magnetic field strength is AT/m

Magnetic flux density (B)

1. The magnetic flux density in any material is defined as the magnetic flux established per unit area of cross-section.

$$B = \text{FLUX} / \text{AREA OF CROSS SECTION} = \phi / A$$

2. The unit of magnetic flux density is wb/m² or TESLA

Relative permeability

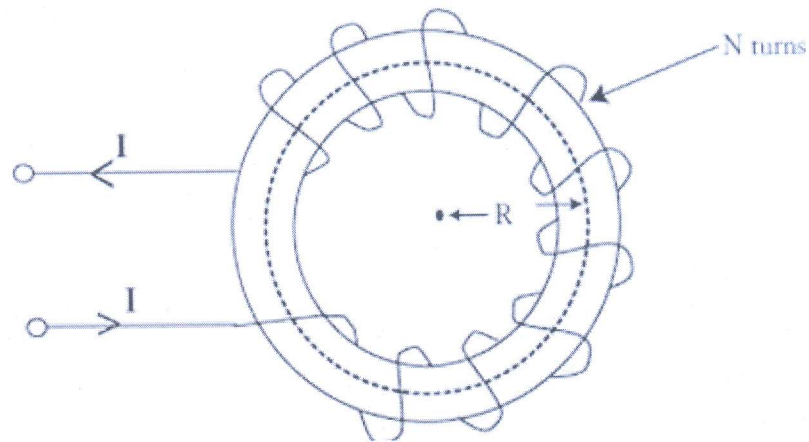
It is defined as the ratio of flux density established in magnetic material to the flux density established in air or vacuum for the same magnetic field strength.

SERIES MAGNETIC CIRCUIT:

➤ A series magnetic circuit is analogous to a series electric circuit. A magnetic circuit is said to be series, if the same flux is flowing through all the elements connected in a magnetic circuit. Consider a circular ring having a magnetic path of 'l' meters, area of cross section 'a' m² with a

Magnetic Circuits

mean radius of 'R' meters having a coil of 'N' turns carrying a current of 'I' amperes wound uniformly as shown in below figure.



The flux produced by the circuit is given by

Magnetic **flux** = $MMF / RELUCTANCE$

$$= NI / S$$

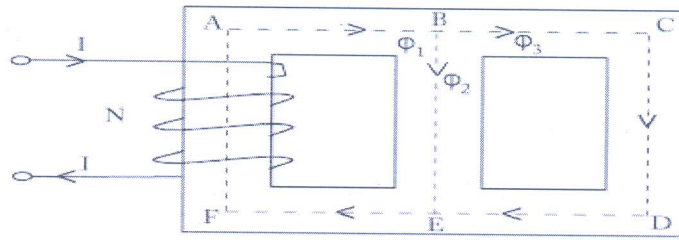
$$= NI / (l / \mu a)$$

In the above equation NI is the MMF of the magnetic circuit, which is analogous to EMF in the electrical circuit.

PARALLEL MAGNETIC CIRCUIT

➤ A magnetic circuit which has more than one path for magnetic flux is called a parallel magnetic circuit. It can be compared with a parallel electric circuit which has more than one path for electric current. The concept of parallel magnetic circuit is illustrated in fig.2. Here a coil of 'N' turns wound on limb 'AF' carries a current of 'I' amperes. The magnetic flux ' ϕ_1 ' set up by the coil divides at 'B' into two paths namely Magnetic flux passes ' ϕ_2 ' along the path 'BE' Magnetic flux passes ' ϕ_3 ' along the path 'BCDE' i.e $\phi_1 = \phi_2 + \phi_3$

Magnetic Circuits



The magnetic paths 'BE' and 'BCDE' are in parallel and form a parallel magnetic circuit. The AT required for this parallel circuit is equal to AT required for any one of the paths.

Let, S_1 = reluctance of path EFAB

S_2 = reluctance of path BE

S_3 = reluctance of path BCDE

Total MMF = MMF for path EFAB + MMF for path BE or path BCD

$$NI = \Phi_1 S_1 + \Phi_2 S_2 = \Phi_1 S_1 + \Phi_3 S_3$$

COMPOSITE MAGNETIC CIRCUIT:

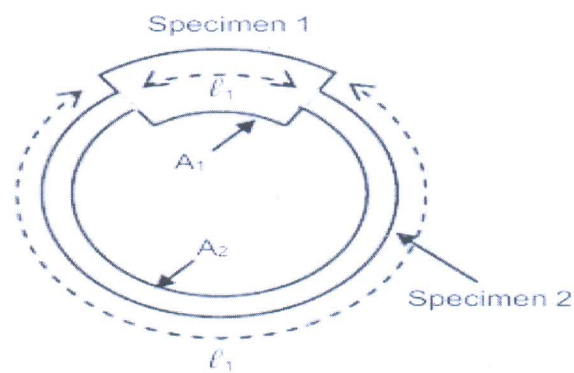
Consider a magnetic circuit which consists of two specimens of iron arranged as shown in figure. Let ℓ_1 and ℓ_2 be the mean lengths of specimen 1 and specimen 2 in meters, A_1 and A_2 be their respective cross sectional areas in square meters, and μ_1 and μ_2 be their respective relative permeability's.

The reluctance of specimen 1 is given as

$$S_1 = \frac{\ell_1}{\mu_0 \mu_1 A_1} \quad (AT/Wb)$$

and that for specimen 2 is

$$S_2 = \frac{\ell_2}{\mu_0 \mu_2 A_2} \quad (AT/Wb)$$



If a coil of N turns carrying a current I is wound on the specimen 1 and if the magnetic flux is assumed to be confined to iron core then the total reluctance is given by the sum of the individual reluctances S_1 and S_2 . This is equivalent to adding the resistances of a series circuit. Thus the total reluctance is given by

$$S = S_1 + S_2 = \frac{\ell_1}{\mu_0 \mu_1 A_1} + \frac{\ell_2}{\mu_0 \mu_2 A_2} \quad AT/Wb$$

And the total flux is given by

$$\Phi = \frac{\text{mmf}}{S} = \frac{NI}{\frac{\ell_1}{\mu_0 \mu_1 A_1} + \frac{\ell_2}{\mu_0 \mu_2 A_2}} \quad \left(\frac{AT}{(AT/Wb)} \Rightarrow Wb \right)$$

Magnetic Circuits

Permeability and $B-H$ curves

For air, or any non-magnetic medium, the ratio of magnetic flux density to magnetizing force is a constant, i.e. $B/H = \text{a constant}$. This constant is μ_0 , the **permeability of free space** (or the magnetic space constant) and is equal to $4\pi \times 10^{-7} \text{ H/m}$, i.e., **for air, or any non-magnetic medium**, the ratio $B/H = \mu_0$ (Although all non-magnetic materials, including air, exhibit slight magnetic properties, these can effectively be neglected.)

$$\text{For all media other than free space, } B/H = \mu_0 \mu_r$$

Where μ_r is the relative permeability, and is defined as

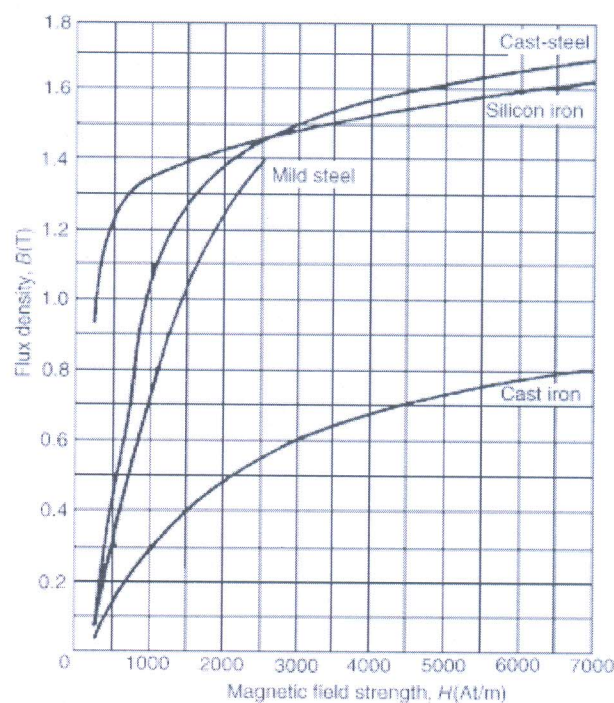
$$\mu_r = \text{flux density in material} / \text{flux density in a vacuum}$$

μ_r varies with the type of magnetic material and, since it is a ratio of flux densities, it has no unit. From its definition, μ_r for a vacuum is 1.

$$\mu_0 \mu_r = \mu, \text{ called the absolute permeability}$$

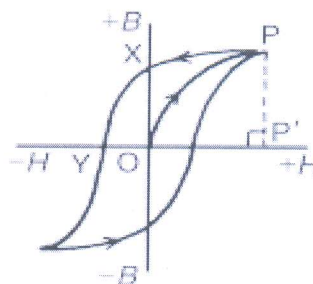
By plotting measured values of flux density B against magnetic field strength H , a **magnetization curve** (or **$B-H$ curve**) is produced. For nonmagnetic materials this is a straight line. Typical curves for four magnetic materials are shown in Figure. The **relative permeability** of a ferromagnetic material is proportional to the slope of the $B-H$ curve and thus varies with the magnetic field strength.

Magnetic Circuits



Hysteresis and hysteresis loss

Hysteresis is the 'lagging' effect of flux density B whenever there are changes in the magnetic field strength H . When an initially unmagnetized ferromagnetic material is subjected to a varying magnetic field strength H , the flux density B produced in the material varies as shown in Figure below, the arrows indicating the direction of the cycle. Figure is known as a **hysteresis loop**.



Figure

Magnetic Circuits

From Figure, distance OX indicates the **residual flux density**, OY indicates the **coercive force**, and PP' is the **saturation flux density**. Hysteresis results in a dissipation of energy which appears as a heating of the magnetic material. **The energy loss associated with hysteresis is proportional to the area of the hysteresis loop.** The area of a hysteresis loop varies with the type of material. The area, and thus the energy loss, is much greater for hard materials than for soft materials. For AC-excited devices the hysteresis loop is repeated every cycle of alternating current. Thus a hysteresis loop with a large area (as with hard steel) is often unsuitable since the energy loss would be considerable. Silicon steel has a narrow hysteresis loop, and thus small hysteresis loss, and is suitable for transformer cores and rotating machine armatures.