SYNERGY INSTITUTE OF ENGINEERING & TECHNOLOGY

Department of Electrical Engineering

LECTURE NOTE

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Degree	**:B. TECH.**
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Branch	**:ELECTRICAL ENGINEERING**
Semester	**:3RD**
Name of Subject	**:NETWORK THEORY**
Subject Code	**:REE3C002**
Subject Credit	**:3**

MODULE1

NETWORKTOPOLOGY

1. **Introduction**:Whenalltheelementsinanetworkarereplacesbylineswithcirclesordotsatbothe nds,configurationiscalledthegraphofthenetwork.

A. **Terminology used in network graph**:-

(i) **Path:-**Asequenceofbranchestraversedingoingfromonenodetoanotheriscalleda path.

(ii) **Node:-**A nodepoint is defined as an end point of a line segment and exits at thejunctionbetweentwobranchesorat theendofanisolatedbranch.

(iii) **Degreeofanode:-**Itistheno.ofbranchesincidenttoit.

2-degree 3-degree

(iv) **Tree:-**It is an interconnected open set of branches which include all the nodes ofthe given graph. In atree of thegraphtherecan'tbeanyclosedloop.

(v) **Treebranch(Twig):-** It isthe branch of atree. It is alsonamed as twig.

(vi) **Tree link(or chord):-**It is the branch of a graph that does not belong to theparticular tree.

(vii) **Loop:-**Thisis the closed contour selectedin a graph.

(viii) **Cut-Set:-**It is that set of elements or branches of a graph that separated two partsof a network. If any branch of the cut-set is not removed, the network remainsconnected. The term cut-set is derived from the property designated by the way bywhichthe network can be dividedintotwoparts.

(ix) **Tie-Set:-**It is a unique set with respect to a given tree at a connected graphcontaining on chord and all of the free branches contained in the free path formedbetween two vertices of the chord.

(x) **Networkvariables:-**Anetworkconsistsofpassiveelementsaswellassourcesofenergy . In order to find out the response of the network the through current andvoltagesacross each branch of the network are tobe obtained.

(xi) **Directed(orOriented)graph:-**Agraphissaidtobedirected(ororiented)whenall the nodes and branches are numbered or direction assigned to the branches byarrow.

(xii) **Subgraph:-**Agraph G_s saidtobesub-graphof agraphGifeverynode of G_s isanodeofGandeverybranchof$G_s$isalsoabranchofG.

(xiii) **Connected Graph:-**When at least one path along branches between every pair ofagraphexits,itiscalledaconnectedgraph.

(xiv) **Incidence matrix:-**Any oriented graph can be described completely in a compactmatrix form. Here we specify the orientation of each branch in the graph and thenodesat whichthisbranch is incident.Thisbranchiscalledincidentmatrix.

When one row is completely deleted from the matrix the remainingmatrixis called a reducedincidencematrix.

(xv) **Isomorphism:-** Itisthepropertybetweentwographssothatbothhavegotsameincidencematrix.

B. Relationbetweentwigsandlinks-

Let N=no.ofnodes

L= total no. of links

B= total no. of branchesNo.of twigs= N-1

Then,L= B-(N-1)

C. PropertiesofaTree-

(i) It consists ofallthe nodes of the graph.

(ii) IfthegraphhasNnodes,thenthetreehas(N-1)branch.

(iii) Therewillbenoclosedpathinatree.

(iv) There can be many possible different trees for a given graph depending on the no.of nodes and branches.

1. FORMATIONOFINCIDENCEMATRIX:-

- Thismatrixshowswhichbranchisincidenttowhichnode.
- Each row of the matrix being representing the corresponding node of thegraph.
- Each column corresponds to a branch.
- If a graph contain N- nodes and B branches then the size of the incidencematrix[A]will be NXB.

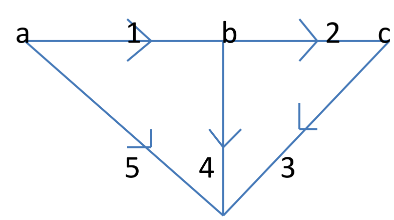
A. Procedure:-

(i) If the branch j is incident at the node I and oriented away from the node, a_{ij} =1.Inotherwords,when a_{ij} =1,branchjleavesawaynodei.

(ii) If branch j is incident at node j and is oriented towards node i, a_{ij} =-1. In otherwords j enters node i.

(iii) Ifbranchjis notincidentat nodei. a_{ij} =0. Thecompletesetofincidencematrixiscalledaugmentedincidencematrix.

<u>Ex-1:-</u>Obtainthe incidence matrixofthe followinggraph.



Node-a:-Branchesconnected are1& 5and bothare awayfrom thenode.

Node-b:- Branches incident at this node are 1,2 &4. Here branch is oriented towardsthe nodewhereasbranches2&4 are directedaway fromthe node.

Node-c:- Branches 2 &3 are incident on this node. Here, branch 2 is oriented towardsthenodewhereasthe branch3is directedawayfromthe node.

Node-d:- Branch 3,4 &5 are incident on the node. Here all the branches are directedtowardsthenode.

So,bra
nch

Node	1	2	3	4	5
1	1	0	0	0	1
[A_i]=2	-1	1	0	1	0
3	0	-1	1	0	0
4	0	0	-1	-1	-1

B. Properties:-

(i) Algebraicsum ofthe columnentriesofan incidencematrix iszero.

(ii) Determinantofthe incidencematrixofaclosedloopiszero.

C. ReducedIncidenceMatrix :-

Ifanyrowofamatrixiscompletelydeleted,thentheremainingmatrixisknownasreducedIncidencematrix. Forthe above example,after deleting row,
we get,

$$[A_i']=\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix}$$

A_i'isthe reduced matrix ofA_i.

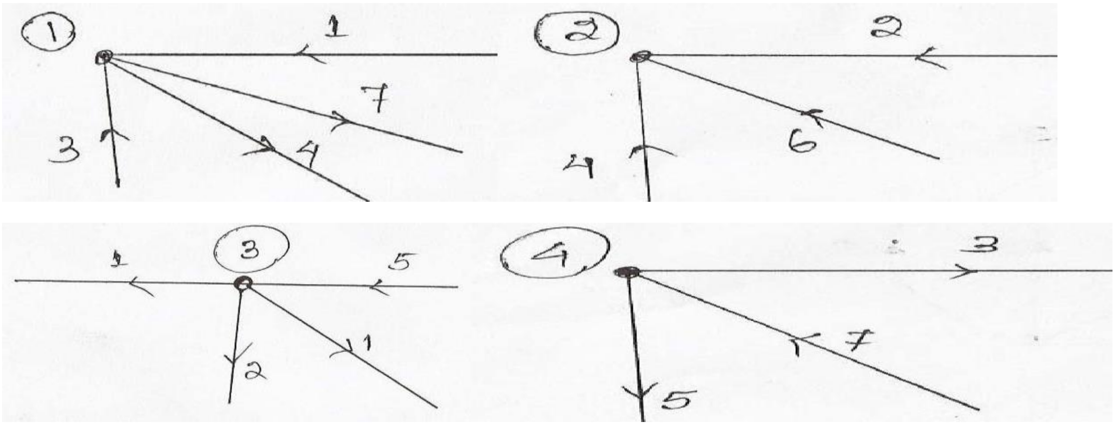
<u>Ex-2</u>: Drawthedirectedgraphforthefollowingincidencematrix.

Branch

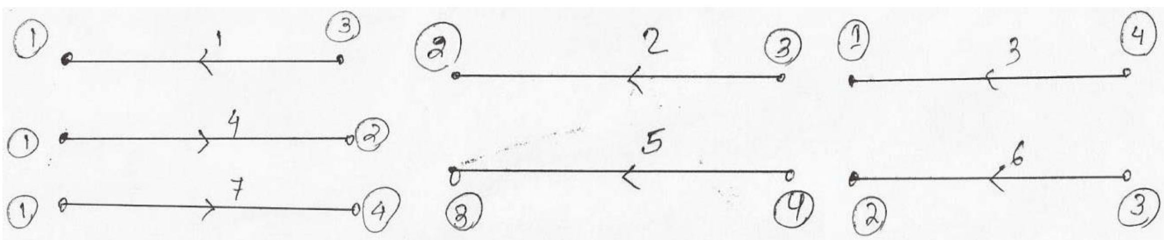
Node	1	2	3	4	5	6	7
1	-1	0	-1	1	0	0	1
2	0	-1	0	-1	0	-1	0
3	1	1	0	0	-1	1	0
4	0	0	1	0	1	0	-1

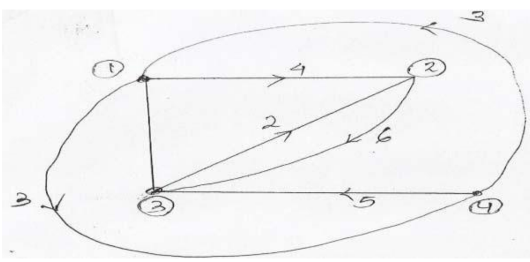
Solution:-

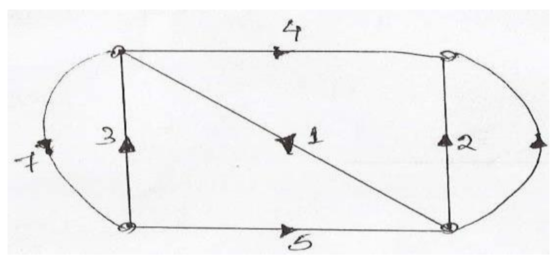
Fromnode



Frombranch







Tie-setMatrix:

		Branch				
		1	2	3	4	5
Loop currents	I_1	1	0	0	1	1
	I_2	-1	-1	1	0	-1

$$Bi = \begin{vmatrix} 1 & 0 & 0 & 1 & 1 \\ -1 & -1 & 1 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & -1 & 0 & 1 \end{vmatrix}$$

Let V_1, V_2, V_3, V_4& V_5 be the voltage of branch 1,2,3,4,5 respectively and j_1, j_2, j_3, j_4, $j_5$arecurrentthrough the branch 1,2,3,4,5 respectively.

So, algebraic sum of branch voltages in a loop is zero.Now,wecan write,

$$V_1+V_4+V_5=0$$

$V_1+V_2-V_3+V_5=0$

Similarly, $j_1=I_1-I_2$ $j_2=-I_2$ $j_3=I_2$ $j_4=I_1$

$j_5=I_1-I_2$

Fundamentalof cut-set matrix:-

A fundamental cut-set of a graph w.r.t a tree is a cut-set formed by onetwig and a set of links. Thus in a graph for each twig of a chosen tree there would be afundamental cutset.

No.of cut-sets=No.of twigs=N-1.

Procedureofobtainingcut-setmatrix:-

(i) Arbitrarilyattreeisselectedinagraph.

(ii) Fromfundamentalcut-setswith eachtwigin thegraphfortheentiretree.

(iii) Assumedirectionsofthecut-setsorientedinthesame directionoftheconcernedtwig.

(iv) Fundamentalcut-setmatrix[Q_{ki}]

Q_{kj} =+1;whenbranch$b_i$hasthesame orientationofthecut-set Q_{kj} =-1;when branch b_i hastheoppositeorientationofthecut-set

Q_{kj} =0; when branch$b_j$isnot in the cut-set

Fundamentalof Tie-set matrix:-

Afundamentaltie-setofagraphw.r.tatreeisaloopformedbyonlyonelinkassociatedwithothertwigs.

No.of fundamentalloops=No. oflinks=B-(N-1)

ProcedureofobtainingTie-setmatrix:-

(i) Arbitrarilyatreeisselectedinthegraph.

(ii) Fromfundamental loops witheach link inthe graph for theentire tree.

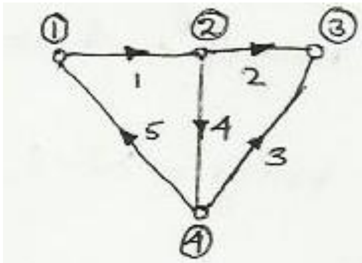
(iii)Assume directions of loop currents oriented in the same direction as that of the link.(iv)From fundamentaltie-setmatrix[b_{ij}] where

b_{ij} =1; when branchb_j is in the fundamental loop i and their reference directions areorientedsame.

b_{ij} =-1; when branchb_j is in the fundamental loop i but, their reference directions areoriented oppositely.

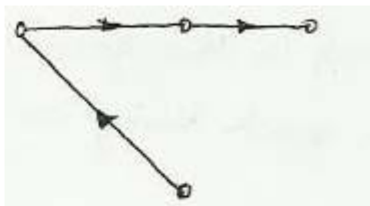
b_{ij} =0;when branchb_j isnot in thefundamental loop i.

Ex-3: Determinethetiesetmatrixofthefollowinggraph.Alsofindtheequationofbranchcurrentandvoltages.



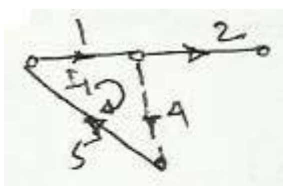
Solution

Tree

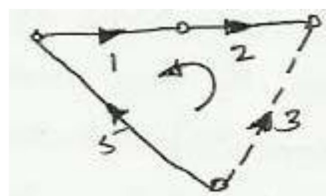


No.ofloops= No.of links=2

Loop 1

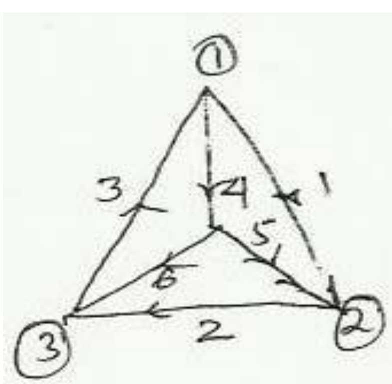


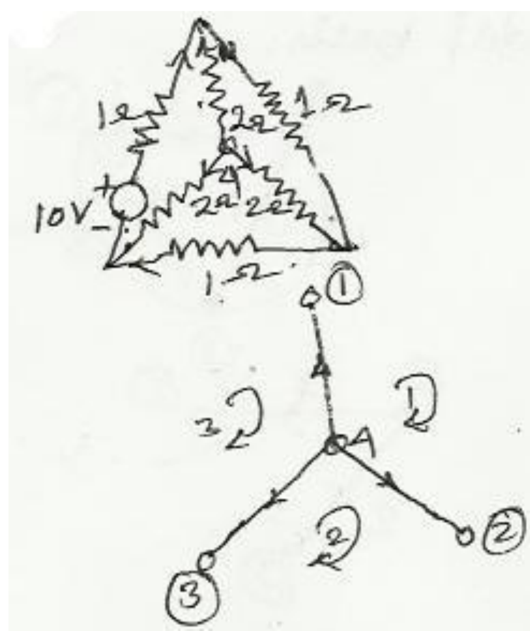
Loop 2



Q1.Drawthegraphandwritedownthetie-setmatrix.Obtainthenetworkequilibriumequationsin matrix formusing KVL.

Solution





Tie-set

	1	2	3	4	5	6
I_1	1	0	0	1	-1	0
I_2	0	1	0	0	1	-1
I_3	0	0	1	-1	0	1

$V_1+V_4-V_5=0 \quad j_1=I_1$

$V_2+V_5-V_6=0 \quad j_2=I_2$

$V_3-V_4+V_6=0 \quad j_3=I_3$

Again, $V_1=e_2-e_2$ $\quad i_4=I_1-I_3$

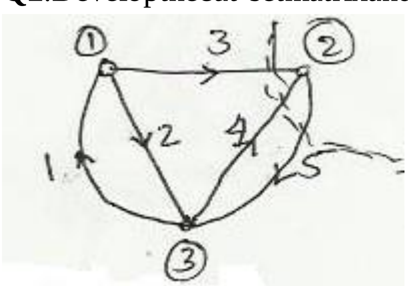
$V_2=e_3-e_2$ $\quad i_5=I_2-$

$I_1 V_4=e_4-e_1$ $\quad i_6=I_3-I_2$

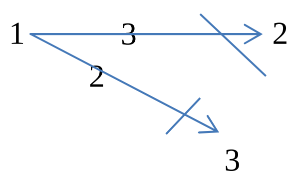
$V_5=e_2-$

$e_4 V_6=e_3-e_4$

Q2.Developthecut-setmatrixandequilibriumequationonnodalbasis.



Ans.

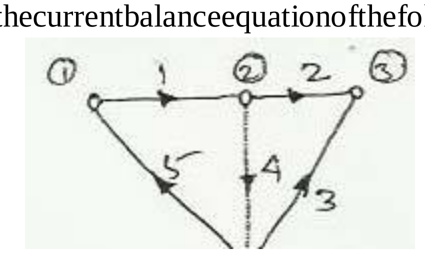


Cutset	1	2	3	4	5
C1	0	0	1	1	-1
C2	-1	1	0	-1	1

$i_3+i_4-i_5=0$

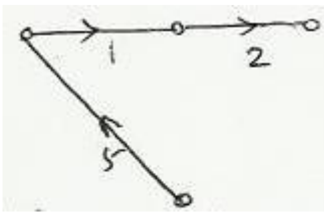
$-i_1+i_2-i_4+i_5=0$

<u>Ex</u>-Determinethecut-setmatrixandthecurrentbalanceequationofthefollowinggraph?

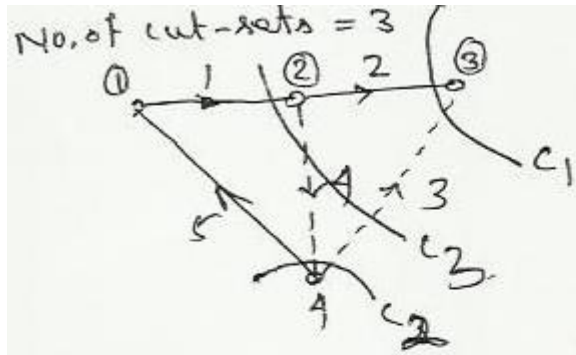


Solution:

Tree



Noof twigs=1, 2, 5



<u>Cut-setmatrix</u>

branch

cut-set	1	2	3	4	5
C1	0	1	1	0	0
C2	0	0	1	-1	1
C3	1	0	1	-1	0

$i_2+i_3=0$

$i_3-i_4+i_5=0$ where, i_1,i_2,i_3,i_4,i_5 arerespectivebranchcurrents.

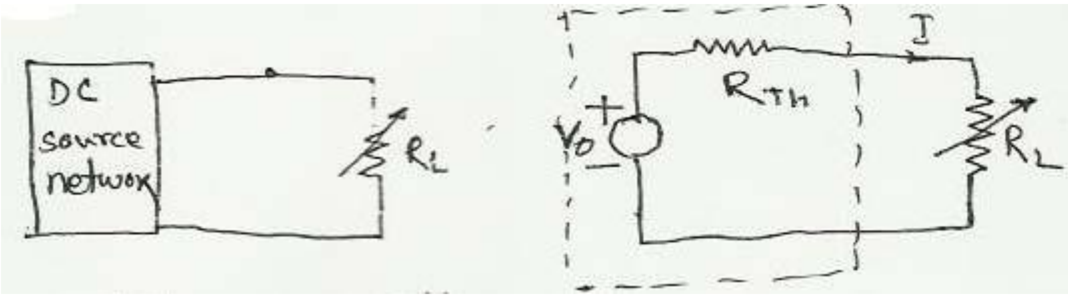
$i_1+i_3-i_4=0$

NETWORKTHEOREMS

1. <u>MaximumPowerTransferTheorem:</u>

A resistance load being connected to a dc network receives maximum power when the load resistance isequal to the internal resistance (Thevenin 's equivalent resistance)of the source network as seen from theloadterminals.

<u>Explanation:</u>



V_0 = Thevenin's voltage

Now, $I = \dfrac{V_0}{R_{TH} + R_L}$

Whilethepowerdeliveredto theresistiveloadis:

$$P_L = I^2 R_L = \left(\frac{V_0}{R_{TH} + R_L}\right)^2 R_L$$

P_L can be maximumised by varying R_L and hence maximum power can be delivered to the loadwhen

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{dP_L}{dR_L} = \frac{1}{[(R_{TH} + R_L)^2]^2}\left[(R_{TH} + R_L)^2 \frac{dV_0^2 R_L}{dR_L} - V_0^2 R_L \frac{d(R_{TH} + R_L)^2}{dR_L}\right]$$

$$= \frac{V_0^2 (R_{TH} + R_L - 2R_L)}{(R_{TH} + R_L)^3} = \frac{V_0^2 (R_{TH} - R_L)}{(R_{TH} + R_L)^3}$$

But

$$\frac{dP_L}{dR_L} = 0$$

$$\Rightarrow V_0^2 (R_{TH} - R_L) = 0$$

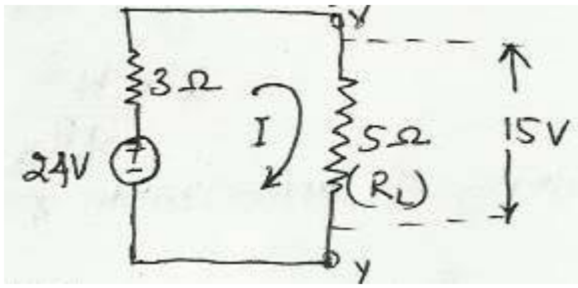
$$\Rightarrow R_{TH} = R_L$$

2. **<u>SubtitutionTheorem:-</u>**

The voltage across and the current through any branch of a dc bilateral network being known, this branchcan be replaced by any combination of elements that will make the same voltage across and currentthroughthe chosen branch.

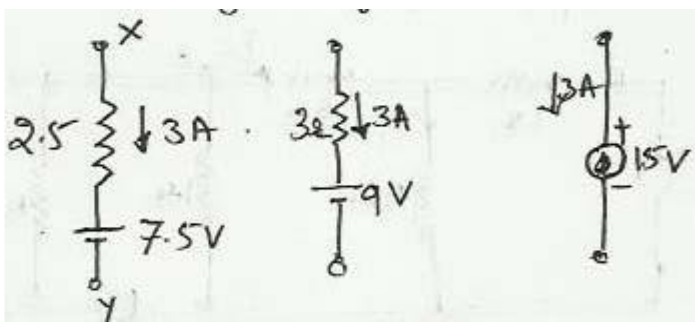
<u>Explanation:</u>

Letusconsiderasimplenetwork asbelow,wherewe taketo seethebranchequivalenceofthe loadresistance R_L.

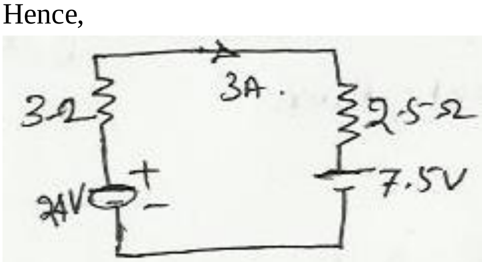


Here, $I = \frac{24}{8} = 3$ Amp

Now,according to superpositionthermothe branchX-Ycanbe replaced byany ofthe followingequivalentbranches.



Hence,



$$P_{max} = \frac{V_0^2 R_{TH}}{(R_{TH} + R_L)^2} = \frac{V_0^2}{4R_{TH}}$$

Totalpowersupplied=powerconsumedby theload+powerconsumedby theveninequivalentresistance

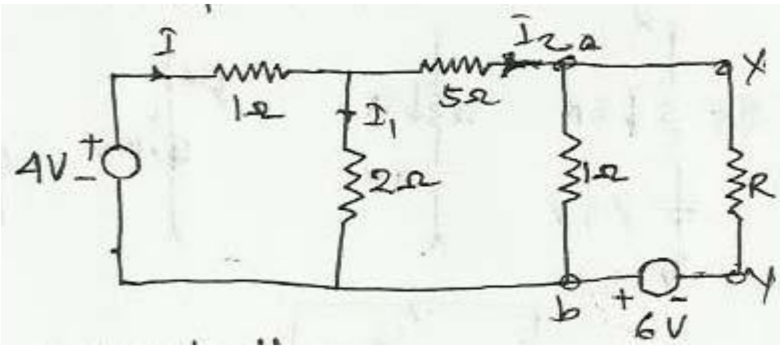
$$= 2 * \frac{V_0^2}{4R_{TH}} = \frac{V_0^2}{2R_{TH}}$$

Nowefficiencyofmaximumpowertransferis:

$$\eta = \frac{P_{max}}{2P_{max}} * 100 = 50\%$$

Example3:

FindthevalueofRinthefollowingcircuitsuchthatmaximumpowertransfertakesplace.whatisheamount ofthispower?



Solution:

When XY isopen ckt;then

$$I = \frac{4}{5/3} = \frac{8}{5}A$$

$$I_2 = \frac{8}{5} * \frac{2}{8} = \frac{2}{5}A$$

$$V_0 = V_{ab} + 6V = \frac{2}{5} * 1 + 6 = \frac{32}{5}V = 6.4V$$

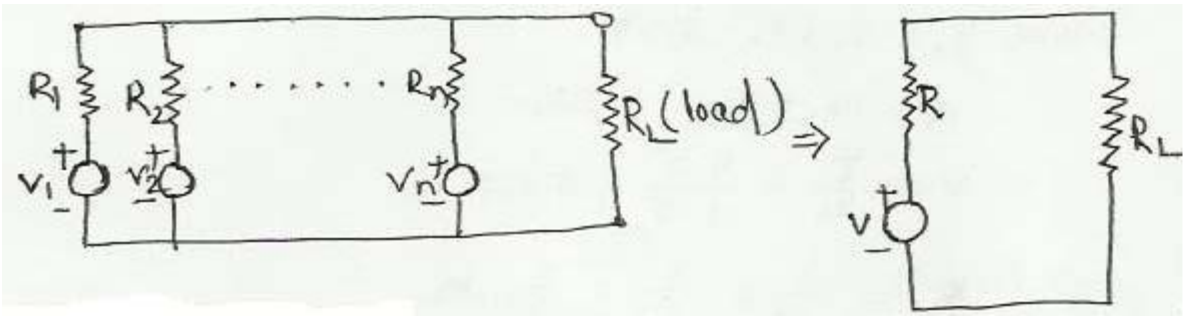
$$R_{,}TH = ((1||2) + 5)||1$$

$$= \frac{\left(\frac{2}{3} + 5\right) * 1}{\left(\frac{2}{3} + 5\right) + 1} = \frac{17/3}{20/3} = \frac{17}{20} = 0.85\Omega$$

$$P_{max} = \frac{V_0^2}{4R_{TH}} = \frac{(6.4)^2}{4 * 0.85} = 12W$$

3.<u>Millman'sTheorem:-</u>

Statement:-Whenanumberofvoltagesources($V_1, V_2, V_3, \dots, V_n$)areinparallelhavinginternalresistances($R_1, R_2, R_3, \dots, R_n$)respectively,thearrangementcanbereplacedbyasingleequivalentvoltagesourceVinserieswithanequivalentseriesresistanceRasgivenbelow.



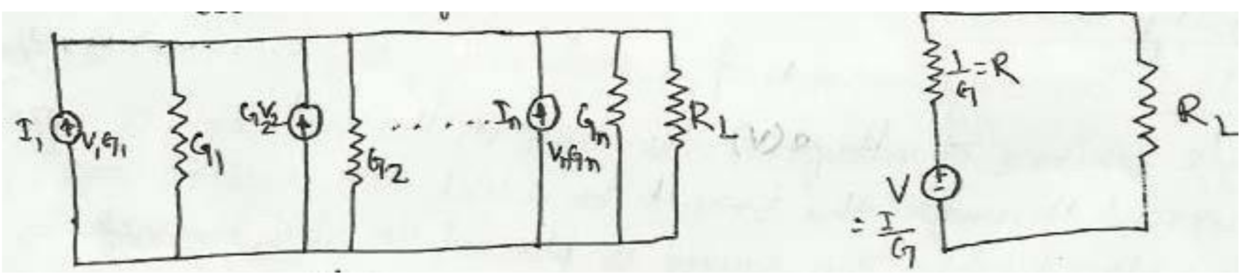
AsperMillmantheorem

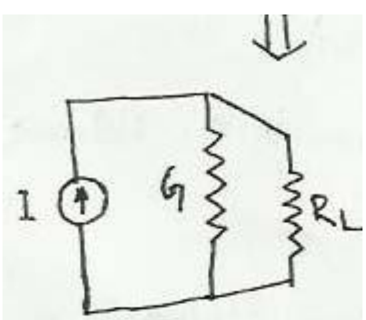
$$V=\frac{\pm V_1G_1 \pm V_sG_s \pm \cdots \dots \dots \dots \pm V_1G_1}{G_1+G_1+\cdots G_n} \quad , \quad R=\frac{1}{G}=\frac{1}{G_1+G_1+\cdots G_n}$$

Where,(G_1 , $G_1, \dots$ G_n)arethe conductancesof ($R_1, R_2, R_3, \dots, R_n$) respectively.

<u>Explanation:-</u>

Letusconsider,thefollowingdcnetwork,afterconvertingtheabovevoltagesourcesintocurrentsource.





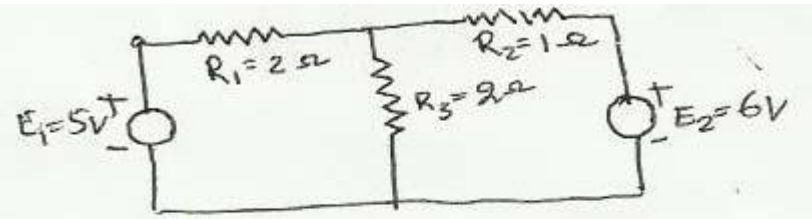
Where, $I = I_1 + I_2 + \cdots I_n$

$G = G_1 + G_2 + \cdots G_n$

$$\rightarrow V = \frac{I}{G} = \frac{I_1 + I_1 + \cdots I_n}{G_1 + G_2 + \cdots G_n} = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm \cdots \ldots \ldots \ldots \pm V_1 G_1}{G_1 + G_1 + \cdots G_n}$$

Example-1

Findcurrentinresistorofthefollowingnetworkbyusingmillman'stheorem.



Solution-

R_1 =2 ohm $\Rightarrow G_1$ =0.5mho E$_1$=5V

$R_2 = 1$ ohm $\Rightarrow G_2$ =1mho E$_2$=6V

R_3 = 2 ohm $\Rightarrow G_3 = 0.5$ mho

$I_1 = E_1 G_1 = 2.5\ A$

$I_2 = E_2 G_2 = 6\ A$

Now, $I = I_1 + I_2$ =8.5A

$G = G_1 + G_2$

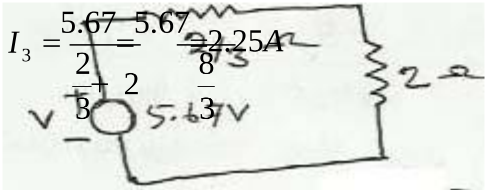
=1.5mhoV=

$\frac{I}{G} = 5.67\ V$

$R = \frac{1}{G}$=0.66ohm

Now,

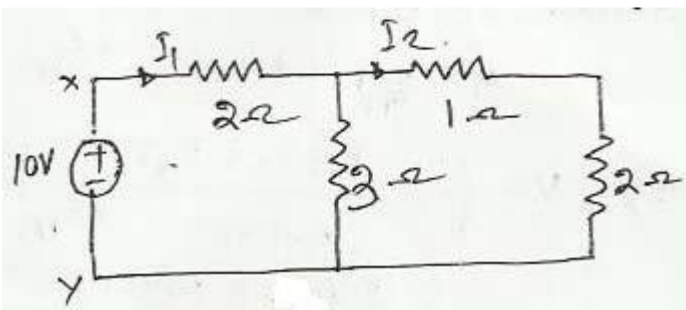
$$I_3 = \frac{5.67}{\frac{2}{3}+2} = \frac{5.67}{\frac{8}{3}} = 2.25A$$



4. **Reciprocating Theorem:-**

Statement: In any branch of a network,the current (I) due to a single source of voltage (V)elsewhere in the network is equal to the current through the branch in which the source wasoriginally placed when the source is placed in the branch in which the current(I) was originallyobtained.

Example:-Showtheapplicationofreciprocitytheoreminthenetwork

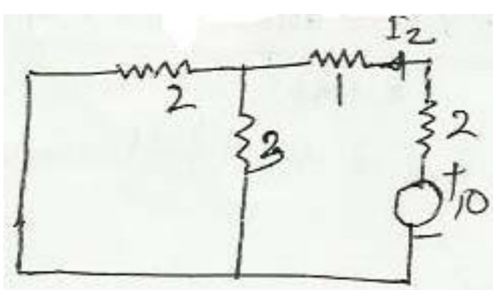


Solution

$R_{eq} = \frac{21}{5}$ =4.2ohm

$I_1 = \frac{50}{21}$ =2.83

$I_2 = 1.43$



$R_{eq} =$ 4.2ohm

$I_2 = \frac{10}{4}.2$ =2.381

$I_1 = 1.43$

Hence,proved.

5. Tellegen'sTheorem:

Statement:Foranygiventime,thesumofpowerdeliveredtoeachbranchofanyelectricalnetwork is zero. Mathematically,

$$\sum_{k=1}^{n} V_k i_k = 0$$

Where,k=k[th]branch

n=total no. of branchesv_k =voltage across k-th branchi_k=currentthrough k-th branch

Explanation:

Let i_{pg}=current through the branch pg=$i_k$$V_{pg}$=votage across p-g=$v_p$-$v_q$=$v_k$So,$v_{pg}i_{pg}$=$v_pi_{pg}$-$v_qi_{pg}$

Similarly,$v_{qp}i_{pq}$=$(v_q-v_p)i_{qp}$[v_{qp}=v_q-v_p=v_k,i_{qp}=-i_{pq}=i_k]Now,

$V_{pq}i_{qp}+v_{qp}i_{qp}=2v_ki_k=[(v_p-v_q)i_{pq}+(v_q-v_p)i_{qp}]$

- $$\sum_{k=1}^{n} vk\, ik = \frac{1}{2\sum_{p=1}^{n}\sum_{q=1}^{n}(vp - vq)ipq}$$

$$=1/2[\sum_{p=1}^{n} vp\left(\sum_{q=1}^{n} ipq\right) - \sum_{q=1}^{n} vq \sum_{p=1}^{n} ip$$

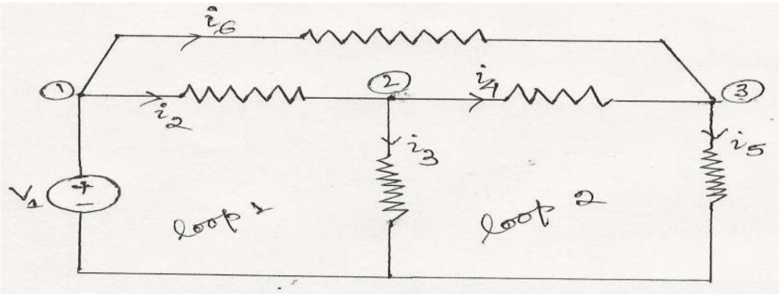
Since $\sum ipq = 0$ atanode

So $\sum_{k=1}^{n} vkik$ =0

Example-4

Check the validity of Tellegen's theorem in the following

network.Assume,V_1=8v,V_2=4v,V_4=2v.AlsoI_1=4A,I_2=2A,I_3=1A



Solution:

Inloop-1;Inloop-2;

$-V_1+V_2+V_3= 0$-$V_3+V_4+V_5=0$

$\rightarrow V_3=V_1-V_2=4v \rightarrow V_5=V_3-V_4$

In loop-3;

$-V_2+V_6-V_4=0$

$\rightarrow V_6=V_2+V_4=6$

At node-1,

$I_1+I_2+I_6=0$

$\rightarrow I_6=-I_1-I_2=-6A$

At node-2

$I_2=I_3+I_4$

$\rightarrow I_4=I_2-I_3=IA$

At node-3

$I_5=I_4+I_6=1-6=-5A$

Summationof powers inthe branches gives;

$$\sum_{b=1}^{b} vb\ ib =V_1I_1+V_2I_2+V_3I_3+V_4I_4+V_5I_5+V_6I_6$$

$=8\times4+4\times2+4\times1+2\times1+2\times(-5)+6\times(-6)$

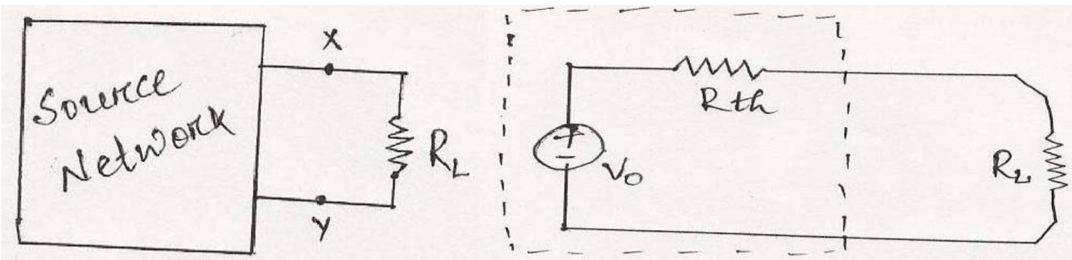
$=32+8+4+2-10-36=0$

Thus,Telegen'stheormis verified

6. <u>CompensationTheorem</u>

Statement:Inalineartime-invariantnetworkwhentheresistances(R)ofanuncoupledbranch,carryingacurrent(I)ischangedby(ΔR),thecurrentinallthebrancheswouldchangesand can be obtained by assuming that an ideal voltage source of (V_C)has been connected[suchthatV_c=I(ΔR)]in serieswiththe resistances.

Explanation:



Here,$I=V_0/R_{th}+R_L$,V_0=Thevenin'svoltage

Letthe load resistances R_L be changed to ($R_L+\Delta R_L$). Since the rest of the circuit remainsunchanged,the thevenin's equivalents network remains the same.

$I'=V_0/R_{Th}+(R_L+\Delta R)$.

Nowthe changein current,

$$\Delta I=I'-I$$

$$=\frac{V_0}{R_{TH}+(R_L+\Delta R_L)}-\frac{V_0}{R_{TH}+R_L}$$

$$=\frac{V_0\{R_{TH}+R_L-(R_{TH}+R_L+\Delta R_L)\}}{(R_{TH}+R_L+\Delta R_L)(R_{TH}+R_L)}$$

$$=-\left[\frac{V_0}{R_{TH}+R_L}\right]\left[\frac{\Delta R_L}{R_{TH}+R_L+\Delta R_L}\right]$$

$$=\frac{-I\Delta R_L}{R_{TH}+R_L+\Delta R_L}=\frac{-V_C}{R_{TH}+R_L+\Delta R_L}$$

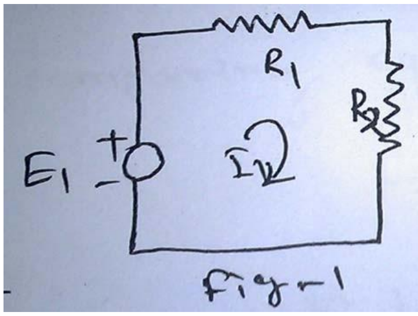
Where, $V_c = I\Delta R_L$ =compensatingvoltage

*note:-

Anyresistance'R'inanetworkcarryingacurrent'I'canbereplacedinanetworkbyavoltagegenerator of zero internalresistanceandemf.(E=-IR)

Example:

In the following network having two resistances R_1 and R_2. The resistance R_2 is replaced by agenerator of emf $E_2 = E_1\frac{R_2}{R_1+R_2}$. Using compensation theorem show that the two circuits areequivalent.



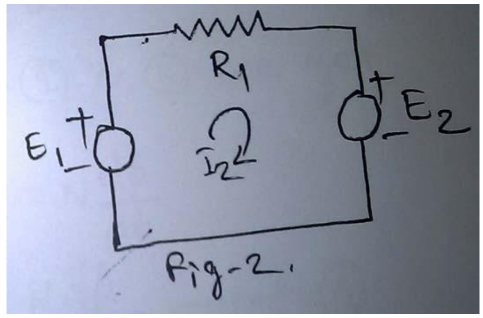


Fig:-1 **Fig:2**

Solution

$I_1 = \frac{E_1}{R_1+R_2}$; $I_2 = \frac{E_1 - E_2}{R_1}$;

So, $I = \frac{E_1 - E_1\frac{R_2}{R_1+R_2}}{R_1}\left[as\ E_2 = -IR_2 = -\frac{E_1R_2}{R_1+R_2}\right]$

$= \frac{E_1}{R_1+R_2} = I_1$

Sothe above two circuits are equivalent.

ANALYSISOFCOUPLEDCIRCUITS

1.SelfInductance:-

Whenacurrentchangesinacircuit,themagneticfluxlinkingthesamecircuitchanges and an emf is induced in the circuit.

Accordingtofaraday'slaw,thisinducedemfisproportionaltothe rateof changeofcurrent.

$$V = L\frac{di}{dt} \qquad (1)$$

Where, L=constant of proportionality called self inductance and its unit is henery.Also the self inductance is given as

$$L = \frac{N\phi}{i} \qquad (2)$$

Where,N=no. of turns of the coil

$\phi = flux\ linkage$

i=currentthroughthecoil

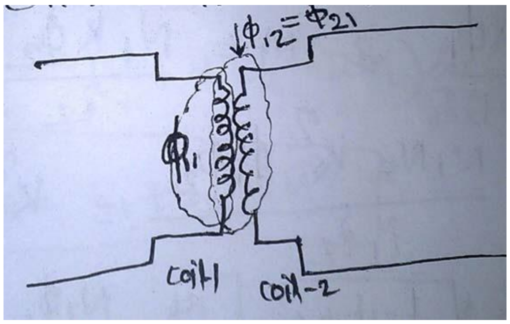
$$V = L\frac{d\left(\frac{N\emptyset}{L}\right)}{dt} = L\frac{1}{L}.N\frac{d\emptyset}{dt} = N\frac{d\emptyset}{dt} \text{------------(3)}$$

Comparingequation1and3weget,

$$V = L\frac{di}{dt} = N\frac{d\phi}{dt}$$

$$\Rightarrow L = N\frac{d\phi}{di} \text{-------------------------------(4)}$$

2.<u>MutualInductance</u>:-Lettwocoilscarrycurrents i_1 and i_2 .Eachcoilwillhaveleakageflux($\emptyset_{L1}$ and for coil 1 $\emptyset_{L2}$ and coil 2) respective as well as mutual flux ($\emptyset_{21}$ and $\emptyset_{12}$ where, thefluxofcoil2linkcoil1orflux of coil 1 links coil 2)



Thevoltageinducedincoil2duetoflux $\emptyset_{12}$ isgivenas

$$V_{L1} = N_2\frac{d\emptyset_{12}}{dt}$$

And $V_{L2} = M\frac{di_1}{dt}$ [faraday'slaw]

Where,M=Mutualinductance

Now, $M\frac{di_1}{dt} = N_2\frac{d\phi_{12}}{dt}$

$$\Rightarrow M = N_2 \frac{d\phi_{12}}{di_1} \text{ ------------------------------ (5)}$$

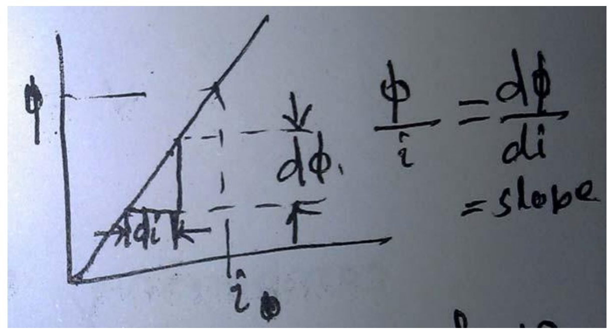
Similarlywecanobtain

$$M = N_1 \frac{d\phi_{21}}{di_2} \text{ ------------------------- (6)}$$

Whenthecoilsarelinkedwithairmedium,thefluxandcurrentarelinearlyrelatedandtheexpressionfor mutual inductance are modified as:

$$M = \frac{N_2\, \emptyset_{12}}{i_1} \text{ ------------------------ (7)}$$

$$M = \frac{N_2\, \emptyset_{21}}{i_2} \text{ .... (8)}$$



***Note:**Mutualinductanceisthebilateral propertyof thelinked coils.

3.<u>Coefficientofcoupling</u>:-Itis definedas thefractionoftotalfluxthatlinks thecoils.

i.e, k-coefficentof coupling= $\frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$

$$\Rightarrow \phi_{12} = k\phi_1 \;\&\; \phi_{21} = k\phi_2$$

So $M = \frac{N_2 k \phi_1}{i_1}$ & $M = \frac{N_1 k \emptyset_2}{i_2}$

Thus, $M^2 = \frac{N_1 N_2 k^2 \phi_1 \phi_2}{i_1 i_2} = k^2 \frac{N_1 \emptyset_1}{i_1} \frac{N_2 \emptyset_2}{i_2}$

$$\Rightarrow M = k\sqrt{L_1 L_2} \text{ ------------------------------------- (9)}$$

$$\left[as\ \frac{N_1\phi_1}{i_1} = L_1 \ \& \frac{N_2\phi_2}{i_2} = L_2\right]$$

4. <u>SeriesConnectionofCoupledcoils:-</u>

Let, two coils of self-inductances L_1 and L_2 are connected in series such that the voltage inducedin coil 1 is V_{L1} and that in coil 2 is V_{L2} while a current I flows through them. Let M_{12} be themutualinductance.

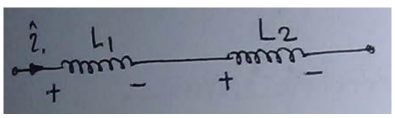
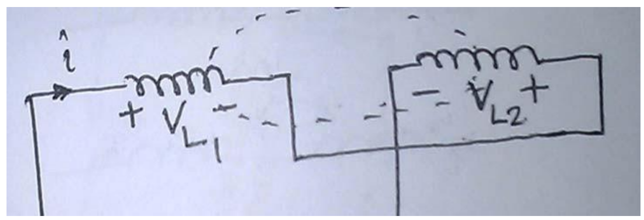


Fig 1



Soforfig 1, **Fig 2**

$$V_{L1}=L_1\frac{di}{dt} + M_{12}\frac{di}{dt} = (L_1 + M_{12})\frac{di}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

$$V_{L2}==L_2\frac{di}{dt} + M_{21}\frac{di}{dt} = (L_2 + M_{21})\frac{di}{dt}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Netvoltage, $V_L=V_{L1}+V_{L2}$

$=(L_1+L_2+2M)di/dt$

Hence, total inductance of the coil is given as, $L=(L_1+L_2+2M)$… ....(12)

Similarlyfor**fig2**

$$V_{L1}=L_1\frac{di}{dt} - M12\frac{di}{dt} = (L_1 - M_{12})\frac{di}{dt}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

$$V_{L2}==L_2\frac{di}{dt} - M21\frac{di}{dt} = (L_2 - M_{21})\frac{di}{dt}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

Netvoltage, $V_L=V_{L1}+V_{L2}$

$=(L_1+L_2-2M)di/dt$...(15)

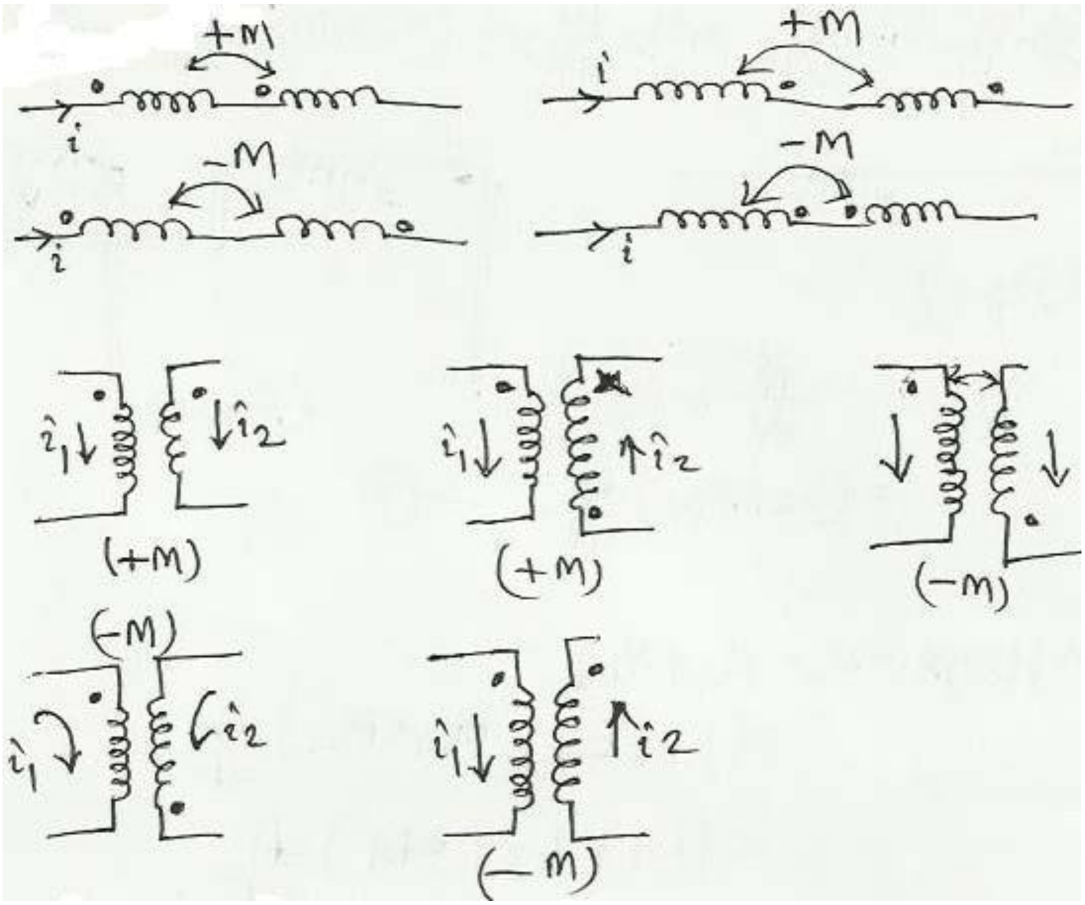
Hence,totalinductanceofthecoilisgivenas,

$L=(L_1+L_2-2M)$… .... (16)

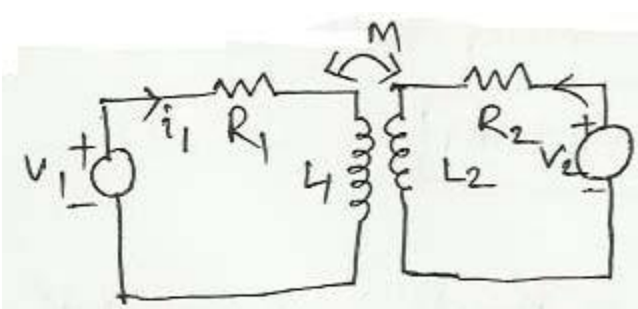
5. <u>DotConvention in Coupled coils:-</u>

To determine the relative polarity of the induced voltage in the coupled coil, the coils are markedwith dots. On each coil, a dot is placed at the terminals which are instantaneous of the samepolarityon the basis ofmutualinductancealone.

<u>SeriesConnection</u>



Modelingofcoupled circuit



$V_1=R_1i_1+L_1\frac{di1}{dt}+M12\frac{di1}{dt}$

$V_2=R_2i_2+\frac{di2}{dt}+M21\frac{di2}{dt}$

$L_2V_1+R_1i_1+j\omega(L1i1 + Mi2) = o$

So, $V_1=Z_{11}i_1+Z_{12}i_2$

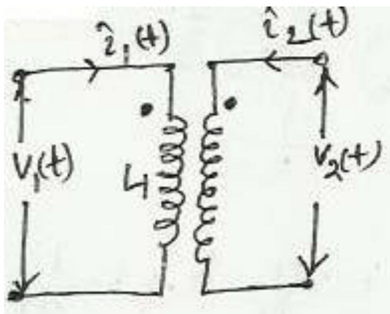
Similarly, $V_2=Z_{21}i_1+Z_{22}i_2$

Electrical Equivalents of magnetically coupled circuits:

In electrical equivalent representation of the circuit, the mutually induced voltages may be shown as controlled voltage source in both the coils. In the frequency domain representation, the operator ($\frac{d}{dt}$) is replaced by "$j\omega$" term.

Example

Draw the equivalent circuit of the following coupled circuit.



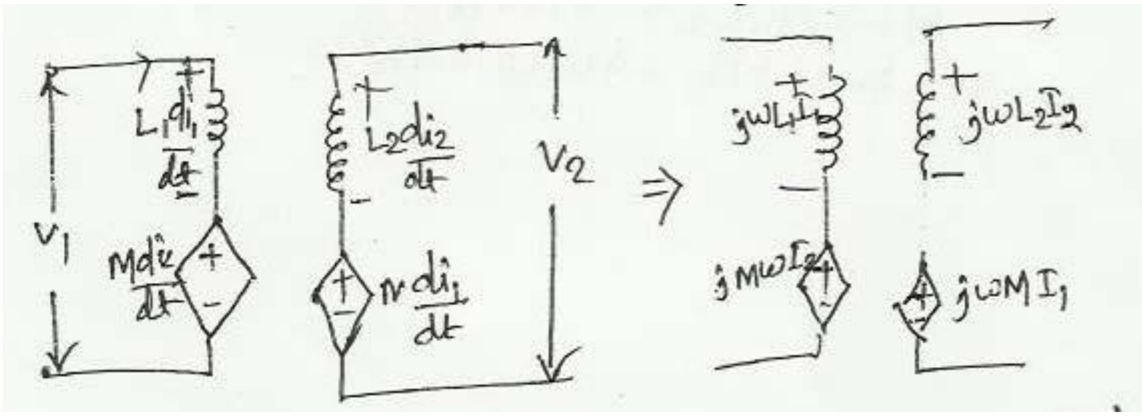
Solution

Voltage equation of both circuits are given as

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

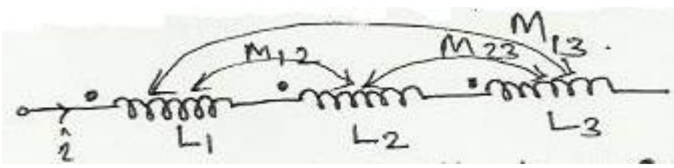
$$v_2(t) = L_1 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}$$

So,



Example

Findthe totalinductanceofthethreeseriesconnectedcoupledcoils.



Solution

Given,L_1=1H,L_2=2H,L_3=5H,M_{12}=0.5H,M_{23}=1H,M_{13}=1HFor coil1

:$L_1+M_{12}+M_{13}$=1+0.5+1=2.5H

Forcoil2:$L_2+M_{12}+M_{23}$=2+0.5+1=3.5HForco

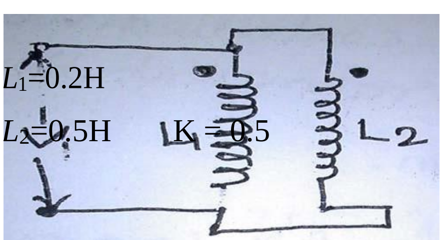
il3:$L_3+M_{23}+M_{13}$=5+1+1=7H

Totalinductanceof circuit = L = $L_1+ M_{12}+ M_{13}+ L_2+M_{12}+M_{23}+, L_3+M_{23}+M_{13}$

=0.5+1.5+3=5H

Example

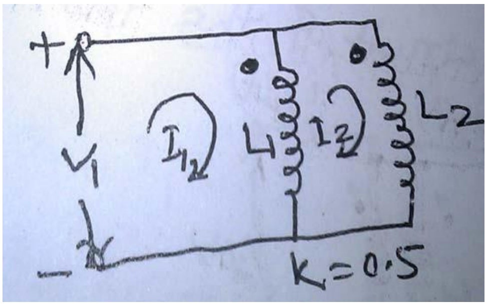
Inthe following coupledcircuit, find theinputimpedanceaswellasthenetinductance.



L_1=0.2H

L_2=0.5H K = 0.5

Solution



In loop 1

$$V_1 = L_1 \frac{d(i_1 - i_2)}{dt} + M\frac{di_2}{dt}$$

$$= j\omega L_1(i_1 - i_2) + j\omega M i_2$$

In loop 2

$$0 = L_1 \frac{d(i_2 - i_1)}{dt} + M\frac{d(i_2 - i_1)}{dt} + L_2\frac{di_2}{dt}$$

$$= j\omega L_1(i_2 - i_1) + j\omega M(i_2 - i_1) + j\omega L_2 i_2$$

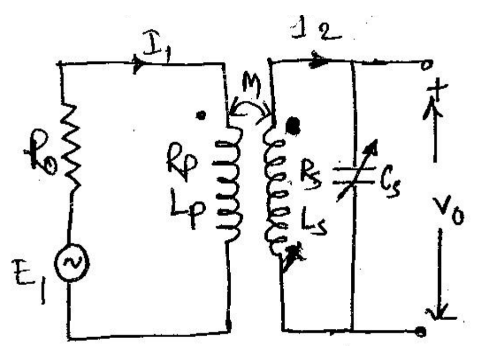
$$M = K\sqrt{L_1 L_2} = 0.158H$$

$$V_1 = jw(.2)I_1 - jw(0.042)I_2$$

TUNEDCOUPLECIRCUITS

A. SingleTunedcouplecircuits.

Inthegivencircuit;



Z_{11}=drivingpointimpedanceatinput

$=R_0+(R_P+j\omega L_p)$

$=R_1+j\omega L_p=R_1+jX_1$

Z_{22}=drivingpointimpedanceatoutput

$=R_S+j\omega L_S-j\omega C_S$

$=R_2+j(\omega L_S-1/\omega C_S)=R_2+jX_2$

E_1=source

voltageV_O=outputvoltageω=I_2

/J ωM

C_S

$Z_{12}=Z_{21}=j$

Theloopequationsaregivenas

$Z_{11}I_1-Z_{12}I_2=E_1$ (mutualfluxopposesselfflux)

$-Z_{21}I_1+Z_{22}I_2=0$

$$I_{12}=\begin{vmatrix} Z_{11} & E_1 \\ -Z_{12} & 0 \end{vmatrix} / \begin{vmatrix} Z_{11} & -Z_{12} \\ -Z_{21} & Z_{22} \end{vmatrix}$$

$=E_1Z_{12}/(Z_{11}Z_{22}-Z_{12}Z_{21})$

$=E_1Z_{12}/(Z_{11}Z_{22}-Z^2_{22})$

$=E_1(j\omega M)/(R_1+Jx_1)(R_2+Jx_2)+\omega^2M^2$

This

gives,V_0=ωI_2/J

C_S

$=E_1M/C_S[R_1R_2+j(R_1X_2+R_2X_1)-X_1X_2+\omega^2M^2]$

ByvaryingCs,foranyspecificvalueofM,tuningcanbe obtainedwhen ω $\frac{1}{\omega}$

ω Ls=Cs.The resonant frequency is given by

r.

Atfreq.ofresonance;$X_2=0,X_1X_2=0$

So,$I_{2res}=E_1\omega_rM/R_1R_2+jR_2X_1+\omega_r^2M^2$

$V_{0res}=E_1M/C_S(R_1R_2+jR_2X_1+\omega_r^2M^2)$

TheaboveequationisvalidforspecificvalueofM,however,M=K$\sqrt{L_sL_p}$. ifKis varied, this willresult in varistor in M. There will be one value of K that will result in a value of M so that V_{0res} ismaximum.This particular value of K iscalled critical coefficient of coupling.

$V_{0res}=E_1/C_S/(R_1R_2/M)+\omega_rM+(jR_2X_1/M)$

V_{0res} to be maximum,

$R_1R_2+jR_2X_1/M= \omega_r$

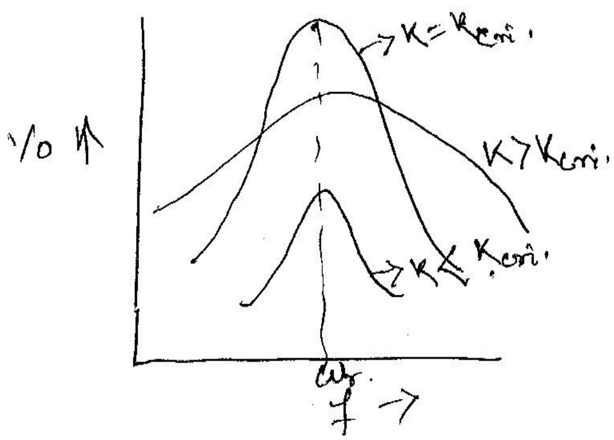
$\omega_r^2 M R_1$

$R_2+jR_2X_1= {}^2M^2$

$\boxed{R_1R_2= \omega_r^2 M^2}$

Therefore, $M= \sqrt{R_1R_2}/\omega_r$

$=K\sqrt{L_sL_p}$

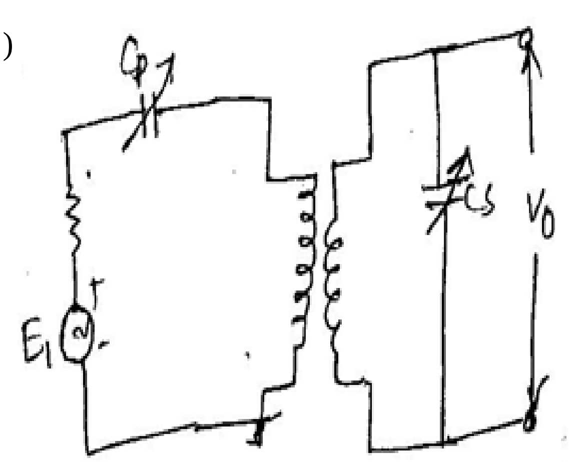


B. DoubleTunedCoupledCircuits:-

Here, $Z_{11} = R_{10} + R_{1P} + j(\omega L_{1P} - \frac{1}{\omega C_P})$

$= R_1 + jX_1$

$Z_{22} = R_{1S} + j(\omega L_{1S} - 1/\omega C_{1S})$



$$I_2 = \frac{E_{11}Z_{12}}{Z_{11}Z_{22} - Z_{12}^2}$$

$$V_0 = \frac{I_2}{j\omega C_s}$$

At Resonance,

$$\omega_r = \frac{1}{\sqrt{L_P C_P}} = \frac{1}{\sqrt{L_S C_S}}$$

$$\mathbf{X_1 = 0\ , X_2 = 0}$$

So, $I_{2res} = \dfrac{E_1 \omega_r M}{R_1 R_2 + \omega_r{}^2 M^2}$

$$V_{0res} = \frac{E_1 M / C_S}{R_1 R_2 + \omega_r{}^2 M^2}$$

LAPLACETRANSFORM

Givenafunction*f(t)*,itsLaplacetransform,denotedby*F(s)*or*L*[*f(t)*],is givenby

$$L[F(t) = F(s) = \int_{\downarrow}(-\infty)^{\uparrow}\infty [f(t)]\ e^{\uparrow}(-st)\,dt$$

The *Laplace transform*is an integral transformation of a function *f* (*t*) from the timedomain intothecomplex frequency domain, giving*F*(*s*).

Properties of

L.T.(i)Multiplication by a

constant:-Let,Kbeaconstant

F(s) be the L.T. of f(t)

Then; $L[kf(t)] = \int_0^{\infty} kf(t)e^{-st}dt = k\int_0^{\infty} f(t)e^{-st}dt = kF(S)$

(ii)Sumand Difference:-

Let $F_1(S)$and$F_2(S)$aretheL.T.ofthefunctions $f_1(t)$&$f_2(t)$respectively.

$$L[f_1(t) \pm f_2(t)] = F_1(S) + F_2(S)$$

(iii) Differentiationw.r.t.time[Time–differentiation]

$$L\left[\frac{df(t)}{dt}\right] = SF(S) - f(0^+)$$

Proof

F(S)= $L[f(t)] = \int_0^\infty f(t)e^{-st}dt$

Let,f(t)=u;then, $\frac{df(t)}{dt}dt = du$ &

$e^{-st}dt = dv \Rightarrow v = \frac{-e^{-st}}{s}$

So, $\int_0^\infty f(t)e^{-st}dt$ = - $\int_0^\infty \frac{-e^{-st}}{s} du + f(t)\left(\frac{-e^{-st}}{s}\right)$

=>F(s)= $\frac{f[(0]^+)}{s}$ + $\frac{1}{s}\int_0^\infty e^{-st}\left[\frac{df(t)}{dt}\right]dt$

=>F(s)= $\frac{f[(0]^+)}{s} + \frac{1}{s}L\left[\frac{df(t)}{dt}\right]$

=> $L\left[\frac{df(t)}{dt}\right] =$ s F(s) $-f(0^+)$

(iv)Integrationby time“t”:-

$L\left[\int_0^\infty f(t)dt\right] = \int_0^\infty \left[\int_0^\infty f(t)dt\right]e^{-st}$ dt

U= $\int_0^\infty f(t)dt$ => $f(t) = \frac{du}{dt} \Rightarrow du = f(t)dt$

dv = e^{-st} dt => v= $\frac{-e^{-st}}{s}$

$L\left[\int_0^\infty f(t)dt\right] = L\int_0^\infty u dv = u[v]_0^\infty - \int_0^\infty v du$

$$\frac{-e^{-st}}{s}\int_0^{\infty}\int_{\infty}^{\infty} f(t)dt - \frac{1}{s}\int_{\infty}^{\infty} f(t)e^{-st}dt$$

$$\frac{1}{s}\left[\int_{\infty}^{\infty} f(t)dt\right]_0^{\infty} + \frac{F(s)}{s}$$

$$\int_0^{\infty}[f(\infty) - f(0)]dt =$$

(v) .Differentiationw.r.toS[frequencydifferentiation]:-

$dF(s)/ds = -L[t.f(t)$

Proof:

$dF(s)/ds = d/ds \int_0^{\infty} [f(t).e^{(-st)}.dt = \int_0^{\infty} [f(t)[(de^{(-st)})/ds]dt = \int_0^{\infty} [f(t)\, e^{(-st)}\, (-t)$

(vi) .Integrationby'S':-

$$\int_s^{\infty} F(s) = L\left[\frac{f(t)}{t}\right]$$

Proof; $\int_s^{\infty} F(s) = \int_0^{\infty}\int_0^{\infty} f(t).e^{-st}.ds.dt = \int_0^{\infty} f(t)\left[\frac{de^{-st}}{-t}\right]_0^{\infty} dt$

$= \int_0^{\infty} f(t)\left[0 - \frac{de^{-st}}{-t}\right]dt = \int_0^{\infty}\frac{f(t)}{-t}.e^{-st}.dt = L\left[\frac{f(t)}{t}\right]$

(vii). ShiftingTheorem:-

(a) L[f(t-1).U(t-a)] $= e^{-as}F(s)$

(b) F(s+a)=L[$e^{-as}f(t)$]

Proof:L[$e^{(-as)}\, f(t)] = e^{(-(a+s)t)}\, f(t).dt = F(s+a)$

(viii). InitialValueTheorem:-Typeequationhere.

f($0^{+}) = (Lt@t \to 0)f(t) = (Lt@s \to \infty)[sF(s)$

proof:sF(s)-f($0^{+}) = \int_0^{\infty} (df(t))/dt.e^{(-st)}.dt$

=>s(s) = f($0^{+}) + \int_0^{\infty} (df(t))/dt.e^{(-st)}.dt$

=> $\underset{s\to\infty}{Lt} sF(s) =$ f($0^{+}) + (Lt@s \to \infty)\int_0^{\infty} df(t)/dt.e^{(-st)}.dt = f(0^{+})$

(ix). <u>Final Value Theorem:-</u>F(

$\infty) = \blacksquare(Lt@t \to \infty) f(t) = \blacksquare(Lt@s \to 0)[sf(s)]$

Proof :- [sf(s) -

f(

$0^+)] = \blacksquare(Lt@s \to 0) \int_{0}^{\infty} df(t)/dt . e^{(-st)} . dt = \int_{0}^{\infty} df(t)/dt . dt = \int_{0}^{\infty} df(t) . dt = f(t)\blacksquare(\infty@0)$

$= f(\infty) - f(0) = f(\infty) = \underset{t \to \infty}{Lt} f(t)$

(x). <u>Theoremofperiodicfunctions:-</u>

Let $f_1(t), f_2(t), f_3(t), \dots \dots be$ the functions described by 1st ,2nd & 3rd ...cycles of the periodicfunction $f(t)$, whosetime periods is T.

$f(t) = f_1(t) + f_2(t) + f_3(t) + \cdots \dots = f_1(t) + f_1(t - T) + f_1(t - 2T)$

$L[f(t)] = F_1(s) + e^{-ST} F_1(s) + e^{-2ST} F_1(s) + \cdots$

$= F_1(s)[1 + e^{(-ST)} + e^{(-2ST)} + \cdots] = F_1(s)$

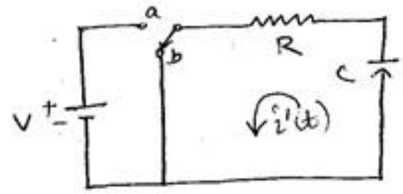
(xi). <u>ConvolutionTheorem:</u>

$L^{-1}[F_1(s)F_2(s)] = f_1(t) * f_2(t) = \int_0^t f_1(t-\tau) f_2(\tau) d\tau$

(xii). <u>TimeScaling:</u>

$L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$

Q4. When connected to a;



$$i(t)R + \frac{1}{C}\int_0^t i(t)\,dt = V$$

$$\Rightarrow R I(s) + \frac{1}{Cs} I(s) = \frac{V}{s}$$

$$\Rightarrow I(s) = \frac{CV}{RCs+1} = \frac{V/R}{s+\frac{1}{RC}}$$

$$\Rightarrow i(t) = \frac{V}{R} e^{-t/RC}, \quad V_R = Ve^{-t/RC}, \quad V_C(s) = \frac{V/RC}{s(s+\frac{1}{RC})} = V(1-e^{-t/RC})$$

under steady state condition;

$$i(t) = 0$$
$$V_R(t) = 0$$
$$V_C(t) = V$$

When connected to b;

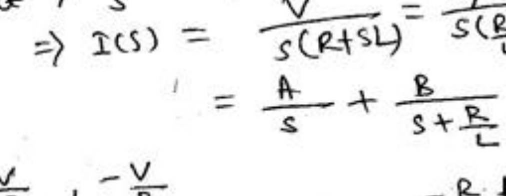
$$0 = R i'(t) + \frac{1}{C}\int_0^t i'(t)\,dt$$

$$\Rightarrow 0 = R I'(s) + \frac{1}{C}\left[\frac{I'(s)}{s} + \frac{i(0^+)}{s}\right]$$

$$= \left(R + \frac{1}{Cs}\right) I'(s) + v_C(0^+) = \left(R + \frac{1}{Cs}\right) I'(s) + V$$

$$\Rightarrow \frac{-V/R}{s+\frac{1}{RC}} = I'(s) \Rightarrow i'(t) = \pm\frac{V}{R} e^{-t/RC}$$

Q5



$R = 1K\Omega$
$L = 1H.$
$C = 0.1\mu F.$
$V = 100V.$

$$V = Ri + L\frac{di}{dt} \Rightarrow \frac{V}{s} = (R+sL)I(s)$$

$$\Rightarrow I(s) = \frac{V}{s(R+sL)} = \frac{V/L}{s(\frac{R}{L}+s)}$$

$$= \frac{A}{s} + \frac{B}{s+\frac{R}{L}}$$

where, $A = \frac{V}{R}, -\frac{V}{R}$

so, $i(t) = \frac{V}{R}\left[1 - e^{-\frac{R}{L}t}\right]$

At steady state, $i(t) = \frac{V}{R} = 0.1A.$

$i(0^+) = 0.1.$

When connected to b;

$$Ri' + L\frac{di'}{dt} + \frac{1}{C}\int_{-\infty}^{t} i'\,dt = 0$$

$$\Rightarrow RI'(s) + sLI'(s) - i'(0^+) + \frac{1}{sC}\left[\int I'(s) + i'(0^+)\right] = 0$$

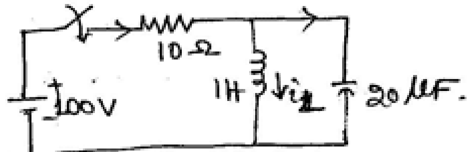
$$\Rightarrow RI'(s) + sLI'(s) + \frac{V}{R} + \frac{I'(s)}{Cs} = 0$$

$$\Rightarrow I'(s)\left[R + SL + \frac{1}{Cs}\right] = \frac{V}{R}.$$

$$\Rightarrow I'(s)\left[100 + s + \frac{1}{0.1\times10^{-6}s}\right] = 0.1$$

$$\Rightarrow I'(s) = \frac{0.1}{s^2 + 100s + 10^5}$$

Q6 In the circuit, the switch 'S' is closed and a steady state is reached in the circuit. At $t=0$, the switch is opened. Find an expression for the current in the inductor, $i_L(t)$.



Soln:-

When switch was closed.

$$100 = 10i + \frac{di_L}{dt} \Rightarrow \quad 10I(s) + sI_L(s) = \frac{100}{s}. \quad \frac{100}{s(s+10)}$$

$$L\frac{di_L}{dt} - \frac{1}{C}\int_0^t (i - i_L)\,dt = 0$$

$$\Rightarrow sI_L(s) - \frac{1}{Cs}\left[I(s) - I_L(s)\right] = 0$$

$$\Rightarrow \left(s + \frac{1}{Cs}\right)I_L(s) = \frac{I(s)}{Cs}$$

$$\Rightarrow I_L(s) = \frac{I(s)}{Cs^2 + 1}$$

$$\text{so, } \left[10 + \frac{s}{Cs^2+1}\right]I(s) = \frac{100}{s}$$

$$\Rightarrow I(s) = \frac{100}{s}\left[\frac{Cs^2+1}{10Cs^2 + s + 10}\right]$$

At steady state

$$i_L(0^+) = \frac{100}{10} = 10\,A.$$

$$V_C(0^+) = 0$$

When switch is open

$$\frac{di_L}{dt} + \frac{1}{C}\int_0^t i_L\,dt = 0.$$

$$\Rightarrow sI_L(s) - i(0^+) + \frac{1}{Cs}\left[I_L(s) + i(0)\right] = 0$$

$$\Rightarrow \left(s + \frac{1}{Cs}\right)I_L(s) - 10 = 0$$

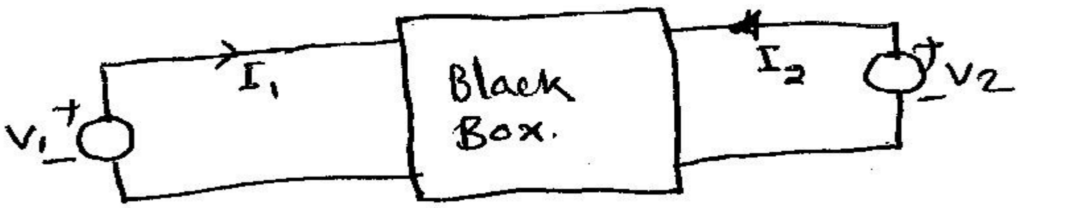
$$\Rightarrow I_L = \frac{10Cs}{Cs^2+1} = \frac{10s}{s^2 + \frac{1}{C}}$$

$$= 10\cos 223.6t.$$

TWOPORTNETWORKFUNCTIONANDRESPONCES

INTRODUCTION

A network having two end ports is known as a two portnetwork. The ports may supply orconsumeelectricalpower.Acomplexnetworkcanberepresentedasatwoportnetworkconstitutestwostationsandablackboxinbetweenthestationasbelow.



The study of the above networkbecomes complicatedas the network present inside theblack box is known so far the techniques has been developed , the two port networksareanalyzedby usingdifferent parameters.

One can imagine the network inside the black boxmay be impedances oradmittances connected in series or parallel randomly. Now applying KVL and KCL we candefine the equations

As

$V_1=$
$Z_{11}I_1+Z_{12}I_2V_2=$
$Z_{21}I_1+Z_{22}I_2$

Or

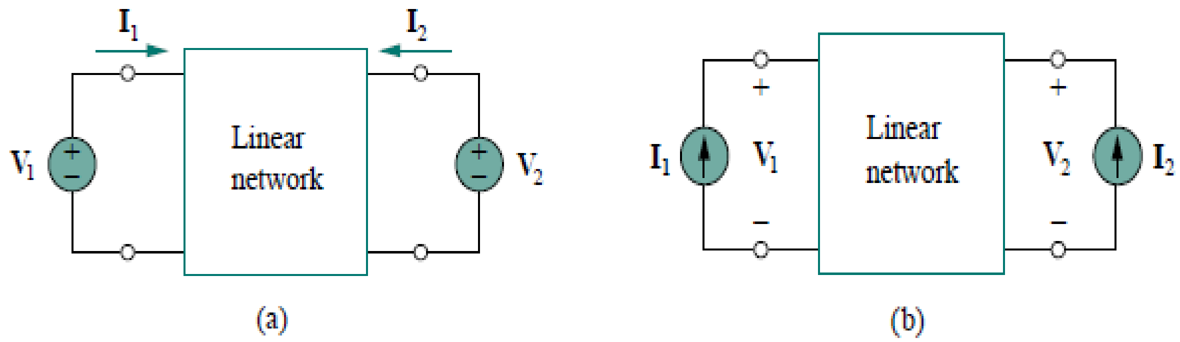
$I_1=Y_{11}V_1+Y_{12}V_2$
$I_2+Y_{21}V_1+Y_{22}V_2$

Z_{11} , Z_{12} , Z_{21} , & Z_{22}--→Z-ParametersY$_{11}$,Y$_{12}$.Y$_{21}$,& Y$_{22}$--→Y-Parameters

IMPEDANCEPARAMETERS:

Impedanceandadmittanceparametersarecommonlyusedinthesynthesisoffilters.Theyarealso useful in the design and analysis of impedance-matching networks and power distributionnetworks.

Atwo-portnetworkmaybevoltage-drivenorcurrent-drivenasshowninFig.



The terminal voltages can be related to the terminal currents as,$V_1=Z_{11}I_1+Z_{12}I_2$

$V_2=Z_{21}I_1+Z_{22}I_2$

orinmatrixformas,

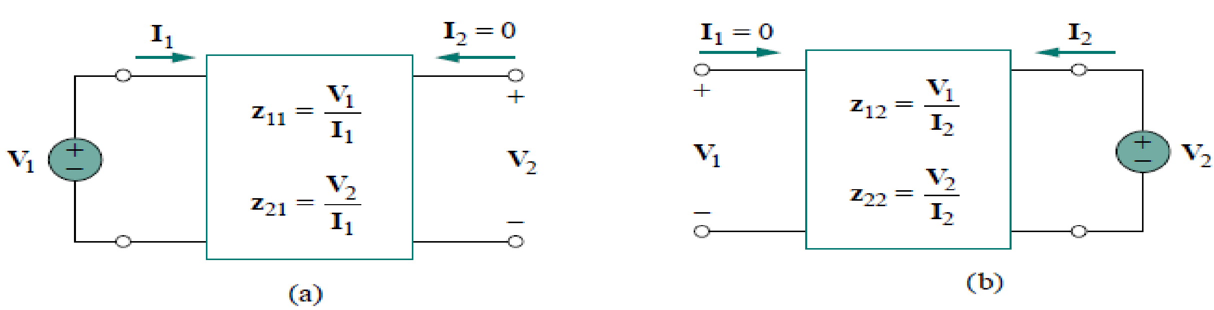
$$\begin{bmatrix}V_1\\V_2\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12}\\Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}I_1\\I_2\end{bmatrix}=[Z]\begin{bmatrix}I_1\\I_2\end{bmatrix}$$

whereth**ez**termsarecalledthe*impedanceparameters*,orsimply*z-parameters*,andhaveunitsofohms.

Thevaluesoftheparameterscan beevaluated bysetting $I_1=0$(inputportopen-circuited)or$I_2=0$(outputportopen-circuited).Thus,

$$Z_{11}=\frac{V_1}{I_1}\Big|_{I_2=0} \quad Z_{21}=\frac{V_2}{I_1}\Big|_{I_2=0}$$

$$Z_{12}=\frac{V_1}{I_2}\Big|_{I_1=0} \quad Z_{22}=\frac{V_2}{I_2}\Big|_{I_1=0}$$



Sincethezparametersareobtainedbyopen-circuitingtheinputoroutputport,theyarealsocalledthe*open-circuit impedance parameters*.

Specifically,

Z_{11}=Open-circuitinputimpedance

Z_{12}=Open-circuittransferimpedancefromport1toport2Z_{21}=

Open-circuit transfer impedance from port 2 to port 1Z_{22}=

Open-circuitoutput impedance

Sometimes Z_{11}andZ_{22}arecalled *driving-pointimpedances*,whileZ_{21}andZ_{12}arecalled *transferimpedances*.

ADMITTANCEPARAMETERS:

In general, for a two port network consisting of 2 loops,

$$I_1=y_{11}V_1+y_{12}V_2 \quad I_2=y_{21}V_1+y_{22}V_2$$

orinmatrixformas,

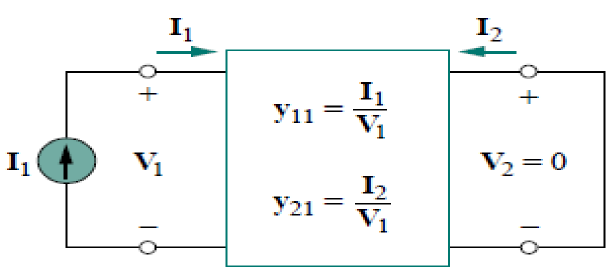
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Where, the **y-**terms are called the *admittance parameters*, or simply *y- parameters*, and haveunitsofsiemens.

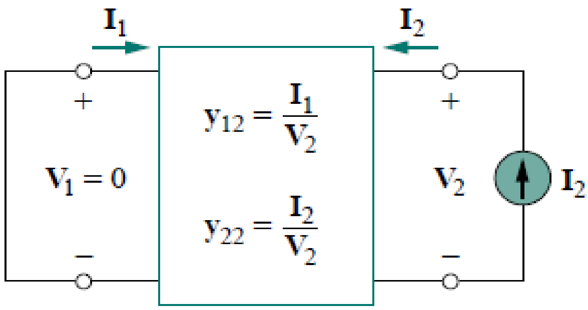
The values of the parameters can be determined by setting V_1= 0 (input port short-circuited) orV_2=0(outputportshort-circuited).Thus,

Now, $y_{11}= \frac{I_1}{V_1}\Big|V_2=0 \quad y_{21}= \frac{I_2}{V_1}\Big|V_2=0$

$y_{12}= \frac{I_1}{V_2}\Big|V_1=0 \quad y_{22}= \frac{I_2}{V_2}\Big|V_1=0$



Circuittofind $Y_{11}$and Y_{21}



CircuittofindY_{12} andY_{22}

Since the *y* parameters are obtained by short-circuiting the input or output port, they are alsocalledthe*short-circuit admittance parameters*.

Specifically,

y$_{11}$=Short-circuitinputadmittance

y$_{12}$= Short-circuit transfer admittance from port 2 to port

1**y**$_{21}$= Short-circuit transfer admittance from port 1 to port

2**y**$_{22}$=Short-circuit output admittance

HYBRIDPARAMETERS:

This hybrid parameters is based on making V_1 and I_2 the dependent variables.Thus,

V_1=$h_{11}I_1$+$h_{12}V_2I_2$

=$h_{21}I_1$ +$h_{22}V_2$

orinmatrixformas,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [h] \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The **h** terms are known as the *hybrid parameters* (or, simply, *h parameters*) because they are ahybrid combination of ratios. They are very useful for describing electronic devices such astransistors.

Thevaluesofthe parametersaredeterminedas,

$$h_{11} = \frac{V_1}{I_1}\Big|V_2=0 \quad h_{12} = \frac{V_1}{V_2}\Big|I_1=0$$

$$h_{21} = \frac{I_2}{I_1}\Big|V_2=0 \quad h_{22} = \frac{I_2}{V_2}\Big|I_1= 0$$

Theparametersh$_{11}$,h$_{12}$,h$_{21}$,andh$_{22}$representanimpedance,avoltagegain,acurrentgain,andanadmittance,respectively.Thisiswhytheyarecalledthehybridparameters.

Tobespecific,

h_{11}= Short-circuit input

impedanceh_{12}= Open-circuit reverse

voltage gainh_{21}= Short-circuit forward

current gainh_{22}=Open-

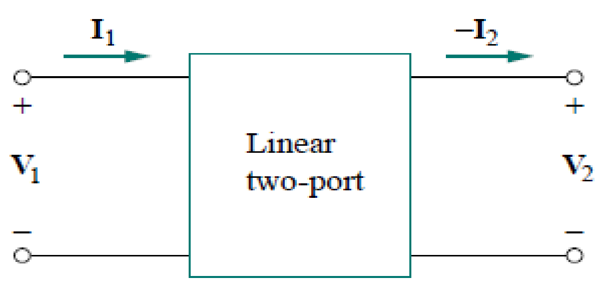
circuitoutputadmittance

The procedure for calculating the *h* parameters is similar to that used for the *z* or *y* parameters.Weapplyavoltageorcurrentsourcetotheappropriateport,short-circuitoropen-circuittheotherport, depending on theparameter of interest, and performregular circuit analysis.

TRANSMISSIONPARAMETERS:

Since there are no restrictions on which terminal voltages and currents should be consideredindependentandwhichshouldbedependentvariables,weexpecttobeabletogeneratemanysets of parameters. Another set of parameters relates the variables at the input port to those at theoutputport.Thus,

$$V_1=AV_2-BI_2$$



$$I_1=CV_2-DI_2$$

or

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V \\ -I_2 \end{bmatrix}$$

The two-port parameters provide a measure of how a circuit transmits voltage and current from asource to a load. They are useful in the analysis of transmission lines (such as cable and fiber),becausetheyexpresssending-endvariables(V_1andI_1)intermsofthereceiving-endvariables(V_2 and $-I_2$). For this reason, they are called *transmission parameters*. They are also known as**ABCD**parameters.

Thetransmissionparametersaredeterminedas,

$$A=\frac{V_1}{V_2}\Big|_{I_2=0} \quad B=-\frac{V_1}{I_2}\Big|_{V_2=0}$$

$$C=\frac{I_1}{V_2}\Big|_{I_2=0} \quad D=-\frac{I_1}{I_2}\Big|_{V_2=0}$$

Thus,thetransmission parametersare called,specifically,

A=Open-circuitvoltageratio

B=Negativeshort-circuittransferimpedance

C=Open-circuittransferadmittance

D=Negativeshort-circuitcurrentratio

Aand**D**are dimensionless, **B** is in ohms, and **C** is in siemens. Since the transmission parametersprovide a direct relationship between input and output variables, they are very useful in cascadednetworks.

InterRelationshipbetweenparameters:

1. Z-parameters in terms of Y-parameters$[Z]=[Y]^{-1}$

$$Z_{11}=\frac{Y_{22}}{\Delta Y} \quad Z_{12}=\frac{-Y_{12}}{\Delta Y} \quad Z_{21}=\frac{-Y_{21}}{\Delta Y} \quad Z_{22}=\frac{Y_{11}}{\Delta Y}$$

Where$\Delta Y=Y_{11}Y_{22}-Y_{12}Y_{21}$

2. Z-parametersintermsofh-parameters

$$Z_{11}=\frac{\Delta h}{h_{22}} \quad Z_{12}=\frac{h_{12}}{h_{22}} \quad Z_{21}=\frac{-h_{21}}{h_{22}} \quad Z_{22}=\frac{1}{h_{22}}$$

Where$\Delta h=h_{11}h_{22}-h_{12}h_{21}$

3. Z-parametersintermsofABCD-parameters

$$Z_{11}=\frac{A}{C} \quad Z_{12}=\frac{AD-BC}{C} \quad Z_{21}=\frac{1}{C} \quad Z_{22}=\frac{D}{C}$$

4. Y-parametersintermsofZ-parameters

$$Y_{11}=\frac{Z_{22}}{\Delta Z} \quad Y_{12}=\frac{-Z_{12}}{\Delta Z} \quad Y_{21}=\frac{-Z_{21}}{\Delta Z} \quad Y_{22}=\frac{Z_{11}}{\Delta Z}$$

Where$\Delta Z=Z_{11}Z_{22}-Z_{12}Z_{21}$

5. Y-parametersintermsofABCD-parameters

$$Y_{11} = \frac{D}{B} \quad Y_{12} = -\frac{AD-BC}{B} \quad Y_{21} = -\frac{1}{B} \quad Y_{22} = \frac{A}{B}$$

6. h-parametersintermsofZ-parameters

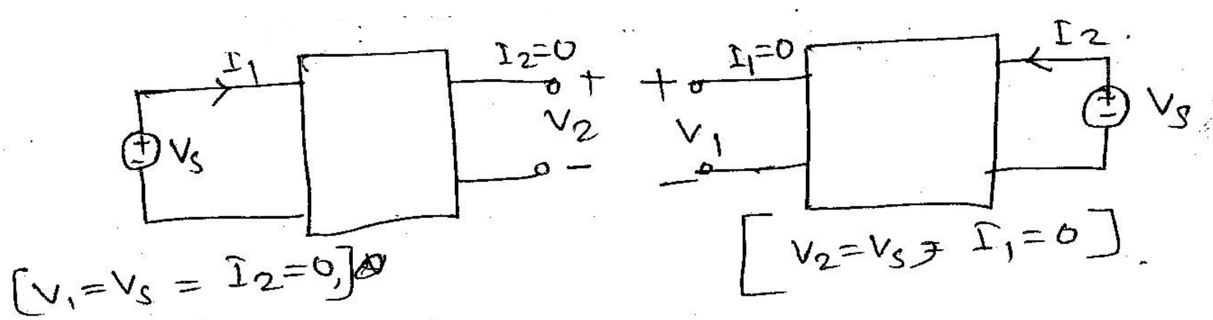
$$h_{11} = \frac{\Delta Z}{Z_{22}} \quad h_{12} = \frac{Z_{12}}{Z_{22}} \quad h_{21} = \frac{-Z_{21}}{Z_{22}} \quad h_{22} = \frac{1}{Z_{22}}$$

7. h-parametersintermsofY-parameters

$$h_{11} = \frac{1}{Y_{11}} \quad h_{12} = -\frac{Y_{12}}{Y_{11}} \quad h_{21} = \frac{Y_{21}}{Y_{11}} \quad Z_{22} = \frac{\Delta Y}{Y_{11}}$$

Conditionof symmetry:-

Atwoportnetworkissaidtobesymmetricaliftheportscanbeinterchangedwithoutportvoltages and currents.



1).IntermsofZ-parameters:-

$$[V]_s/I_1|_{I_2=0} = Z_{11}$$

$$V_s/I_2|_{I_1=0} = Z_{22}$$

So, $Z_{11} = Z_{22}$

2). IntermsofY-parameters:-

$$I_1 = Y_{11}V_s + Y_{12}V_2$$
$$0 = Y_{21}V_s + Y_{22}V_2$$

So,

$$I_1 = Y_{11}V_s + Y_{12}\left\{\frac{-y_{21}}{y_{22}}\right\}V_s$$

$$\Rightarrow \frac{V_s}{I_1} = \frac{y_{22}}{y_{11}y_{22} - y_{21}y_{12}}$$

$$0 = y_{11}V_1 + y_{12}V_s$$

$$I_2 = y_{21}V_1 + y_{22}V_s$$

$$\frac{V_s}{I_s} = \frac{y_{11}}{y_{11}y_{22} - y_{21}y_{12}}$$

$so, \quad y_{11} = y_{22}$

3). <u>IntermsofABCD-parameters:-</u>

$$V_s = AV_2$$
$$I_1 = CV_2$$
$$then, \frac{V_s}{I_1} = \frac{A}{C}$$

Again,

$$V_1 = AV_2 \quad BI_2$$
$$0 = CV_2 - DI_2$$
$$\frac{V_s}{I_2} = \frac{D}{C}$$

So, $\frac{A}{C} = \frac{D}{C}$

So, A=D

<u>Conditionof reciprocity:-</u>

A two port network is said to be reciprocal, if the rate of excitation to response is invariant to aninterchange of the position of the excitation and response in the network. Network containingresistors,capacitorsandinductors are generallyreciprocal.

1) <u>IntermsofZ- parameters:-</u>

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$Now. \; V_s = Z_{11}I_1 - Z_{12}I_2'$$

$$0 = Z_{21}I_1 + Z_{22}I_2'$$

$$I_2' = \frac{V_s Z_{21}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

Similarly,

$$0 = -Z_{11}I_1' + Z_{12}I_2$$

$$V_s = -Z_{21}I_1' + Z_{22}I_2$$

$$hence, I_1' = \frac{V_s Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

Comparing I_2' and I_1' we get,

$$Z_{12} = Z_{21}$$

2) Interms of Y- parameters:-

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

So,

$$I_2' = -Y_{21}V_S$$

$$I_1' = -Y_{12}V_S$$

$$so, Y_{21} = Y_{12}$$

3) Interms of ABCD-parameters:-

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

So, $V_S = BI_2'$

$$I_2' = \frac{V_S}{B}$$

$$I_1 = DI_2'$$

Similarly,

$$0 = AV_S - BI_2$$

$$-I_1' = C\,V_S - DI_2 = CV_S - D\frac{A}{B}V_S$$

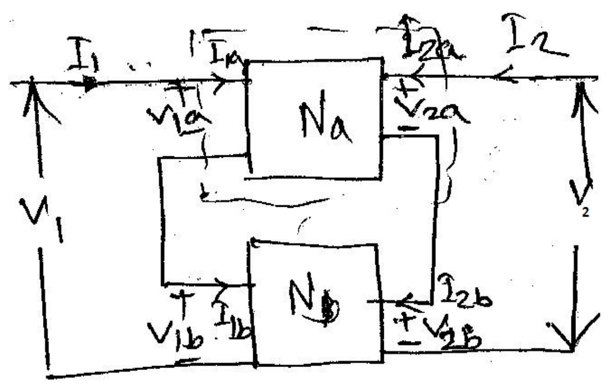
$$\rightarrow I_1' = \frac{AD - BC}{B}V_S$$

$So, I_2' = I_1'$

$\Rightarrow AD - BC = 1$

<u>SeriesConnection:</u>

The fig. shows a series connection of twotwo-port networks Na and Nb with opencircuit Z-parameters Za and Zb respectively.FornetworkNa,



$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

Similarly,fornetworkNb,

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

Then,theirseriesconnectionrequiresthat

$I_1 = I_{1a} = I_{1b} I_2 = I_{2a} = I_{2b}$

$V_1 = V_{1a} + V_{1b} V_2 = V_{2a} + V_{2b}$

Now, $V_1 = V_{1a} + V_{1b}$

$= (Z_{11a} I_{1a} + Z_{12a} I_{2a}) + (Z_{11b} I_{1b} + Z_{12b} I_{2b})$

$= (Z_{11a} + Z_{11b}) I_1 + (Z_{12a} + Z_{12b}) I_2$

Similarly, $V_2 = V_{2a} + V_{2b} = (Z_{21a} + Z_{21b}) I_1 + (Z_{22a} + Z_{22b}) I_2$

So,inmatrix formthe Z-parametersoftheseriesconnectedcombinednetworkcanbewrittenas,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Where, $Z_{11} = Z_{11a} + Z_{11b}$

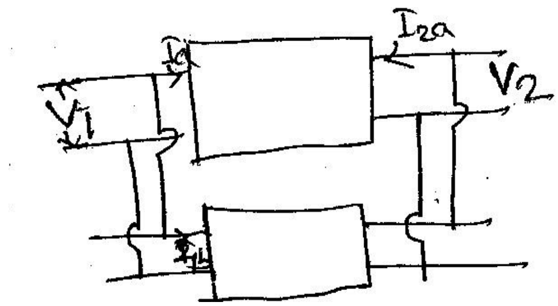
$Z_{12} = Z_{12a} + Z_{12b}$

$Z_{21} = Z_{21a} + Z_{21b}$

$Z_{22} = Z_{22a} + Z_{22b}$

So, $[Z] = [Z_a] + [Z_b]$

<u>ParallelConnection:</u>



Here,

$V_1 = V_{1a} = V_{1b}$

$V_2 = V_{2a} = V_{2b}$

$I_1 = I_{1a} + I_{1b}$

$- Y_{11a}V_{1a} + Y_{12a}V_{2a} + Y_{11b}V_{1b} + Y_{12b}V_{2b}$

$I_2 = I_{2a} + I_{2b}$

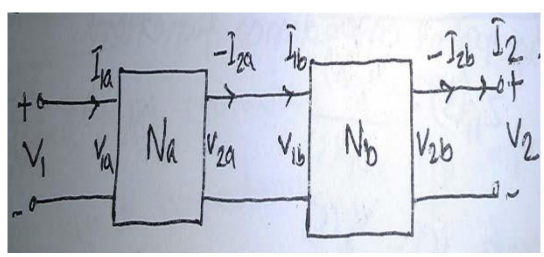
$= Y_{21a}V_{1a} + Y_{22a}V_{2a} + Y_{21b}V_{1b} + Y_{22b}V_{2b}$

So,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11a} + Y_{11b} & Y_{12a} + Y_{12b} \\ Y_{21a} + Y_{21b} & Y_{22a} + Y_{22b} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$\rightarrow [Y] = [Y_a] + [Y_b]$

<u>CascadeConnection:</u>



Now,

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

Then,theircascadeconnectionrequiresthat

$I_1 - I_{1a} - I_{2a} - I_{1b} I_{2b} - I_2$

$V_1 = V_{1a}V_{2a} = V_{1b}V_{2b} = V_2$

So, $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$