# SYNERGY INSTITUTE OF ENGINEERING & TECHNOLOGY
## Department of Electrical Engineering

### LECTURE NOTE (Academic Session 2022-23)

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**Section**          :All          **Name of Subject**      :Basic Electrical Engineering

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# MODULE-I

## D.C NETWORKS

**Kirchoff's Laws:-**

### Kirchoff's current law or point law (KCL)

Statement:- In any electrical network, the algebraic sum of the currents meeting at a point is zero.

$\Sigma I = 0$.............................. at a junction or node

Assumption:- Incoming current = positive

Outgoing current = negative

### Kirchoff's voltage law or mesh law (KVL)

Statement:- The algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the emfs in that path is zero.

$\Sigma IR + \Sigma emf = 0$........................................ round the mesh

Assumption:- i) Rise in voltage (If we go from negative terminal of the battery to positive terminal) = positive

ii) Fall in voltage (If we go from positive terminal of the battery to negative terminal) = negative

iii) If we go through the resistor in the same direction as current then there is a fall in potential. Hence this voltage is taken as negative.

iv)If we go through the resistor against the direction of current then there is a rise in potential. Hence this voltage drop is taken as positive.

**Example**:- Write the loop equation for the given circuit below
(**Supplementary exam 2004**)
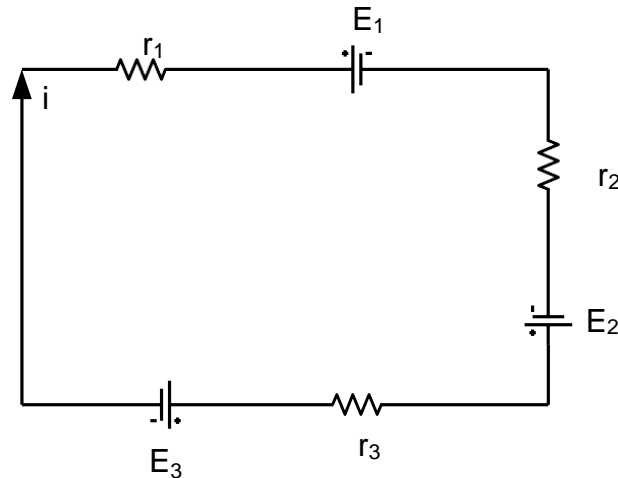


Solution: Apply KVL to the loop,

$$-ir_1 - E_1 - ir_2 + E_2 - ir_3 - E_3 = 0$$

$$\Rightarrow E_1 - E_2 + E_3 = -ir_1 - ir_2 - ir_3$$

$$\Rightarrow E_1 - E_2 + E_3 = -i\left(r_1 + r_2 + r_3\right)$$

## MAXWELL'S LOOP CURRENT METHOD (MESH ANALYSIS)

Statement:- This method determines branch currents and voltages across the elements of a network. The following process is followed in this method:-

- Here, instead of taking branch currents (as in Kirchoff's law) loop currents are taken which are assumed to flow in the clockwise direction.
- Branch currents can be found in terms of loop currents
- Sign conventions for the IR drops and battery emfs are the same as for Kirchoff's law.
- This method is easier if all the sources are given as voltage sources. If there is a current source present in a network then convert it into equivalent voltage source.

**Explanation**:-

Consider a network as shown in Fig. below. It contains two meshes. Let $I_1$ and $I_2$ are the mesh currents of two meshes directed in clockwise.
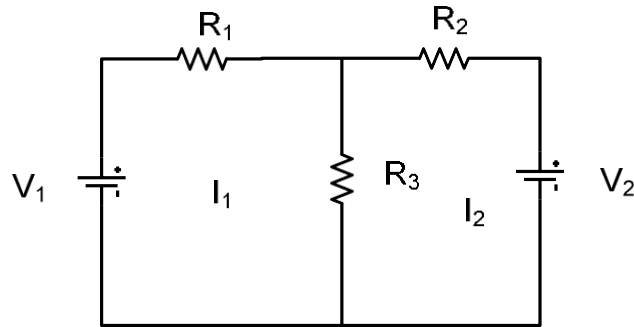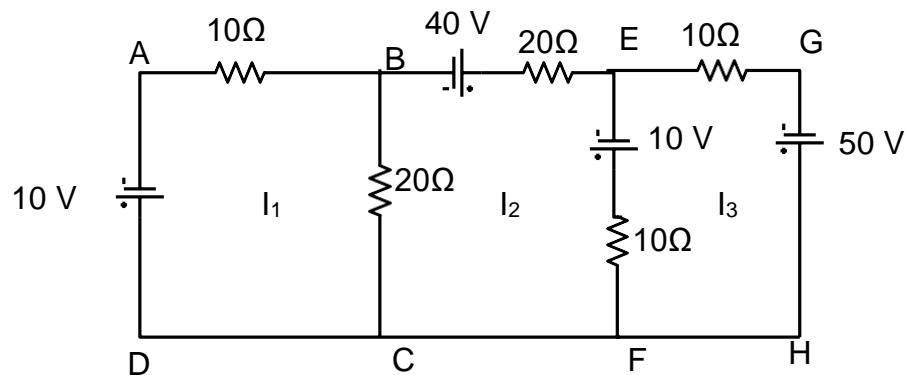


Apply KVL to mesh-1,

$$V_1 - I_1 R_1 - (I_1 - I_2) R_3 = 0$$

Apply KVL to mesh-2,

$$-I_2 R_2 - V_2 - (I_2 - I_1) R_3 = 0$$

When we consider mesh-1, the current $I_1$ is greater than $I_2$. So, current through $R_3$ is $I_1 - I_2$. Similarly, when we consider mesh-2, the current $I_2$ is greater than $I_1$. So, current through $R_3$ is $I_2 - I_1$.

**Example**: Find $I_1$, $I_2$ and $I_3$ in the network shown in Fig below using loop current method



Solution:- For mesh ABCDA,

$$-I_1 \times 10 - (I_1 - I_2) \times 20 - 10 = 0$$

$$\Rightarrow 3I_1 - 2I_2 = -1 \qquad\qquad (1)$$

For mesh BEFCB,

$$40 - I_2 \times 20 + 10 - (I_2 - I_3) \times 10 - (I_2 - I_1) \times 20 = 0$$

$$\Rightarrow 2I_1 - 5I_2 + I_3 = -5 \qquad\qquad (2)$$

For mesh EGHFE,

$$-10I_3 + 50 - (I_3 - I_2) \times 10 - 10 = 0$$

$$\Rightarrow I_2 - 2I_3 = -4 \qquad\qquad (3)$$

Equation (2) x 2 + Equation (3)

$$4I_1 - 9I_2 = -14 \qquad (4)$$

Solving eq$^n$ (1) & eq$^n$ (4)

$$I_1 = 1 \text{ A}, \ I_2 = 2 \text{ A}, \ I_3 = 3 \text{ A}$$

## NODAL ANALYSIS

**Statement**:- This method determines branch currents in the circuit and also voltages at individual nodes.

The following steps are adopted in this method:-

- Identify all the nodes in the network.

- One of these nodes is taken as reference node in at zero potential

- The node voltages are measured w.r.t the reference node

- KCL to find current expression for each node

- This method is easier if all the current sources are present. If any voltage source is present, convert it to current source

- The number of simultaneous equations to be solved becomes (n-1) where 'n' is the number of independent nodes.

**Explanation**:-
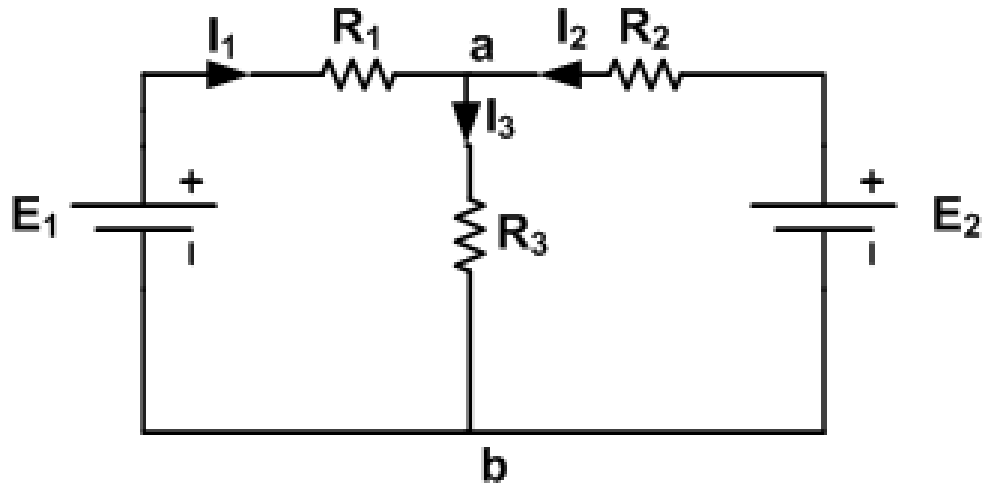


At node 'a'    $I_1+I_2 = I_3$

By ohms law,    $I_1 = \dfrac{E_1 - V_a}{R_1}, I_2 = \dfrac{E_2 - V_a}{R_2}, I_3 = \dfrac{V_a}{R_3}$

Therefore,    $V_a \left[ \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} \right] - \dfrac{E_1}{R_1} - \dfrac{E_2}{R_2} = 0$

or,    $V_a \left[ \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} \right] - \dfrac{E1}{R_1} - \dfrac{E2}{R_2} = 0$

or,    $V_a \left[ \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} \right] - \dfrac{E1}{R_1} - \dfrac{E2}{R_2} = 0$

Hence,

- Node voltage multiplied by sum of all the conductance connected to this node. This term is positive

- The node voltage at the other end of each branch (connected to this node multiplied by conductance of this branch). This term is negative.
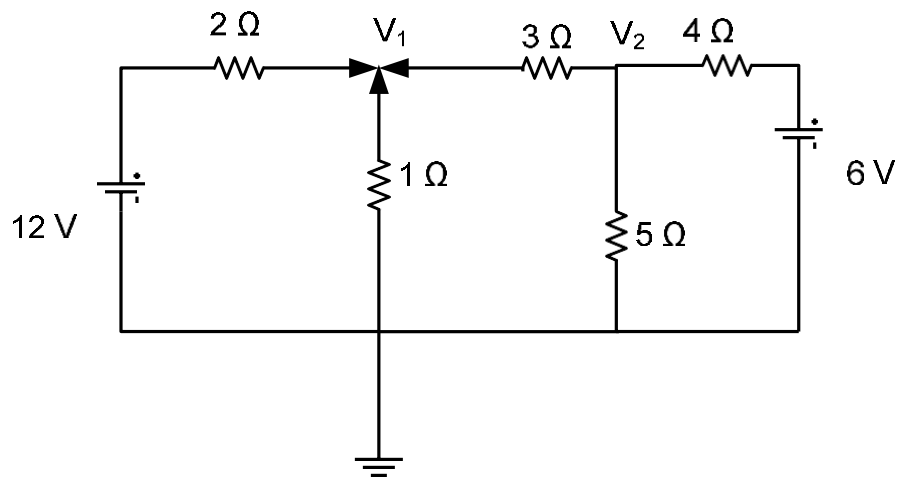
**Example**:- Use nodal analysis to find currents in the different branches of the circuit shown below.

(Supplementary Exam July- 2004)
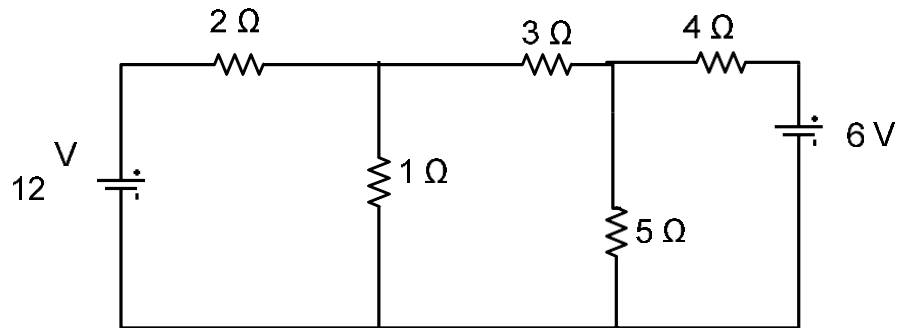


Solution:-

Let $V_1$ and $V_2$ are the voltages of two nodes as shown in Fig below



Applying KCL to node-1, we get

$$\frac{12 - V_1}{2} + \frac{0 - V_1}{1} + \frac{V_2 - V_1}{3} = 0$$
$$\Rightarrow 36 - 3V_1 - 6V_1 + 2V_2 - 2V_1 = 0$$
$$\Rightarrow -11V_1 + 2V_2 = 36...............(1)$$

Again applying KCL to node-2, we get:-

$$\frac{V_1 - V_2}{3} + \frac{0 - V_2}{5} + \frac{6 - V_2}{4} = 0$$

$$\Rightarrow 20V_1 - 47V_2 + 90 = 0$$

$$\Rightarrow 20V_1 - 47V_2 = -90 \ldots \ldots \ldots \ldots (2)$$

Solving Eq (1) and (2) we get $V_1 = 3.924$ Volt and $V_2 = 3.584$ volt

Current through 2 Λ resistance $= \dfrac{12 - V_1}{2} = \dfrac{12 - 3.924}{2} = 4.038$ A

Current through 1 Λ resistance $= \dfrac{0 - V_1}{1} = -3.924$ A

Current through 3 Λ resistance $= \dfrac{V_1 - V_2}{3} = 0.1133$ A

Current through 5 Λ resistance $= \dfrac{0 - V_2}{5} = -0.7168$ A

Current through 4 Λ resistance $= \dfrac{6 - V_2}{4} = 0.604$ A

As currents through 1Λ and 5Λ are negative, so actually their directions are opposite to the assumptions.

## STAR-DELTA CONVERSION

**Need**:- Complicated networks can be simplified by successively replacing delta mesh to star equivalent system and vice-versa.

In delta network, three resistors are connected in delta fashion (Δ) and in star network three resistors are connected in wye (Y) fashion.
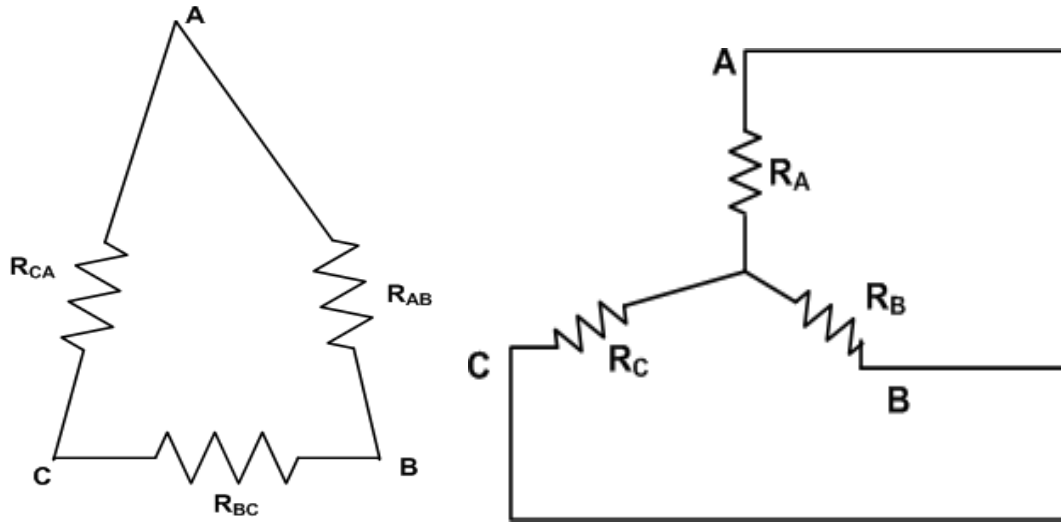
Fig. 1.4.1.

a) Delta connection                    b) Star connection

**Delta to Star Conversion**:- From Fig. 1.4.1 (a), Δ : Between A & B, there are two parallel path.

Resistance between terminal A & B = $\dfrac{R_{AB}\left(R_{BC}+R_{CA}\right)}{R_{AB}+R_{BC}+R_{CA}}$

From Fig. 1.4.1 (b), STAR: Between A & B two series resistances are there $R_A + R_B$. So, terminal resistances have to be the same.

$$R_A + R_B = \frac{R_{AB}\left(R_{BC}+R_{CA}\right)}{R_{AB}+R_{BC}+R_{CA}}\dots\dots\dots(1)$$

$$R_B + R_C = \frac{R_{BC}\left(R_{CA}+R_{AB}\right)}{R_{AB}+R_{BC}+R_{CA}}\dots\dots\dots(2)$$

$$R_C + R_A = \frac{R_{CA}\left(R_{AB}+R_{BC}\right)}{R_{AB}+R_{BC}+R_{CA}}\dots\dots\dots(3)$$

Eq {(1)-(2)}+(3) & Solving,-

$$R_A = \frac{R_{AB} \times R_{CA}}{R_{AB}+R_{BC}+R_{CA}}\dots\dots\dots(4)$$

$$R_B = \frac{R_{AB} \times R_{BC}}{R_{AB}+R_{BC}+R_{CA}}\dots\dots\dots(5)$$

$$R_C = \frac{R_{CA} \times R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \dots\dots\dots\dots(6)$$

**Easy way to remember**:-

$$Any\ arm\ of\ star\ connection = \frac{\text{Pr}\ oduct\ of\ two\ adjacent\ arms\ of\ delta}{sum\ of\ arms\ of\ delta}$$

## **Star to Delta conversion**

Eq {(1) X (2)}+(2) X (3)+ (3) X (1) & Simplifying,-

$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B}$$

**Easy way to remember**:- *Resistance between two terminals of delta = sum of star resistance connected to those terminals + product of the same to resistance divided by the third resistance.*

**Example(delta to star):-** Convert the following Delta Resistive Network into an equivalent Star Network.
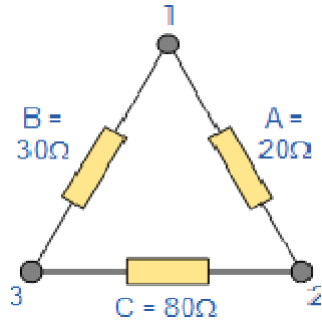


$$Q = \frac{AC}{A+B+C} = \frac{20 \times 80}{130} = 12.31\Omega$$

$$P = \frac{AB}{A+B+C} = \frac{20 \times 30}{130} = 4.61\Omega$$

$$R = \frac{BC}{A+B+C} = \frac{30 \times 80}{130} = 18.46\Omega$$

## NETWORK THEOREMS

- SUPERPOSITION THEOREM
- THEVENIN'S THEOREM
- NORTON'S THEOREM
- MAXIMUM POWER TRANSFER THEOREM

1. **Superposition theorem**

   Statement:- In a network of linear resistances containing more than one generator (or source of emf), the current which flows at any point is the sum of all the currents which would flow at that point if each generator were considered separately and all the other generators replaced for the time being by resistances equal to their internal resistance.

**Example**:- By means of superposition theorem, calculate the currents in the network shown.
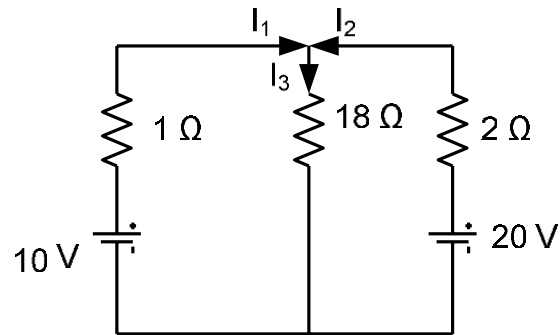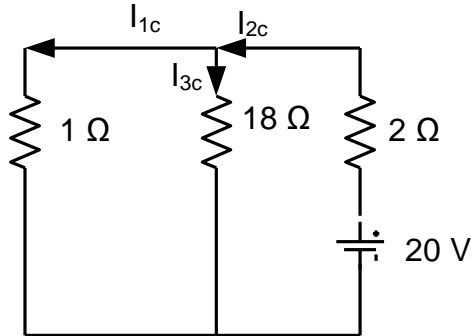


Step 1. <u>Considering 10 V battery</u>



$$R_{eq} = \frac{2 \times 18}{2 + 18} + 1 = 2.8 \, \Lambda$$
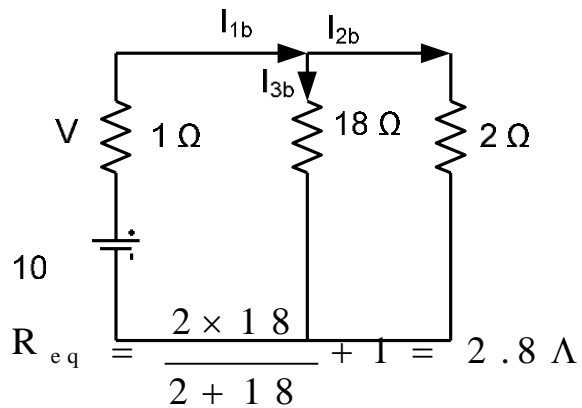
$$I_{1b} = \frac{10}{2.8} = 3.57 \text{ A}$$

$$I_{2b} = 3.57 \times \frac{18}{20} = 3.21 \text{ A}$$

$$I_{3b} = I_{1b} - I_{2b} = 0.36 \text{ A}$$

Step 2. Considering 20 V battery



$$R_{eq} = \frac{1 \times 18}{1 + 18} + 2 = 2.95 \, \Lambda$$

$$I_{2c} = \frac{20}{2.95} = 6.78 \, A$$

$$I_{1c} = 6.78 \times \frac{18}{19} = 6.42 \, A$$

$$I_{3b} = I_{2c} - I_{1c} = 0.36 \, A$$

Step 3. Results

$$I_1 = I_{1b} - I_{1c} = 3.57 - 6.42 = -2.85A$$
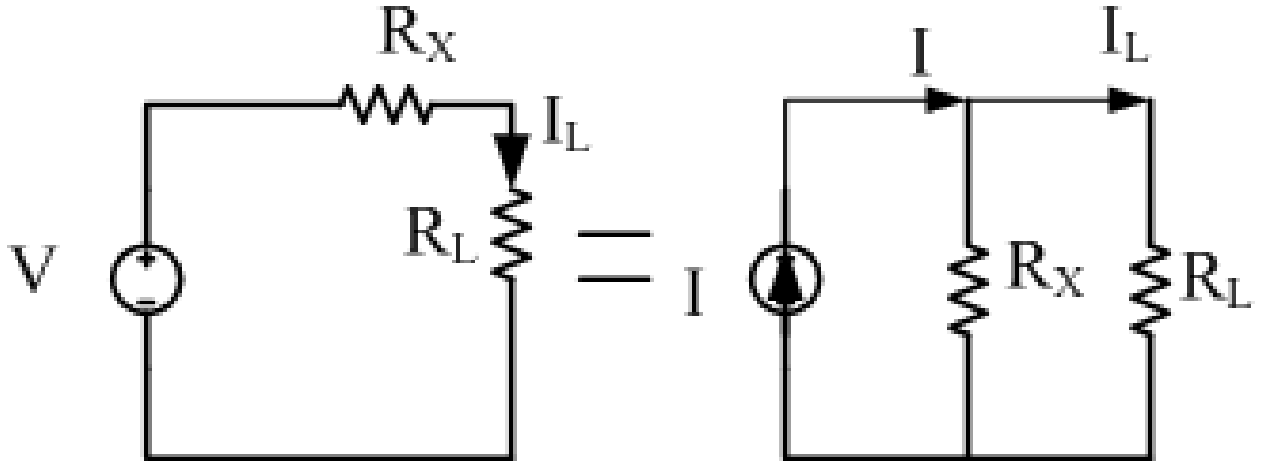$$I_2 = I_{2c} - I_{2b} = 6.78 - 3.21 = 3.57A$$
$$I_3 = I_{3b} + I_{3c} = 0.36 + 0.36 = 0.72A$$

2. **SOURCE CONVERSION**:-
   **Statement**: A voltage source (V) with a series resistance (R) can be converted to a current source (I=V/R) with a parallel resistance (R) and vice-versa.

**Proof**:-



$$I_L = \frac{V}{R_X + R_L} \quad (1)$$

$$I_L = I \frac{R_X}{R_X + R_L} \quad (2)$$

From Eq. (1) & (2)

$$V = IR_X \quad (3)$$

- **STATEMENT**: The two circuits are said to be electrically equivalent if they supply equal load currents with the same resistance connected across their terminals.

- voltage source having a voltage V and source resistance Rx can be replaced by I(= V/Rx) and a source resistance Rx in parallel with current source.

- Current source I and source resistance Rx can be replaced by a voltage source V (=IRx) and a source resistance Rx in series with V.

3. **THEVENIN'S THEOREM**:-
   **Statement**:- Any pair of terminals *AB* of a linear active network may be replaced by an equivalent voltage source in series with an equivalent

resistance $R_{th}$. The value of $V_{th}$ (called the Thevenin's voltage) is equal to potential difference between the terminals *AB* when they are open circuited, and $R_{th}$ is the equivalent resistance looking into the network at *AB* with the independent active sources set to zero i.e with all the independent voltage sources short-circuited and all the independent current sources open-circuited.

Example:- Find $V_X$ by first finding $V_{TH}$ and $R_{TH}$ to the left of A-B
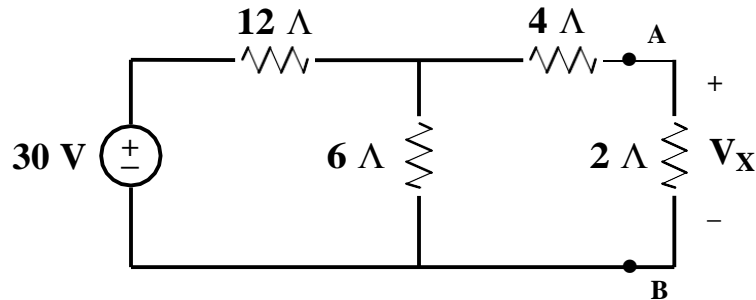


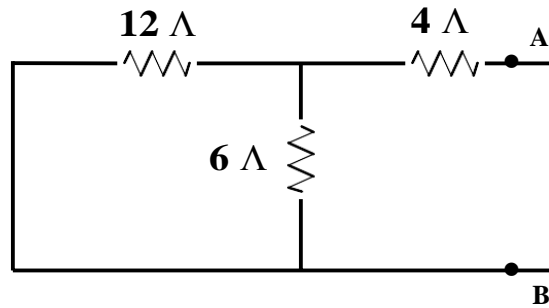Solution:- step1. First remove everything to the right of A-B.
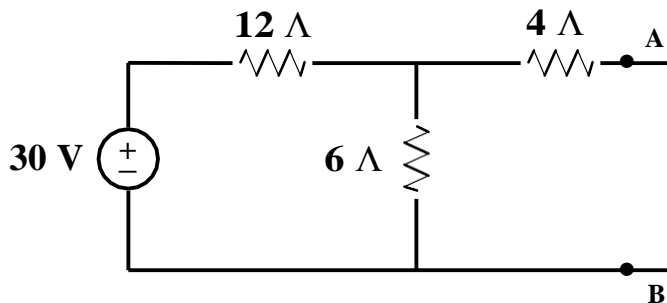


$$V_{AB} = \frac{(30)(6)}{6 + 12} = 10\,V$$

Notice that there is no current flowing in the 4 Λ resistor (A-B) is open. Thus there can be no voltage across the resistor.
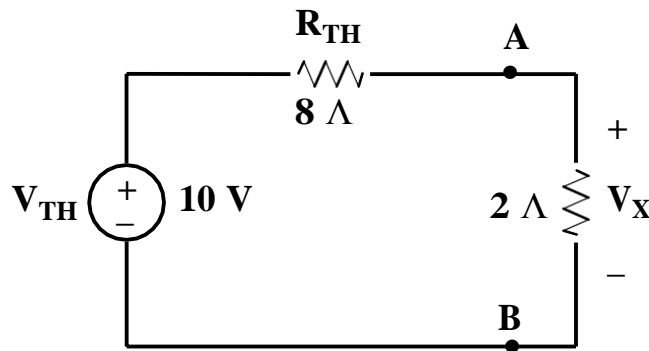
Step 2. To find $R_{th}$

We now deactivate the sources to the left of A-B and find the resistance seen looking in these terminals.

$$R_{TH} = 12\|6 + 4 = 8\ \Lambda$$

Step 3. To find $V_x$

After having found the Thevenin circuit, we connect this to the load in order to find $V_X$.



$$V_X = \frac{(10)(2)}{2+8} = 2V$$

4. **NORTON'S THEOREM**:
   **Statement**: Any two terminal linear active network (containing independent voltage and current sources), may be replaced by a constant current source $I_N$ in parallel with a resistance $R_N$, where $I_N$ is the current flowing through a short circuit placed across the terminals and $R_N$ is the equivalent resistance of the network as seen from the two terminals with all sources replaced by their internal resistance.

**Example**: Find the Norton equivalent circuit to the left of terminals A-B for the network shown below. Connect the Norton equivalent circuit to the load and find the current in the 50 Λ resistor.

10 A

20 Λ   40 Λ   A

+ 50 V   60 Λ   50 Λ
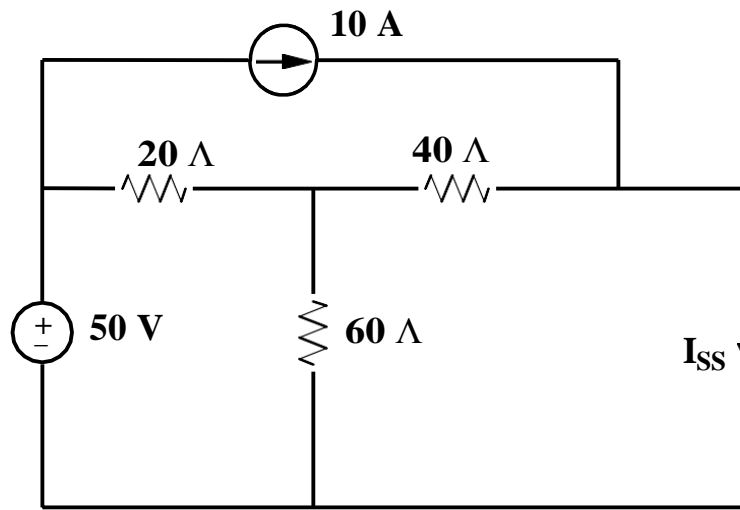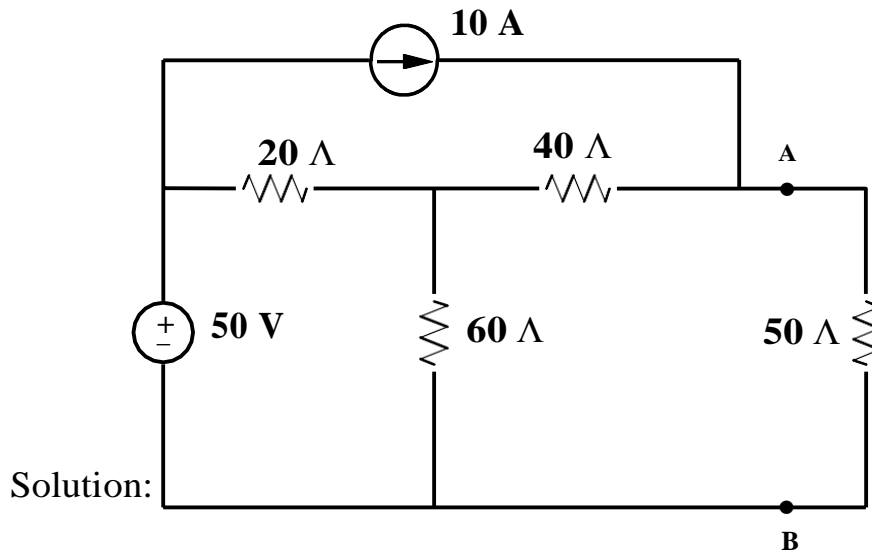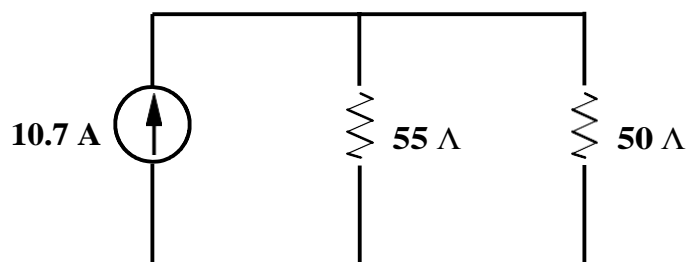
Solution:

B

10 A

20 Λ   40 Λ

+ 50 V   60 Λ   $I_{SS}$

Fig. Circuit to find $I_{NORTON}$

$$I_{SS} = 10.7\,A$$

It can also be shown that by deactivating the sources, We find the resistance looking into terminals A-B is $R_N = 55\,Λ$

$R_N$ and $R_{TH}$ will always be the same value for a given circuit. The Norton equivalent circuit tied to the load is shown below.

10.7 A   55 Λ   50 Λ

## 5. <u>__MAXIMUM POWER TRANSFER THEOREM__</u>:

- **Statement**: For any power source, the maximum power transferred from the power source to the load is when the resistance of the load $R_L$ is equal to the equivalent or input resistance of the power source ($R_{in} = R_{Th}$ or $R_N$).
- The process used to make $R_L = R_{in}$ is called impedance matching.

**Explanation**:
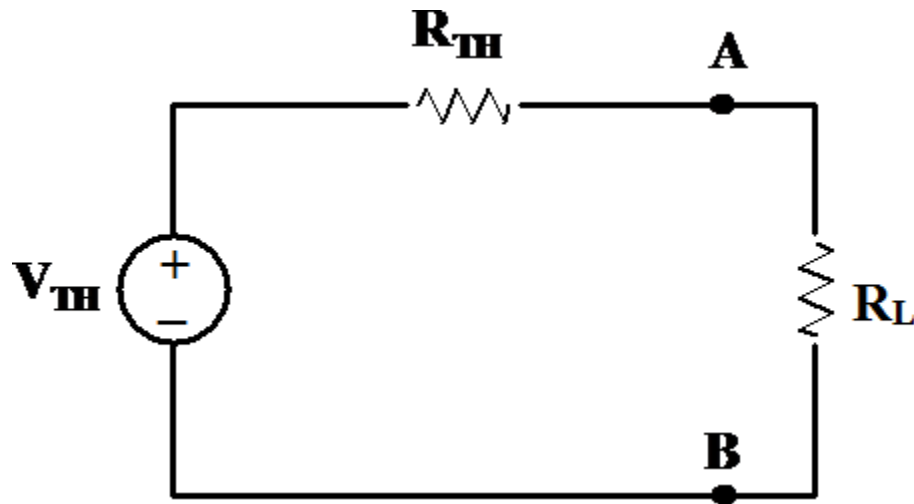


$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

$$P_L = I^2 R_L = \frac{V_{TH}^2 R_L}{(R_{TH} + R_L)^2}$$

For $P_L$ to be maximum, $\frac{dP_L}{dR_L} = 0$

Or, $R_L = R_{TH}$

So, Maximum power drawn by $R_L = I^2 R_L = \frac{V_{TH}^2 R_L}{(2R_L)^2} = \frac{V_{TH}^2}{4R_L}$

Power supplied by the source $= \frac{V_{TH}^2}{(R_{TH} + R_L)}$

# TRANSIENTS

**Statement**: Sudden change in electrical circuit.

- Amplitude dies out and frequency is more
- Transient disturbances are produced whenever:-
  - An apparatus or circuit is suddenly connected to or disconnected from the supply.
  - A circuit is shorted
  - There is a sudden change in the applied voltage from one finite value to another.
  - *Transients are produced whenever different circuits are suddenly switched on or off from the supply voltage.*

  Resultant current consists of two parts:-

- A final steady state or normal current

- A transient current superimposed on the steady state current

  - Transient currents are not driven by any part of the applied voltage but are entirely associated with changes in the stored energy in inductors and capacitors.
  - Since there is no stored energy in resistors, there are no transients in pure resistive circuit.

  ## Transient in R-L Series circuit:-
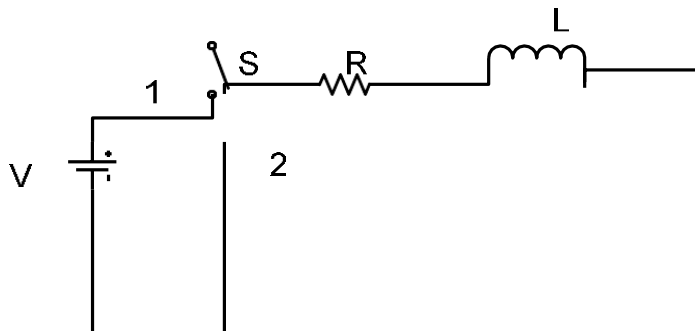
  

  When Switch 'S' is connected to '1',

$$V = R\,i + L\,\frac{d\,i}{d\,t}$$

$$i = i_{ss} + i_{tr}$$

$$i_{ss} = \frac{V}{R}$$

$$i_{tr} = R\,i + L\,\frac{d\,i}{d\,t} = 0$$

$$\frac{d\,i}{d\,t} + \frac{R}{L}\,i = 0$$

$$\frac{d\,i}{d\,t} = \frac{-R}{L}\,i$$

$$\frac{d\,i}{i} = \frac{-R}{L}\,d\,t$$

$$\ln i = \frac{-R}{L}\,t\,; i_{tr} = K\,e^{\frac{-R}{L}t}$$

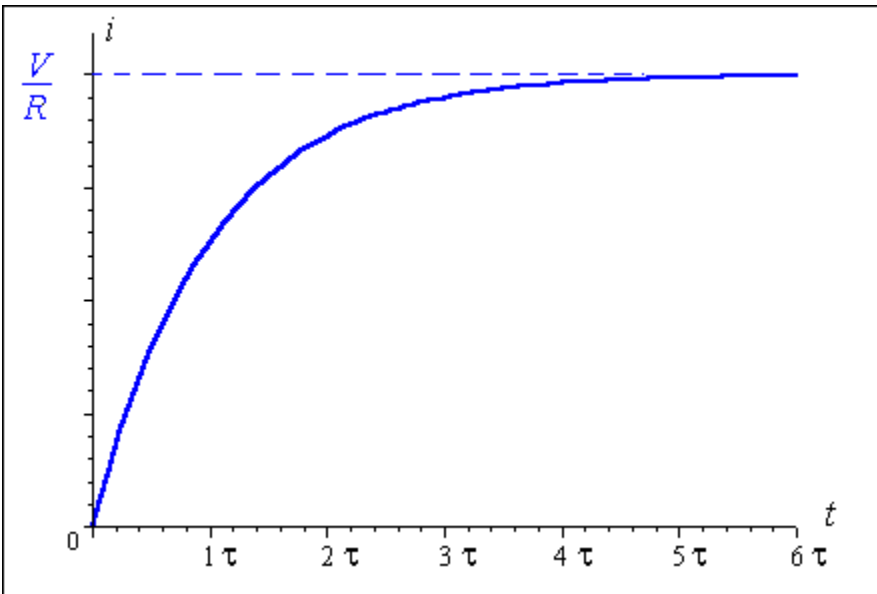$$i_{ss} = \frac{V}{R}$$

$$i_{tr} = K\,e^{\frac{-R}{L}t}$$

$$i = \frac{V}{R} + K\,e^{\frac{-R}{L}t}$$

At t=0, i=0 So,

$$0 = \frac{V}{R} + K$$

$$K = \frac{-V}{R}$$

$$i = \frac{V}{R}\left(1 - e^{\frac{-R}{L}t}\right)$$

$\lambda = \dfrac{L}{R}$ is called time constant and $\dfrac{R}{L}$ is called damping coefficient of the circuit

$$V_R = iR = V\left(1 - e^{\frac{-t}{\lambda}}\right)$$

Emf of self inductance is $-L\dfrac{di}{dt} = i\,R_t$

If $t = \lambda$, then $i_t = I_0\, e^{-1} = I_0 e^{-1} = I_0/e = I_0/2.718 = 0.37\, I_0$

Hence, time period of a circuit is the time during which the transient current decrease to 0.37 of its initial value.

**Transient in R-C  Series Circuit**:

Consider an ac circuit containing a resistor of resistance R ohms and a capacitor of capacitance C farad across an a.c source of rms voltage V volts as shown in Fig. below:-
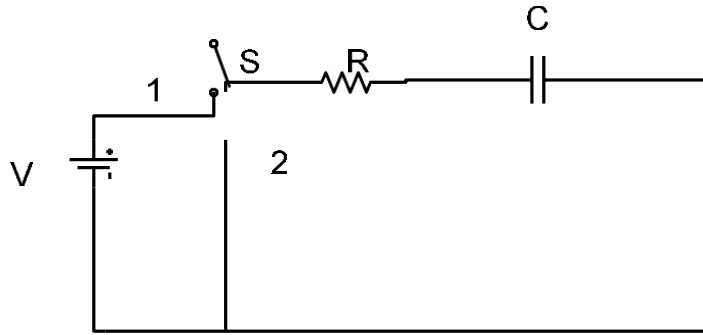
## Charging of RC

$V = V_R + V_C$

When switch is connected to '1' (charging):-

$V = V_R + V_c$

$V = iR + \dfrac{1}{C} \int i \, dt$

$R \dfrac{di}{dt} + \dfrac{i}{C} = 0$

$\dfrac{di}{dt} + \dfrac{1}{RC} i = 0$

$i = K \, e^{\frac{-t}{RC}}$

At $\quad t = 0^+ \; ; i = I_0 \, e^{\frac{-t}{\tau}}$

$K = \dfrac{V}{R} \; ; \; So, \; i = \dfrac{V}{R} \, e^{\frac{-t}{RC}}$

$V_R = iR = I_0 \, e^{\frac{-t}{\tau}} R = V \, e^{\frac{-t}{\tau}}$

$V_c = \dfrac{1}{C} \int i \, dt = \dfrac{1}{C} \int\limits_0^t I_0 \, e^{\frac{-t}{\tau}}$

$V_c = \dfrac{1}{C} I_0 \, (-\tau) \left[ \dfrac{-t}{\tau} \right]_0^t = \dfrac{1}{C} \dfrac{V}{R} (-RC)$

$V_c = -V \left( e^{\frac{-t}{\tau}} - e_0 \right)$

$V_c = V \left( 1 - e^{\frac{-t}{\tau}} \right)$
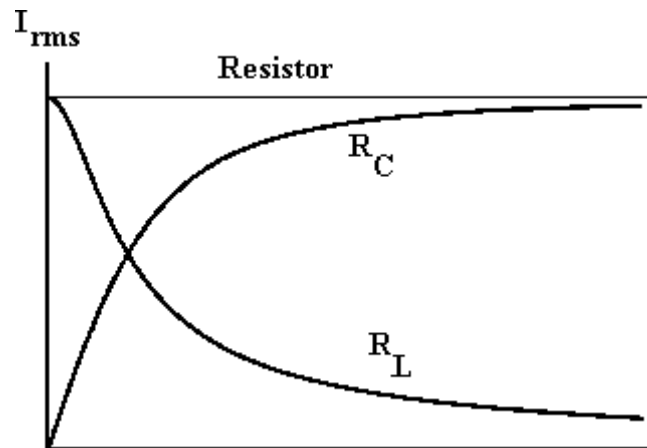
## Discharging of RC

When connected to '2' in the Fig. above,

$$R\,i + \frac{1}{C}\int i\,d\,t = 0$$

$$R\,\frac{d\,i}{d\,t} + \frac{i}{C} = 0$$

$$i = K\,e^{\frac{-t}{RC}}$$

$$At\,t = 0; i = \frac{-V}{R} \quad (\text{voltage across capacitor starts discharging in opposite direction to the original current direction})$$

$$i = -I_0\,e^{\frac{-t}{RC}} = -I_0\,e^{\frac{-t}{\tau}}$$

## Transient in R-L-C Series Circuit



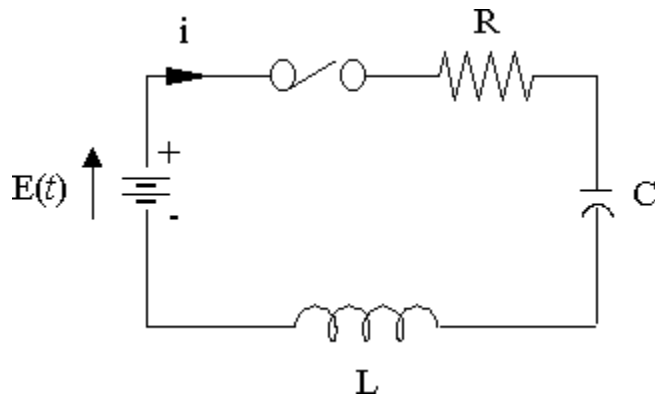- Two types of energy:- Electromagnetic and electrostatic. So any sudden change in the conditions of the circuit involves redistribution of these two energies.
- Transient current produced due to this redistribution may be unidirectional and decaying oscillatory.

From the above Fig,

$$iR + L\frac{di}{dt} + \frac{1}{C}\int id\,t = V$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

$$\alpha \pm \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha \pm \beta = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$\alpha \pm \beta = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\alpha = \frac{-R}{2L} \quad and \quad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Solution of differential equation is:-

$$i = K_1 e^{P1t} + K_2 e^{P2t}$$

Roots are: $\alpha + \beta = P_1 ; \alpha - \beta = P_2$

$K_1 \& K_2$ depends on boundary condition

Case 1: High loss circuit:- $\left(\dfrac{R}{2L}\right)^2 > \dfrac{1}{LC}$ i.e Overdamped

In this case, $\beta$ is positive real quantity. Hence $P_1$ and $P_2$ are real but unequal.

$$i = K_1 e^{P1\,t} + K_2 e^{P2\,t}$$
$$i = K_1 e^{(\alpha + \beta\,)t} + K_2 e^{(\alpha - \beta\,)t}$$
$$i = K_1 e^{\alpha\,t} e^{\beta\,t} + K_2 e^{\alpha\,t} e^{-\beta\,t}$$
$$i = e^{\alpha\,t} \left[ K_1 e^{\beta\,t} + K_2 e^{-\beta\,t} \right]$$

The expression of 'i' is over damped transient non-oscillatory current.

CASE 2:- Low-loss circuit: $\left(\dfrac{R}{2L}\right)^2 < \dfrac{1}{LC}$ i.e Underdamped

In this case, $\beta$ is imaginary. Hence roots are complex conjugate

$$P_1 = \alpha + j\beta ; P_2 = \alpha - j\beta$$
$$i = K_1 e^{P1t} + K_2 e^{P2t}$$
$$i = K_1 e^{(\alpha + j\beta\,)t} + K_2 e^{(\alpha - j\beta\,)t}$$
$$i = K_1 e^{\alpha\,t} e^{j\beta\,t} + K_2 e^{\alpha\,t} e^{-j\beta\,t}$$
$$i = e^{\alpha\,t} \left[ K_1 e^{j\beta\,t} + K_2 e^{-j\beta\,t} \right]$$

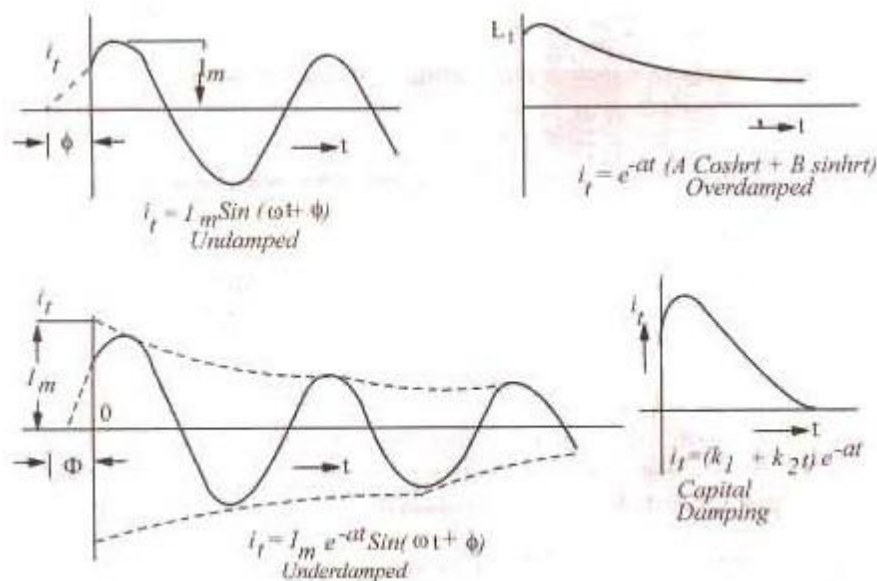The expression of 'i' is damped oscillatory

CASE 3: $\left(\dfrac{R}{2L}\right)^2 = \dfrac{1}{LC}$ i.e Critical damping

In this case $\beta = 0$, Hence roots $P_1 \& P_2$ are real and equal.

$P_1 = \alpha + 0 = \alpha \; ; P_2 = \alpha - 0 = \alpha$

$i = K_1 e^{\alpha t} + K_2 t e^{\alpha t}$

 The above expression is of critical damping because current is reduced to almost zero in the shortest possible time.



$i_t = I_m Sin(\omega t + \phi)$
Undamped

$i_t = e^{-at}(A\,Coshrt + B\,sinhrt)$
Overdamped

$i_t = I_m\, e^{-at} Sin(\omega t + \phi)$
Underdamped

$i_t = (k_1 + k_2 t)e^{-at}$
Capital Damping

**Example**: A coil having a resistance of 2Λ and an inductance of 1 H is switched on to a 10 V D.C supply. Write down the expression of current i(t) in the coil as a function of time

Ans: R = 2Λ, L=1 H, V=10 V

Time constant $(\tau)$ = L/R=1/2=0.5 sec

Steady current = V/R = 10/2= 5 A

$$i(t) = \frac{V}{R}\left(1 - e^{\frac{-t}{\tau}}\right)$$

$$i(t) = 5\left(1 - e^{\frac{-t}{0.5}}\right)\,A$$
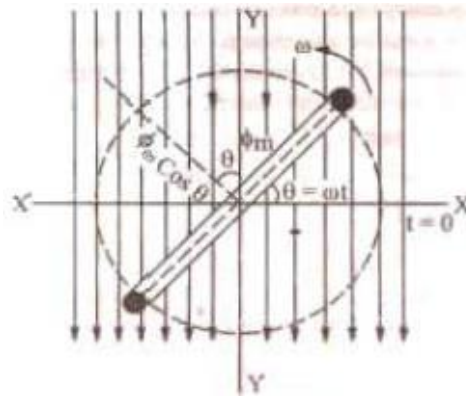
# SINGLE PHASE A.C CIRCUIT

## Single phase EMF generation:

Alternating voltage may be generated

1) By rotating a coil in a magnetic field
2) By rotating a magnetic field within a stationary coil

The value of voltage generated depends upon
1) No. of turns in the coil  2) field strength  3) speed

## Equation of alternating voltage and current



N= No. of turns in a coil

$\Phi_m$= Maximum flux when coil coincides with X-axis

$\omega$= angular speed (rad/sec) $= 2\pi f$

At $\theta=\omega t$,      $\Phi$= flux component $\perp$ to the plane $=\Phi_m \cos \omega t$

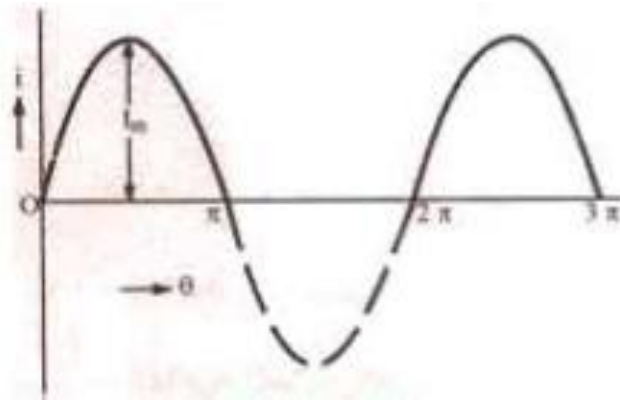According to the Faraday's law of electromagnetic induction,

$$e = -N \frac{d\varphi}{dt} = -N \frac{d}{dt} \varphi_m \cos\omega t = \omega N \varphi_m \sin\omega t \ \ldots\ldots\ldots \ (1)$$

Now, e is maximum value of $E_m$ , when $\sin\theta = \sin 90° = 1$.

i.e $E_m = \omega N \Phi_m$ .......................................................... (2)

From Eq$^n$ (1) & (2), e = $E_m$ Sin ωt    volt

Now, current (i) at any time in the coil is proportional to the induced emf (e) in the coil. Hence, i = $I_m$ Sin ωt amp



## A.C terms:

- Cycle:- A complete set of positive and negative values of an alternating quantity is known as cycle.
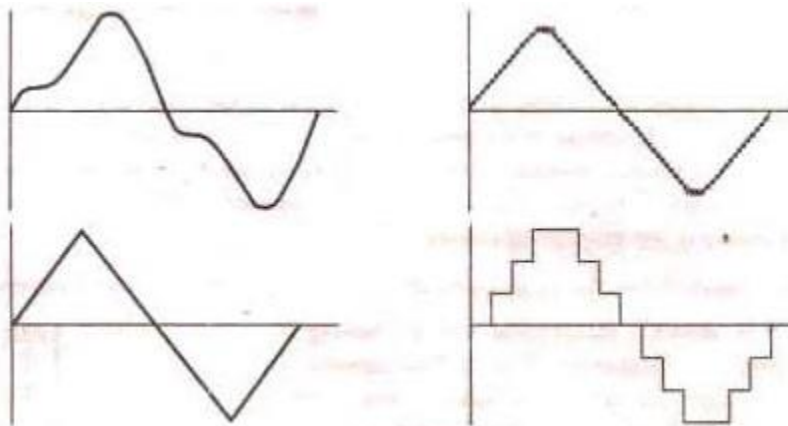


- Time period: The time taken by an alternating quantity to complete one cycle is called time T.
- Frequency: It is the number of cycles that occur in one second. f = 1/T
  f = PN/120 where, P= No. of poles, N= Speed in rpm
- Waveform: A curve which shows the variation of voltage and current w.r.t time or rotation.

- Phase & Phase difference:



$$e_A = E_{mA} \sin \omega t$$
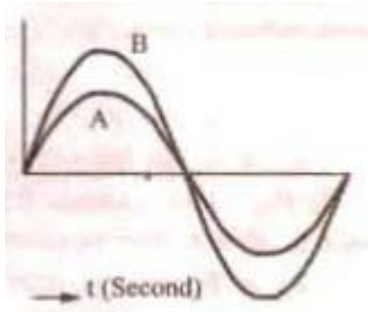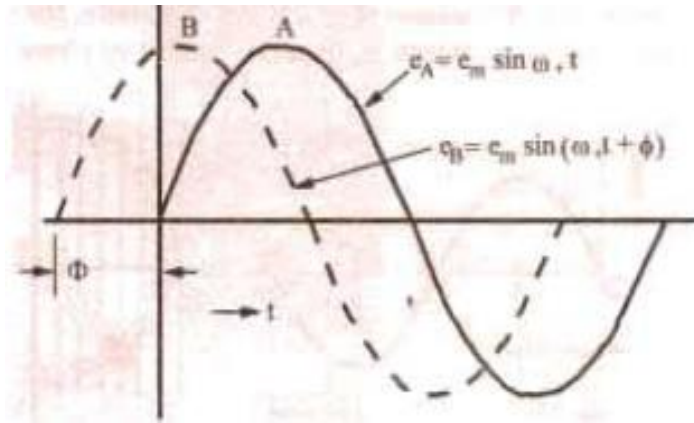
<u>In phase:</u> $e_B = E_{mB} \sin \omega t$

<u>Out of phase:</u> i) B leads A



$$e_A = e_m \sin \omega_+ t$$
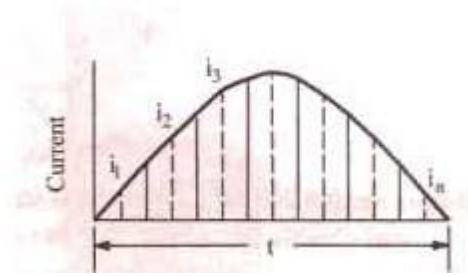$$e_B = e_m \sin(\omega_+ t + \phi)$$

$$e_A = E_m \sin \omega t$$

Phase difference Φ. $e_B = E_{mB} \sin(\omega t + \alpha)$

ii) A leads B or B lags A

$$e_A = E_m \sin \omega t$$
$$e_B = E_m \sin(\omega t - \alpha)$$

## Root mean Square (RMS) or effective or virtual value of A.C:-



$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + \dots + i_n^2}{n}} = \text{Square root of the mean of square of the instantaneous currents}$$

- It is the square root of the average values of square of the alternating quantity over a time period.

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2 (\omega t) d(\omega t)}$$

## Average Value (or mean value):

- It is the arithmetic sum of all the instantaneous values divided by the number of values used to obtain the sum

$$I_{av} = \frac{i_1 + i_2 + \dots + i_n}{n}$$

$$I_{av} = \frac{1}{T} \int_0^T i(\omega t) d(\omega t)$$

**Form factor ($K_f$):-** is the ratio of rms value to average value of an alternating quantity. ($K_f = I_{rms}/I_{av}$)

**Peak factor ($K_a$) or crest factor:-** is the ratio of peak (or maximum) value to the rms value of alternating quantity ($Ka = I_{max}/I_{rms}$)

**Example**: An alternating current varying sinusoidally with a frequency of 50 Hz has an RMS value of 20 A. Write down the equation for the instantaneous value and find this value a) 0.0025 sec b) 0.0125 sec after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A?

Ans:
$$I_m = 20\sqrt{2} = 28.2\ A$$
$$\omega = 2\pi \times 50 = 100\pi\ rad/s$$

The equation of the sinusoidal current wave with reference to point O as zero time point is

i= 28.2 sin 100πt Ampere

Since time values are given from point A where voltage has positive and maximum value, the equation may itself be referred to point A. In this case, equation becomes

i= 28.2 cos 100πt
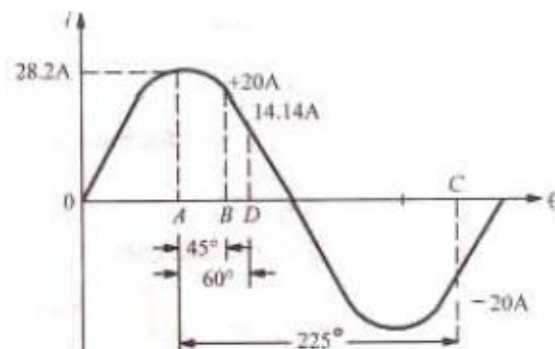


i)      When t= 0.0025 second
         i = 28.2 cos 100π X 0.0025 ................angle in radian
          = 28.2 cos 100 X 180 X 0.0025................angle in degrees
          = 28.2 cos 45° = 20 A .............................point B

ii)     When t = 0.0125 sec
         I =28.2 cos 100 X 180 X 0.0125
          = 28.2 cos 225° = 28.2 X (-1/√2)
          = -20 A........................................ point C

iii)    Here  i = 14.14 A

$14.14 = 28.2 \cos 100 \times 180\, t$

$\cos 100 \times 180\, t = \frac{1}{2}$

Or, $100 \times 180\, t = \cos^{-1}(1/2) = 60°$, t=1/300 sec.................. point D

## Phasor & Phasor diagram:
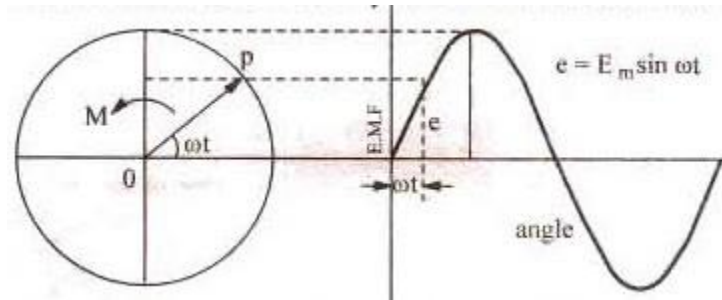


**Phasor**: Alternating quantities are vector (i.e having both magnitude and direction). Their instantaneous values are continuously changing so that they are represented by a rotating vector (or phasor). A phasor is a vector rotating at a constant angular velocity

**Phasor diagram**: is one in which different alternating quantities of the same frequency are represented by phasors with their correct phase relationship
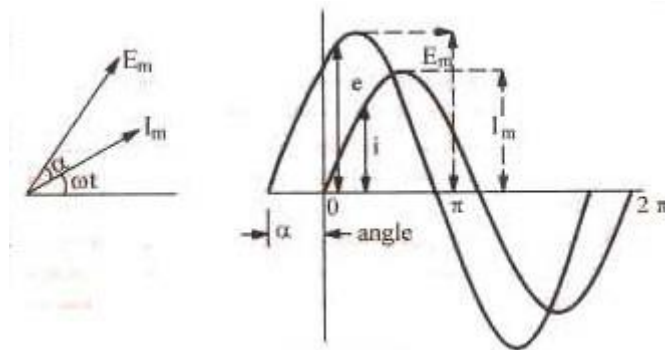


**Points to remember:**
1. The angle between two phasors is the phase difference
2. Reference phasor is drawn horizontally
3. Phasors are drawn to represent rms values
4. Phasors are assumed to rotate in anticlockwise direction
5. Phasor diagram represents a "still position" of the phasors in one particular point

## A.C through pure ohmic resistance only
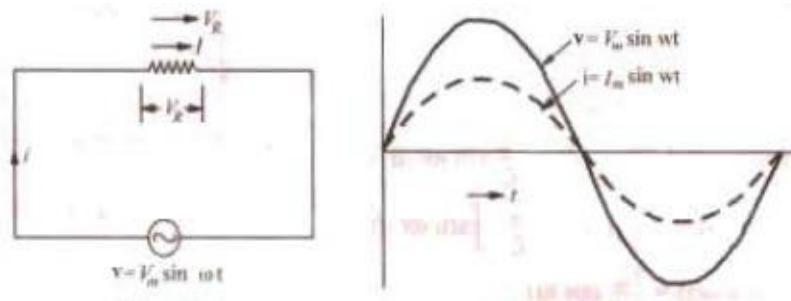


$$v = iR \text{ or } i = \frac{v}{R} = \frac{V_m}{R} \sin\omega t \ \left(\text{in phase}\right)$$

## A.C through pure inductance only

$$v = L$$
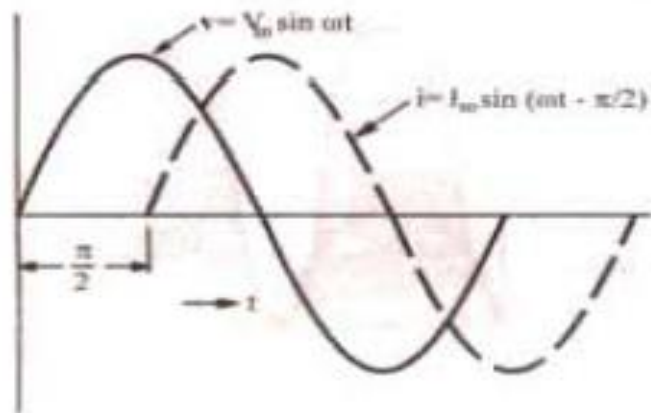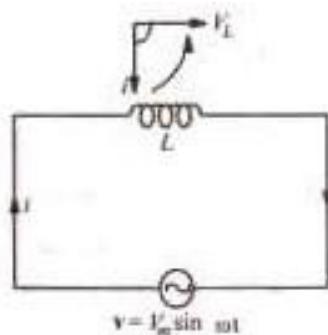


$$\frac{di}{dt} = V_m \sin\omega t$$

$$i = \frac{V_m}{L} \int \sin\omega t$$

$$i = -\frac{V_m}{\omega L} \cos\omega t$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \quad (\text{current lags by } 90^{\circ})$$

$$\omega L = 2\pi f L = X_L = \text{inductive reactance(in } \Omega)$$

## A.C through pure Capacitance only
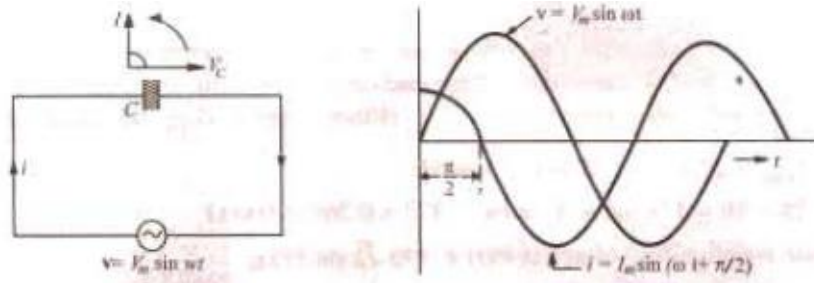


$$i = C\frac{dv}{dt} = C\frac{d}{dt}(V_m \sin \omega t)$$

$$= \omega C V_m \cos \omega t$$

$$= \omega C V_m \sin\left(\omega t + \frac{\pi}{2}\right) = \frac{V_m}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= I_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad (current\ leads\ by\ 90°)$$

$$\frac{1}{\omega C} = X_C = \frac{1}{2\pi fC} = capacitive\ reactance\ (in\ \Lambda)$$

**'j' operator:** j is a operator which rotates a vector by 90° in anticlockwise direction

$$j^2 = -1 \;;j = \sqrt{-1}$$

Note: 'i' is used for current hence 'j' is used to avoid confusion

**Mathematical representation of vectors:**

1. Rectangular or Cartesian form :- $\vec{V} = a \pm jb$

2. Polar form : $\vec{V} = V\angle\pm\theta$

3. Trignometrical form : $\vec{V} = V(\cos\theta \pm j\sin\theta)$

4. Exponential form : $\vec{V} = V e^{\pm j\theta}$

**Note**: rectangular form is best suited for addition and subtraction & polar form is best suited for multiplication and division

**IMPEDANCE:**

In quantitative terms, it is the complex ratio of the voltage to the current in an alternating current (AC) circuit. Impedance extends the concept of resistance to AC circuits, and possesses both magnitude and phase, unlike resistance, which has only n agnitude. When a circuit is driven with direct current (DC), there is no distinction between impedance and resistance; the latter can be thought of as impedance with zero phase angle.
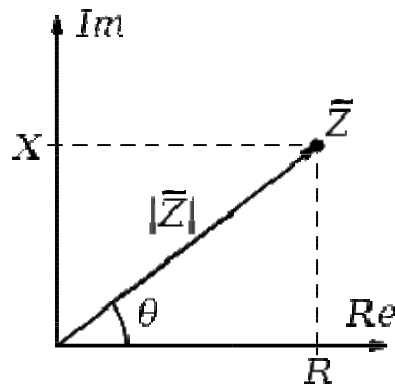


Where X=Total reactance of the network (Both inductive and capacitive)

R=Resistance of the ₁etwork in ohm.

$\theta$= Phasor angle in deɡree/Radian.

Note:

    I.    If $\theta$ =0 degree then the load is purely**Resistive**.

    II.    If $\theta$=-90 degree then the load is purely **inductive**.(lagging)

    III.    If $\theta$=90 degree then the load is purely **capacitive**.(leading)

$$Z=R+jX$$

Where Z=impedance of the electrical network in ohm.

R=Resistance of the network in ohm.

X=Reactance of the electrical network in ohm.

**Admittance:**

In electrical engineering, admittance is a measure of how easily a circuit or device will allow a current to flow. It is defined as the inverse of impedance. The SI unit of admittance is the siemens (symbol S).

Admittance is defined as:

$$Y = 1/Z$$

Where

Y is the admittance, measured in siemens
Z is the impedance, measured in ohms

The synonymous unit mho, and the symbol ℧ (an upside-down uppercase omega Ω), are also in common use.

Resistance is a measure of the opposition of a circuit to the flow of a steady current, while impedance takes into account not only the resistance but also dynamic effects (known as reactance). Likewise, admittance is not only a measure of the ease with which a steady current can flow, but also the dynamic effects of the material's susceptance to polarization:

$$Y = G + j\,B$$

Where

   Y is the admittance, measured in siemens.

   G is the conductance, measured in siemens.

   B is the susceptance, measured in siemens.

## AC Equivalent Circuits:



Impedances in series add:

Impedances in parallel behave like resistors in parallel:

Admittances in parallel add:

Admittances in series behave like conductances in series:
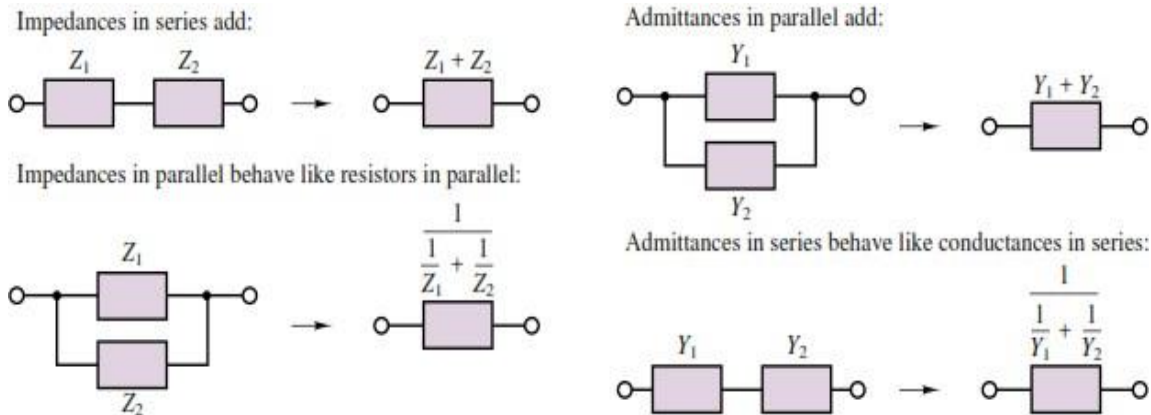
1. Impedances in series add together to give the equivalent impedance while the admittance in parallel add together to give the equivalent admittance.

2. Impedances in parallel gives equivalent impedance by reciprocating the reciprocal sum of the impedances and to obtain the equivalent admittance in series same procedure has to be followed.

## Instantaneous and Average Power

The most general expressions for the voltage and current delivered to an arbitrary load are as follows:

$$v(t) = V \cos(\omega t - \theta_V)$$

$$i(t) = I \cos(\omega t - \theta_I)$$

Since the instantaneous power dissipated by a circuit element is given by the product of the instantaneous voltage and current, it is possible to obtain a general expression for the power dissipated by an AC circuit element:

$p(t) = v(t)i(t) = V\ I\ \cos(\omega t)\ \cos(\omega t - \theta)$

It can be further simplified with the aid of trigonometric identities to yield

$p(t) = V\ I/2\cos(\theta) + V\ I/2\ \cos(2\omega t - \theta)$

where $\theta$ is the difference in phase between voltage and current

The average power corresponding to the voltage and current signal can be obtained by integrating the instantaneous power over one cycle of the sinusoidal signal. Let $T = 2\pi/\omega$ represent one cycle of the sinusoidal signals. Then the average power, Pav, is given by the integral of the instantaneous power,

p(t), over one cycle:

$$P_{av} = \frac{1}{T} \int_0^T p(t)\ dt$$

$$= \frac{1}{T} \int_0^T \frac{V\ I}{2} \cos(\theta)\ dt + \frac{1}{T} \int_0^T \frac{V\ I}{2} \cos(2\omega t - \theta)\ dt$$

$$P_{av} = \frac{V\ I}{2} \cos(\theta) \qquad \text{Average power}$$

since the second integral is equal to zero and $\cos(\theta)$ is a constant.

Instantaneous and average power
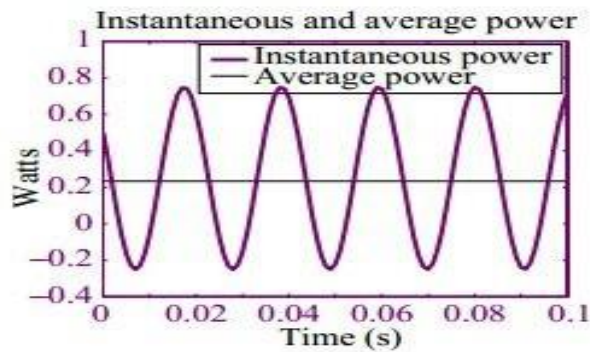
In phasor notation, the current and voltage are given by

$$\mathbf{V}(j\omega) = Ve^{j0}$$
$$\mathbf{I}(j\omega) = Ie^{-j\theta}$$

impedance of the circuit elementdefined by the phasor voltage and currentto be

$$Z = \frac{V}{I}e^{-j(\theta)} = |Z|e^{j\theta_z}$$

The expression for the average power using phasor notation

$$P_{av} = \frac{1}{2}\frac{V^2}{|Z|}\cos\theta = \frac{1}{2}I^2|Z|\cos\theta$$

## Power Factor

The phase angle of the load impedance plays a very important role in the absorption of power by load impedance. The average power dissipated by an AC load is dependent on the cosine of the angle of the impedance. To recognize the importance of this factor in AC power computations, the term $\cos(\theta)$ is referred to as the power factor (pf). Note that the power factor is equal to 0 for a purely

inductive or capacitive load and equal to 1 for a purely resistive load; in every other case, $0 < pf < 1$. If the load has an inductive reactance, then $\theta$ is positive and the current lags (or follows) the voltage. Thus, when $\theta$ and Q are positive, the corresponding power factor is termed lagging. Conversely, a capacitive load will have a negative Q, and hence a negative $\theta$. This corresponds to a leading power factor, meaning that the load current leads the load voltage. A power factor close to unity signifies an efficient transfer of energy from the AC source to the load, while a small power factor corresponds to inefficient use of energy. Two equivalent expressions for the power factor are given in the following:

$$pf = \cos(\theta) = \frac{P_{av}}{\tilde{V}\tilde{I}} \qquad \text{Power factor}$$

where $\tilde{V}$ and $\tilde{I}$ are the rms values of the load voltage and current.

## Complex Power

The expression for the instantaneous power may be further expanded to provide further insight into AC power. Using trigonometric identities, we obtain the

$$p(t) = \frac{\tilde{V}^2}{|Z|}[\cos\theta + \cos\theta \cos(2\omega t) + \sin\theta \sin(2\omega t)]$$
$$= \tilde{I}^2|Z|[\cos\theta + \cos\theta \cos(2\omega t) + \sin\theta \sin(2\omega t)]$$
$$= \tilde{I}^2|Z|\cos\theta(1 + \cos(2\omega t)) + \tilde{I}^2|Z|\sin\theta \sin(2\omega t)$$

following expressions:
Recalling the geometric interpretation of the impedance Z

$|Z| \cos\theta = R$ and $|Z|\sin\theta = X$

are the resistive and reactive components of the load impedance, respectively. On the basis of this fact, it becomes possible to write the instantaneous power as:

$$p(t) = \tilde{I}^2 R(1 + \cos(2\omega t)) + \tilde{I}^2 X \sin(2\omega t)$$
$$= \tilde{I}^2 R + \tilde{I}^2 R \cos(2\omega t) + \tilde{I}^2 X \sin(2\omega t)$$

Since Pav corresponds to the power absorbed by the load resistance, it is also called the real power, measured in units of watts (W). On the other hand, Q takes the name of reactive power, since it is associated with the load reactance. The units of Q are volt-amperes reactive, or VAR. Note that Q represents an exchange of energy between the source and the reactive part of the load; thus, no net power is gained or lost in the process, since the average reactive power is zero. In general, it is desirable to minimize the reactive power in a load.

The computation of AC power is greatly simplified by defining a fictitious but very useful quantity called the complex power, S:

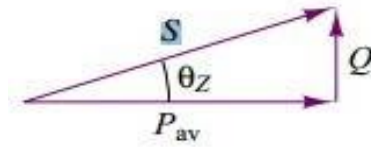$$S = \tilde{\mathbf{V}} \tilde{\mathbf{I}}^*$$

where the asterisk denotes the complex conjugate You may easily verify that this definition leads to the convenient expression

$$S = \tilde{V}\tilde{I}\cos\theta + j\tilde{V}\tilde{I}\sin\theta = \tilde{I}^2 R + j\tilde{I}^2 X = \tilde{I}^2 Z$$

or

$$S = P_{av} + jQ$$

The complex power S may be interpreted graphically as a vector in the complex S plane

$$|S| = \sqrt{P_{av}^2 + Q^2} = \tilde{V} \cdot \tilde{I}$$

$$P_{av} = \tilde{V}\tilde{I} \cos \theta$$

$$Q = \tilde{V}\tilde{I} \sin \theta$$

The magnitude of S, |S|, is measured in units of volt-amperes (VA) and is called apparent power, because this is the quantity one would compute by measuring the rms load voltage and currents without regard for the phase angle of the load.The complex power may also be expressed by the product of the square of the rms current through the load and the complex load impedance:

$$S = \tilde{I}^2 Z$$

or

$$\tilde{I}^2 R + j\tilde{I}^2 X = \tilde{I}^2 Z$$

or, equivalently, by the ratio of the square of the rms voltage across the load to the complex conjugate of the load impedance:

$$S = \frac{\tilde{V}^2}{Z^*}$$
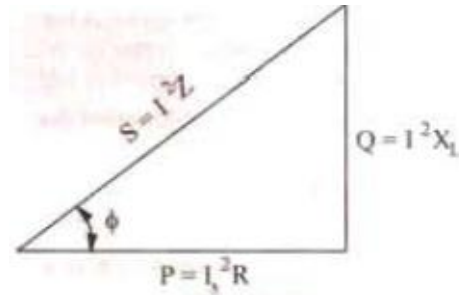
## Active, Reactive and Apparent Power



Fig. Power Triangle

$$S^2 = P^2 + Q^2$$

$$S = P + jQ$$

- **Apparent power, S**: is the product of rms values of the applied voltage and circuit current. It is also known as wattless (idle) component
  $S=VI=IZx\ I=I^2Z$     volt-amp
- **Active power or true power, P**: is the power which actually dissipated in the circuit resistance. It is also known as wattful component of power.
  $P=I^2R=I^2Z\cos\Phi=VI\cos\Phi$    watt
- **Reactive power, Q**:- is the power developed in the reactance of the circuit.
  $Q=I^2X=I^2Z\sin\Phi=VI\sin\Phi$    VAR

Example: In a particular R-L series circuit a voltage of 10 V at 50 Hz produces a current of 700 Ma while the same voltage at 75 Hz produces 500 mA. What are the values of R and L in the circuit.
Ans. i)

$$Z=\sqrt{R^2+(2\pi\times50L)^2}=\sqrt{R^2+98696L^2}$$

$$V=IZ \text{ or } 10=700\times10^{-3}\sqrt{(R^2+98696L^2)}$$

$$\sqrt{(R^2+98696L^2)}=10/700\times10^{-3}=100/7$$

$$\text{or } R^2+98696L^2=10000/49.................(i)$$

ii) In the second case

$$Z = \sqrt{R^2 + (2\pi \times 75\,L)^2} = \sqrt{R^2 + (222066\,L^2)}$$

$$10 = 500 \times 10^{-3}\sqrt{(R^2 + 222066\,L^2)}$$

$$\sqrt{(R^2 + 222066\,L^2)} = 20$$

$$R^2 + 222066\,L^2 = 400 \ldots\ldots\ldots\ldots\ldots(ii)$$

subtracting eq(i) from eq(ii), we get

$$222066\,L^2 - 98696\,L^2 = 400 - (10000/49)$$

$$123370\,L^2 = 196$$

$$L = 0.0398\,H = 40\,mH$$

Substituting this value of L in eq(ii),we get

$$R^2 + 222066(0.398)^2 = 400$$

$$R = 6.9\,\Omega$$

## **Introduction to resonance in series & parallel circuit**

**Resonance:**

Definition: An AC circuit is said to be in resonance when the circuit current is in phase with the applied voltage. So, the power factor of the circuit becomes unity at resonance and the impedance of the circuit consists of only resistance.

Series Resonance: In R-L-C series circuit, both $X_L$ and $X_C$ are frequency dependent. If we vary the supply frequency then the values of $X_L$ and $X_C$ varies. At a certain frequency called resonant frequency ($f_r$), $X_L$ becomes equal to $X_C$ and series resonance occurs.

At series resonance, $X_L = X_C$

$2\pi f_r L = 1/2\pi f_r C$

$f_r = 1/2\pi\sqrt{LC}$

Impedance of RLC series circuit is given by:

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} \qquad (\text{Since}, X_L = X_C)$$

$$Z = \sqrt{R^2}$$

$$Z = R$$

$$\cos\varphi = \frac{R}{Z} = \frac{R}{R} = 1$$

**Properties of series resonance**:-

In series resonance,

- The circuit impedance Z is minimum and equal to the circuit resistance R.
- The circuit current I= V/Z = V/R and the current is maximum
- The power dissipated is maximum, $P = V^2/R$
- Resonant frequency is fr = $1/2\pi\sqrt{LC}$
- Voltage across inductor is equal and opposite to the voltage across capacitor
- Since power factor is 1, so zero phase difference. Circuit behaves as a purely resistive circuit.

**Example**: A series RLC circuit having a resistance of 50Λ, an inductance of 500 mH and a capacitance of 400 μF, is energized from a 50 Hz, 230 V, AC supply. Find a) resonant frequency of the circuit b) peak current drawn by the circuit at 50 Hz and c) peak current drawn by the circuit at resonant frequency

Ans.

a) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{500\times10^{-3}\times400\times10^{-6}}} = 11.25$ Hz

b) $R = 50\Omega$

$X_L = \omega L = 2\pi\times50\times500\times10^{-3} = 157\Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi\times50\times400\times10^{-6}} = 7.9\Omega$

$X = X_L - X_C = 157 - 7.9 = 149.1\Omega$

$Z = \sqrt{R^2 + X^2} = \sqrt{50^2 + 149.1^2} = 157.26\Omega$

Peak supply voltage, $V_m = \sqrt{2}\ V_{rms} = \sqrt{2}\ (230) = 325.26$ V

Hence peak current at 50Hz $I_m = \dfrac{V_m}{Z} = \dfrac{325.26}{157.26} = 2.068$

c) At resonance, $Z_0 = R = 50\Omega$

So, peak current during resonance, $I_{mo} = \dfrac{V_m}{R} = \dfrac{325.26}{50} = 6.5025$A

**Parallel resonance**:

**Points to remember:**
- Net susceptance is zero, i.e $1/X_C = X_L/Z^2$

$X_L \times X_C = Z^2$

Or $L/C = Z^2$

- The admittance equals conductance
- Reactive or wattless component of line current is zero
- Dynamic impedance = L/CR Λ

- Line current at resonance is minimum and V/L/CR but is in phase with the applied voltage
- Power factor of the circuit is unity

# THREE PHASE AC CIRCUIT

## Three phase EMF Generation:-
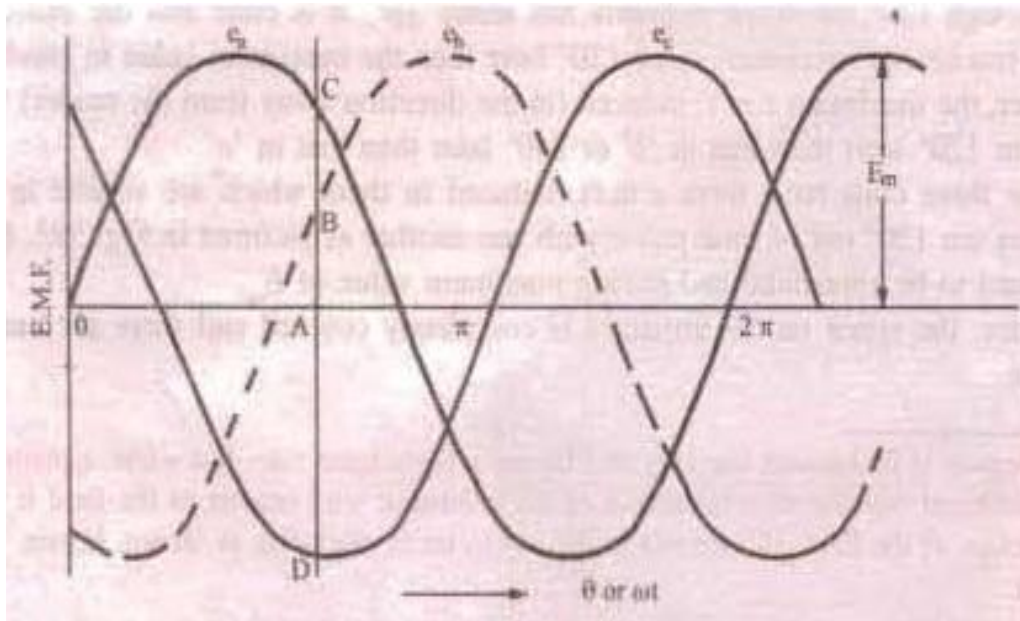


If the 3-coil windings $W_1$, $W_2$ and $W_3$ arranged at 120° apart from each other on the same axis are rotated, then the emf induced in each of them will have a phase difference of 120°. In other words if the emf (or current) in one winding ($w_1$) has a phase of 0°, then the second winding ($w_2$) has a phase of 120° and the third ($w_3$) has a phase of 240°.

## Star (Y) connection:-
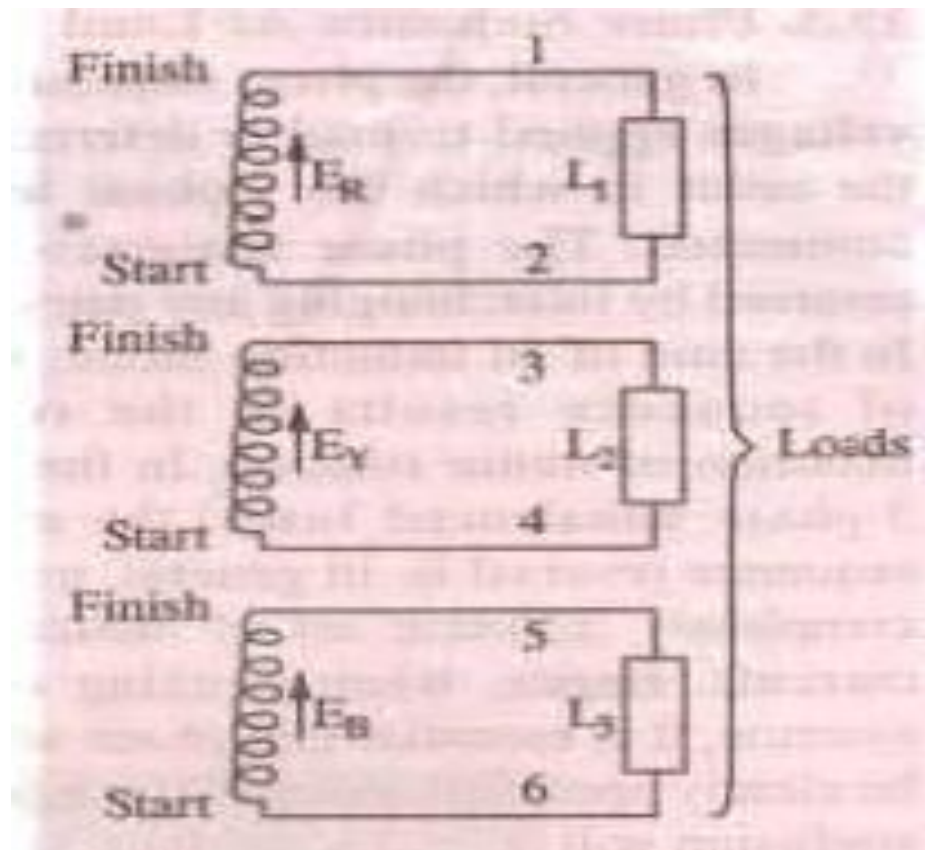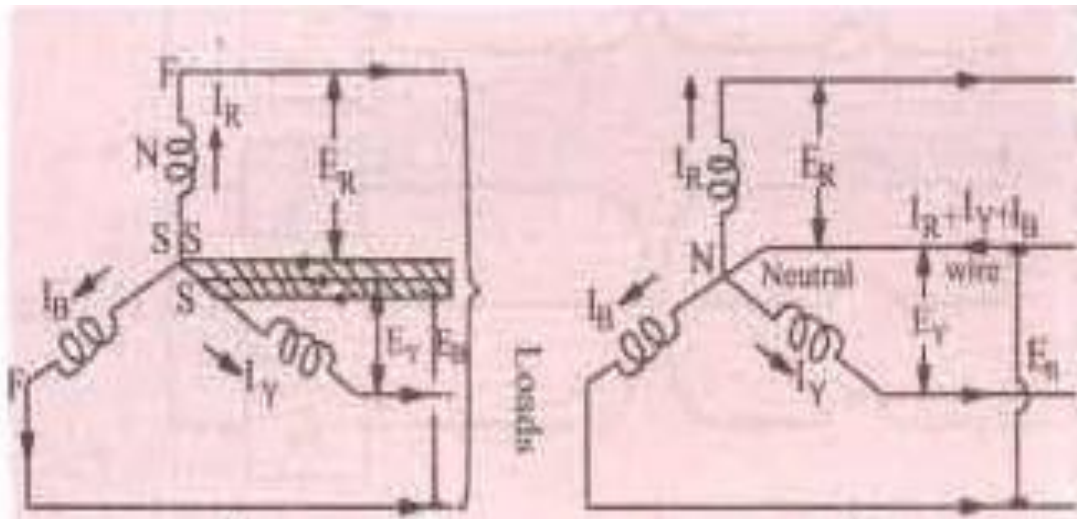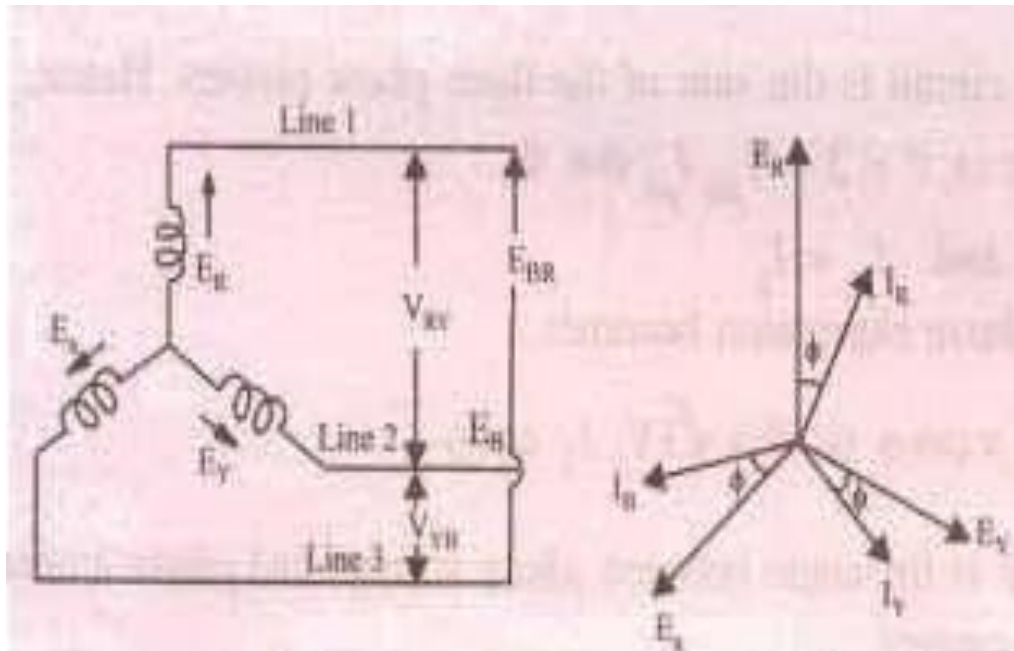
Phasor diagram:-
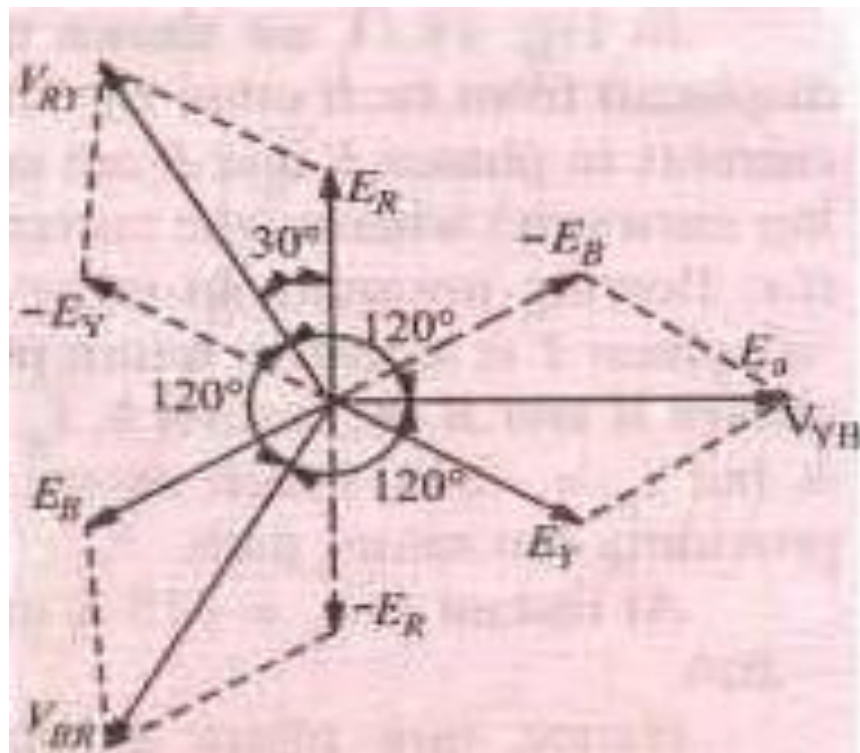


Here, $E_R$, $E_Y$, $E_B$ are phase voltages and $V_{RY}$, $V_{YB}$, $V_{BR}$ are line voltages

$$V_{RY} = \sqrt{E_R{}^2 + E_Y{}^2 + 2E_R E_Y \cos 60^\circ}$$

$$= \sqrt{E_R{}^2 + E_R{}^2 + 2E_R E_R \cos 60^\circ}$$

$$= \sqrt{3} E_R$$

Hence,

- Line voltage $= \sqrt{3}$ x phase voltage
- Line current $=$ phase current
- Line voltages are also 120° apart
- Line voltage are 30° ahead of respective phase voltages
- The angle between line voltage and line current is (30°+Φ)

Power: Total power $= 3$ x phase power

$= 3$ x $V_{ph}$ x $I_{ph}$ x cosΦ

$= \sqrt{3}$ $V_L I_L$ cosΦ

Φ is the angle between phase voltage and current

**Example**: A balanced star connected load of (8+j6)Λ per phase is connected to a balanced 3-phase 400 V supply. Find the line current, power factor, power and total volt-amperes.

Ans.
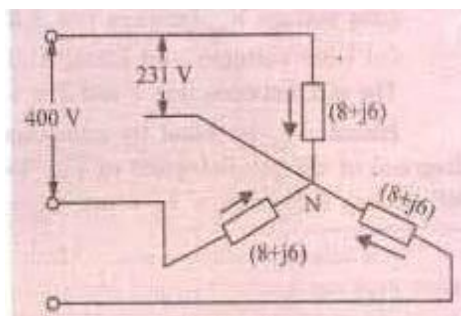
$$Z_{ph} = \sqrt{8^2 + 6^2} = 10\,\Lambda$$

$$V_{ph} = \frac{400}{\sqrt{3}} = 231\ V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{10} = 23.1\ A$$

(i) $I_L = I_{ph} = 23.1$ A

(ii) p.f $= \cos\Phi = \dfrac{R_{ph}}{Z_{ph}} = \dfrac{8}{10} = 0.8$(lag)

(iii) Power $P = \sqrt{3}\ V_L\,I_L\,\cos\Phi$

$= \sqrt{3} \times 400 \times 23.1 \times 0.8$

$= 12,800$ W [Also, $P = 3 I_{ph}{}^2 R_{ph} = 3(23.1)^2 \times 8 = 12,800$ W]

(iv) Total volt-amperes,

$S = \sqrt{3}\ V_L\,I_L = \sqrt{3} \times 400 \times 23.1 = 16,000$ VA

**Delta-connection**:
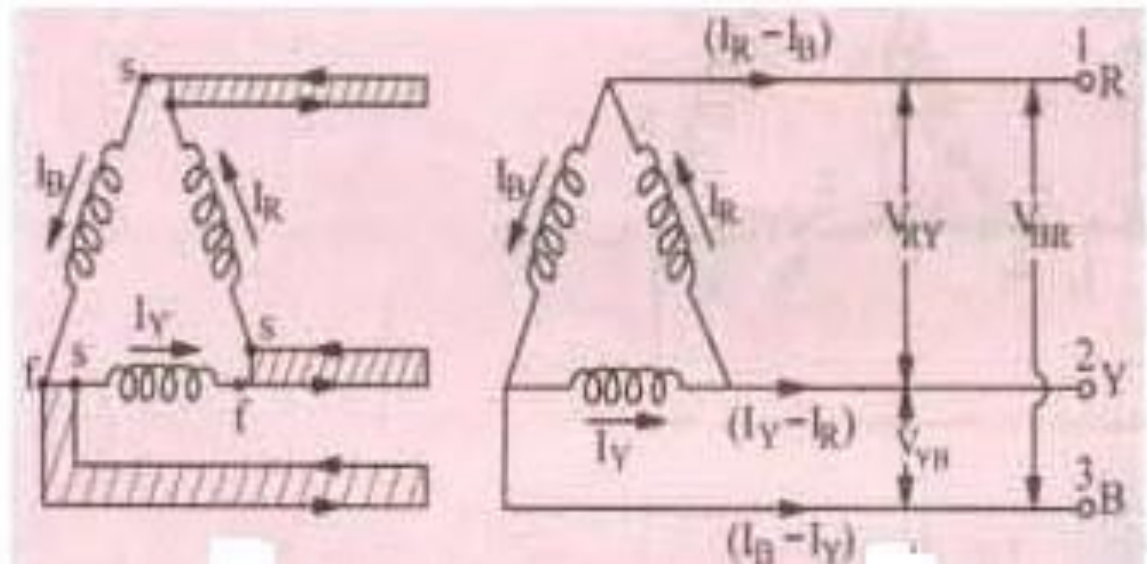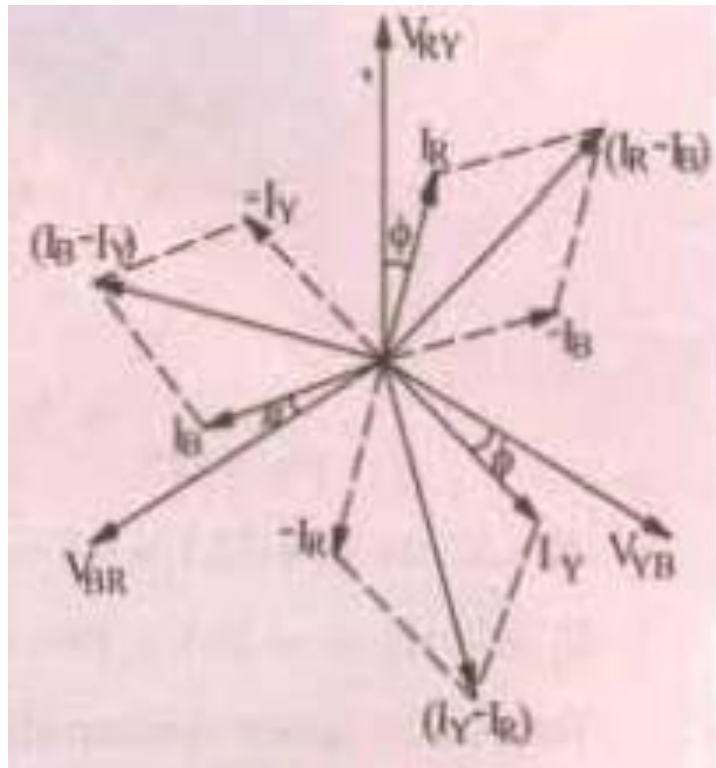
Fig. Phasor Diagram

$$I_L = I_R - I_B$$

$$I_L = \sqrt{I_R^2 + I_B^2 + 2I_R I_B \cos 60^\circ} = \sqrt{I_R^2 + I_R^2 + 2I_R I_R \cos 60^\circ} = \sqrt{3} I_R$$

Hence,

- Line current $=\sqrt{3}$ phase current
- Line voltage = phase voltage
- Line currents are also 120° apart
- Line currents are 30° behind the respective phase currents
- Angle between line current and line voltage is 30°+Φ

Power: Total power = 3 x phase power

$$= 3 \times V_{ph} I_{ph} \cos\Phi$$

$$= 3 \times V_L \times I_L/\sqrt{3} \times \cos\Phi$$

$$= \sqrt{3} V_L I_L \cos\Phi$$

**Note**: For both star and delta system:

Active & True power = $\sqrt{3}\ V_L\ I_L\ \cos\Phi$

Reactive power = $\sqrt{3}\ V_L\ I_L\ \sin\Phi$

Apparent power = $\sqrt{3}\ V_L\ I_L$

# MODULE-II

**Magnetic Circuits**:

**The Magnetic Field and Faraday's Law:**

Magnetic fields are generated by electric charge in motion, and their effect is measuredbytheforcetheyexertonamovingcharge.Asyoumayrecallfromprevious physics courses, the vector force f exerted on a charge of q moving at velocity u in the presence of a magnetic field with flux density B is given by

$f = qu \times B$

Where the symbol $\times$ denotes the (vector) cross product. If the charge is moving at a velocity u in a direction that makes an angle $\theta$ with the magnetic field, then the magnitude of the force is given by

$f = quB\sin\theta$

and the direction of this force is at right angles with the plane formed by the vectors B and u.
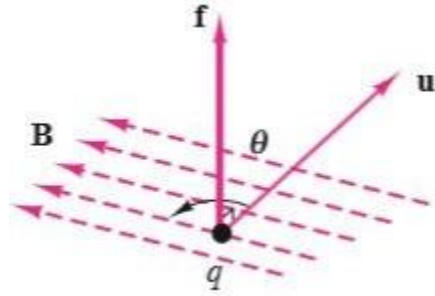
The magnetic flux φ is then defined as the integral of the flux density over some surface area.

$$\varphi = \int_A B \, dA \quad \text{in webers}$$

$$\Rightarrow \phi = B.A$$

**Faraday's law** states that a time-varying flux causes an induced electromotive force, or emf

$$e = -\frac{d\phi}{dt}$$

In practical applications, the size of the voltages induced by the changing magnetic field can be significantly increased if the conducting wire is coiled many times around, so as to multiply the area crossed by the magnetic flux lines many times over. For an N-turn coil with cross-sectional area A, for example, we have the emf

$$e = N \frac{d\phi}{dt}$$

When N-turn coil linking a certain amount of magnetic flux, then the flux linkage

$$\lambda = N\phi$$

$$\Rightarrow e = \frac{d\lambda}{dt}$$

The relation between flux linkage and current is gven by $\lambda = Li$

so that the effect of a time-varying current was to induce a transformer voltage across an inductor coil, according to the expression $v = L\frac{di}{dt}$

$L$ is the self-inductance which measures the voltage induced in a circuit by magnetic field generated by a current flowing in the same circuit.

**Self and mutual Inductance:**
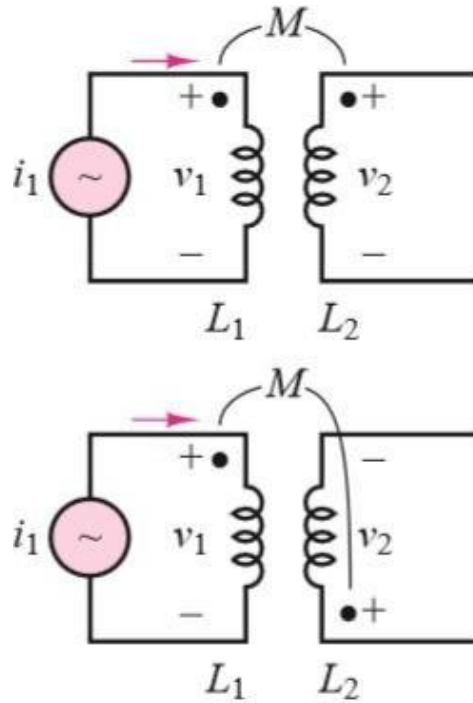


The figure shown above depicts a pair of coils one of which, $L_1$ is excited by a current $i_1$ and therefore develops a magnetic field and a resulting induced voltage $v_1$.

The second coil, $L_2$, is not energized by a current, but links some of the flux generated by current $i_1$ around $L_1$ because of its close proximity to the first coil. The magnetic coupling between the coils established by virtue of their proximity is described by a quantity called mutual inductance and defined by the symbol M. The mutual inductance is defined by the equation

$$v_2 = M \frac{di_1}{dt}$$

The dots shown in the two drawings indicate the polarity of the coupling between the coils. If the dots are at the same end of the coils, the voltage induced in coil 2 by a current in coil 1 has the same polarity as the voltage induced by the same current in coil 1; otherwise, the voltages are in opposition, as shown in the lower part of Figure. Thus, the presence of such dots indicates that magnetic coupling is present between two coils. It should also be pointed out that if a current (and therefore a magnetic field) were present in the second coil, an additional voltage would be induced across coil 1. The voltage induced across a coil is, in general, equal to the sum of the voltages induced by self-inductance and mutual inductance.

As already discussed $v = L \frac{di}{dt}$ with $L$ constant

then $e = N \frac{d\phi}{dt}$

the inductance is given by $L = \frac{N\phi}{i} = \frac{\lambda}{i}$

This expression implies that the relationship between current and flux in a magnetic structure is linear, but due to the properties of ferromagnetic materials the flux-current relationship is nonlinear.

**Ampere's Law:**

Ampere's law forms a counter part to Faraday's law. Both the laws explains the relationship between electricity and magnetism. Ampere's law states that the magnetic field intensity H in the vicinity of a conductor is related to the current carried by the conductor; thus Ampère's law establishes a dual relationship with Faraday's law.

In the previous section, we described the magnetic field in terms of its flux density Band flux $\varphi$. To explain Ampère's law and the behaviour of magnetic materials, we need to define a relationship between the magnetic field intensity H and the flux density B. These quantities are related by

$B = \mu H = \mu_r \mu_0 H \, Wb/m^2$ or T

where the parameter $\mu$ is a scalar constant for a particular physical medium which is the permeability of the medium. The permeability of a material can be factored as the product of the permeability of freespace $\mu_0 = 4\pi \times 10-7$ H/m, and the relative permeability $\mu_r$, which varies greatly according to the medium.7 H/m, and the relative permeability $\mu_r$ which varies greatly according to the medium. For example, for air and for most electrical conductors and insulators, $\mu_r$ is equal to 1. For ferromagnetic materials, $\mu_r$ can take values in the hundreds or thousands. The size of $\mu_r$ represents a measure of the magnetic properties of the material.

Ampère's law states that the integral of the vector magnetic field  intensity H

around a closed path is equal to the total current linked by the closed path $i$:

$$\int H.dl = \sum i$$

where dl is an increment in the direction of the closed path. If the path is in the

same direction as the direction of the magnetic field, we can use scalar quantities to

state that

$$\int Hdl = \sum i$$

Direction of the magnetic field intensity H is determined by the familiar right-hand

rule. This rule states that if the direction of current $i$ points in the direction of the

thumb of one's right hand, the resulting magnetic field encircles the conductor in

the direction in which the other four fingers would encircle it.



By the right-hand rule, the current, $i$, generates a magnetic field intensity, H, in the direction shown.
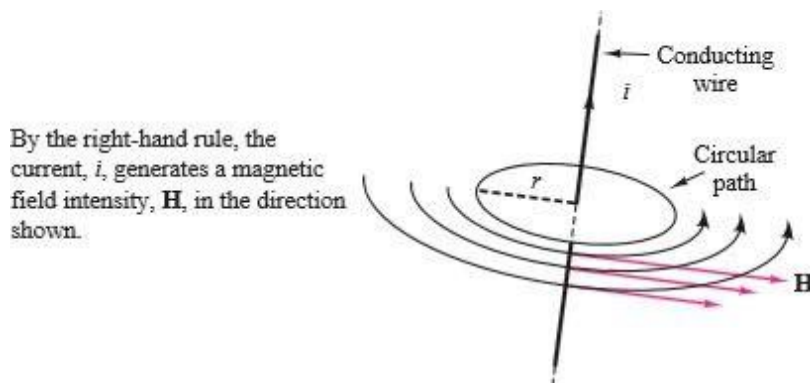
Therefore for the closed-path integral becomes equal to H · 2πr, since the path and the magnetic field are in the same direction, and the magnitude of the magnetic field intensity is given by

$$H = \frac{i}{2\pi r}$$

**Magnetic circuit:**

To analyse the operation of electromagnetic devices the approximation is taken that a mean path for the magnetic flux and that the corresponding mean flux density is approximately constant over the cross-sectional area of the magnetic structure. When a coil is wound around a core with cross-sectional area A will have flux density as $B = \frac{\phi}{A}$, where area is assumed to be perpendicular to the direction of the flux lines. The field intensity obtained to be $H = \frac{B}{\mu} = \frac{\phi}{A\mu}$

mmf of the coil can be represented by the product of magnetic field intensity and the length of the magnetic field

$$F = N.i = H.l$$

$$\Rightarrow F = \phi \frac{l}{\mu A}$$

The term $\dfrac{l}{\mu A}$ is known as reluctance of magnetic circuit.

The relation between inductance and reluctance derived as

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N}{i}\frac{Ni}{R} = \frac{N^2}{R} \ \text{H}$$

In many magnetic structures and in rotating machines air gaps are very common. The effect of air gap is to break the continuity of the high-permeability path for the flux, adding a high reluctance component to the equivalent circuit. The situation is analogous to adding a very large series resistance to a series electric circuit. In this case the basic concept of reluctance still applies, although now two different permeabilities must be taken into account.

Hence reluctance of the air gap is $R_g = \dfrac{l_g}{\mu_0 A_g}$
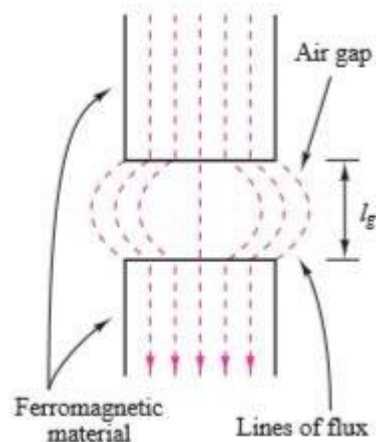
Where $R_g$ = reluctance of air gap

$\mu_0$ = permeability

$A_g$ = cross-sectional area of the air-gap in the given structure

$l_g$ = length of air gap

$A_g$ is different from the other cross-sectional area of the structure because of the phenomenon known as Fringing as they cross an air gap.



**Magnetic materials and B-H curves:**

The relationship between the magnetic flux density B and the associated field intensity H is expressed by $B = \mu H$ ,where $\mu$ = permeability of magnetic material.

From the above expression flux density increases in proportion to field intensity upto a saturation point reaches. But in general all magnetic material shows a nonlinear B-H curve, depending upon the value of permeability, which can be better explained by eddy currents and hysteresis. Eddy current caused by any time-varying flux in the core material. It will induce a voltage , and therefore current. The induced voltage will cause eddy current, which depends on the resistivity of the core. Hysteresis is another loss mechanism in magnetic materials. It shows a

complex behaviour related to the magnetization properties of the material which can be shown as
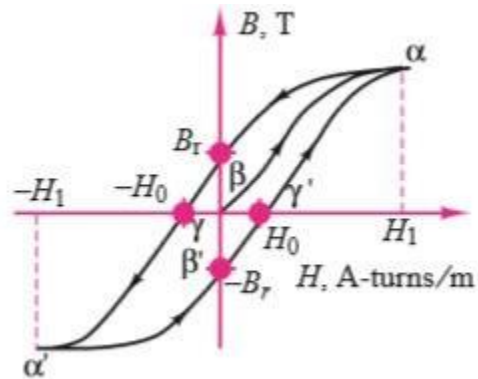


Here the core has been energized for some time ,with a field intensity of $H_1$ A-turns/m. as the current decreases curve follow from the point $\alpha$ to the point $\beta$. At this point mmf is zero to bring the flux density to zero ,mmf is further deceased until the field intensity reaches to $-H_0$. As mmf value is made more negative , the curve eventually reaches to the point $\alpha'$. The excitation current is now increased, the magnetization curve will follow the path $\alpha' = \beta' = \gamma'= \alpha$, and finally returns to the original point of B-H curve.

**Hysteresis loss:-** During the complete cycle, the magnets within the magnetic material try to align first in one way and then in reverse way. The tendency to turn around of elementary magnets give rise to mechanical stresses in the magnetic material, which in turn produces heat  which is a waste form  of energy. The

dissipitated heat energy during the cycle of magnetization is given by the area within the hysteresis loop and is called hysteresis loss.

**Hysteresis power loss =**
$$P_h = KfB_{max}^x V$$

Where,  K= Hysteresis coefficient

   f= frequency of magnetization

   V= volume of the material ($m^3$)

   $B_{max}$ = Maximum flux density (wb/$m^2$)

   x = 1.5-2.5

- **Steinmetz law:-**
$$P_h = \eta \ fB_{max}^{1.6} V$$

Where, $\eta$= Steinmetz constant or hysteresis coefficient

f= frequency of magnetization

V= volume of the material ($m^3$)

$B_{max}$ = Maximum flux density (wb/$m^2$)

$B_m$ lies between 0.1 to 1.2 wb/$m^2$ , when B is not between 0.1

**Eddy current loss:** During the cycle of magnetization, the change in flux density induces an emf in the core of an electromagnet. The effect sets up small locally circulating currents called eddy currents. These currents are of no practical significance but produce heat which means some loss of energy. This loss of energy is called eddy current loss.

$$P_e = K_e t^2 f^2 V$$

**Eddy current loss:-**

Where, Ke= Eddy current constant

t= thickness of the lamination of the pole core and armature

B= Flux density

F= Frequency

V= Volume of iron subject to change of flux

**Points to Remember:**

- The eddy current loss can be minimized by using thin laminated cores.

- Use of laminations increase the resistance of eddy current path and thereby reduces its magnitude

- The hysteresis loss can be minimized by choosing the material having low hysteresis coefficient. e.g:- silicon steel:- 1.91

- The hysteresis and eddy current losses are together known as iron loss or core loss.

- For any machine , Bm and f are also nearly constant. Hence these are also called constant loss.

- Hysteresis loss = xy(area of B/H loop)

  where, x and y are scales of B& H.

- Unit of hysteresis loss is $J/m^3/cycle$ or $watt/m^3$

**Permeability:** Every substance possesses a certain power of conducting magnetic lines of force (iron is better conductor for magnetic lines of force than air). Permeability of a material is its conducting power for magnetic lines of force. It is the ratio of flux density (B) produced in a material to the magnetic field strength i.e

$\mu = B/H$

**Absolute & Relative Permeability:** For measuring relative permeability, vaccum or free space is chosen as the reference medium.

Absolute permeability $\mu_0 = 4\pi \times 10^{-7}$ H/m

Now, take any medium other than vaccum. If its relative permeability as compared to vaccum is $\mu_r$ , then $\mu = \mu_0 \mu_r$

In other words, $\mu_r$ indicates the extent to which the given material is a better conductor of magnetic flux than air.

**Reluctance: (S)** is a measure of the opposition offered by a magnetic circuit to the setting up of flux.

S=MMF/$\Phi$ ,   $S = l/ \mu_0 \mu_r$ A [unit- AT/wb]

**Example:** The hysteresis loop of an iron ring was found to have an area of 10 cm$^2$ on a scale of 1 cm = 1000 AT/m (X-axis); 1 cm = 0.2 wb/ m$^2$ (Y- axis). The ring

has a mean length of 100 cm and cross-sectional area of 5 cm. Compute the hysteresis loss in watts for a frequency of 50 Hz.

**Ans.** Area of hysteresis loop = 10 x 0.2 x 1000 AT-wb/m³ = 2000 AT-wb/m³

  Hysteresis loss = 2000 x 50 = 10,0000 J/m³/s [f= 50 cycle/s]

  Volume = 1 x 5 x 10⁻⁴ m³

  Hysteresis loss = 100000 x 1 x 5 x 10⁻⁴ W = 50 W

## TRANSFORMER

One of the more common magnetic structures in everyday applications is the transformer. An ideal transformer is a device that can step an AC voltage up or down by a fixed ratio, with a corresponding decrease and increase in current. A simple magnetic transformer is shown as below
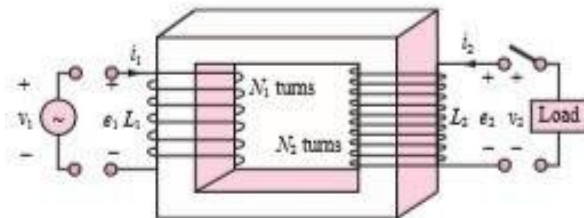


Here coil $L_1$ represents the input side of the transformer or primary winding of it, where as the coil $L_2$ is the output coil or secondary winding ; both winding are wound around the same magnetic structure. The operation of a transformer requires a time-varying current; if a time-varying voltage is applied to the primary

side of the transformer, a corresponding current will flow in L1;this current acts as an mmf and causes a (time-varying) flux in the structure. But the existence of a changing flux will induce an emf across the secondary coil! Without the need for a direct electrical connection ,the transformer can couple a source voltage at the primary to the load; the coupling occurs by means of the magnetic field acting on both coils. Thus, a transformer operates by converting electric energy to magnetic, and then back to electric energy. When a time-varying voltage source is connected to the input side, then by Faraday's law, a corresponding time-varying flux dφ/dt is established in coil L1:

$$e_1 = N_1 \frac{d\phi}{dt} = v_1$$

Due to flux an emf induced across the secondary coil is

$$e_2 = N_2 \frac{d\phi}{dt} = v_2$$

So the relation between the input and output voltage is $\frac{v_2}{v_1} = \frac{N_2}{N_1}$

As mmf in transformer remains same though out the core  i.e.

$$i_1 N_1 = i_2 N_2$$

$$\frac{i_2}{i_1} = \frac{N_1}{N_2}$$

Here N1 and N2 are the primary and secondary turns, respectively. As the ideal transformer does not dissipate any power, since

$$v_1 i_1 = v_2 i_2$$

Another important performance characteristic of a transformer is its power efficiency

Power efficiency $\eta$ = Output power/ Input power

**Definition:**

- It is a static device used for the purpose of transferring electrical energy from one circuit to another at same frequency but at different voltage (or current or both).
- It is used for raising or lowering the voltage of an a.c. supply with corresponding decrease or increase in current.
- It is an a.c. device

**<u>Some more aspects of transformer:-</u>**

It consists of two windings, primary and secondary wound on a common laminated magnetic core.
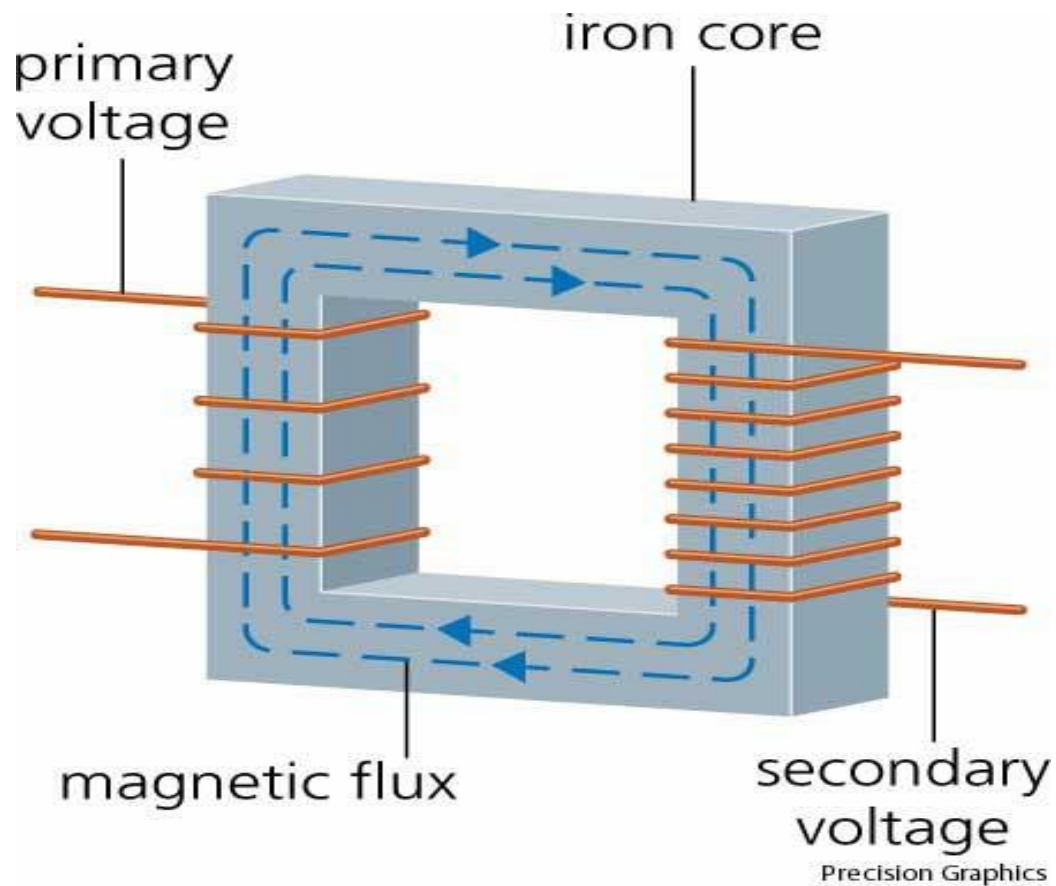
The winding connected to the a.c. source is called primary winding and one connected to the load is called secondary winding.

$V_1$ is applied to primary. Depending upon $N_1$ & $N_2$, $E_2$ is induced in the secondary. This $E_2$ causes a secondary current $I_2$ consequently terminal voltage $V_2$ appears across the load.

If $V_1 > V_2$, it is called a step up transformer.

If $V_1 < V_2$, it is called a step down transformer.

Two types-core type and shell type

Core type: In core type the winding surrounds the steel core. The core consists of two vertical logs limbs with 2 horizontal section called yokes.

To keep the leakage flux to a minimum , half of each winding is placed on each lag of core. The low voltage winding is placed adjacent to the steel core and high voltage winding is placed outside to reduce the insulating material required.

Shell type: In shell type transformer, steel core surrounds windings.LV and HV windings are wound over central lump.

Core type is used for high voltage and shell type is used for low voltage.

Core Type and Shell Type Transformer Winding
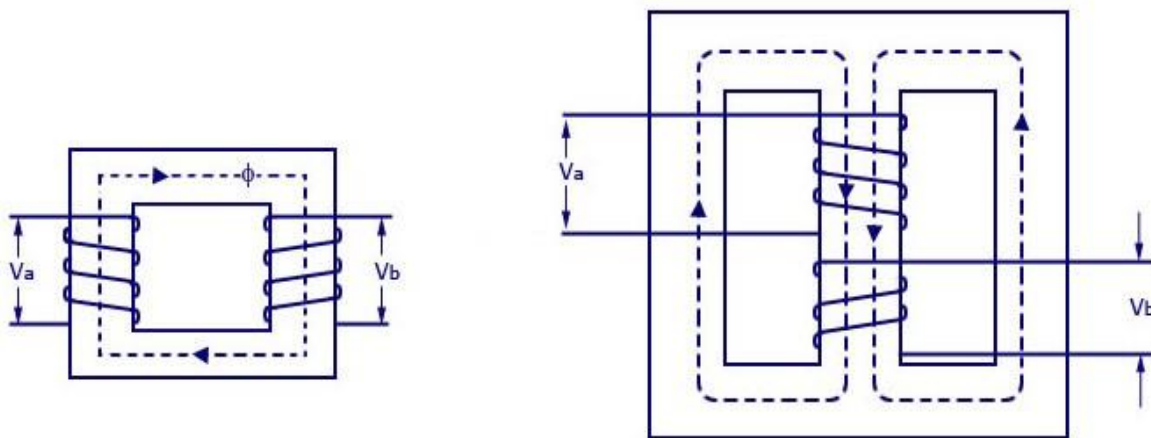


## PRINCIPLE OF OPERATION

It is based on the principle of mutual induction i.e. whenever the amount of magnetic flux linked with the coil charges, and emf is induced in the coil.

Whenever alternating voltage $V_1$ is applied to the primary winding,$I_0$ (exciting current) flows which sets up $\emptyset$ in magnetic core. This flux links with both windings and $E_1$ & $E_2$ are induced.

From Lenz's law, $E_1 = -V_1$

If $N_2 > N_1$ then $E_2 > E_1$ and it becomes step up transformer

If $N_2 < N_1$ then $E_2 < E_1$ and it becomes step down transformer

Here $E_2$ is in phase opposite to $V_1$.

If the secondary is open circuited, then the terminal voltage $V_2$ at secondary is equal in magnitude and in phase with the induced emf at the secondary, i.e. $E_2 = V_2$.

## IDEAL TRANSFORMER

An imaginary transformer which has the following properties

1- Primary and secondary winding resistance are negligible, hence no resistive voltage drop.

2- leakage flux and leakage inductance are zero. There is no reactive voltage drop in the windings.

3- power transformer efficiency is 100% i.e. there are no hysteresis loss, eddy current loss or heat loss due to resistance.

4- permeability of the core is infinite so that it requires zero mmf to create flux in the core.

Power In the primary= power in the secondary .

$E_1 I_1 = E_2 I_2$

$I_1 / I_2 = E_2 / E_1 = N_2 / N_1 = K = V_2 / V_1$

1 – when transferring resistance or reactance from primary to secondary, multiply it by $K^2$

2- when transferring resistance or reactance from secondary to primary, divide it by $K^2$

3- Transferring voltage or current, only K is used.

a) Any voltage V in primary becomes KV in secondary.

b) Any voltage V in secondary becomes V/Kin primary.

c) Any current I in primary becomes I /K in secondary.

d) Any current I in secondary becomes KI in primary.

e) A resistance R in primary $K^2R$ in secondary.

f) A resistance R in secondary becomes $R/K^2$.

**EMF Equation**:

$$\varphi = \varphi_m \sin w \, t$$

$$e_1 = -N_1 \frac{d\varphi}{dt} = -N_1 \frac{d}{dt}\left(\varphi_m \sin wt\right)$$

$$e_1 = -N_1 w \varphi_m \cos w \, t = -N_1 \times 2\pi f \varphi_m \cos wt$$

$$e_1 = N_1 2\pi f \varphi_m \sin\left(wt\text{-}90^\circ\right) \qquad \left[E_{m1} = 2\pi f N_1 \varphi_m\right]$$

$$\text{R.M.S value of } E_1 \text{ is: } E_1 = \frac{E_{m1}}{\sqrt{2}} = \frac{2\pi f N_1 \varphi_m}{\sqrt{2}} = 4.44 f N_1 \varphi_m$$

$$E_2 = 4.44 f N_2 \varphi_m$$

**Voltage Transformation Ratio**:

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

• $N_2/N_1$ is known as voltage transformation ratio and represented by K.

• If $N_2 > N_1$ or K>1 then step up transformer

- If $N_1 > N_2$ or $K < 1$ then step down transformer
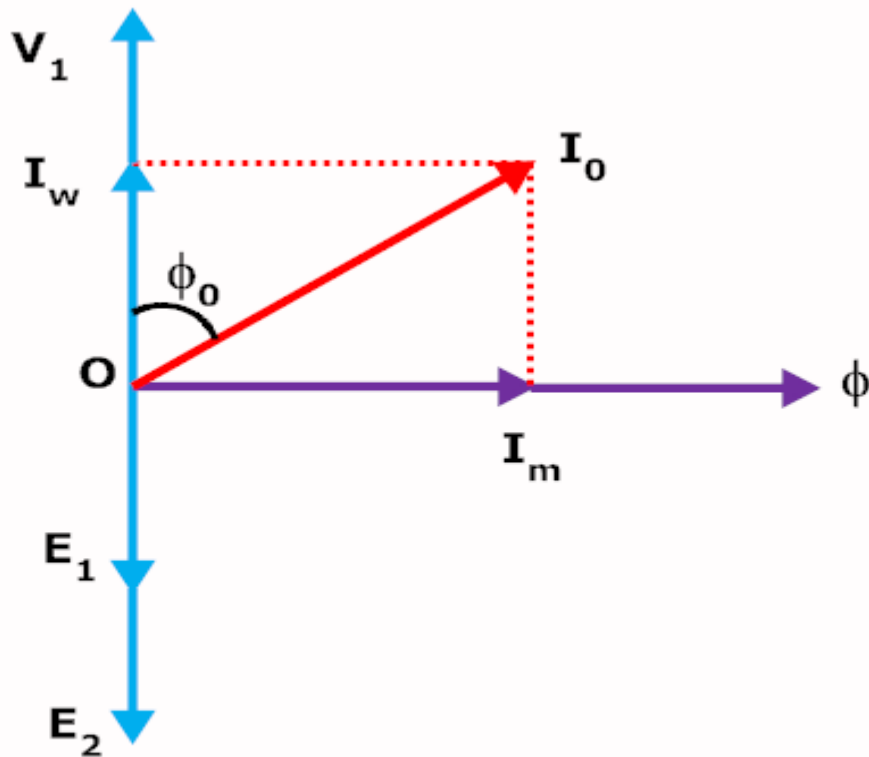
**Practical Transformer on no load:**



Figure 4: Phasor diagram of practical transformer on no load

- A transformer is said to be on no load if its primary winding is connected to AC supply and secondary is open. i.e secondary current is zero

- When an A.C voltage is applied to primary, a small current $I_0$ flows in primary.

- $I_0$ = N0-load current

- $I_m$ = magnetizing current. It magnetizes the core and sets flux. So, in phase with it.

- $I_m$ is called the reactive or wattless component of no load current

- $I_w$ produces eddy current and hysteresis losses in the core and very small copper loss in primary. It is called active or wattful component of no load current.

- $I_w$ is in phase with the applied voltage ($V_1$) at the primary.

- No load current $I_0$ is small. So drops in $R_1$ and $X_1$ on primary side are very small. At no load $V_1 = E_1$.

- No load primary copper loss ($I_0^2 R_1$) is very small. So, no load primary input power is equal to iron loss

$I_w = I_0 \cos\varphi_0$ , $I_m = I_0 \sin\varphi_0$ , $I_0 = \sqrt{I_m^2 + I_w^2}$

No load power factor, $\cos\varphi_0 = \dfrac{I_w}{I_0}$

No load input power (active power)$=V_1 I_0 \cos\varphi_0$ ,
No load reactive power $=V_1 I_0 \sin\varphi_0$

(a)
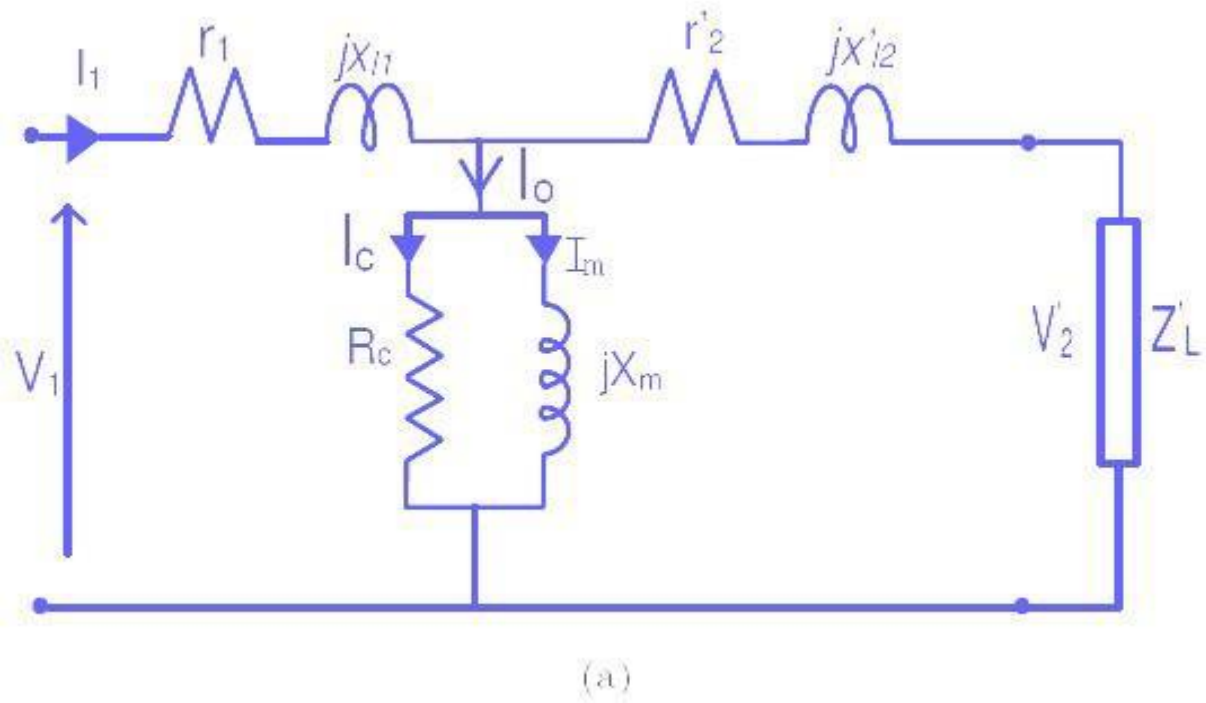
- Winding resistance

- Leakage reactance

- Iron losses

➤ Depends on supply frequency,

➤ Maximum flux density in the core

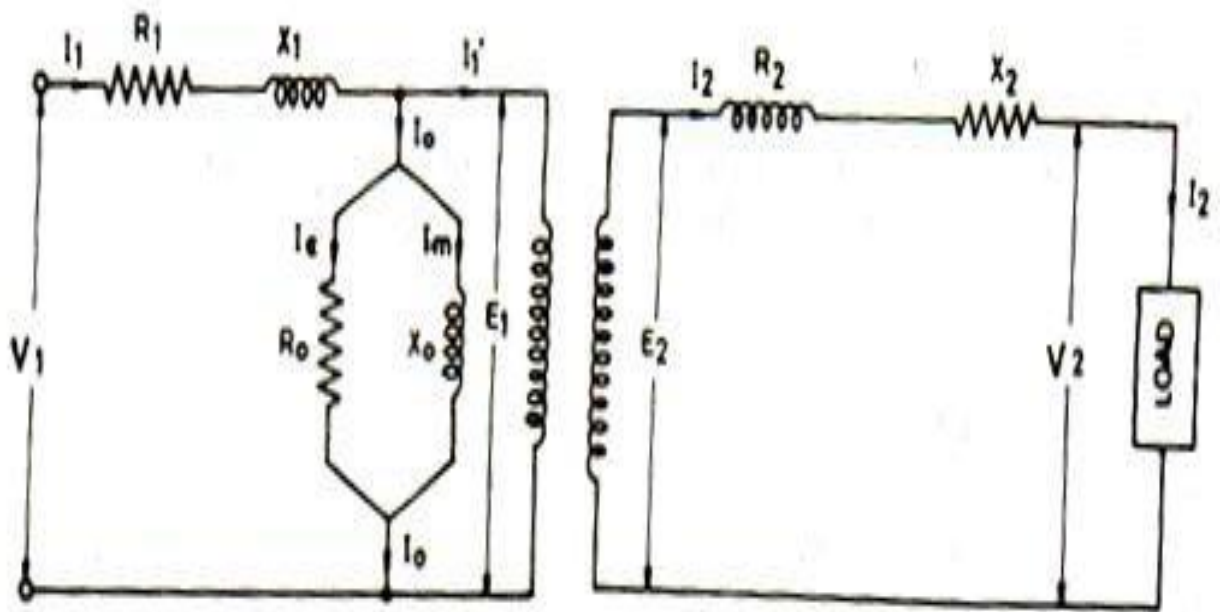➤ Volume of the core

- Impedance reflection and power transform:-

$$(I_P)^2 R_P = I_S^2 R_S$$

$$R_P = \left(\frac{I_S}{I_P}\right)^2 R_S \quad [\text{since, } N_P I_P = N_S I_S; \frac{I_P}{I_S} = \frac{N_S}{N_P} = K]$$

$$R_P = \frac{1}{K^2} R_S$$

$$\text{Similarly,} X_P = \frac{1}{K^2} X_S$$

## Equivalent Circuit of a loaded transformer:



**Example**: The primary winding of a single phase transformer is connected to a 220 V, 50 Hz supply. The secondary winding has 2000 turns. If the maximum value of the core flux is 0.003 wb, determine i) the number of turns on the primary winding ii) the secondary induced voltage

Ans.

$E_1 = 220 \text{ V}, f = 50 \text{ Hz}$

$N_2 = 2000, \varphi_m = 0.003 \text{ wb}$

i) $E_1 = 4.44 \, f \varphi_m N_1$

$$N_1 = \frac{E_1}{4.44 \, f \varphi_m} = \frac{220}{4.44 \times 50 \times 0.003} = 330$$

ii) $E_2 = 4.44 \, f \varphi_m N_2 = 4.44 \times 50 \times 0.003 \times 2000 = 1332$

# INTRODUCTION TO ELECTRIC MACHINES

The operation of the three major classes of electric machines—DC, synchronous, and induction—first is described as intuitively as possible. The second part of the chapter is devoted to a discussion of the applications and selection criteria for the different classes of machines. The emphasis of this chapter is on explaining the properties of each type of machine, with its advantages and disadvantages with regard to other types; and on classifying these machines in terms of their performance characteristics and preferred field of application.

## ROTATING ELECTRIC MACHINES

The range of sizes and power ratings and the different physical features of rotating machines are such that the task of explaining the operation of rotating machines in a single chapter may appear formidable at first. Some features of rotating machines, however, are common to all such devices. This introductory section is aimed at explaining the common properties of all rotating electric machines. We begin our discussion with reference to Figure 14.1, in which a hypothetical rotating machine is depicted in a cross-sectional view. In the figure, a box with a cross inscribed in it indicates current flowing into the page, while a dot represents current out of the plane of the page.
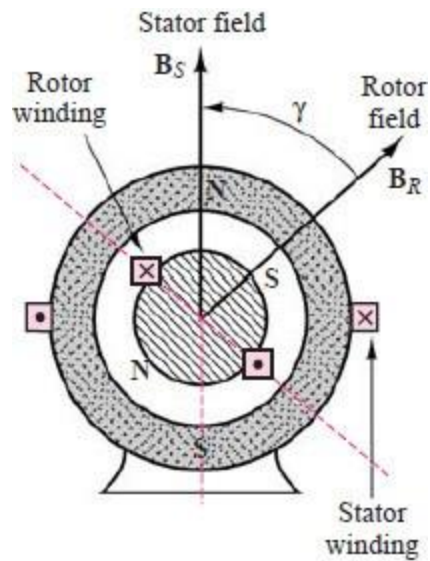
**Figure 14.1** A rotating electric machine

In Figure 14.1, we identify a **stator,** of cylindrical shape, and a **rotor,** which, as the name indicates, rotates inside the stator, separated from the latter by means of an air gap. The rotor and stator each consist of a magnetic core, some electrical insulation, and the windings necessary to establish a magnetic flux (unless this is created by a permanent magnet). The rotor is mounted on a bearing-supported shaft, which can be connected to *mechanical loads* (if the machine is a motor) or to a *prime mover* (if the machine is a generator) by means of belts, pulleys, chains, or other mechanical couplings. The windings carry the electric currents that generate the magnetic fields and flow to the electrical loads, and also provide the closed loops in which voltages will be induced.

**BASIC CLASSIFICATION OF ELECTRIC MACHINES**

An immediate distinction can be made between different types of windings characterized by the nature of the current they carry. If the current serves the sole

purpose of providing a magnetic field and is independent of the load, it is called a *magnetizing,*or excitation, current, and the winding is termed a **field winding.** Field currents are nearly always direct current (DC) and are of relatively low power, since their only purpose is to magnetize the core (recall the important role of high-permeability cores in generating large magnetic fluxes from relatively small currents). On the other hand, if the winding carries only the load current, it is called an **armature.** In DC and alternating-current (AC) synchronous machines, separate windings exist to carry field and armature currents. In the induction motor, the magnetizing and load currents flow in the same winding, called the *input winding,* or *primary;* the output winding is then called the *secondary*. As we shall see, this terminology, which is reminiscent of transformers, is particularly appropriate for induction motors.

It is also useful to classify electric machines in terms of their energy conversion characteristics. A machine acts as a **generator** if it converts mechanical energy from a prime mover, say, an internal combustion engine, to electrical form. Examples of generators are the large machines used in power generating plants, or the common automotive alternator. A machine is classified as a **motor** if it converts electrical energy to mechanical form. The latter class of machines is probably of more direct interest to you, because of its widespread application in engineering practice. Electric motors are used to provide forces and torques to generate motion in countless industrial applications. Machine tools, robots, punches, presses, mills, and propulsion systems for electric vehicles are but a few examples of the application of electric machines in engineering.

Note that in Figure 14.1 we have explicitly shown the direction of two magnetic fields: that of the rotor $\mathbf{B}_R$ and that of the stator $\mathbf{B}_S$. Although these fields are generated by different means in different machines (e.g., permanent magnets, alternating currents, direct currents), the presence of these fields is what causes a rotating machine to turn and enables the generation of electric power. In particular, we see that in Figure 14.1 the north pole of the rotor field will seek to align itself with the south pole of the stator field. It is this magnetic attraction force that permits the generation of torque in an electric motor; conversely, a generator exploits the laws of electromagnetic induction to convert a changing magnetic field to an electric current. To simplify the discussion in later sections, we now introduce some basic concepts that apply to all rotating electric machines. Referring to Figure 14.2, we note that for all machines the force on a wire is given by the expression

$$\mathbf{f} = iw\mathbf{l} \times \mathbf{B}$$

Where $iw$ is the current in the wire, $\mathbf{l}$ is a vector along the direction of the wire, and ×denotes the cross product of two vectors. Then the torque for a multi-turn coil becomes

$$T = KBiw\sin \alpha$$

Where

$B$ = magnetic flux density caused by stator field

$K$ = constant depending on coil geometry

$\alpha$ = angle between $\mathbf{B}$ and normal to plane of coil
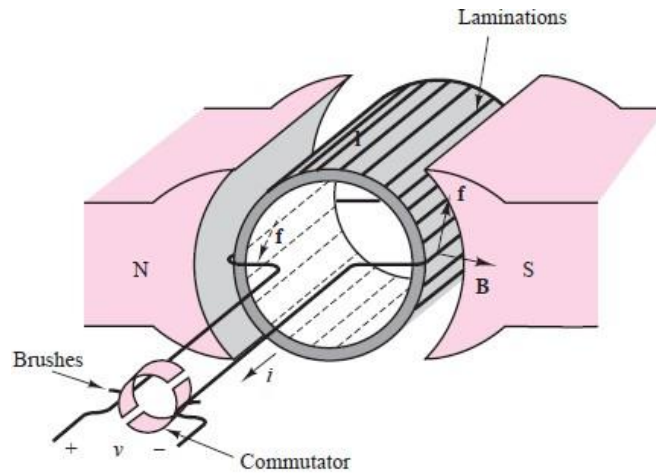
**Figure 14.2** Stator and rotor fields and the force acting on a rotating machine

In the hypothetical machine of Figure 14.2, there are two magnetic fields: one generated within the stator, the other within the rotor windings. Either (but not both) of these fields could be generated by a current or by a permanent magnet. Thus, we could replace the permanent-magnet stator of Figure 14.2 with a suitably arranged winding to generate a stator field in the same direction. If the stator were made of a toroidal coil of radius $R$ (see Chapter 13), then the magnetic field of the stator would generate a flux density $B$, where

$$B = \mu H = \mu \frac{Ni}{2\pi R}$$

and where $N$ is the number of turns and $i$ is the coil current. The direction of the torque is always the direction determined by the rotor and stator fields as they seek to align to each other (i.e., counterclockwise in the diagram of Figure 14.1). It is important to note that Figure 14.2 is merely a general indication of the major features and characteristics of rotating machines. A variety of configurations exist, depending on whether each of the fields is generated by a current in a coil or by a

permanent magnet and whether the load and magnetizing currents are direct or alternating.

## DC MACHINE

**FUNDAMENTAL:**

Emf induced in coil $\quad\quad\quad e = -N \, d\Phi/dt$ volt

Emf induced in a conductor $\quad e = Blv$ volt

Force developed in a conductor $F = BIL$ N

Electrical energy $\quad\quad\quad\quad E_n = VIt$ Whour

Electrical power (Active) $P = VI$ Watt

Resistance of conductor $R = \rho l/a$ Ohm

Eqn. of current by Ohm's law, $I = V/R$ A

Magnetizing force, $H = NI/L$ AT/m

Flux density, $B = \Phi/A$

Electrical field intensity, $E = V/d$ V/m

Current density, $J = I/a$ A/m$^2$

**INTRODUCTION :**

DC machine is a device which converts mechanical energy into electrical energy and vice versa.

When the device acts as a generator (or dynamo), mechanical energy is converted into electrical energy.

On the other hand, when the device acts as a motor, electrical energy is converted into mechanical energy.
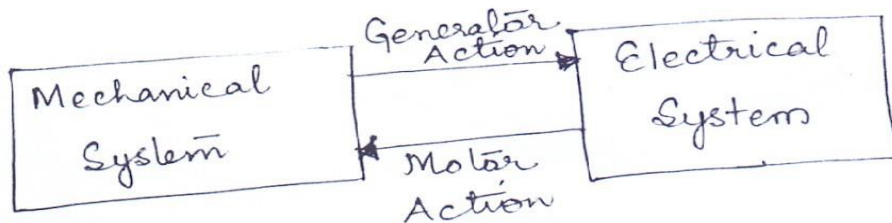


## ESSENTIAL FEATURES:

There is no real difference between a dc. motor and dc generator either from the point of view of the essential components or of the fundamental principles involved.

In the generator there is a conversion of mechanical to electrical energy while in the motor there is the reverse conversion of electrical to mechanical energy. The structure difference is merely one of the enclosure. Most generators work in sheltered situations e.g. in power stations and an open type of construction is thus possible, the advantage being that all the parts are in consequence, easily accessible. Motors often work in exposed

Situations, where there are dirt as damp, chemical fumes, explosive gases and so on, and it may therefore be necessary to enclose all the working parts.

## CONSTRUCTION:

| STATOR | ROTOR |
|---|---|
| Yoke/Frame | Armature core |
| Field magnet (pole core &pole shoe) | Armature winding/ conductor |
| Main field winding | Commutator & Brushes |
| Interpoles/commutating poles | Shaft |
| Interpoles winding | Bearings |

STATOR & ROTOR

All conventional electrical machines consist of a stationary member called the stator separated by an air gap from a rotating member called rotor.

In d.c machines the stator usually consists of salient poles with coils wound round them so as to produce a magnetic field.

The rotor is familiarly called the armature and consists of a series of coils located in slots around its periphery and connected to a commutator.

Yoke/Frame

Yoke is the outer frame dc m/c. It carries the magnetic flux provided by the pole and acts as a protecting shield for the entire machine. In small generators yokes are made of cast iron whereas in large machine cast sheet is used.
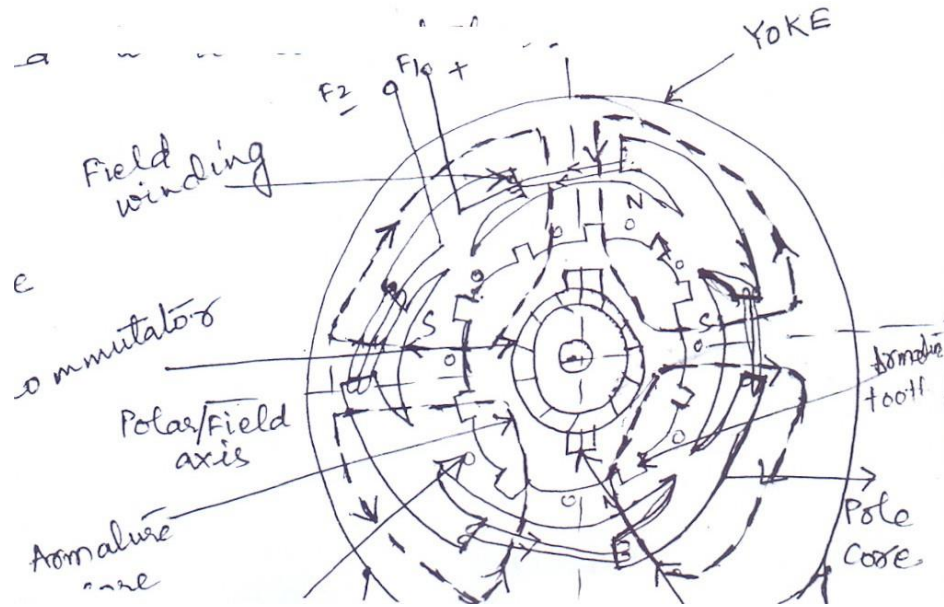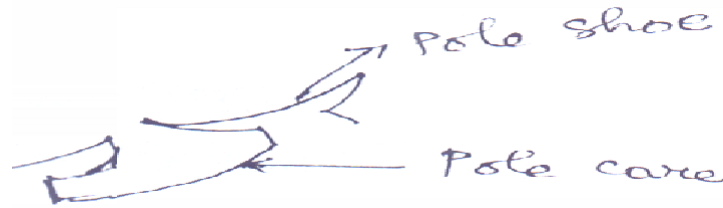
Field magnet :

It is a strong permanent magnet (in case of a small dynamo) or an electromagnet (in case of large dynamo) of intense magnetic field.

Pole core & Pole shoe : The field magnet has two parts

Pole core (pole pieces)

Pole shoes.

Pole core is made of cast steel or cast iron with laminated pole shoes screwed on to the holes in the yoke.

Pole shoes spread out the flux in the air gap and reduce the reluctance of the magnetic path due to its large cross -section.

Poles shoes support the exciting coils.

Pole coils (field winding)

- It is otherwise known as exciting winding
- These are fine copper wire (or strip) wound around the pole pieces
- The flux produced by the winding is cut by the revolving armature

ARMATURE CORE:

- It houses armature coils in the slots. It is cylindrical or drum shaped.
- Armature is placed in between the two poles of field magnet and is rotated about its central axis mechanically (by a prime mover)
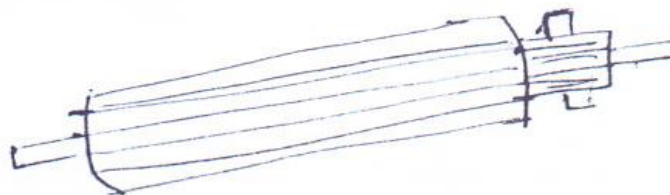


- It is usually made of circular sheet steel disc or laminations.

- Thickness of laminations is of the order of 0.5mm. Perforations exist in these laminations to provide axial flow of air through the armature for cooling purposes.
- Armature core is keyed to the shaft.
- During rotation it cuts the magnetic flux of the field magnets.
- It also provides a path of very low reluctance to the flux from north pole to south pole.

ARMATURE WINDING:

- It is made up of copper.
- It consists of large no. of insulated coils, each coil having one or more turns.
- Armature conductor are placed in armature slots.

Commutator:

- It converts the alternating current produced in the armature conductors into direct current.
- It consists of wedge shaped copper segments (Insulated from either side to form a ring. The number of segments of commutator equals the no. of armature coils.
- It facilitates the collection of current from the armature conductors.

Brushes

- They are fixed on the commutator by pressure springs.
- They are usually made of a high grade carbon or graphite and are in the shape of rectangular block.

- These are housed in a brush holder.
- They collect the current from the commutator and finally they pass on the EMF generated to load.

## Bearing

- The armature shaft is supported at the commutator end on ball or roller bearings which are packed in hard oil.
- They provide quieter operation, together with reduced wear and tear.
- Ball bearings are frequently employed for their reliability.
- Roller bearings are preferable for heavy duties.

## Interpoles

- In addition to the main poles, the dc machines are usually fitted with auxiliary poles placed between the main poles and they are called interpoles, the purpose of which is to provide better commutation.
- It minimizes sparking at the commutator.

## **Types of Armature Winding**

These are of two types : Lap winding
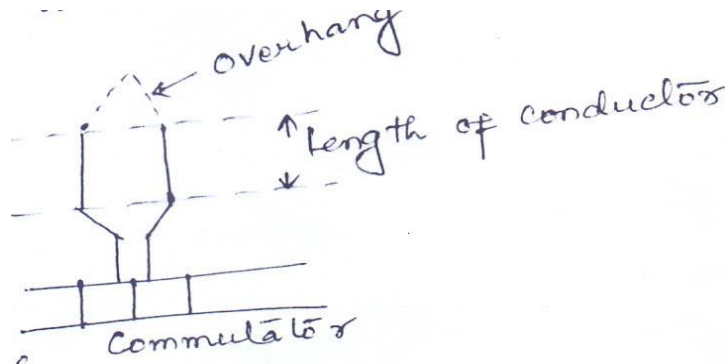
Wave winding

## Pole Pitch

- It is defined as the no of armature conductors per pole.
- If there are 80 conductors and 8 poles, pole pitch is $80/8 = 10$.

Conductor

- It is defined as the length of a wire lying the magnetic field as shown in fig1.

Coil

- Two conductors with their end connections shown in fig1 from one coil.



- Coil may be single turn coil or multi-turn coil.
- End connections are called overhang.

Coil Pitch

- It is defined as the distance between two sides of a coil in terms of armature slots.
- It is defined as the periphery of the armature between two sides of a coil.
- If the coil pitch is equal to the pole pitch, the winding is said to be full pitched, otherwise it is half-pitch

Pitch:

It is defined as the distance between the second conductor one coil and the first conductor of the next coil as shown in fig.2 & 3.
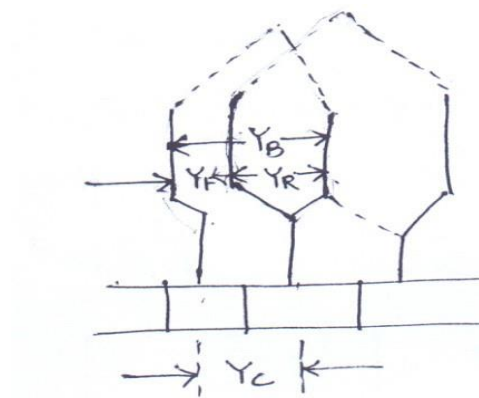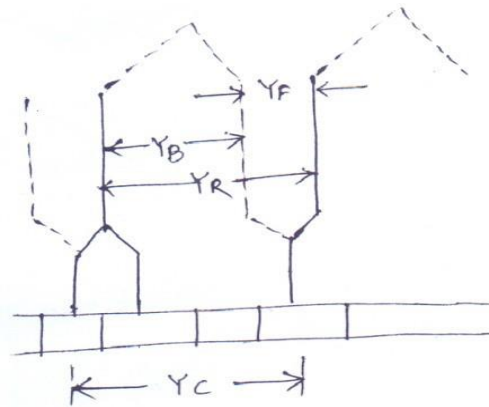
Fig 2. Lap wdg

Fig 3. Wave winding

Back Pitch :

It is defined as the distance measured in terms of armature conductors that a coil advances on the back side of the armature as shown in fig.2 & 3.

Resultant Pitch: ($Y_R$)

It is defined as the distance between the beginning of one coil and the beginning of the next coil or

lap winding $\quad$ $Y_R = Y_B - Y_F$

wave winding $\quad$ $Y_R = Y_B + Y_F$

Coil Pitch ($Y_c$):

It is defined as the distance between the beginning of one coil to the beginning of the next coil as shown in fig 2 & 3 for lap and wave winding respectively.

Lap winding:

To get these type foe dg following rules are commonly used: winding must be full pitched i.e.

$$Y_B + Y_F = \text{Pole Pitch}$$

1. To place the coil properly on the armatures, the front pitch as well as back pitch must be odd.
2. No. of commutator segment = no. of coils.
3. winding must close upon itself.
4. $Y_c = \pm 1$

   For Progressive or right handed winding

   $Y_F = Z/P - 1$  and  $Y_B = Z/P + 1$

   For Retrogressive or left handed winding

   $Y_F = Z/P + 1$  and  $Y_B = Z/P - 1$

Wave Winding:

1. $Y_B$ & $Y_F$ are odd and of opposite sign.
2. $Y_B$ & $Y_F$ are nearly equal to the pole pitch and differ by 2.
3. $Y_C = (\text{No. of commutator bars} \pm 1) / \text{No. of pairs of pole}$

Comparison:

| Lap winding | Wave winding |
|---|---|
| No. of parallel paths = no. of poles | No of parallel path = 2 |
| No. of brushes sets = no. of poles | No. of brushes sets = 2 |
| Used for high current & low voltage | Used for low current & high voltage |

## PRINCIPLE OF OPERATION OF DC GENERATOR:

**Principle:**

Faradays law of electromagnetic induction according to which emf is induced in the conductors which drives the current through the conductor.

Fig below shows a single turn generator.

The coil is rotated in anticlockwise direction with the help of prime mover.

Fig. shows the different instants of the induced emf due to different positions of the coil.

Magnetic field is produced either by a permanent magnet or an electromagnet energised by the DC supply.

When the angle $\theta = 0°$ , the coils is perpendicular to the magnetic field and the instantaneous component of velocity of conductors ab and cd is parallel to the magnetic field .
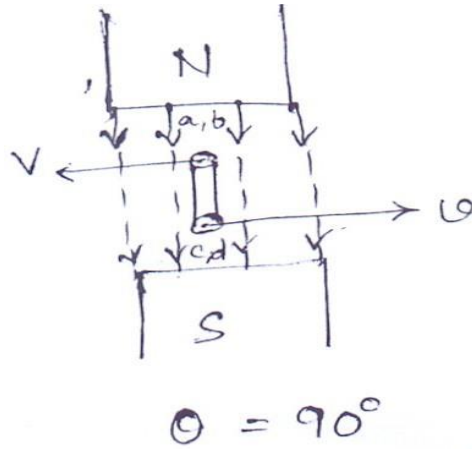


fig 1 .
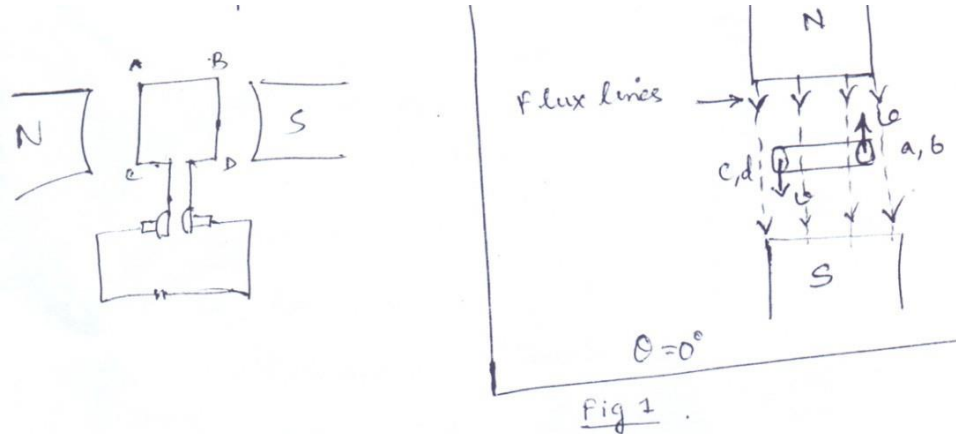
The flux linked with the coil is maximum but the rate of change of flux is minimum. Hence emf cannot be generated . Therefore current flows through the conductors.
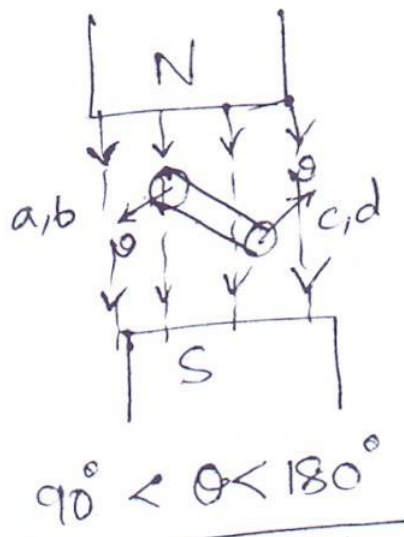
When 0 <θ<90°, due to the rotation of the coil in the anticlockwise direction, the velocity of the conductor has two components that is vsinθ and vcosθ . Therefore the conductors will cut the flux due to vsinθ. Hence emf is induced in the conductors and current flows in the conductors.



$$0° < \theta < 90°$$
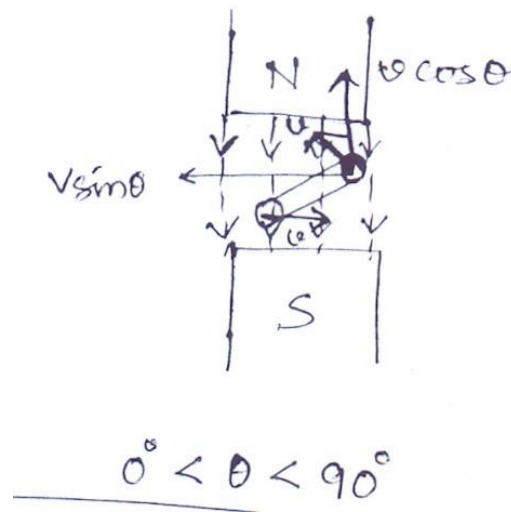
If when θ = 90°, coil is parallel to the magnetic field and the instantaneous component of velocity of conductors ab and cd is perpendicular to the magnetic field. The flux linked with the coil is minimum but the rate of change of flux is maximum. Hence emf generated is maximum . Therefore maximum current flows through the conductors.

$\Theta = 90°$

When the coil rotates from $\theta = 90°$ to $\theta = 180°$, the components of velocity that is $\sin \theta$ of conductors perpendicular to the field decreases and hence emf is induced in the conductor and current flow in the conductors.



$90° < \theta < 180°$

When $\theta = 180°$, the coil is perpendicular to the magnetic field and the instantaneous components of velocity of conductors ab and cd is parallel to the magnetic field.
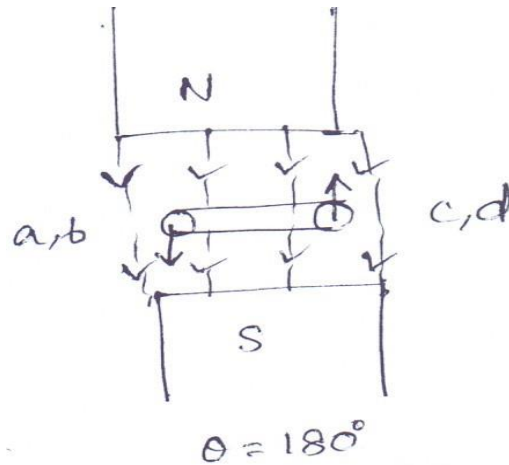
$\theta = 180°$

The flux linked with the coil is maximum but the rate of change of flux is minimum. Hence emf cannot be generated. Therefore the current flows through the conductor.

When 180°,$< \theta < 270°$ due to rotation of the coil in the anticlockwise direction, the velocity of the conductors has two components that is vsin $\theta$ and vcos $\theta$ and the direction is opposite to the case when $0^0 < \theta < 90^0$ . Therefore the conductor have the flux due to vsin .



$180 < \theta < 270°$
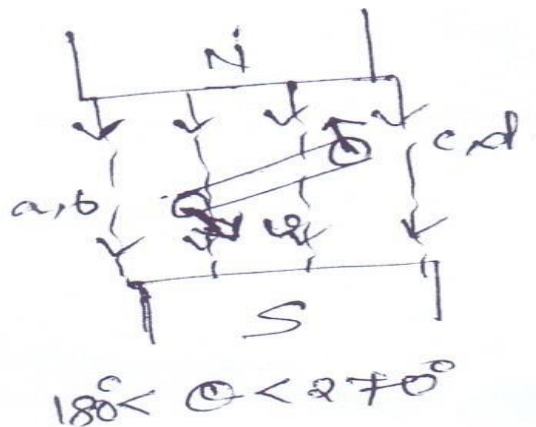
Hence emf is induced in the conductors in the direction opposite to the cases of $0^0 < \theta < 90^0$ and current flows in the conductors in the direction opposite to the $0^0 < \theta < 90^0$. Similarly the other case till $\theta = 360^0$.
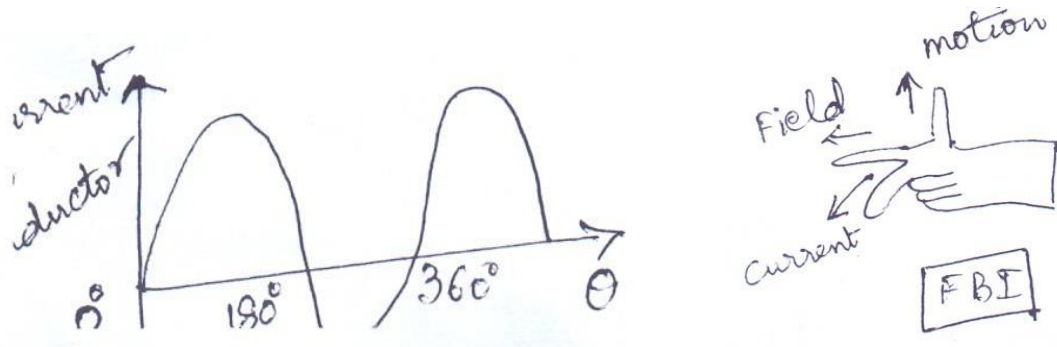


The direction of flow of current through the conductor can be found by Flemings right hand rule. To have a unidirectional current communicator is used.

Fig below shows a split ring where two segments A and B are separated by an insulating material. Let the conductor 1 be connected to segment A and conductor 2 be connected to segment B.
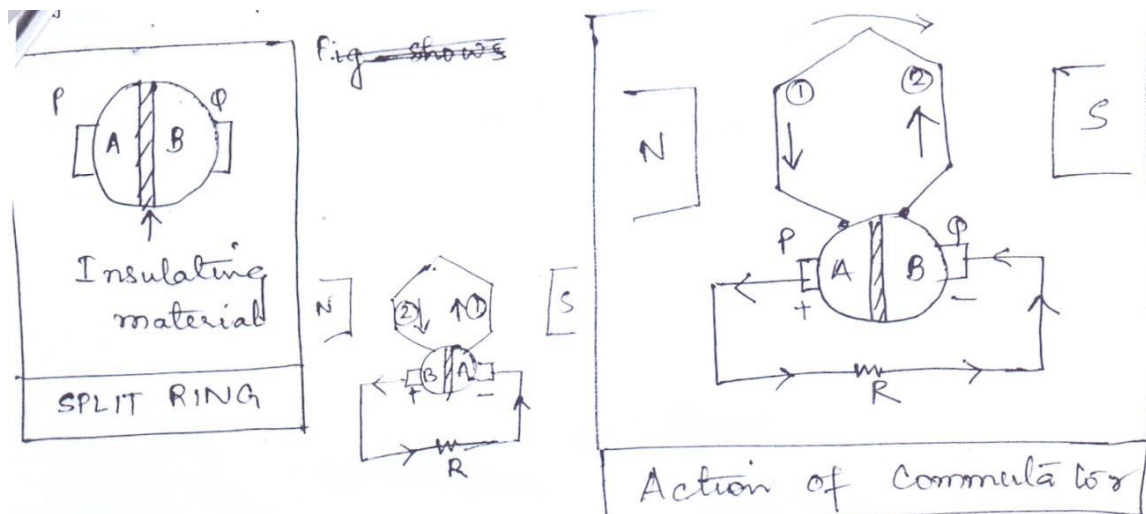


The conductor 1 carries current in the downward direction while the conductor 2 carries current in the upward direction and current through the resistance R flows

from P to Q. After one half of rotation, conductor 1 and 2 come under the influence of south and north pole respectively.

The split ring is mounted on the shaft and it rotates with the armature. The split ring will reverse this positions with the reversing of the position of the conductors, but the direction of current flow through the conductor remains unaltered because the P & Q are stationary.

## Equivalent circuit of a DC Machine
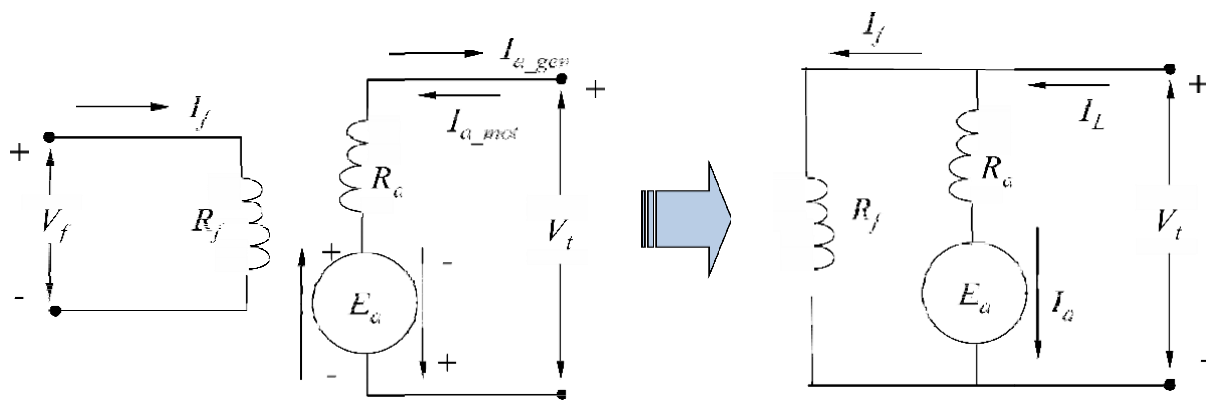


$$V_f = I_f R_f$$
$$V_t = E_a \pm I_a R_a$$

## Generated EMF and Electromagnetic Torque

$$V_f = I_f R_f$$
$$V_t = E_a \pm I_a R_a$$

Motor: $V_t > E_a$

Generator: $V_t > E_a$

Voltage generated in the armature circuit due the flux of the stator field current:

$$E_a = K_a \, \phi_d \, \omega_m$$

$K_a$: design constant

Electromagnetic torque:

$$T_e = K_a \, \phi_d \, I_a$$

$$P_{em} = E_a \, I_a = T_e \omega_m$$

**Types of DC Machines**

Both the armature and field circuits carry direct current in the case of a DC machine.

Self-excited DC machine: when a machine supplies its own excitation of the field windings. In this machine, residual magnetism must be present in the ferromagnetic circuit of the machine in order to start the self-excitation process.

Separately-excited DC machine: The field windings may be separately excited from an eternal DC source.

Shunt Machine: armature and field circuits are connected in parallel. Shunt generator can be separately-excited or self-excited.

Series Machine: armature and field circuits are connected in series.

## Separately-Excited and Self-Excited DC Generators
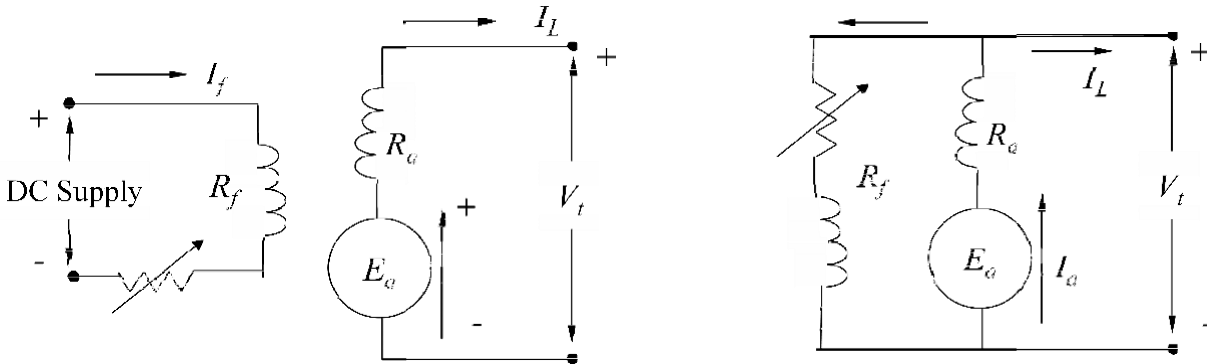


**Example:** A 100-kW, 250-V DC shunt generator has an armature resistance of 0.05 W and field circuit resistance of 60 W. With the generator operating at rated voltage, determine the induced voltage at (a) full load, and (b) half-full load.

Ans. (a) At full load,

$V_t = E_a - I_a R_a$

$I_f = 250/60 = 4.17$ A

$I_{L\_FL} = 100,000/250 = 400$ A

$I_a = I_{L\_FL} + I_f = 400 + 4.17 = 404.17$ A

$E_a = V_t + I_a R_a = 250 + 404.17*0.05 = 270.2$ V

(b) At half load,

$I_f = 250/60 = 4.17$ A

$I_{L\_HL} = 50,000/250 = 200$ A

$I_a = I_{L\_HL} + I_f = 200 + 4.17 = 204.17$ A

$E_a = V_t + I_a R_a = 250 + 204.17*0.05 = 260.2$ V

# MODULE-III (10 HOURS)

## AC MACHINES

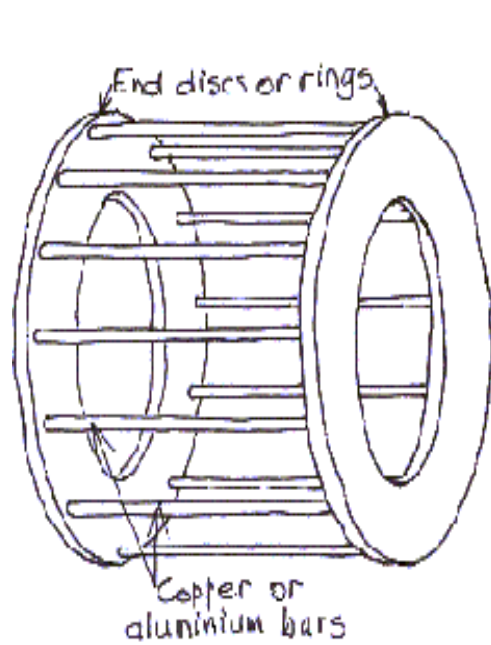AC machines have large advantages over DC machines. They are,

- Since basic generation of electricity is in the form of AC voltage, no converting device is necessary.

- Suitable for large ratings (upto more than 500 MW)

- Cost of an AC machine for same power and voltage rating is less than that of DC machines

- Since transformers are used in AC only, AC generation is must for commercial purpose operation.
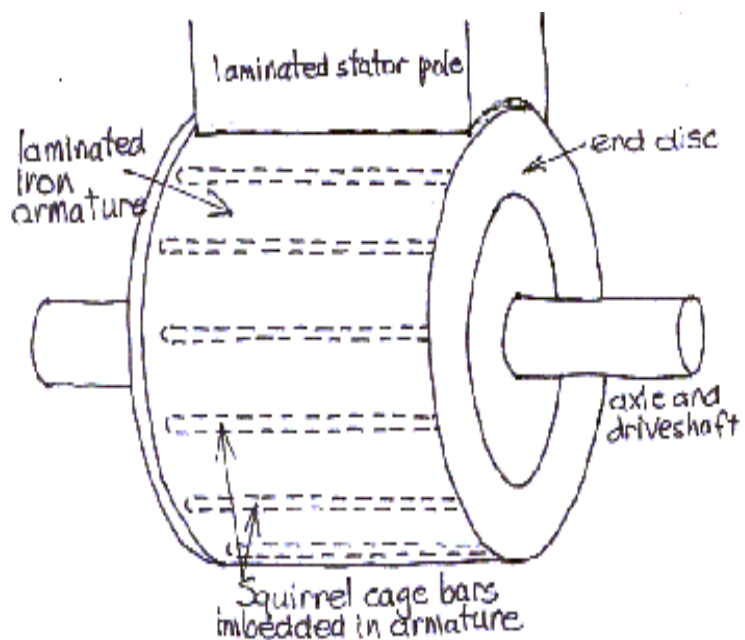
AC machines are broadly classified as:-

- Synchronous machine

➢ Operates at constant speed and constant frequencies under steady state condition

➢ Generator known as alternator

➢ Faradays laws of electromagnetic induction

➢ Does not have commutator

➢ EMF is generated from mutual interaction between conductors and magnetic flux.

- Asynchronous machine

➢ Operates with variable speed

➢ Induction machine

➢ Induction motor application in air conditioning, industrial use (rolling mills)

➢ Induction generator application in wind generating system :- convert wind energy to electrical energy at constant frequency irrespective of wind speed
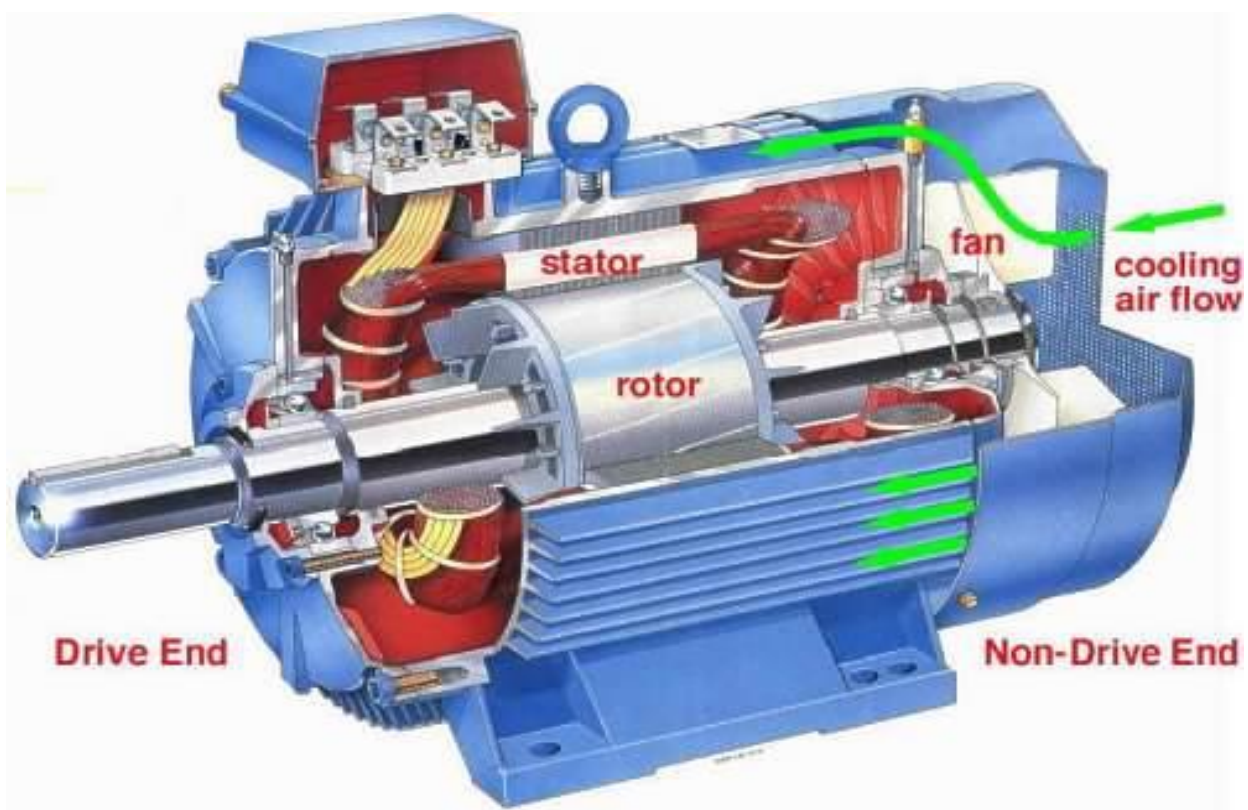
**Three phase Induction Motor:**

- Three-phase induction motors are the most common and frequently encountered machines in industry
    - simple design, rugged, low-price, easy maintenance
    - wide range of power ratings: fractional horsepower to 10 MW
    - run essentially as constant speed from no-load to full load
    - Its speed depends on the frequency of the power source
- Construction:-
- **Stator** :- three phase winding in star or delta
- **Rotor** :-
- **Squirrel cage rotor**
- rotor windings are shorted internally
- **Slip ring or phase wound rotor**
- rotor windings are short circuited externally through three slip rings
- Rotor windings are short circuited
- Slip (S) :- difference between the speed of rotor (N) and the speed of rotating magnetic field (Ns). It is expressed in % as:

    S= [ (Ns – N)/ Ns] x 100

- Slip varies from syn. Speed (Ns) to 1 (When the rotor is at rest)

End discs or rings

"Squirrel cage" rotor

Copper or aluminium bars

laminated stator pole

laminated iron armature

end disc

axle and driveshaft

Squirrel cage bars imbedded in armature

Induction motor rotor

stator

rotor

fan

cooling air flow

Drive End

Non-Drive End

Slip rings

Shaft

Brush

R    Y    B

External star
connected rheostat

Star connected
rotor winding

Rotor
frame

FRAME

ROTOR

STATOR

BRUSHES

MOTOR
SHAFT

SLIP RINGS

BRUSHES

END SHIELDS
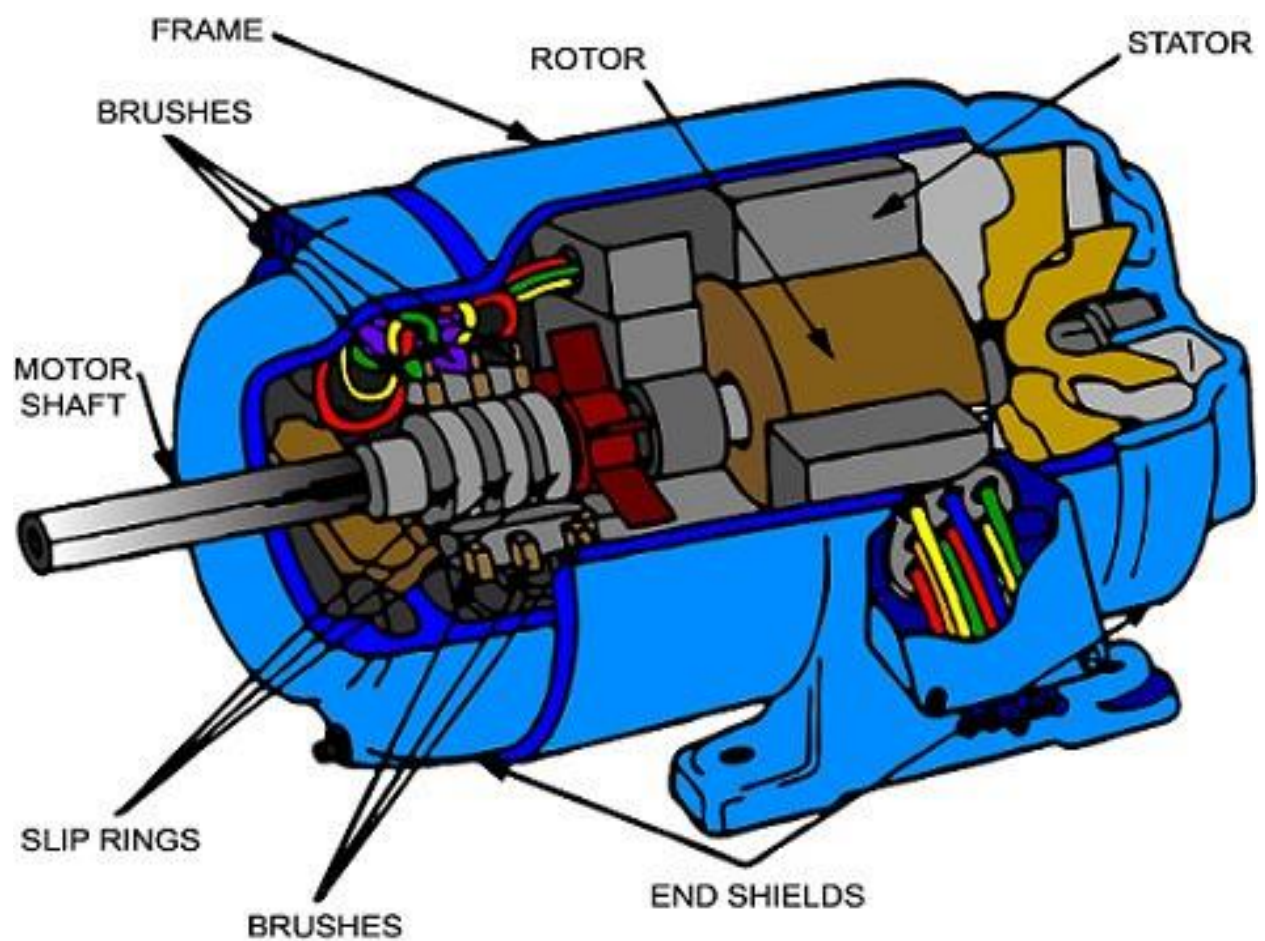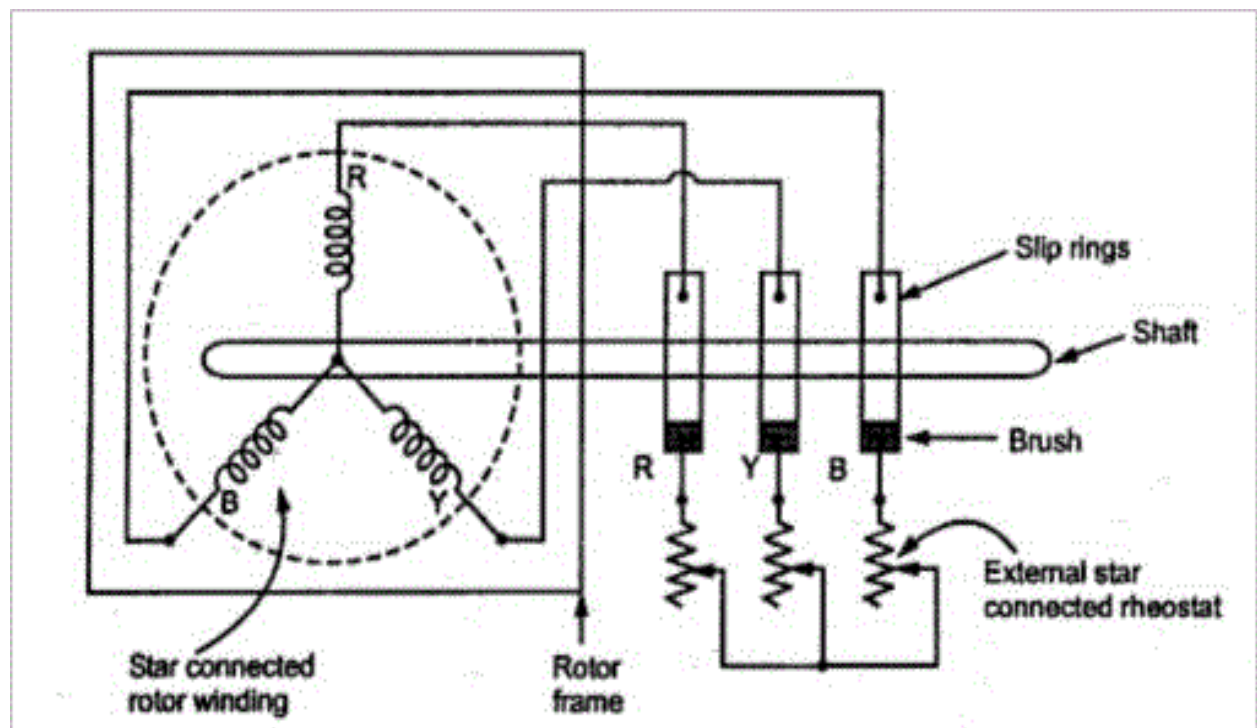
### Rotating Magnetic Field

- Balanced three phase windings, i.e. mechanically displaced 120 degrees form each other, fed by balanced three phase source

- A rotating magnetic field with constant magnitude is produced, rotating with a speed

$$n_{sync} = \frac{120 f_e}{P} \quad rpm$$

Where $f_e$ is the supply frequency and

$P$ is the no. of poles and $n_{sync}$ is called the synchronous speed in *rpm* (revolutions per minute)
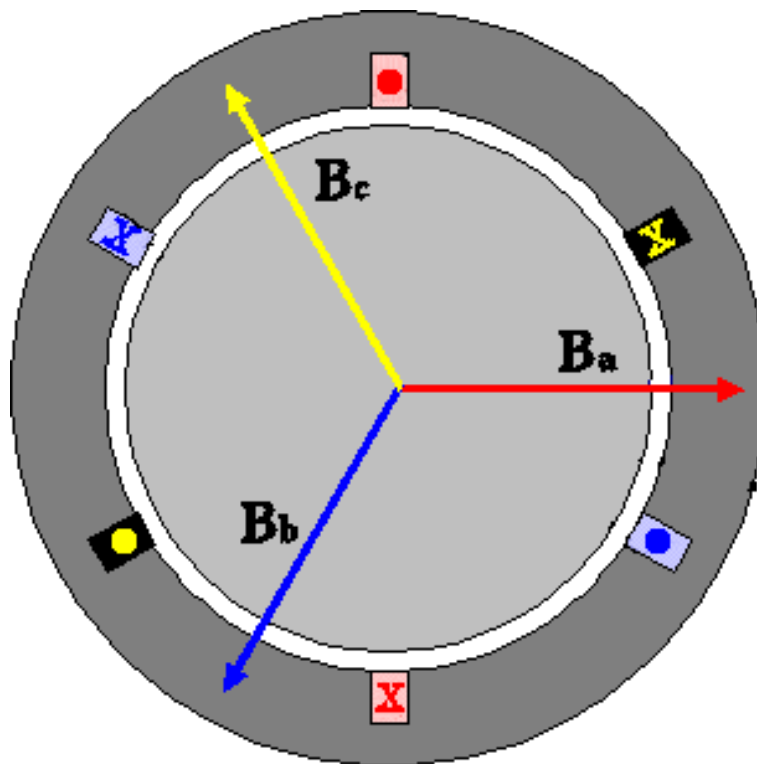
**Principle of operation:**

- This rotating magnetic field cuts the rotor windings and produces an induced voltage in the rotor windings

- Due to the fact that the rotor windings are short circuited, for both squirrel cage and wound-rotor, and induced current flows in the rotor windings

- The rotor current produces another magnetic field

A torque is produced as a result of the interaction of those two magnetic fields

$$\tau_{ind} = kB_R \times B_s$$

Where $\tau_{ind}$ is the induced torque and $B_R$ and $B_S$ are the magnetic flux densities of the rotor and the stator respectively

**Slip**: The difference between synchronous speed Ns and actual speed N of the rotor is known as slip.

$$\% \ Slip = \frac{N_S - N}{N_S} \times 100\%$$

Sometimes, Ns- N is called slip speed

**Frequency of rotor (f′) :**

$$N_S - N = \frac{120 f'}{P}$$

$$N_S = \frac{120 f}{P}$$

$$\frac{N_S - N}{N_S} = \frac{f'}{f}$$

$$f' = sf$$

**Torque Equation:**

$$T_{EM} = \frac{60}{2\pi N_S} * rotor \ .input$$

Unit- synchronous watt

## TORQUE- SLIP OR TORQUE-SPEED CHARACTERISTICS

$$T_{EM} = \frac{180}{2\pi N_S} * \frac{SE_2{}^2 R_2}{R_2{}^2 + (SX_2)^2}$$

For a constant supply voltage, E2 is also constant. So we can write torque equation as:-

$$T \propto \frac{S R_2}{R_2{}^2 + (S X_2)^2}$$
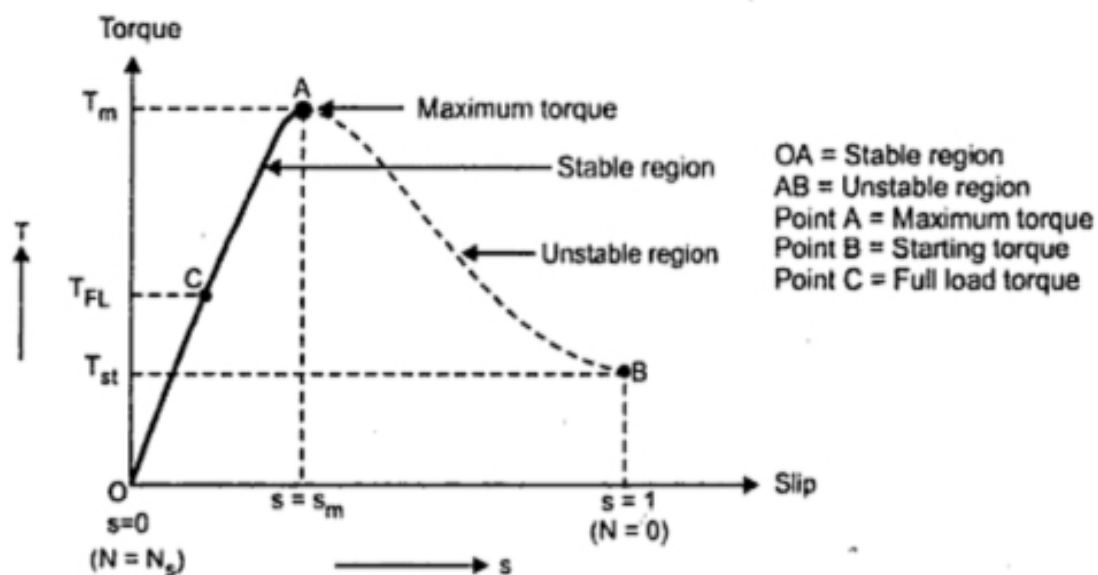
As R2 is constant.

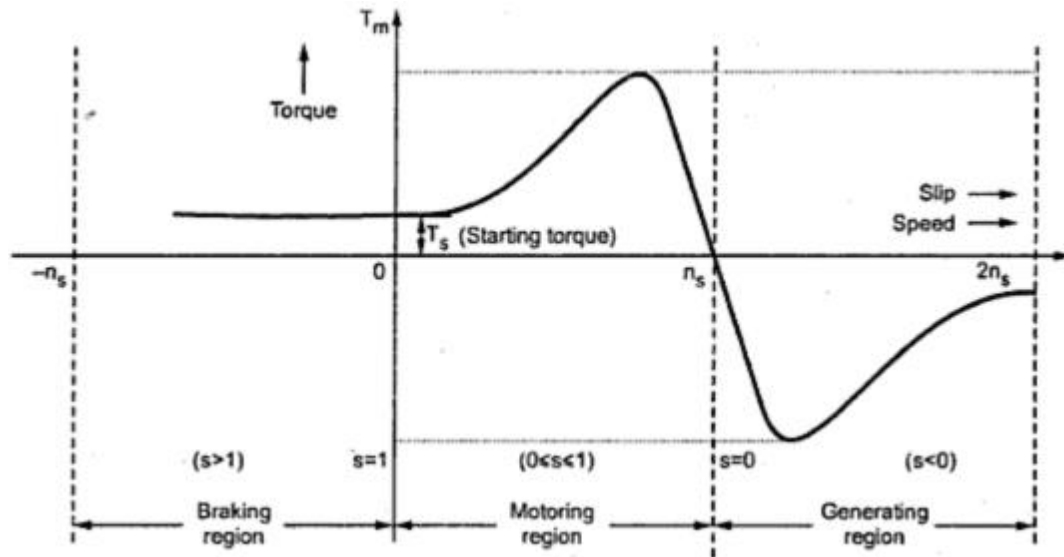1) HIGH SLIP REGION –
   Here S is high

So,

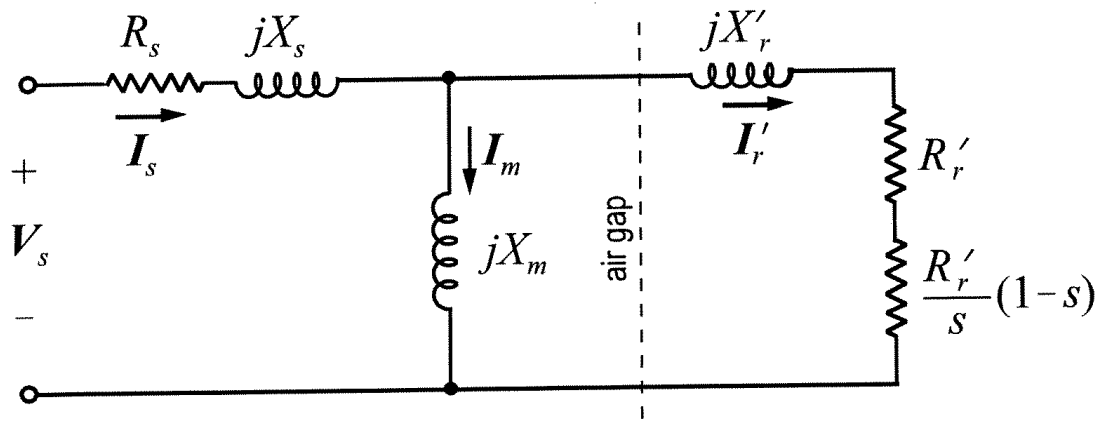$$T \propto \frac{SR_2}{(SX_2)^2} \propto \frac{1}{S}$$

As R2 and X2 are constants

$$T_{Full\ Load} < T_M$$



Torque

$T_m$ — Maximum torque

Stable region

$T_{FL}$

$T_{st}$

Unstable region

OA = Stable region
AB = Unstable region
Point A = Maximum torque
Point B = Starting torque
Point C = Full load torque

$s = s_m$

$s = 1$
$(N = 0)$

$s=0$
$(N = N_s)$

Slip

$s$

## EQUIVALENT CIRCUIT



### Introduction to 1-phase induction motor:

- Used for domestic application
- 1-phase IM are fractional KW motors
  - Rotor – squirrel cage
  - Stator – distributed

### Types of 1-phase IM:

- Resistance split phase IM
- Capacitor split phase IM
  - Capacitor start IM

- o Capacitor start capacitor run IM
- o Capacitor run IM

## TEXT BOOK

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[3]. C.L. Wadhwa, *"Electrical Engineering",* New Age International Publishers.

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[5]. S. Parker Smith: *"Problems in Electrical Engineering"* Asia Publications.