SYNERGY INSTITUTE OF ENGINEERING & TECHNOLOGY
Department of Mechanical Engineering
LECTURE NOTE (Academic Session 2023-2024)

Name of Faculty: Mrs. Sushri sangita Mohapatra

Degree : B.TECH **Admission Batch: 2021-22**

Branch : ME **Semester : VI**

Subject Code: RME6C001 **Subject Credit : 3**

Name of Subject : Design of Machine Elements

Theories of Failure

THEORIES OF FAILURE UNDER STATIC LOADING

It has already been discussed in the previous chapter that strength of machine members is based upon the mechanical properties of the materials used. Since these properties are usually determined from simple tension or compression tests, therefore, predicting failure in members subjected to uniaxial stress is both simple and straight-forward. But the problem of predicting the failure stresses for members subjected to bi-axial or tri-axial stresses is much more complicated. In fact, the problem is so complicated that a large number of different theories have been formulated. The principal theories of failure for a member subjected to bi-axial stress are as follows:

- Maximum principal (or normal) stress theory (also known as Rankine's theory).
- Maximum shear stress theory (also known as Guest's or Tresca's theory).
- Maximum principal (or normal) strain theory (also known as Saint Venant theory).
- Maximum strain energy theory (also known as Haigh's theory).
- Maximum distortion energy theory (also known as Hencky and Von Mises theory).

Since ductile materials usually fail by yielding *i.e.* when permanent deformations occur in the material and brittle materials fail by fracture, therefore the limiting strength for these two classes of materials is normally measured by different mechanical properties. For ductile materials, the limiting strength is the stress at yield point as determined from simple tension test and it is, assumed to be equal in tension or compression. For brittle materials, the limiting strength is the ultimate stress in tension or compression.

1. Maximum principal stress theory (Rankine's theory)

According this theory failure or yielding occurs at a point in a member when the maximum principal or normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test. Since the limiting strength for ductile materials is yield point stress and for brittle materials (which do not have well defined yield point) the limiting strength is ultimate stress, therefore according to the above theory, taking factor of safety (F.S.) into consideration, the maximum principal or normal stress σ_{t1} in a bi-axial stress system is given by

$$\sigma_{t1} = \frac{\sigma_{yt}}{F.S.}, \text{ for ductile materials}$$

$$= \frac{\sigma_u}{F.S.}, \text{ for brittle materials}$$

where σ_{yt} = Yield point stress in tension as determined from simple tension test, and

σ_u = Ultimate stress.

Since the maximum principal or normal stress theory is based on failure in tension or compression and ignores the possibility of failure due to shearing stress, therefore it is not used for ductile materials. However, for brittle materials which are relatively strong in shear but weak in tension or compression, this theory is generally used.

2. Maximum shear stress theory (Guest's or Tresca's theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum shear stress in a bi-axial stress system reaches a value equal to the shear stress at yield point in a simple tension test. Mathematically,

$$\tau_{max} = \tau_{yt}/F.S. \qquad ...(i)$$

where τ_{max} = Maximum shear stress in a bi-axial stress system,

τ_{yt} = Shear stress at yield point as determined from simple tension test, and

$F.S.$ = Factor of safety.

Since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension, therefore the equation (*i*) may be written as

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.}$$

This theory is mostly used for designing members of ductile materials.

3. Maximum principal strain theory (Saint Venant theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a bi-axial stress system reaches the limiting value of strain (i.e. strain at yield point) as determined from a simple tensile test. The maximum principal (or normal) strain in a bi-axial stress system is given by

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \cdot E}$$

∴ According to the above theory,

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \cdot E} = \varepsilon = \frac{\sigma_{yt}}{E \times F.S.} \qquad ...(i)$$

where σ_{t1} and σ_{t2} = Maximum and minimum principal stresses in a bi-axial stress system,

ε = Strain at yield point as determined from simple tension test,

$1/m$ = Poisson's ratio,

E = Young's modulus, and

$F.S.$ = Factor of safety.

From equation (*i*), we may write that

$$\sigma_{t1} - \frac{\sigma_{t2}}{m} = \frac{\sigma_{yt}}{F.S.}$$

4. Maximum strain energy theory (Haigh's theory)

According to this theory, the failure or yielding occurs at a point in a member when the strain energy per unit volume in a bi-axial stress system reaches the limiting strain energy (i.e. strain energy at the yield point) per unit volume as determined from simple tension test.

We know that strain energy per unit volume in a bi-axial stress system,

$$U_1 = \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right]$$

and limiting strain energy per unit volume for yielding as determined from simple tension test,

$$U_2 = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

According to the above theory, $U_1 = U_2$.

$$\therefore \quad \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

or

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

5. Maximum distortion energy theory (also known as Hencky and Von Mises theory)

According to this theory, the failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (i.e. distortion energy at yield point) per unit volume as determined from a simple tension test. Mathematically, the maximum distortion energy theory for yielding is expressed as

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

Example 1:

The load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5 kN. Find the diameter of bolt required according to 1. Maximum principal stress theory; 2. Maximum shear stress theory; 3. Maximum principal strain theory; 4. Maximum strain energy theory; and 5. Maximum distortion energy theory. Take permissible tensile stress at elastic limit = 100 MPa and poisson's ratio = 0.3.

Given : $P_{t1} = 10$ kN ; $P_s = 5$ kN ; $\sigma_{t(el)} = 100$ MPa $= 100$ N/mm^2 ; $1/m = 0.3$

Let d = Diameter of the bolt in mm.

∴ Cross-sectional area of the bolt,

$$A = \frac{\pi}{4} \times d^2 = 0.7854\ d^2\ \text{mm}^2$$

We know that axial tensile stress,

$$\sigma_1 = \frac{P_{t1}}{A} = \frac{10}{0.7854\ d^2} = \frac{12.73}{d^2}\ \text{kN/mm}^2$$

and transverse shear stress,

$$\tau = \frac{P_s}{A} = \frac{5}{0.7854\ d^2} = \frac{6.365}{d^2}\ \text{kN/mm}^2$$

1. ***According to maximum principal stress theory***

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \ldots(\because \sigma_2 = 0)$$

$$= \frac{12.73}{2\,d^2} + \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} + \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4 + 4}\right]$$

$$= \frac{6.365}{d^2}\left[1 + \frac{1}{2}\sqrt{4 + 4}\right] = \frac{15.365}{d^2}\ \text{kN/mm}^2 = \frac{15\,365}{d^2}\ \text{N/mm}^2$$

According to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{t(el)} \quad \text{or} \quad \frac{15\,365}{d^2} = 100$$

∴ $d^2 = 15\,365/100 = 153.65$ or $d = 12.4$ mm **Ans.**

2. ***According to maximum shear stress theory***

We know that maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \ldots(\because \sigma_2 = 0)$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right] = \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4 + 4}\right]$$

$$= \frac{9}{d^2}\ \text{kN/mm}^2 = \frac{9000}{d^2}\ \text{N/mm}^2$$

According to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{t(el)}}{2} \quad \text{or} \quad \frac{9000}{d^2} = \frac{100}{2} = 50$$

∴ $d^2 = 9000 / 50 = 180$ or $d = 13.42$ mm **Ans.**

3. *According to maximum principal strain theory*

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \frac{15\,365}{d^2}$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{12.73}{2\,d^2} - \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} - \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{6.365}{d^2}\left[1 - \sqrt{2}\right] = \frac{-\,2.635}{d^2}\ \text{kN/mm}^2$$

$$= \frac{-\,2635}{d^2}\ \text{N/mm}^2$$

We know that according to maximum principal strain theory,

$$\frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{mE} = \frac{\sigma_{t(el)}}{E} \quad \text{or} \quad \sigma_{t1} - \frac{\sigma_{t2}}{m} = \sigma_{t(el)}$$

$$\therefore \quad \frac{15\,365}{d^2} + \frac{2635 \times 0.3}{d^2} = 100 \quad \text{or} \quad \frac{16\,156}{d^2} = 100$$

$d^2 = 16\,156 / 100 = 161.56$ or $d = 12.7$ mm **Ans.**

4. *According to maximum strain energy theory*

We know that according to maximum strain energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-\,2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-\,2635}{d^2} \times 0.3 = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{24.3 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{2430}{d^4} = 1 \quad \text{or} \quad \frac{26\,724}{d^4} = 1$$

$\therefore$ $d^4 = 26\,724$ or $d = 12.78$ mm **Ans.**

5. *According to maximum distortion energy theory*

According to maximum distortion energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\ 365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - 2 \times \frac{15\ 365}{d^2} \times \frac{-2635}{d^2} = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{80.97 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\ 600}{d^4} + \frac{694}{d^4} + \frac{8097}{d^4} = 1 \quad \text{or} \quad \frac{32\ 391}{d^4} = 1$$

$\therefore$ $d^4 = 32\ 391$ or $d = 13.4$ mm **Ans.**

Example 2:

A mild steel shaft of 50 mm diameter is subjected to a bending moment of 2000 N-m and a torque T. If the yield point of the steel in tension is 200 MPa, find the maximum value of this torque without causing yielding of the shaft according to 1. the maximum principal stress; 2. The maximum shear stress; and 3. the maximum distortion strain energy theory of yielding.

Given: $d = 50$ mm ; $M = 2000$ N-m $= 2 \times 10^6$ N-mm ; $\sigma_{yt} = 200$ MPa $= 200$ N/mm^2

1. *According to maximum principal stress theory*

We know that section modulus of the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(50)^3 = 12\ 273 \text{ mm}^3$$

$\therefore$ Bending stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{2 \times 10^6}{12\ 273} = 163 \text{ N/mm}^2$$

and shear stress due to the torque,

$$\tau = \frac{16\ T}{\pi\ d^3} = \frac{16\ T}{\pi\ (50)^3} = 0.0407 \times 10^{-3}\ T \text{N/mm}^2$$

$$\ldots\left[\because T = \frac{\pi}{16} \times \tau \times d^3\right]$$

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\ \tau^2}\right]$$

$$= \frac{163}{2} + \frac{1}{2}\left[\sqrt{(163)^2 + 4\ (0.0407 \times 10^{-3}\ T)^2}\right]$$

$$= 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} \text{ N/mm}^2$$

Minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{163}{2} - \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\, T)^2}\right]$$

$$= 81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} \text{ N/mm}^2$$

and maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\, T)^2}\right]$$

$$= \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} \text{ N/mm}^2$$

We know that according to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{yt} \qquad \text{...(Taking } F.S. = 1)$$

We know that according to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{yt} \qquad \text{...(Taking } F.S. = 1)$$

$$\therefore \quad 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} = 200$$

$$6642.5 + 1.65 + 10^{-9}\, T^2 = (200 - 81.5)^2 = 14\,042$$

$$T^2 = \frac{14\,042 - 6642.5}{1.65 \times 10^{-9}} = 4485 \times 10^9$$

or $T = 2118 \times 10^3$ N-mm = 2118 N-m **Ans.**

2. *According to maximum shear stress theory*

We know that according to maximum shear stress theory,

$$\tau_{max} = \tau_{yt} = \frac{\sigma_{yt}}{2}$$

$$\therefore \quad \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} = \frac{200}{2} = 100$$

$$6642.5 + 1.65 \times 10^{-9}\, T^2 = (100)^2 = 10\,000$$

$$T^2 = \frac{10\,000 - 6642.5}{1.65 \times 10^{-9}} = 2035 \times 10^9$$

$\therefore$ $T = 1426 \times 10^3$ N-mm = 1426 N-m **Ans.**

3. *According to maximum distortion strain energy theory*

We know that according to maximum distortion strain energy theory

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \sigma_{t1} \times \sigma_{t2} = (\sigma_{yt})^2$$

$$\left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2}\right]^2 + \left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2}\right]^2$$

$$- \left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2}\right]\left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2}\right] = (200)^2$$

$$2\left[(81.5)^2 + 6642.5 + 1.65 \times 10^{-9}\,T^2\right] - \left[(81.5)^2 - 6642.5 + 1.65 \times 10^{-9}\,T^2\right] = (200)^2$$

$$(81.5)^2 + 3 \times 6642.5 + 3 \times 1.65 \times 10^{-9}\,T^2 = (200)^2$$

$$26\,570 + 4.95 \times 10^{-9}\,T^2 = 40\,000$$

$$T^2 = \frac{40\,000 - 26\,570}{4.95 \times 10^{-9}} = 2713 \times 10^9$$

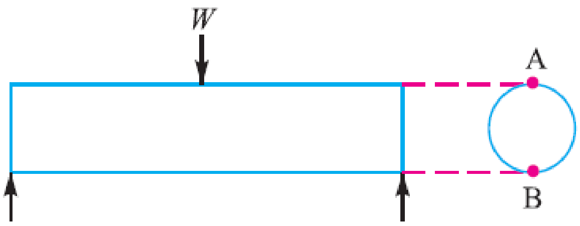
$$\therefore \quad T = 1647 \times 10^3 \text{ N-mm} = 1647 \text{ N-m}$$ **Ans.**

STRESS DUE TO VARIABLE LOADING CONDITIONS

A few machine parts are subjected to static loading. Since many of the machine parts (such as axles, shafts, crankshafts, connecting rods, springs, pinion teeth etc.) are subjected to variable or alternating loads (also known as fluctuating or fatigue loads).

Completely Reversed or Cyclic Stresses

Consider a rotating beam of circular cross-section and carrying a load W, as shown in the figure. This load induces stresses in the beam which are cyclic in nature. A little consideration will show that the upper fibres of the beam (i.e. at point A) are under compressive stress and the lower fibres (i.e. at point B) are under tensile stress. After half a revolution, the point B occupies the position of point A and the point A occupies the position of point B. Thus the point B is now under compressive stress and the point A under tensile stress. The speed of variation of these stresses depends upon the speed of the beam. From above we see that for each revolution of the beam, the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile or vice versa, are known as completely reversed or cyclic stresses.



Fatigue and Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals. In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig. 2, is rotated in a fatigue testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle. This is represented by a time-stress diagram as shown in Fig. 3. A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in Fig. 4. A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig. 4, the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as endurance or fatigue limit (σe). It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 10^7 cycles).It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term endurance strength may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions. We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig. 3. In actual practice, many machine members undergo different range of stress than the completely reversed stress. The stress verses time diagram for fluctuating stress having values σmin and σmax is shown in Fig. 5. The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σv. The following relations are derived from Fig. 5:

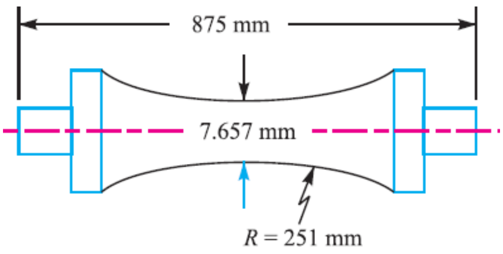


Fig. 2 standard specimen

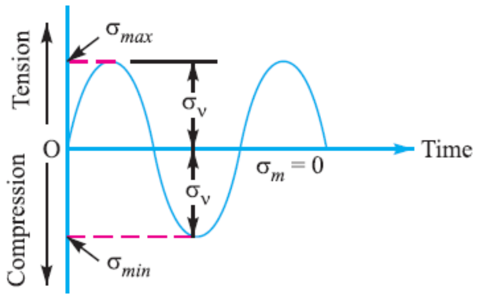


Fig. 3 completely reversed stress

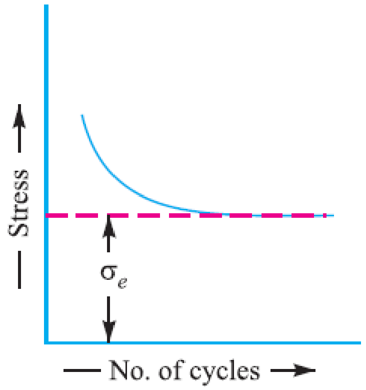


Fig. 4 Endurance/fatigue limit

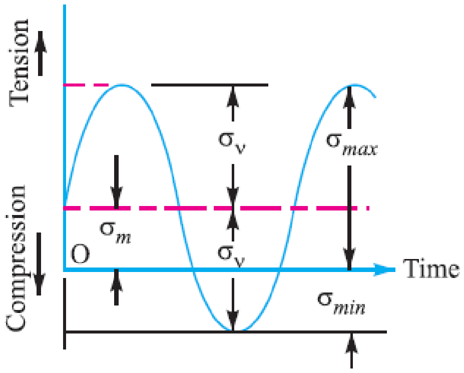


Fig. 5 fluctuating stress

Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

EFFECT OF MISCELLANEOUS FACTORS ON ENDURANCE LIMIT

Corrected endurance limit for variable bending load $\boxed{\sigma_e^{'} = \sigma_e.K_b.K_{sr}.K_{sz}}$ (Page 4.14, Design data handbook by Jalaludeen)

Corrected endurance limit for variable axial load $\boxed{\sigma_e^{'} = \sigma_e.K_a.K_{sr}.K_{sz}}$ (Page 4.14, Design data handbook by Jalaludeen)

Corrected endurance limit for variable torsional load $\boxed{\sigma_e^{'} = \sigma_e.K_s.K_{sr}.K_{sz}}$ (Page 4.14, Design data handbook by Jalaludeen)

Where,

K_a = load correction factor for revered axial load

K_b = load correction factor for revered bending load

K_{sr} = surface finish factor

K_{sz} = size factor

σ_e = endurance limit/fatigue stress

Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for faliure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

And $\sigma_e \approx 0.5\sigma_u \approx 0.85\sigma_y$ (for steel) (Page 4.14, Design data handbook by Jalaludeen)

Stress Concentration

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called stress concentration. It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc. In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in Figure 6. A little consideration will show that the nominal stress in the right and left hand sides will be uniform but in the region where the cross-section is changing, a re-distribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

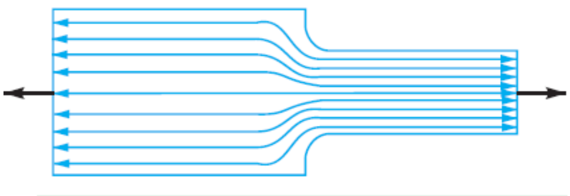


Figure 6 stress concentration

Theoretical Stress Concentration Factor (K_t)

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area. Mathematically, theoretical or form stress concentration factor,

$$K_t = \frac{\text{Maximum stress}}{\text{Nominal stress}}$$

The value of K_t depends upon the material and geometry of the part. (Table 4.9 to 4.16, Design data handbook by Jalaludeen).

Methods of Reducing Stress Concentration

1. By providing fillets as shown in Fig. 7

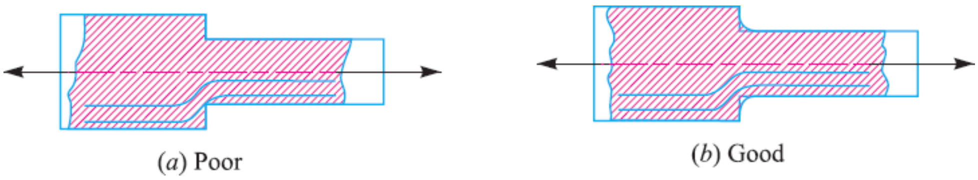


Figure 7 Fillets to improve stress concentration

2. By providing notches shown in Fig. 8

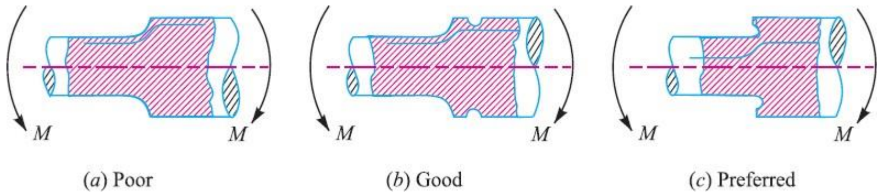


Figure 8 Notches to improve stress concentration

Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Notch Sensitivity

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term notch sensitivity is applied to this behaviour. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material.

When the notch sensitivity factor q is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$$q = \frac{K_f - 1}{K_t - 1}$$

or $$K_f = 1 + q\,(K_t - 1) \qquad \text{...[For tensile or bending stress]}$$

and $$K_{fs} = 1 + q\,(K_{ts} - 1) \qquad \text{...[For shear stress]}$$

(Page 4.14, Design data handbook by Jalaludeen)

Where,

K_t = Theoretical stress concentration factor for axial or bending loading, and

K_{ts} = Theoretical stress concentration factor for torsional or shear loading.

Combined Steady and Variable Stress

The failure points from fatigue tests made with different steels and combinations of mean and variable stresses are plotted in Fig. 9 as functions of variable stress (σ_v) and mean stress (σ_m).

The most significant observation is that, in general, the failure point is little related to the mean stress when it is compressive but is very much a function of the mean stress when it is tensile. In practice, this means that fatigue failures are rare when the mean stress is compressive (or negative). Therefore, the greater emphasis must be given to the combination of a variable stress and a steady (or mean) tensile stress.

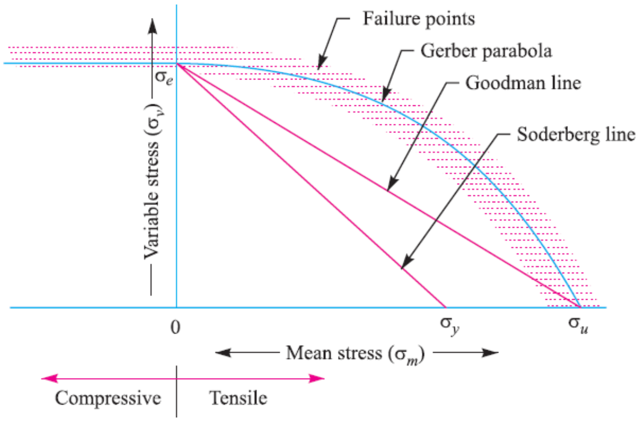


Figure 9 Combined mean and variable stress

There are several ways in which problems involving this combination of stresses may be solved, but the following are important from the subject point of view :

1. Goodman method, and 2. Soderberg method.

<u>Goodman Method for Combination of Stresses</u>

A straight line connecting the endurance limit (σ_e) and the ultimate strength (σu), as shown by line AB in Fig. 10, follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

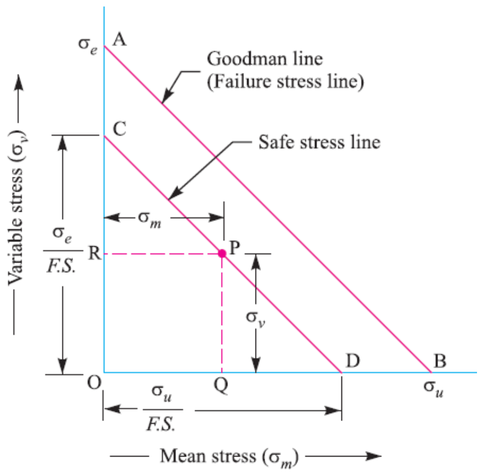


Figure 10 Goodman method

σ_u is called Goodman's failure stress line. If a suitable factor of safety (F.S.) is applied to endurance limit and ultimate strength, a safe stress line *CD* may be drawn parallel to the line

AB. Let us consider a design point P on the line *CD*. Now from similar triangles *COD* and *PQD*,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD} \qquad \ldots(\because\ QD = OD - OQ)$$

$$\therefore \qquad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_u / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u}\right]$$

or

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} \qquad \ldots(i)$$

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads. Since many machine and structural parts that are subjected to fatigue loads contain regions of high stress concentration, therefore equation (i) must be altered to include this effect. In such cases, the fatigue stress concentration factor (K_f) is used to multiply the variable stress (σ_v). The equation (i) may now be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad \ldots(ii)$$

where $F.S.$ = Factor of safety,
σ_m = Mean stress,
σ_u = Ultimate stress,
σ_v = Variable stress,
σ_e = Endurance limit for reversed loading, and
K_f = Fatigue stress concentration factor.

(Equation 4.50, pp. 4.14, Design data handbook by Jalaludeen)

Considering the load factor, surface finish factor and size factor, the equation (ii) may be written as

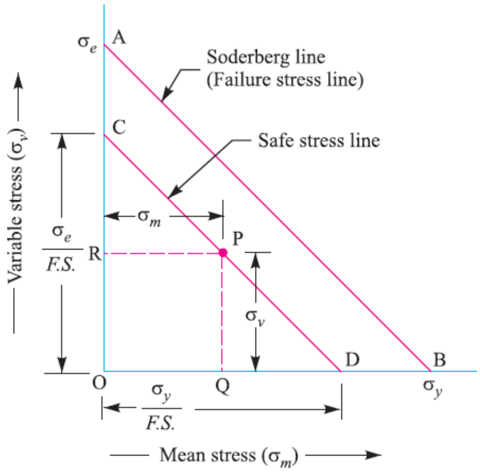
$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_b \times K_{sur} \times K_{sz}} \qquad \ldots(iii)$$

$$= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \qquad \ldots(\because\ \sigma_{eb} = \sigma_e \times K_b \text{ and } K_b = 1)$$

where K_b = Load factor for reversed bending load,
K_{sur} = Surface finish factor, and
K_{sz} = Size factor.

Soderberg Method for Combination of Stresses

A straight line connecting the endurance limit (σ_e) and the yield strength (σ_y), as shown by the line *AB* in Fig. 11, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength. the line AB connecting σe and σy, as shown in Fig. 11, is called Soderberg's failure stress line. If a suitable factor of safety (F.S.) is applied to the endurance limit and yield strength, a safe stress line *CD* may be drawn parallel to the line *AB*. Let us consider a design point P on the line *CD*. Now from similar triangles *COD* and



PQD,

Figure 11 Soderberg method

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD}$$

$$= 1 - \frac{OQ}{OD}$$

$$\ldots (\because QD = OD - OQ)$$

$$\therefore \quad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_y / F.S.}$$

or

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_y / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_y}\right]$$

$$\therefore \quad \frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

(Equation 4.51, pp. 4.14, Design data handbook by Jalaludeen)

(DESIGN OF JOINTS)

Riveted Joint

Often small machine components are joined together to form a larger machine part. Design of joints is as important as that of machine components because a weak joint may spoil the utility of a carefully designed machine part. Mechanical joints are broadly classified into two classes viz., non-permanent joints and permanent joints. Non-permanent joints can be assembled and dissembled without damaging the components. Examples of such joints are threaded fasteners (like screw-joints), keys and couplings etc.

Permanent joints cannot be dissembled without damaging the components. These joints can be of two kinds depending upon the nature of force that holds the two parts. The force can be of mechanical origin, for example, riveted joints, joints formed by press or interference fit etc, where two components are joined by applying mechanical force. The components can also be joined by molecular force, for example, welded joints, brazed joints, joints with adhesives etc. Not until long ago riveted joints were very often used to join structural members permanently. However, significant improvement in welding and bolted joints has curtained the use of these joints. Even then, rivets are used in structures, ship body, bridge, tanks and shells, where high joint strength is required.

Rivets and riveting

A Rivet is a short cylindrical rod having a head and a tapered tail. The main body of the rivet is called shank (see figure 2.1).

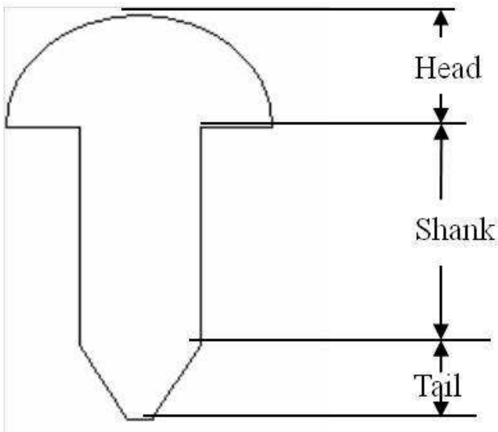


Fig. 2.1 Rivets and its parts

According to Indian standard specifications rivet heads are of various types. Rivets heads for general purposes are specified by Indian standards IS: 2155-1982 (below 12 mm diameter) and IS: 1929-1982 (from 12 mm to 48 mm diameter). Rivet heads used for boiler works are specified by IS: 1928-1978. To get dimensions of the heads see any machine design handbook.

Riveting is an operation whereby two plates are joined with the help of a rivet. Adequate mechanical force is applied to make the joint strong and leak proof. Smooth holes are drilled (or punched and reamed) in two plates to be joined and the rivet is inserted. Holding, then, the head by means of a backing up bar as shown in figure 2.2, necessary force is applied at the tail end with a die until the tail deforms plastically to the required shape. Depending upon whether the rivet is initially heated or not, the riveting operation can be of two types: (a) cold riveting is done at ambient temperature and (b) hot riveting rivets are initially heated before applying force. After riveting is done, the joint is heat-treated by quenching and tempering. In order to ensure leak-proofness of the joints, when it is required, additional operation like caulking is done.

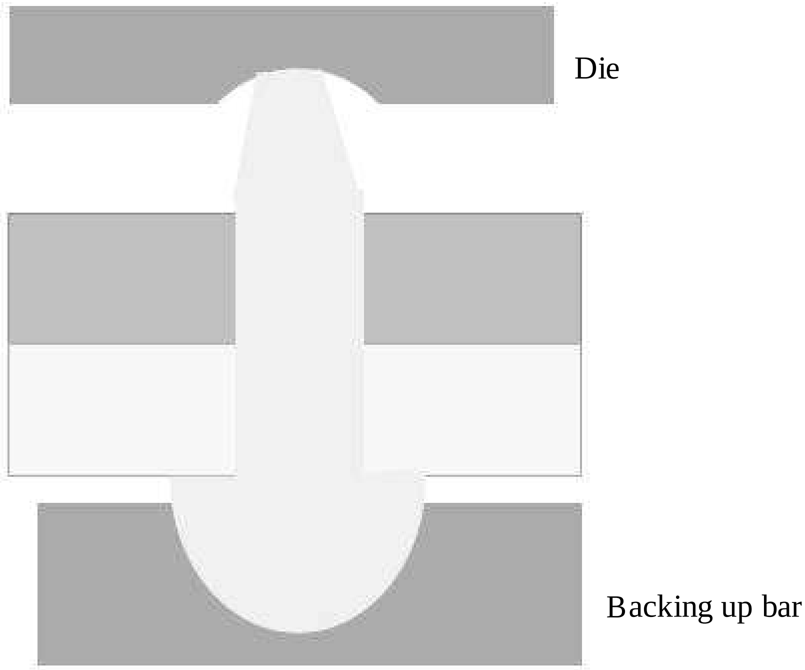


Fig. 2.2 Riveting operation

Types of rivet joints

Riveted joints are mainly of two types

1. Lap joints
2. Butt joints

Lap joints

The plates that are to be joined are brought face to face such that an overlap exists, as shown in figure 10.1.3. Rivets are inserted on the overlapping portion. Single or multiple rows of rivets are used to give strength to the joint. Depending upon the number of rows the riveted joints may be classified as single riveted lap joint, double or triple riveted lap joint etc. When multiple joints are used, the arrangement of rivets between two neighbouring rows may be of two kinds. In chain riveting the adjacent rows have rivets in the same transverse line. In zig-zag riveting, on the other hand, the adjacent rows of rivets are staggered.

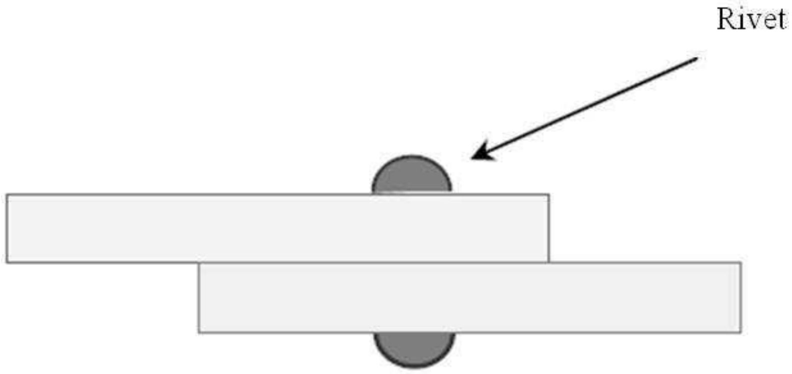


Fig. 2.3 Lap joint

But joints

In this type of joint, the plates are brought to each other without forming any overlap. Riveted joints are formed between each of the plates and one or two cover plates. Depending upon the number of cover plates the butt joints may be single strap or double strap butt joints. A single strap butt joint is shown in figure 2.4. Like lap joints, the arrangement of the rivets may be of various kinds, namely, single row, double or triple chain or zigzag.

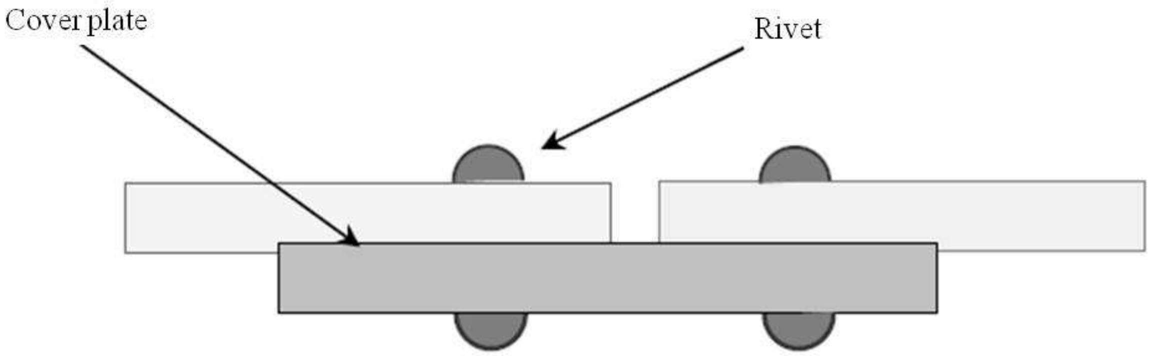


Fig. 2.4 Butt joint

Important terms used in rivet joints

Few parameters, which are required to specify arrangement of rivets in a riveted joint are as follows:

a) Pitch: This is the distance between two centers of the consecutive rivets in a single row. (usual symbol p)

b) Back Pitch: This is the shortest distance between two successive rows in a multiple riveted joint. (usual symbol b_p)

c) Diagonal pitch: This is the distance between the centers of rivets in adjacent rows of zigzag riveted joint. (usual symbol p_d)

d) Margin or marginal pitch: This is the distance between the centre of the rivet hole to the nearest edge of the plate. (usual symbol m)

These parameters are shown in figure 2.5.

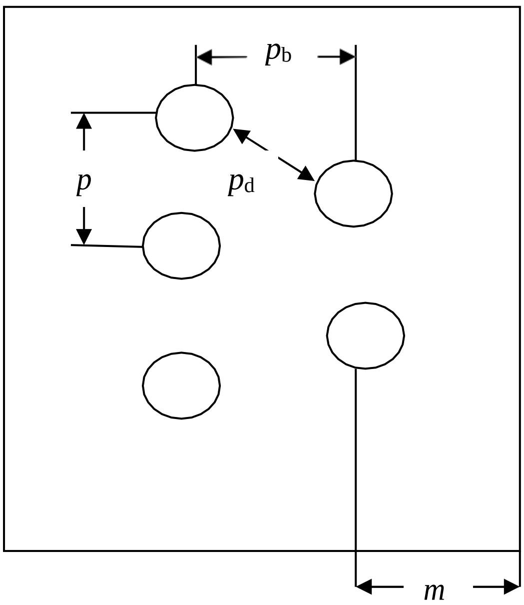


Fig. 2.5 Important design parameters of riveted joints

Modes of failure of rivet joints

(1) *Tearing of the plate at the edge*: Figure 2.6 shows the nature of failure due to tearing of the plate at the edge.

Such a failure occurs due to insufficient margin. This type of failure can be avoided by keeping margin, $m = 1.5d$, where d is the diameter of the rivet.

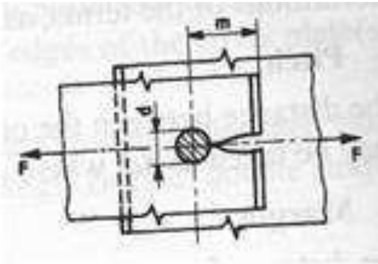


Fig. 2.6 Tearing of the plate at the edge

(2) *Tearing of the plate across a row of rivets*: In this, the main plate or cover plates may tear-off across a row of rivets, as shown in Fig. 2.7. Considering one pitch length,

Tearing strength per pitch length, $F_t = \sigma_t (p - d)t$ (2.1)

Where, σ_t = permissible tensile stress for the plate material; p = pitch; d = diameter of the rivet; t = thickness of the plate.

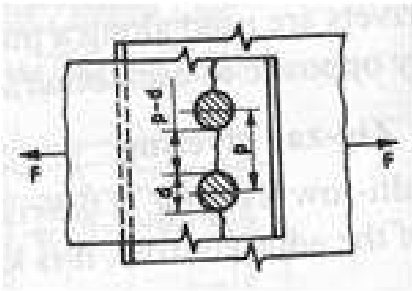
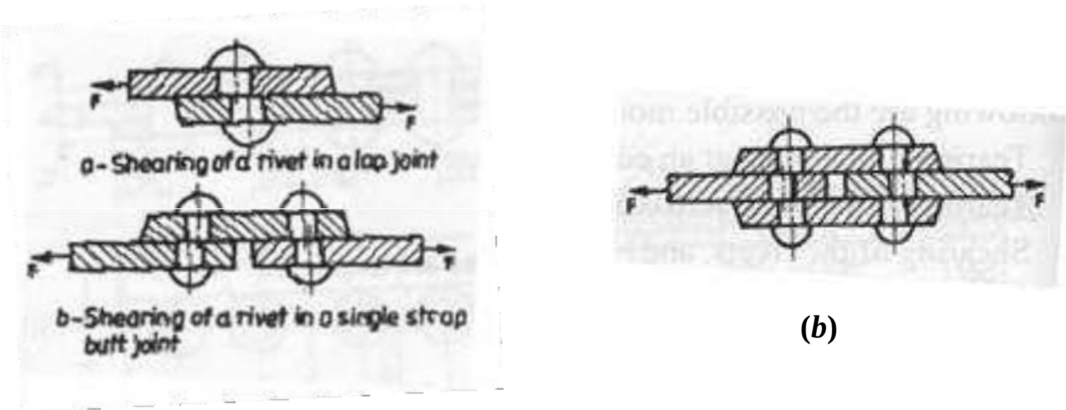


Fig. 2.7 Tearing of the plate across a row of rivets

(3) *Shearing of rivets*: Rivets are in single shear (Fig. 2.8*a*)in lap joints and in double shear in double strap butt joints (Fig. 2.8*b*). Considering one pitch length,



(***a***) (***b***)

Fig. 2.8 Shearing of rivets

Shearing resistance per pitch length, $F_s = \frac{\pi d^2}{4} n\tau$ in single shear (2.2)

$= 2 \times \frac{\pi d^2}{4} \times n\tau$ in double shear (2.3)

Where, n = number of rivets per pitch length

(4) *Crushing of rivets (plates)*: When the joint is loaded, compressive stress is induced over the contact area between rivet and the plate (Fig. 2.9).

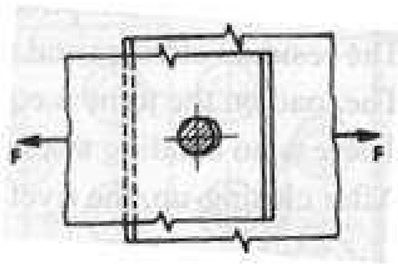


Fig. 2.9 Crushing of a rivets

The contact area is given by the projected area of the contact. Considering one pitch length,

Crushing resistance per pitch length, $F_c = ndt\sigma_c$ (2.4)

Where, n = number of rivets per pitch length; σ_c = permissible compressive stress.

Note: Number of rivets under crushing is equal to the number of rivets under shear.

Efficiency of a riveted joint

The efficiency of a riveted joint is defined as the ratio of the strength of the joint (least of calculated resistances) to the strength of the solid plate.

Efficiency of a riveted joint, $\eta = \frac{F_t, F_s, or F_c (least)}{pt\sigma_t}$ (2.5)

Where, ' $pt\sigma_t$ ' is the strength of the solid plate per pitch length.

Design of boiler joints

In general, for longitudinal joint, butt joint is adopted, while for circumferential joint, lap joint is preferred.

Design of longitudinal butt joint

1. *Thickness of the plate*: The thickness of the boiler shell is determined, by using thin cylinder formula, i.e.

$$t = \frac{P_i d_i}{2\sigma_t \eta} + 1mm \quad (2.6)$$

Where, 1 mm is the allowance for corrosion; P_i = internal steam pressure; d_i = internal diameter of the boiler shell; σ_t = permissible stress of the shell material.

2. *Diameter of rivets*: The diameter of the rivets may be determined from the empirical relation, $d = 6\sqrt{t}$ for $t \geq 8$mm

Note: (1) The diameter of rivet should not be less than the plate thickness.

(2) If the plate thickness is less than 8 mm, the diameter of the rivet is determined by equating the shearing resistance of the rivet to its crushing resistance.

3. *Pitch of the rivets*: The pitch of the rivets may be obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets. However, it should be noted that,

   (i) the pitch of the rivets should not be less than 2*d*.

   (ii) the maximum value of the pitch, for a longitudinal joint is given by,

   $P_{\max} = ct + 41.28mm$ where 'c' is a constant.

   Note: If the pitch of the rivets obtained by equating the tearing resistance to the shearing resistance is more than P_{max}, then the value of P_{max} can be adopted.

4. *Row (transverse) pitch*:

(i) For equal number of rivets in more than one row for lap joint or butt joint, the row pitch should not be less than, $0.33p + 0.67d$ for zig-zag riveting and 2*d*, for chain riveting.

(ii) For joints in which the number of rivets in the rows is half the number of rivets in the inner rows, and if the inner rows are chain riveted, the distance between the outer row and the next row, should not be less than, $0.33p + 0.67d$ or 2*d*, whichever is greater. The distance between the rows in which there are full number of rivets, should not be less than 2*d*.

(iii) For joints in which the number of rivets in outer row is half the number of rivets in inner rows, and if the inner rows are zig-zag riveted, the distance between the outer row and the next row, should not be less than, $0.2p + 1.15d$. The distance between the rows in which there are full number of rivets (zig-zag), should not be less than, $0.165p + 0.67d$.

Note: p is the pitch of the rivets in the outer row.

5. *Thickness of butt straps*: The thickness of butt strap(s) is given by, (in no case it should not be less than 10 mm).

   $t_1 = 1.125t$, for ordinary single butt strap (chain riveting)

$= 1.125t\left(\frac{p-d}{p-2d}\right)$, for a single butt strap, with alternate rivets in the outer rows omitted

$= 0.625t$, for ordinary double straps of equal width (chain riveting)

$= 0.625t\left(\frac{p-d}{p-2d}\right)$, for double straps of equal width, with alternate rivets in the outer rows omitted

When two unequal widths of butt straps are employed, the thickness of butt straps are given by, $t_1 = 0.75t$, for wide strap on the inside and $t_2 = 0.625t$, for narrow strap on the outside.

Note: The thickness of butt strap, in no case, shall be less than 10 mm.

6. *Margin*: The margin '*m*' is generally followed as 1.5*d*.

Design of circumferential lap joint

1. *Diameter of rivets*: It is usual practice to adopt the rivet diameter and plate thickness, same as those used for longitudinal joint.
2. Number of rivets: The rivets are in single shear, since lap joint is used for circumferential joint.

   Total number of rivets to be used for the joint,

$$N = \frac{\text{steam load}}{\text{Shear strength of one}} = \left(\frac{\pi d_i^2}{4} \times p_i\right) / \left(\frac{\pi d^2}{4}\tau\right) = \left(\frac{d_i}{d}\right)^2 \times \frac{p_i}{\tau}$$

   Where, d_i = inner diameter of boiler; d = rivet diameter; τ = allowable shear strength of rivet material

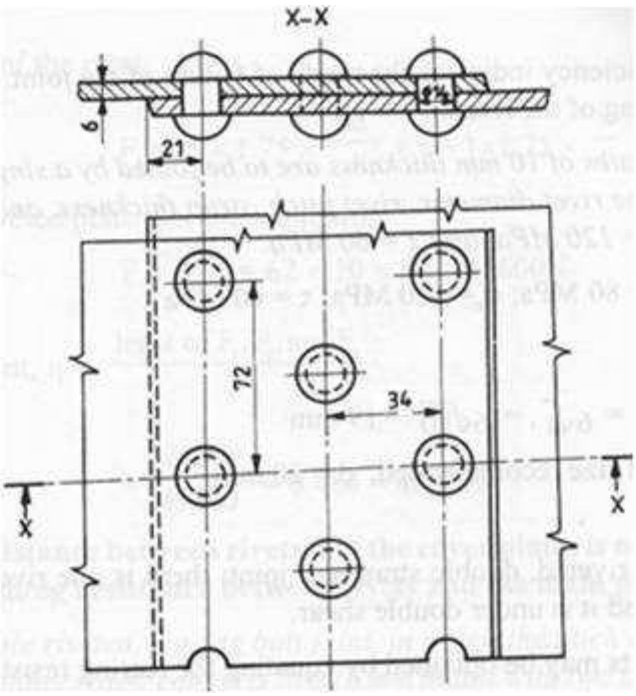
3. *Pitch of rivets*: In general, the efficiency of the circumferential joint may be taken as 50% of the tearing efficiency of the longitudinal joint. If intermediate circumferential joints are used, the strength of the seam should not be less than 62% of the strength of the undrilled plate. Knowing the (tearing) efficiency of the circumferential joint, the pitch of the rivets can be obtained from,

$$\text{Efficiency, } \eta = \frac{(p-d)t\sigma_t}{pt\sigma_t} = \frac{p-d}{p}$$

4. *Number of rows*: Number of rivets per row, $n = \frac{\pi(d_i + t)}{p}$

$$\text{Number of rows, } Z = \frac{\text{Total number of rivets}}{\text{No. of rivets per row}}$$

5. *Selection of the type of joint*: After determining the number of rows, the type of joint (single riveted, double riveted etc.) may be decided.
6. *Row (back) pitch and margin*: The proportions suggested for longitudinal joint, may be followed for the circumferential joint as well.

Example problem – 1: Design a triple riveted lap joint, to join two plates of 6 mm thick. The allowable stresses are: σ_t = 80 MPa, σ_c = 100 MPa, and τ = 60 MPa. Calculate the rivet diameter, rivet pitch, and distance between the rows of rivets. Use zig-zag riveting. State how the joint will fail.	**Reference**
Solution: As the thickness of the plate is less than 8 mm, the diameter of the rivet may be determined by equating the shearing resistance to the crushing resistance. Further, as the joint is triple riveted zig-zag lap joint, there will be three rivets per pitch length (Fig. 2.10) and are under single shear, and same number of rivets under crushing.	Design Data Book by K. Mahadevan & K. B. Reddy
 **Fig. 2.10**	
Shearing resistance, $F_s = \frac{\pi d^2}{4} n\tau = 3\times\frac{\pi d^2}{4}\times 60 = 141.37d^2$ (i)	Table-5.3b; Page-68A;
Crushing resistance, $F_c = ndt\sigma_c = 3\times d\times 6\times 100 = 1800d$ (ii)	**Reference**

Equating equations (i) and (ii) we have, $141.37d^2 = 1800d$

$$d = 12.73 \text{ mm}$$

The nearest standard diameter of the rivet recommended, d = 14 mm

Pitch of the rivets: The pitch of the rivets may be obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.

Tearing resistance, $F_t = \sigma_t (p - d)t = (p - 14) \times 6 \times 80 \text{ N}$ (iii)

Page-66, Eq. 5.31b

Shearing resistance, $F_s = n \times \frac{\pi d^2}{4} \times \tau = 3 \times \frac{\pi 14^2}{4} \times 60 = 27708.8 \text{ N}$ (iv)

Equating equations (i) and (ii) we have, $(p - 14) \times 480 = 27708.8$

$$p = 71.73 \text{ mm, say 72 mm.}$$

Distance between the rows of rivets, p_b (or p_t) = $0.33\,p + 0.67d = 33.14 mm$, say 34 mm.

Mode of failure of the joint:

Tearing efficiency = $\frac{p-d}{p} = \frac{72-14}{72}$ = 0.801 = 80.1%

Crushing efficiency = $\frac{ndt\sigma_c}{pt\sigma_t} = \frac{3 \times 14 \times 6 \times 100}{72 \times 6 \times 80}$ = 0.802 = 80.2%

The lowest efficiency indicates the mode of failure of the joint. In the present case, the joint will fail by crushing of the rivets.

Example problem – 2: A double riveted, zig-zag butt joint, in which the pitch of the rivets in the outer row is twice that in the inner rows; connects two 16 mm plates with two cover plates, each 12 mm thick. Determine the diameter of the rivets and pitch of the rivets and pitch of the rivets, if the working stresses are: σ_t = 100 MPa, σ_c = 150 MPa, and τ = 75 MPa.

Reference

Solution:

Diameter of the rivet:

Diameter of the rivet, $d = 6\sqrt{t} = 24 mm$

Design Data Book by K. Mahadevan & K. B. Reddy

Pitch of the rivets:

Let p_o = pitch of the rivets in the outer row

P_i = pitch of the rivets in the outer row

The pitch of the rivets may be obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.

Referring Fig. 2.11, since the pitch in the outer row is twice the pitch of inner row; for one pitch length in the outer row, there are three rivets, which are under double shear.

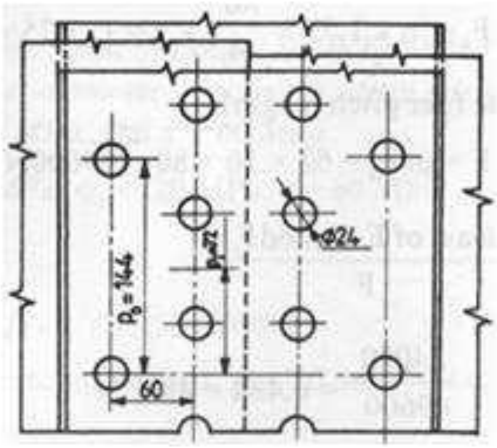


Fig. 2.11- Double riveted, double strap, zig-zag butt joint

Tearing resistance, $F_t = (p-d)t\sigma_t = (p_0 - 14)\times 16\times 100 = (p_o - 24)\times 1600$ N (i)

Shearing resistance, $F_s = n\times 1.875\times \frac{\pi d^2}{4}\tau$, assuming that the rivets under double shear are 1.875 times as strong as those under single shear = $3\times 1.875\times \frac{\pi}{4}\times 24^2\times 75 = 190851.8$ N (ii)

Equating equations (i) and (ii) we have, $(p_o - 24)\times 1600 = 190851.8$

p_o = 143.3, say 144 mm

Pitch of rivets in the inner row, $p_i = \frac{p_o}{2} = \frac{144}{2}$ = 72 mm

Distance between the rows of rivets:

Page-66, Eq. 5.33a

For zig-zag riveting, the row (back) pitch, pb $\geq 0.2p_o + 1.15d \geq 0.2\times 144 + 1.15\times 24 \geq 56.4$ mm.

A back/row pitch of 60 mm may be recommended.

Welded joints

Welded joints and their advantages:

Welding is a very commonly used permanent joining process. Thanks to great advancement in welding technology, it has secured a prominent place in manufacturing machine components. A welded joint has following advantages:

(i) Compared to other type of joints, the welded joint has higher efficiency. An efficiency > 95 % is easily possible.

(ii) Since the added material is minimum, the joint has lighter weight.

(iii) Welded joints have smooth appearances.

(iv) Due to flexibility in the welding procedure, alteration and addition are possible.

(v) It is less expensive.

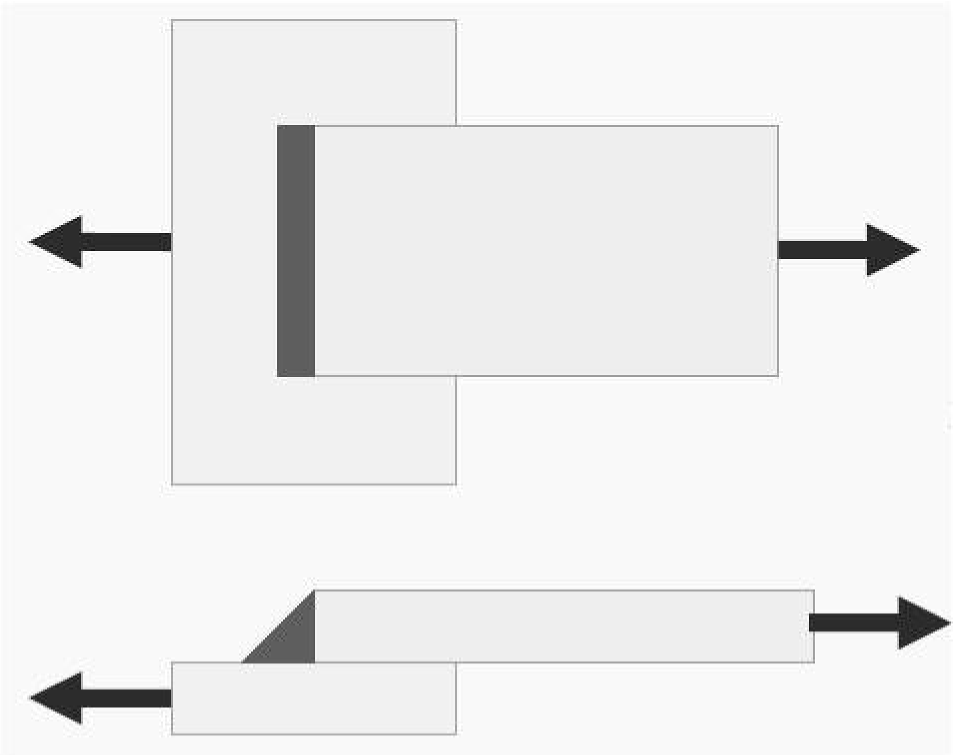
(vi) Forming a joint in difficult locations is possible through welding.

The advantages have made welding suitable for joining components in various machines and structures.

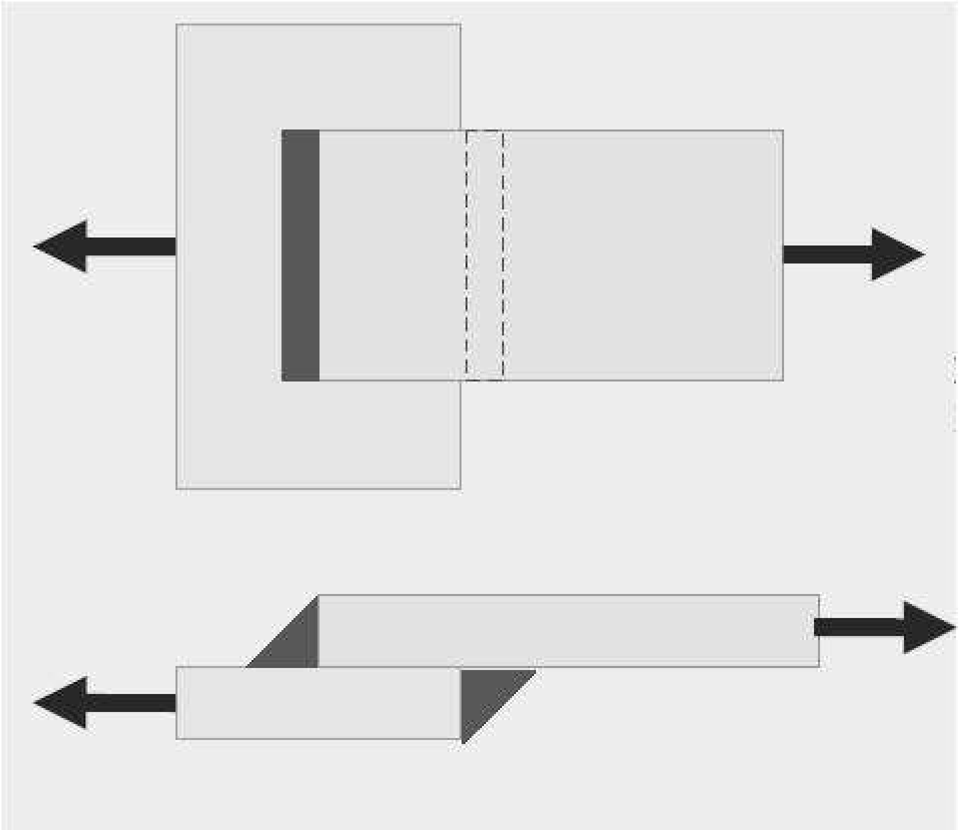
Types of welded joints

Welded joints are primarily of two kinds

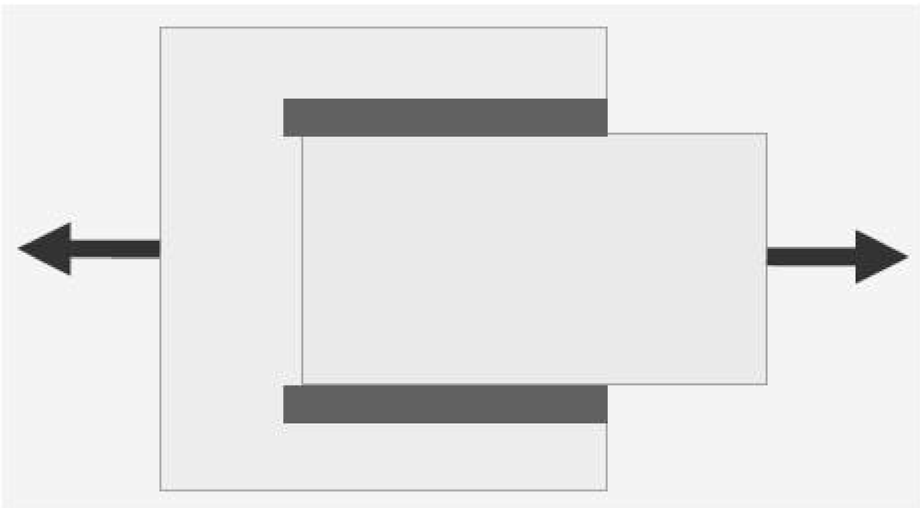
a) *Lap or fillet joint*: obtained by overlapping the plates and welding their edges. The fillet joints may be single transverse fillet, double transverse fillet or parallel fillet joints (see figure 2.12).



Single transverse lap joint



Double transverse lap joint



Parallel lap joint

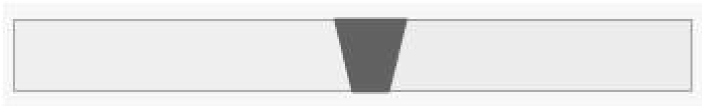
Fig. 2.12 – Different types of lap joint

b) *Butt joints*: formed by placing the plates edge to edge and welding them. Grooves are sometimes cut (for thick plates) on the edges before welding. According to the shape of the grooves, the butt joints may be of different types, e.g.,

- Square butt joint
- Single V-butt joint, double V-butt joint
- Single U-butt joint, double U-butt joint
- Single J-butt joint, double J-butt joint
- Single bevel-butt joint, double bevel butt joint

These are schematically shown in figure 2.13.

Square butt joint



Single – V butt joint

Double – V butt joint

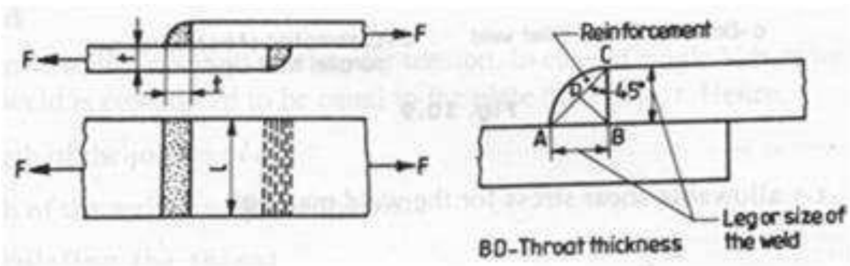
Fig. 2.13 – Different types of butt joints

Strength of welds: in-plane loading

There are different forms of welded joints, subjected to in-plane loading under tension.

1. **Transverse fillet weld**

Figure 2.14*a* shows a double transverse fillet weld under tension. It is assumed that the section of the weld is an isosceles right angled triangle, ABC, i.e. 45° fillet weld (Fig. 2.14b).



(a) **(b)**

Fig. 2.14 – Double transverse fillet weld

The length of each side (AB=BC) is known as leg length or size of the weld. The minimum cross-sectional dimension, BD (at 45º from the plate surface or edge) is termed as throat thickness. Transverse fillet welds are assumed to fail in tension across the throat.

Let t = thickness of the plate or size of the weld

l = length of the weld

σ_t = allowable tensile stress for the weld material

From the geometry of Fig. 2.14b,

Throat thickness, BD(=h) = $t\sin 45^{\circ} = \frac{t}{\sqrt{2}}$

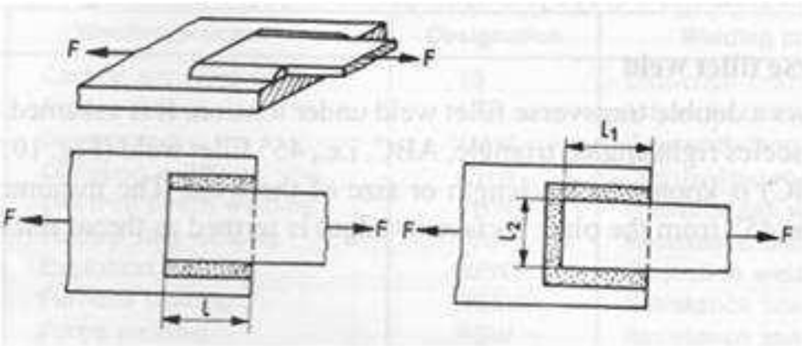
Resisting throat area = $hl = \frac{tl}{\sqrt{2}}$

Tensile strength of the joint = $\frac{tl}{\sqrt{2}}\sigma_t$, for single fillet

$$= \frac{2tl}{\sqrt{2}}\sigma_t = \sqrt{2}tl\sigma_t \text{ , for double fillet}$$

2. **Parallel fillet weld**

Figure 2.15*a* shows a double parallel fillet weld under tension. Parallel fillet welds are assumed to fail in shear across the throat.



(a) Double parallel fillet weld **(b) Combination of transverse and parallel fillet weld**

Fig. 2.15

Let τ = allowable shear stress for the weld material

Resisting throat area = $\frac{tl}{\sqrt{2}}$

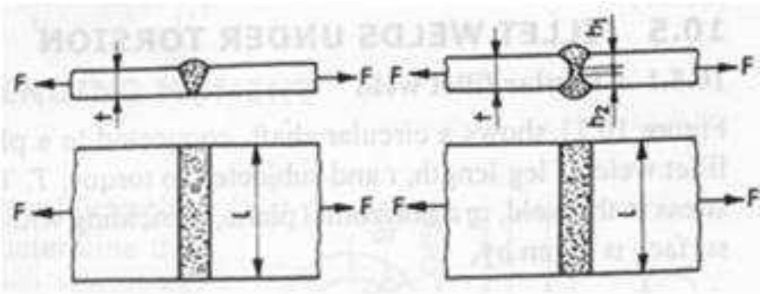
Shear strength of the joint = $\frac{tl}{\sqrt{2}}\tau$, for single fille

(Tensile strength)

$$= \frac{2tl}{\sqrt{2}}\tau = \sqrt{2}tl\tau \text{ , for double fillet}$$

3. **Butt weld**

Fig. 2.15a shows a single V-butt joint under tension.



(a) Single-V butt joint (b) Double-V butt joint

Fig. 2.15 – Butt joints under tension

In case of single V-butt weld, the throat thickness of the weld is considered to be equal to the plate thickness, t. Hence, tensile strength of the joint = $tl\sigma_t$

Where, l = length of the weld = width of the plate.

Figure 2.15b shows a double V-butt joint under tension.

Let h_1 = throat thickness at the top

h_2 = throat thickness at the bottom

Then tensile strength of the joint = $(h_1 + h_2)l\sigma_t$

4. **Fillet welds under torsion**

Circular fillet weld: Figure 2.16 shows a circular shaft, connected to a plate, by a fillet weld of leg length, t and subjected to torque, T. The shear stress in the weld, in a horizontal plane, coinciding with the pate surface is given by,

$$\tau = \frac{T \times d/2}{J}$$

Where,

$$J = \pi t d \left(\frac{d}{2}\right)^2$$

$$\tau = \frac{T \times d/2}{\pi t d \left(\frac{d}{2}\right)^2} = \frac{2T}{\pi t d^2}$$

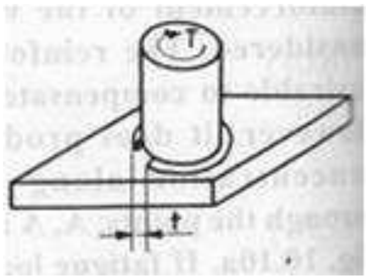


Fig. 2.16

The maximum value of the shear stress occurs in the weld throat, the length of which is $\frac{t}{\sqrt{2}}$

Therefore,

$$\tau_{\max} = \frac{2T\sqrt{2}}{\pi t d^2} = \frac{2.83T}{\pi t d^2}$$

Long adjacent fillet welds: Fig 2.17 shows a vertical plate, connected to a horizontal plate by two identical fillet welds, and subjected to torque, *T* about the vertical axis of the joint.

Let l = length of the joint

T = leg length of the weld

The effect of the applied torque is to produce shear stress, varying from zero at the axis and maximum at the plate ends (This is similar to the variation of normal stress over the depth of a beam, subjected to bending).

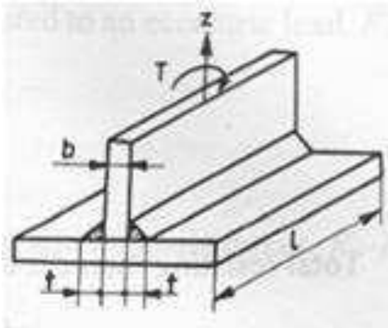


Fig. 2.17

The torsional shear stress, induced at the plate ends, and in a horizontal plane, coinciding with the top surface of the horizontal plane, is given by,

$$\tau_{max} = \frac{3T\sqrt{2}}{tl^2} = \frac{4.2T}{tl^2}$$

5. **Fillet welds under bending moment**

Annular fillet weld: Figure 2.18 shows one example of an annular fillet weld, subjected to bending moment, *M*. To determine the maximum bending stress induced in the joint; let us consider a small element of the weld, at an angle, *θ*, subtending an angle, *dθ* at the centre of the shaft.

Area of the element = *r.dθ.t*

Where, t = size of the weld

Normal force acting on the weld element,

$$dF = r \times d\theta \times t \times \sigma_t$$

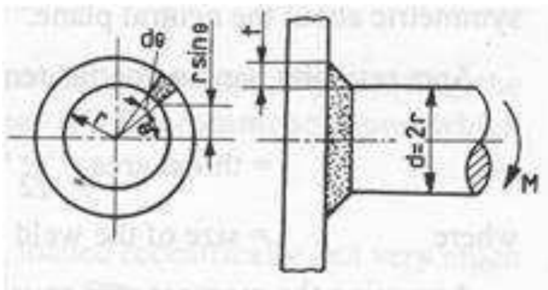


Fig. 2.18

Since the normal stress in the element is proportional to the distance from the neutral plane,

$$\frac{\sigma_{t\,max}}{r} = \frac{\sigma_t}{r\sin\theta}$$

Where, σ = normal (bending) stress induced in the weld element

σ_{tmax} = maximum bending stress

$\sigma_t = \sigma_{t\,max}\sin\theta$

Moment due to the force, *dF* about the neutral plane,

$$= dF \times r\sin\theta$$

$$= r \cdot d\theta \cdot t \cdot \sigma_t \cdot rdin\theta$$

$$= r \cdot d\theta \cdot t \cdot \sigma_{t\,max} \cdot \sin\theta \cdot r\sin\theta$$

$$= r^2 t\sigma_{t\max} \cdot \sin^2\theta \cdot d\theta$$

Total resisting moment offered by weld $= \int_0^{2\pi} r^2 t\sigma_{t\max} \cdot \sin^2\theta \cdot d\theta$

$$= r^2 t\sigma_{t\max}\pi = \text{external moment, } M$$

Therefore, $M = r^2 t\sigma_{t\max}\pi$

or $\sigma_{t\max} = \dfrac{M}{r^2 t\pi}$

Considering the throat area, for evaluation of the stress,

$$\sigma_{t\max} = \frac{M}{\left(\frac{d}{2}\right)^2 \times \frac{t}{\sqrt{2}} \times \pi} = \frac{5.66M}{d^2\pi t}$$

Parallel fillet weld:

Figure 2.19 shows a double parallel fillet weld, subjected to bending moment, M. The joint is symmetric about the neutral plane.

Area resisting bending on the tensile (compressive) side,

$$= \text{throat area} = \frac{t}{\sqrt{2}} \times l$$

Where, t = size of the weld

Assuming the moment arm equal to (b+t),

Resisting moment $= \dfrac{t}{\sqrt{2}} \times l(b+t)\sigma_t$

Where, b = thickness of the plate

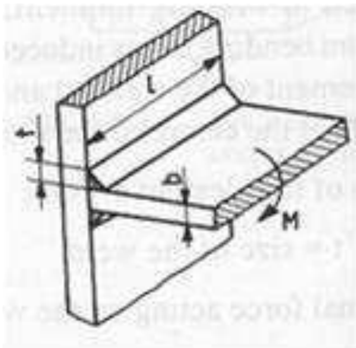


Fig. 2.19

Therefore, $M = \dfrac{t}{\sqrt{2}} \times l(b+t)\sigma_t$

And $\sigma_t = \dfrac{\sqrt{2}M}{tl(b+t)}$

6. Welded joints under eccentric loading

Case – I

Figure 2.20 shows a T-joint, with double parallel fillet weld, subjected to an eccentric load, F at a distance, e

Let t = size of the weld

l = length of the weld and b = thickness of the plate

To analyse the effect of the eccentric load, F, introduce two equal and opposite forces, F_1 – F_2 such that $F_1 = F_2 = F$, as shown in Fig. 2.20.

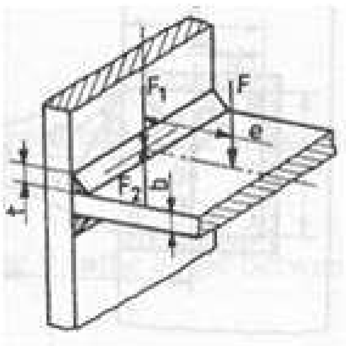


Fig. 2.20

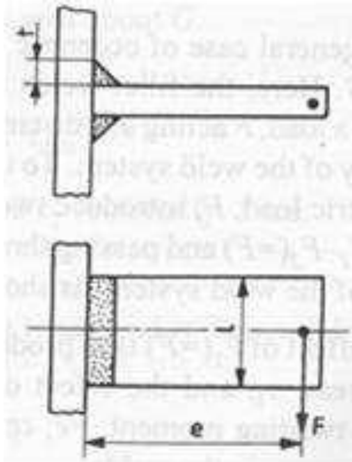


Fig. 2.21

The effect of F_1 (=F) is to produce transverse shear stress, given by,

$$\tau = \frac{F}{2 \times \frac{t}{\sqrt{2}} \times l} = \frac{F}{\sqrt{2}tl}$$

Where, $\frac{t}{\sqrt{2}}$ = throat thickness (h)

The effect of F - F_2 (F - F) is to produce bending moment, M, given by, Fe.

Bending stress induced due to M is,

$$\sigma_b = \frac{\sqrt{2}M}{tl(b+t)} = \frac{\sqrt{2}Fe}{tl(b+t)}$$

The resultant (maximum) normal stress is given by,

$$\sigma_{max} = \sqrt{\sigma_b^2 + \tau^2} = \frac{F}{tl(b+t)} \times \sqrt{2e^2 + \frac{(b+t)^2}{2}}$$

Case – II

Figure 2.21 shows a T-joint with double parallel fillet weld, loaded eccentrically, but very much different from that of the joint as shown in Fig. 2.20.

Let F = load; e = eccentricity; t = leg length; l = length of the weld

Similar to previous case, to analyse the effect of the eccentric load, F, introduce two equal and opposite forces, F_1 - F_2 such that $F_1 = F_2 = F$, as shown in Fig. 2.21.

The effect of $F_1 = F$ is to produce transverse shear stress, given by,

$$\tau = \frac{F}{2 \times \frac{t}{\sqrt{2}} \times l} = \frac{F}{\sqrt{2}tl}$$

Where, $\frac{t}{\sqrt{2}}$ = throat thickness

The effect of F - F_2 (F - F) is to produce bending moment, M, given by, Fe.

Bending stress induced due to M is,

$$\sigma_b (= \sigma_t = \sigma_c) = \frac{M}{Z}$$

Where, $$Z = 2\times\frac{1}{6}\left(\frac{t}{\sqrt{2}}\right)l^2 = \frac{\sqrt{2}}{6}tl^2 = \frac{tl^2}{4.242}$$

$$\sigma_b = \frac{4.242Fe}{tl^2}$$

The resultant (maximum) normal stress is given by,

$$\sigma_{max} = \sqrt{\sigma_b^2 + \tau^2} = \frac{0.707F}{tl} \times \sqrt{1 + \left(\frac{6e}{l}\right)^2}$$

Case – III

A more general case of eccentric loading is shown in Fig. 2.22. Here, the fillet welds are subjected to the action of a load, F acting at a distance, e from the centre of gravity of the weld system. To understand the effect of eccentric load, F; introduce two equal and opposite forces, F_1 - F_2 (=F) and passing through G, the centre of gravity of the weld system, as shown.

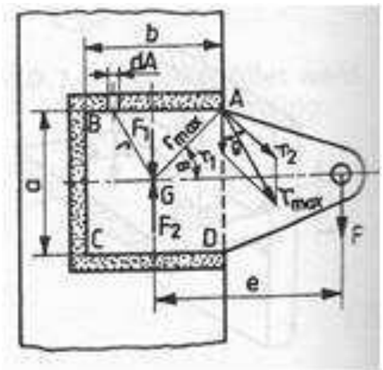


Fig. 2.22

The effect of F_1(=F) is to produce direct or primary shear stress, τ_1, and the effect of F - F_2 (=F-F) is to produce twisting moment, Fe; resulting in secondary shear stress, τ_2 in the welds.

Primary shear stress, $\tau_1 = \frac{\sqrt{2}F}{tl}$

Where, t = size of the weld

l = total length of the weld $\approx a + 2b$

Considering bending action, the shear stress induced is proportional to the distance of the weld section from G. Obviously, it is maximum at the corners of the weld.

Let τ_2 = maximum secondary shear stress at, say corner, A. Then from Fig. 2.22,

$$\frac{\tau}{r} = \frac{\tau_2}{GA} = \frac{\tau_2}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}}$$

$$\tau = 2\tau_2 \times \frac{r}{\sqrt{a^2 + b^2}}$$

Where, τ is the secondary shear stress at distance, r from G.

The moment of the shear force on the weld element of area, dA and at distance, r from G is,

$$dM = \tau \cdot dA \cdot r = \frac{2\tau_2 r^2 \cdot dA}{\sqrt{a^2 + b^2}}$$

Total resisting moment due to the welds AB, BC, CD shall be equal to the external (applied) twisting moment, Fe.

$$Fe = \frac{2\tau_2}{\sqrt{a^2 + b^2}} \left[\int_A^B r^2 \cdot dA + \int_B^C r^2 \cdot dA + \int_C^D r^2 \cdot dA \right] = \frac{2\tau_2}{\sqrt{a^2 + b^2}} \cdot J$$

Where $J = \sum r^2 \cdot dA$ = polar moment of inertia of the throat area about G.

$$\tau_2 = \frac{Fe \times \sqrt{a^2 + b^2}}{2I_G} = \frac{Fe}{J} \times r_{max}$$

The resultant stress, τ_{max} is obtained by adding τ_1 and τ_2 vectorially. Thus,

$$\tau_{max} = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \cos\theta}$$

Where θ is the angle between primary and secondary shear loads, and is obtained from,

$$\cos\theta = \frac{b}{\sqrt{a^2 + b^2}}$$

Example Problem-1: Figure 2.23 shows a cylindrical rod of 50 mm diameter, welded to a flat plate. The cylindrical fillet weld is loaded eccentrically, by a force of 10 kN acting at 200 mm from the welded end. If the size of the weld is 20 mm, determine the maximum normal stress in the weld.	**Reference**
Solution: Let h = throat thickness = $\frac{t}{\sqrt{2}}$ Referring Fig. 2.23, let us introduce two equal and opposite forces, F_1 - F_2 and	Design Data Book by K. Mahadevan

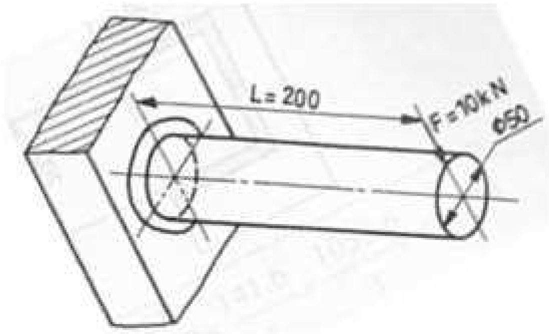
parallel to F, and passing through the centre of the rod at the fixed end such that $F_1 = F_2 = F$. Effect of F_1 (=F) is to produce transverse shear stress, τ. | & K. B. Reddy

Throat area, $A = \pi dh = \pi \times 50 \times \frac{t}{\sqrt{2}} = \pi \times 50 \times \frac{20}{\sqrt{2}} = 2221.8 mm^2$

Transverse shear stress,

$\tau = \frac{F}{A} = \frac{10 \times 1000}{2221.8} = 4.5$ N/mm^2

Effect of $F_1 - F_2$ (=F - F) is to produce bending moment, M, given by,



$M = FL = 10 \times 1000 \times 200 = 20 \times 10^5$ N-mm

Fig. 2.23

For a circular fillet weld, section modulus, Z is given by,

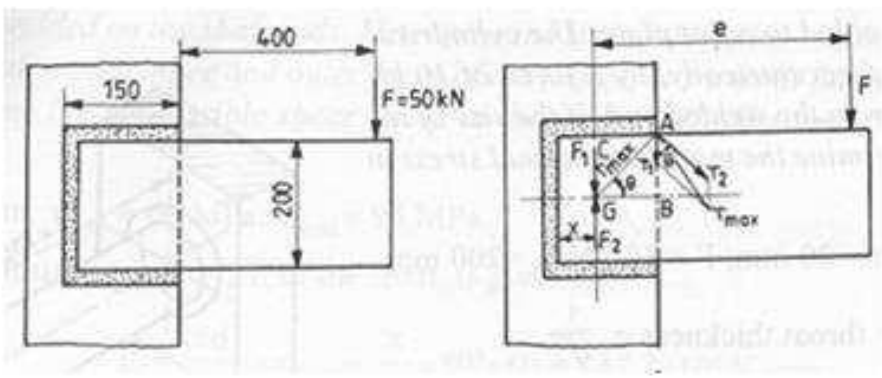
$$Z = \frac{\pi t d^2}{5.66} = \frac{\pi \times 20 \times 50^2}{5.66} = 2.7752.6 mm^3$$

Bending stress, $\sigma_b = \frac{M}{Z} = \frac{20 \times 10^5}{27752.6} = 72.1 \text{N/mm}^2$

Resultant (maximum) normal stress,

$$\sigma_b = \sqrt{\tau^2 + \sigma_b^2} = \sqrt{4.5^2 + 72.1^2} = 72.24 \text{ N/mm}^2$$

Example Problem-2: Figure 2.24*a* shows an eccentrically loaded welded joint. Determine the fillet weld size. Allowable shear stress in the weld is 80 MPa.



(a) ***(b)***

Fig. 2.24

Solution: Given data: F= 50kN; b = 200 mm; l = 150 mm; τ = 80 MPa

Let t = size of the weld; h = throat thickness = $\frac{t}{\sqrt{2}}$

The distance of the centre of gravity, G from the left edge of the plate, x is given by,

$$x = \frac{l^2}{2l + b} = \frac{150^2}{2 \times 150 + 200} = 45\,\text{mm}$$

Eccentrically, $e = 400 + (150 - x) = 400 + (150 - 45) = 505$ mm

Polar moment of inertia of the weld throat about G,

$$J = \frac{t}{\sqrt{2}}\left[\frac{(b + 2l)^3}{12} - \frac{l^2 (b + l)^2}{b + 2l}\right]$$

$$= \frac{t}{\sqrt{2}}\left[\frac{(200 + 2 \times 150)^3}{12} - \frac{150^2 \times (200 + 150)^2}{(200 + 2 \times 150)}\right] = 3468.3 \times 10^3\, t\ \text{mm}^4$$

Maximum radius of the weld, GA = r_{max} =

$$\sqrt{AB^2 + AC^2} = \sqrt{100^2 + 105^2} = 145\,\text{mm}$$

$$\cos\theta = \frac{GB}{GA} = \frac{105}{145} = 0.724$$

Throat area of the weld, $A = (b + 2l) \times \frac{t}{\sqrt{2}} = 353.6t\ \text{mm}^2$

Referring Fig. 2.24b, let us introduce two equal and opposite forces, F_1 - F_2 through G, and parallel to F such that $F_1 = F_2 = F$.

The effect of F_1 (= F) is to produce primary shear stress.

Primary shear stress, $\tau_1 = \frac{F}{A} = \frac{50 \times 1000}{353.6t} = \frac{141.4}{t}$ N/mm^2

The effect of $F - F_2$ (=F-F) is to produce moment, Fe; inducing secondary shear stress. Maximum secondary shear stress,

$$\tau_2 = \frac{Fe}{J} \times r_{max} = \frac{50 \times 1000 \times 505}{3468.3 \times 10^3 \times t} \times 145 = \frac{1055.6}{t}$$

Resultant (maximum) shear stress,

$$\tau_{max} = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2\cos\theta}$$

$$= \sqrt{\left(\frac{141.6}{t}\right)^2 + \left(\frac{1055.6}{t}\right)^2 + 2 \times \frac{141.6}{t} \times \frac{1055.6}{t} \times 0.724}$$

$$= \frac{1162.1}{t}$$

$$80 = \frac{1162.1}{t}$$

$$t = 14.5\,\text{mm}$$

Design of Bolted Joints

Threaded fasteners

Bolts, screws and studs are the most common types of threaded fasteners. They are used in both permanent and removable joints.

Bolts: They are basically threaded fasteners normally used with nuts.

Screws: They engage either with a preformed or a self made internal thread.

Studs: They are externally threaded headless fasteners. One end usually meets a tapped component and the other with a standard nut.

There are different forms of bolt and screw heads for a different usage. These include bolt heads of square, hexagonal or eye shape and screw heads of hexagonal, Fillister, button head, counter sunk or Phillips type. These are shown in Figs. 2.25 and 2.26.

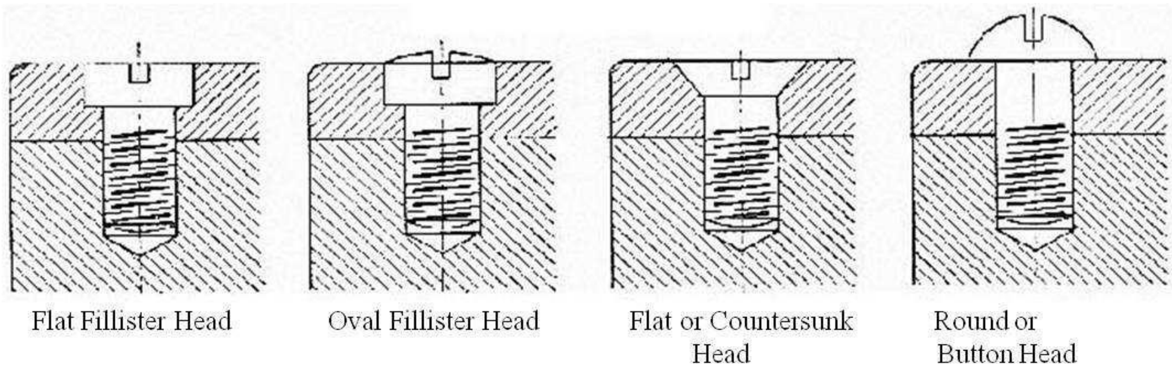


Fig. 2.25 – Types of screw heads

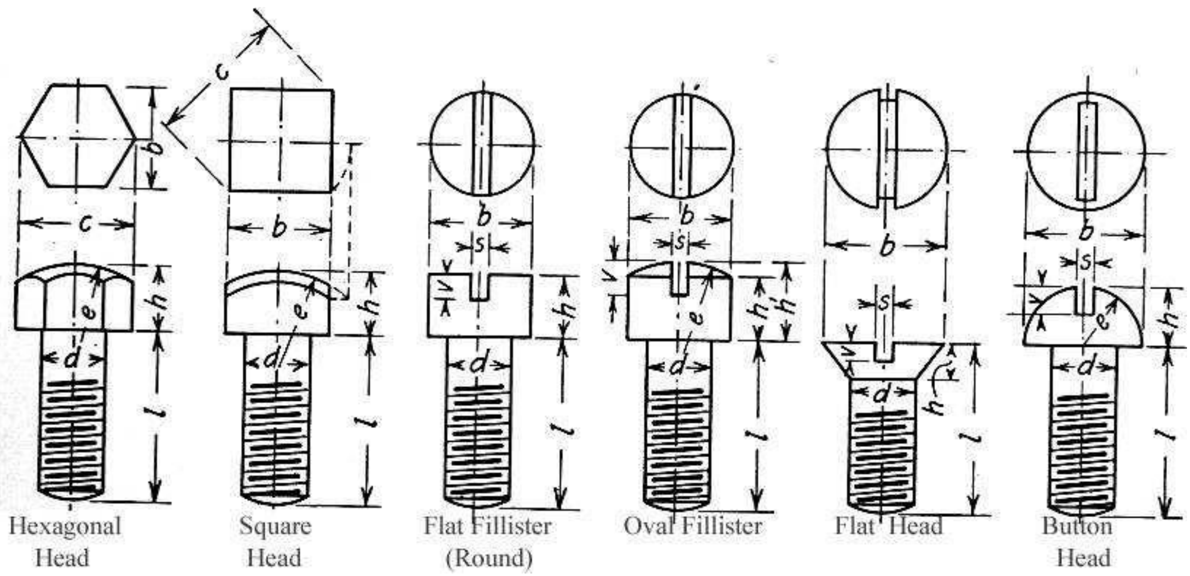


Fig. 2.26 – Types of bolt heads

Tapping screws

These are one piece fasteners which cut or form a mating thread when driven into a preformed hole. These allow rapid installation since nuts are not used.

There are two types of tapping screws. They are known as **thread forming** which displaces or forms the adjacent materials and **thread cutting** which have cutting edges and chip cavities which create a mating thread.

Set Screws

These are semi permanent fasteners which hold collars, pulleys, gears etc on a shaft. Different heads and point styles are available. Some of them are shown in **Fig. 2.27**.

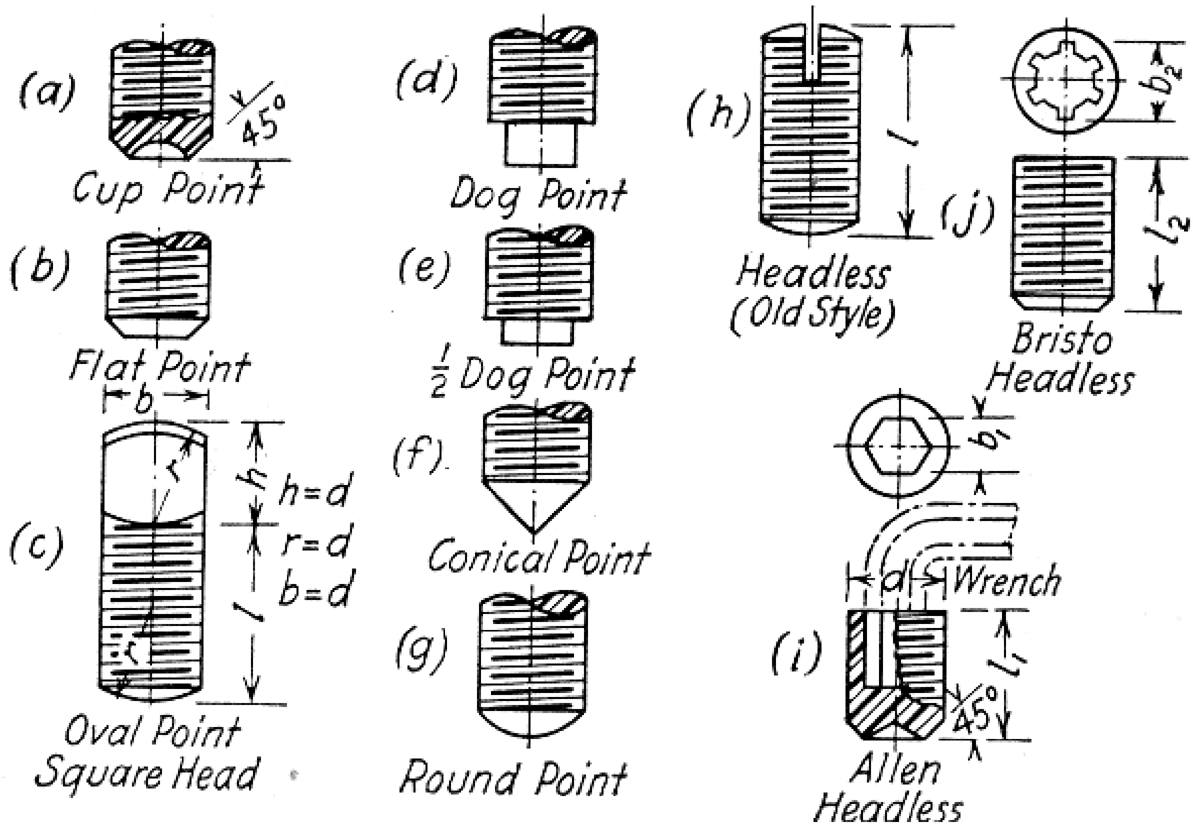


Fig. 2.27 – Different types of set screws

Thread forms

Basically when a helical groove is cut or generated over a cylindrical or conical section, threads are formed. When a point moves parallel to the axis of a rotating cylinder or cone held between centers, a helix is generated. Screw threads formed in this way have two functions to perform in general: (a) to transmit power – Square, ACME, Buttress, Knuckle types of thread forms are useful for this purpose. (b) to secure one member to another- V-threads are most useful for this purpose.

Some standard forms are shown in **Fig. 2.28.**

V-threads are generally used for securing because they do not shake loose due to the wedging action provided by the thread. Square threads give higher efficiency due to a low friction. This is demonstrated in **Fig. 2.29.**

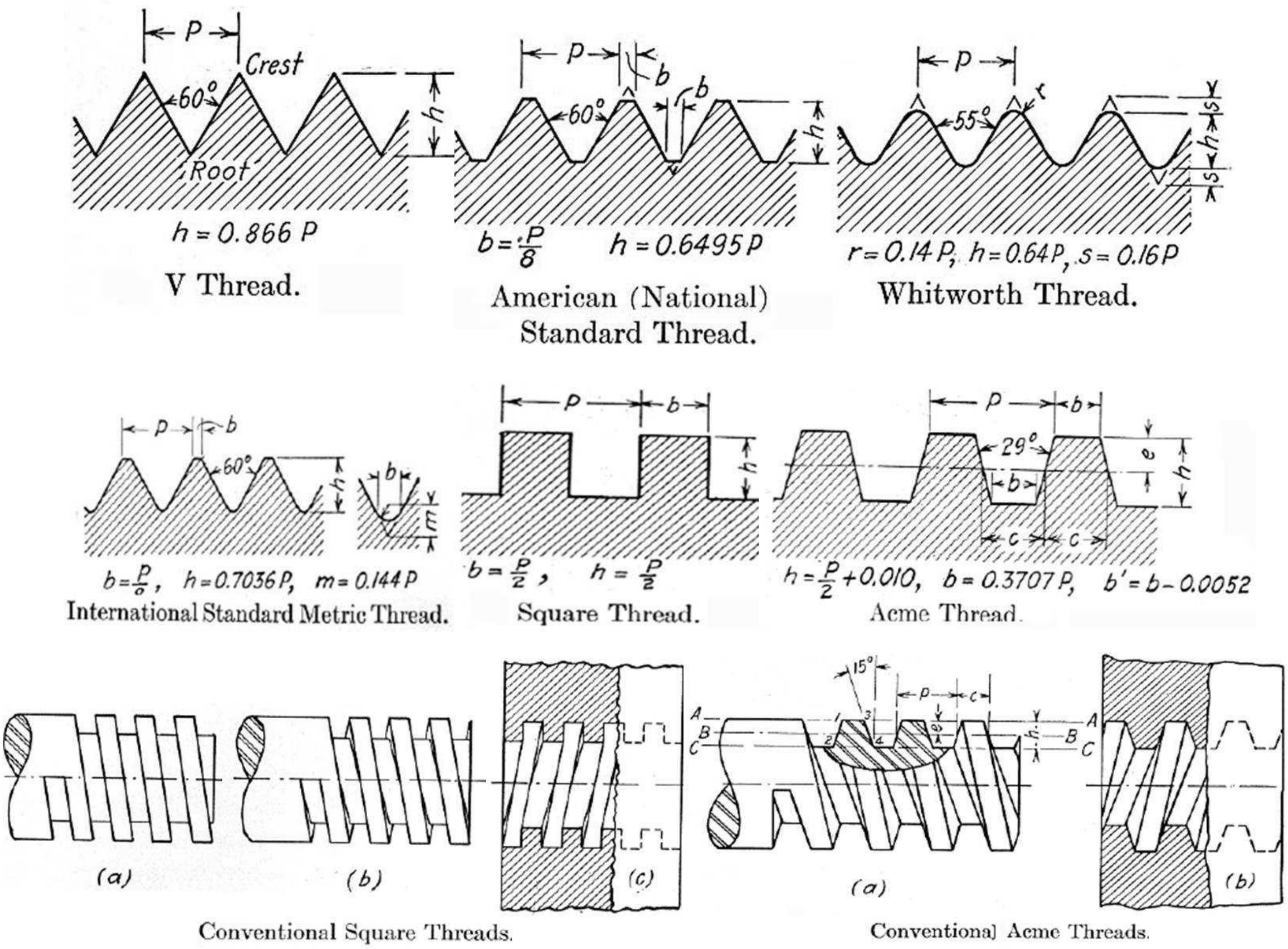


Fig. 2.28 – Different types of thread forms

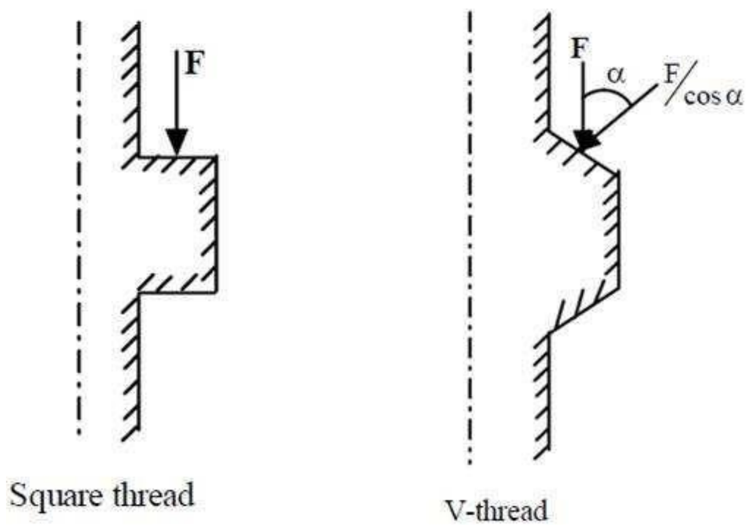


Fig. 2.29 – Loading on square and V-threads

Design of bolted joints

Stresses in screw fastenings

It is necessary to determine the stresses in screw fastening due to both static and dynamic loading in order to determine their dimensions. In order to design for static loading both initial tightening and external loadings need be known.

Initial tightening load

When a nut is tightened over a screw following stresses are induced:

(a) Tensile stresses due to stretching of the bolt

(b) Torsional shear stress due to frictional resistance at the threads.

(c) Shear stress across threads

(d) Compressive or crushing stress on the threads

(e) Bending stress if the surfaces under the bolt head or nut are not perfectly normal to the bolt axis.

(a) Tensile stress

Since none of the above mentioned stresses can be accurately determined bolts are usually designed on the basis of direct tensile stress with a large factor of safety. The initial tension in the bolt may be estimated by an empirical relation P1=284 d kN, where the nominal bolt diameter d is given in mm. The relation is used for making the joint leak proof. If leak proofing is not required half of the above estimated load may be used. However, since initial stress is inversely proportional to square of the diameter $\sigma = \dfrac{284d}{\frac{\pi}{4}d^2}$, bolts of smaller diameter such as M16 or M8 may fail during initial tightening. In such cases torque wrenches must be used to apply known load. The torque in wrenches is given by $T = C P_1 d$ where, C is a constant depending on coefficient of friction at the mating surfaces, P is tightening up load and d is the bolt diameter.

***(b)* Torsional shear stress**

This is given by $\tau = \dfrac{16T}{\pi d_c^3}$ where T is the torque and d_c the core diameter. We may relate torque T to the tightening load P_1 in a power screw configuration (**figure-2.30**) and taking collar friction into account we may write

$$T = P_1 \frac{d_m}{2}\left(\frac{1+\mu\pi d_m \sec\alpha}{\pi d^m - \mu L \sec\alpha}\right) + \frac{P_1 \mu_c d_{cm}}{2}$$

where d_m and d_{cm} dare the mean thread diameter and mean collar diameter respectively, and μ and μ_c are the coefficients of thread and collar friction respectively and α is the semi thread angle. If we consider that $d_{cm} = \dfrac{d_m + 1.5d_m}{2}$, then we may write $T = C\, P_1\, d_m$ where C is a constant for a given arrangement. As discussed earlier, similar equations are used to find the torque in a wrench.

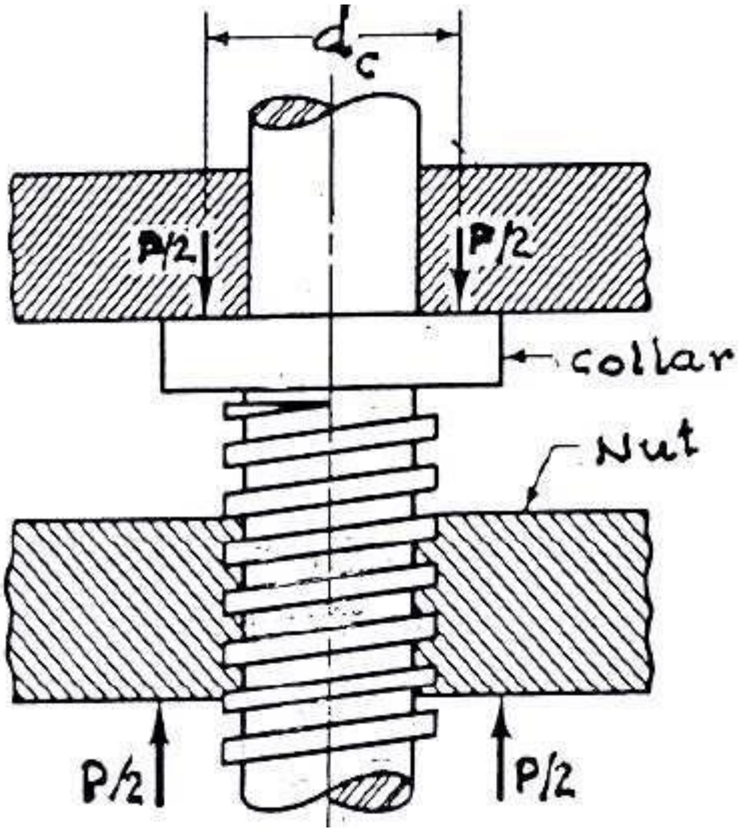


Fig. 2.30 – A typical power screw configuration

(c) **Shear stress across the threads**

This is given by $\tau = \dfrac{3P}{\pi d_c b n}$ where d_c is the core diameter and b is the base width of the thread and n is the number of threads sharing the load.

(d) **Crushing stress on threads**

This is given by $\sigma_c = \dfrac{P}{\frac{\pi}{4}(d_o^2 - d_c^2)n}$ where d_o and d_c are the outside and core diameters as shown in Fig. 2.30.

(e) **Bending stress**

If the underside of the bolt and the bolted part are not parallel as shown in Fig. 2.31, the bolt may be subjected to bending and the bending stress may be given by $\sigma_B = \frac{xE}{2L}$ where x is the difference in height between the extreme corners of the nut or bolt head, L is length of the bolt head shank and E is the young’s modulus.

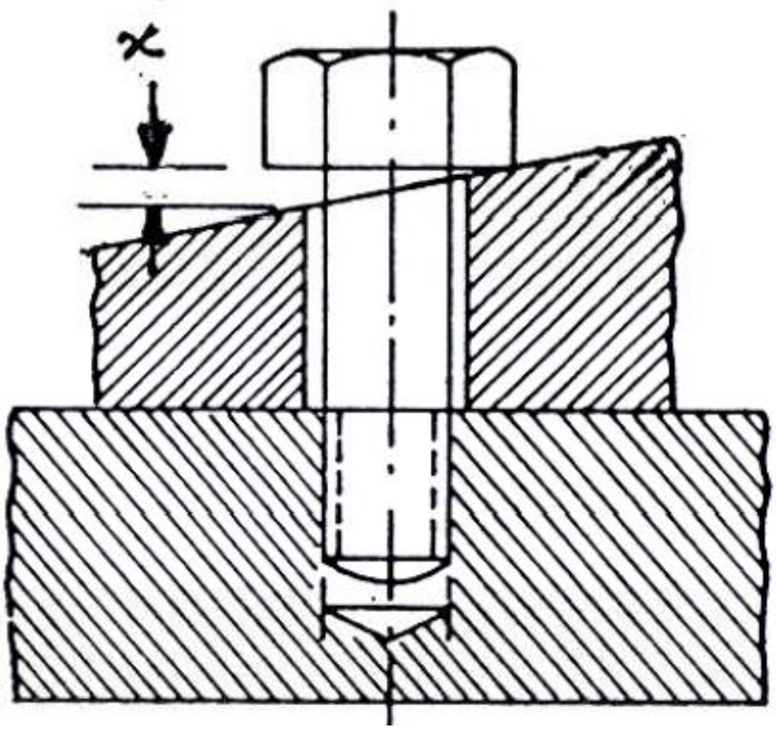


Fig. 2.31 - Development of bending stress in a bolt

Combined effect of initial tightening load and external load

When a bolt is subjected to both initial tightening and external loads i.e. when a preloaded bolt is in tension or compression the resultant load on the bolt will depend on the relative elastic yielding of the bolt and the connected members.

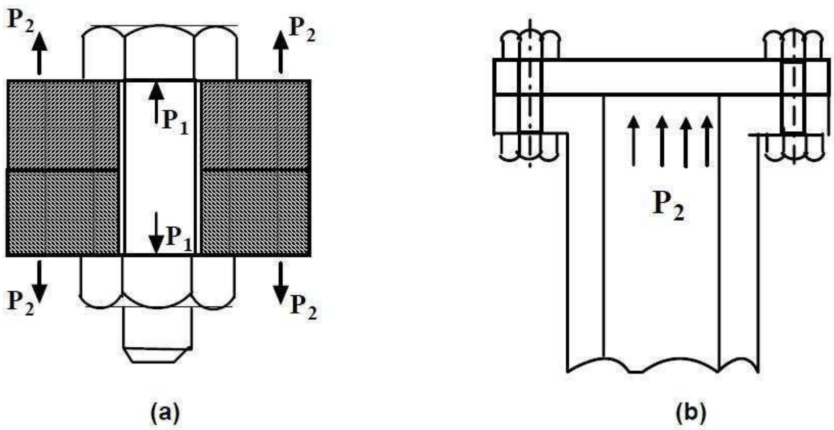


Fig. 2.32 - A bolted joint subjected to both initial tightening and external load

This situation may occur in steam engine cylinder cover joint for example. In this case the bolts are initially tightened and then the steam pressure applies a tensile load on the bolts. This is shown in Fig. 2.32a and b.

Initially due to preloading the bolt is elongated and the connected members are compressed. When the external load P is applied, the bolt deformation increases and the compression of the connected members decrease. Here, P_1 and P_2 in Fig. 2.32a are the tensile loads on the bolt due to initial tightening and external load respectively. The increase in bolt deformation is given by $\delta_B = \frac{P_b}{K_b}$ and decrease in member compression is $\delta_C = \frac{P_c}{K_c}$ where, P_b is the share of P_2 in bolt, and P_c is the share of P_2 in members, K_b and K_c are the stiffnesses of bolt and members. If the parts are not separated then $\delta_b = \delta_c$ and this gives, $\frac{P_b}{K_b} = \frac{P_c}{K_c}$. Therefore, the resultant load on bolt is P+KP. Sometimes connected members may be more yielding than the bolt and this may occurs when a soft gasket is placed between the surfaces. Under these circumstances $K_b >> K_c$ or $\frac{K_c}{K_b} << 1$ and this gives $K \approx 1$. Therefore the total load $P = P_1 + P_2$ Normally K has a value around 0.25 or 0.5 for a hard copper gasket with long through bolts. On the other hand if $K_c >> K_b$, K approaches zero and the total load P equals the initial tightening load. This may occur when there is no soft gasket and metal to metal contact occurs. This is not desirable. Some typical values of the constant K are given in Table 2.1.

Table 2.1

Type of joint	**K**
Metal to metal contact with through bolt	0-0.1
Hard copper gasket with long through bolt	0.25-0.5
Soft copper gasket with through bolts	0.75
Soft packing with through bolts	0.75-1.00
Soft packing with studs	1.00

Cotter Joint

A cotter is a flat wedge-shaped piece of steel as shown in Fig. 2.33. This is used to connect rigidly two rods which transmit motion in the axial direction, without rotation. These joints may be subjected to tensile or compressive forces along the axes of the rods. Examples of cotter joint connections are: connection of piston rod to the crosshead of a steam engine, valve rod and its stem etc.

A typical cotter joint is as shown in Fig. 2.34. One of the rods has a socket end into which the other rod is inserted and the cotter is driven into a slot, made in both the socket and the rod. The cotter tapers in width (usually 1:24) on one side only and when this is driven in, the rod is forced into the socket. However, if the taper is provided on both the edges it must be less than the sum of the friction angles for both the edges to make it self locking i.e. $\alpha_1 + \alpha_2 < \varphi_1 + \varphi_2$ where α_1 , α_2 are the angles of taper on the rod edge and socket edge of the cotter respectively and φ_1, φ_2 are the corresponding angles of friction. This also means that if taper is given on one side only then $\alpha < \varphi_1 + \varphi_2$ for self locking. Clearances between the cotter and slots in the rod end and socket allows the driven cotter to draw together the two parts of the joint until the socket end comes in contact with the cotter on the rod end.

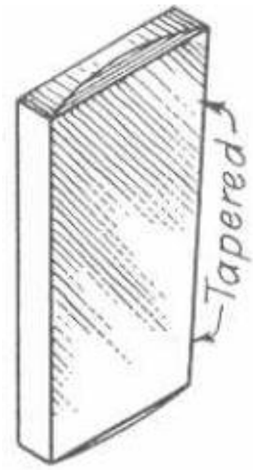


Fig. 2.33

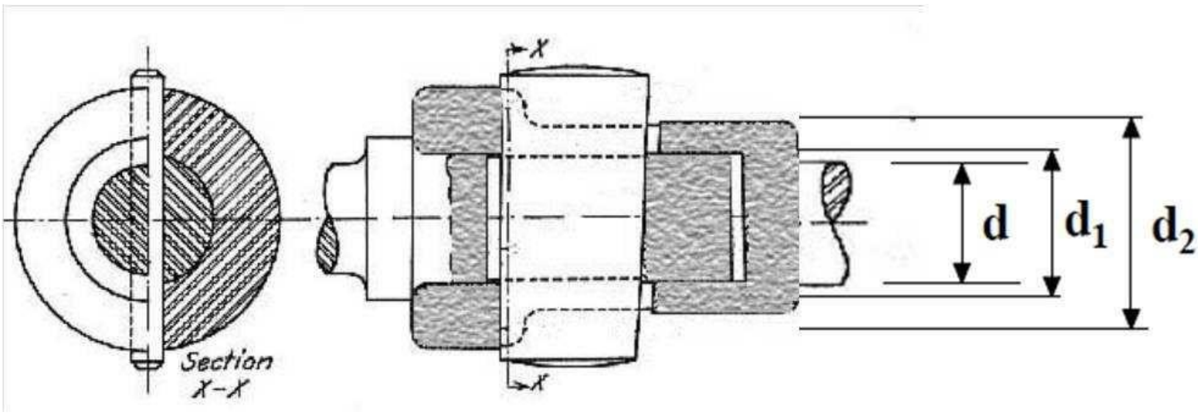


Fig. 2.34 – Cross-sectional views of a typical cotter joint

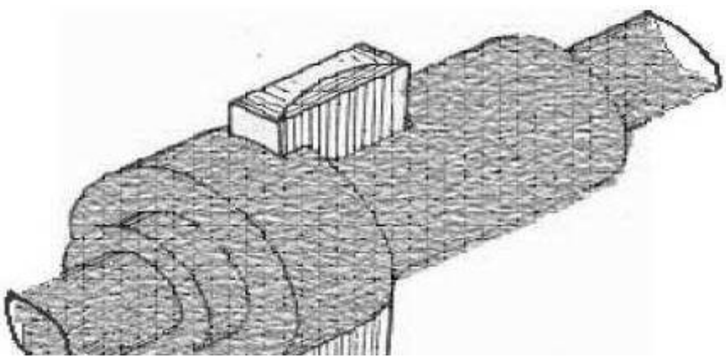


Fig. 2.35 – An isometric view of a typical cotter joint

Design of a cotter joint:

If the allowable stress in tension, compression and shear for the socket, rod and cotter be σ_t, σ_c, and τ respectively, assuming that they are all made of the same material, we may write the following failure criteria:

1. Tension failure of the rod at diameter *d* (Fig. 2.36)

$$\frac{\pi}{4} d^2 \sigma_t = P$$

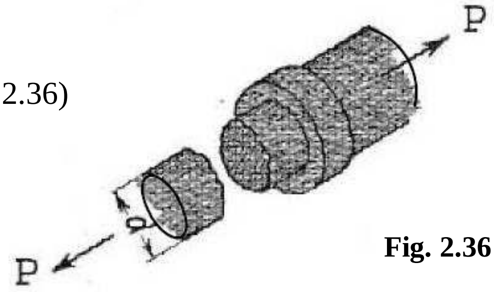


Fig. 2.36

2. Tension failure of the rod across slot (Fig. 2.37)

$$\left(\frac{\pi}{4} d_1^2 - d_1 t\right)\sigma_t = P$$

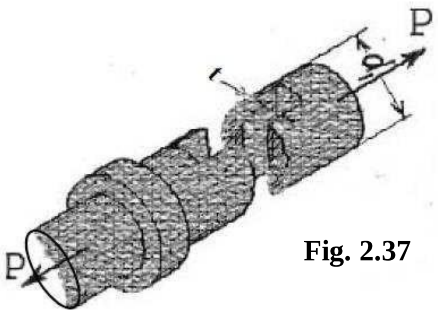


Fig. 2.37

3. Tension failure of the socket across slot (Fig. 2.38)

$$\left(\frac{\pi}{4}\left(d_2^2 - d_1^2\right) - \left(d_2 - d_1\right)t\right)\sigma_t = P$$

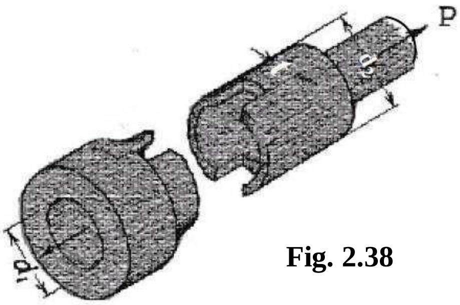


Fig. 2.38

4. Shear failure of cotter (Fig. 2.39)

$$2bt\tau = P$$

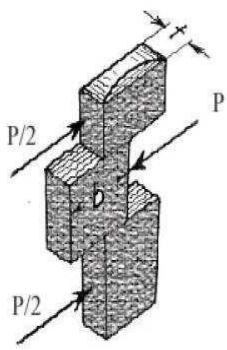
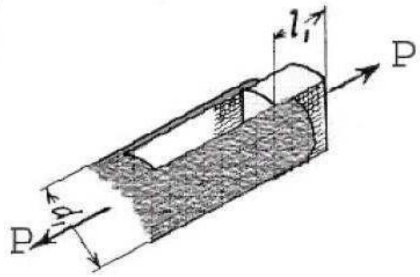


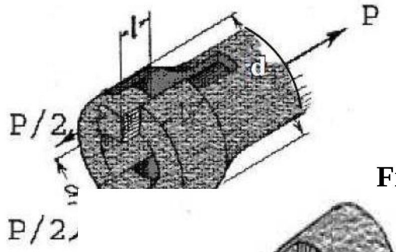
Fig. 2.39



5. Shear failure of the rod end (Fig. 2.40)

$$2l_1d_1\tau = P$$

Fig. 2.40



6. Shear failure of socket end (Fig. 2.41)

$$2l(d_2 - d_1)\tau = P$$

Fig. 2.41

7. Crushing failure of rod or cotter (Fig. 2.42)

$$d_1t\sigma_c = P$$

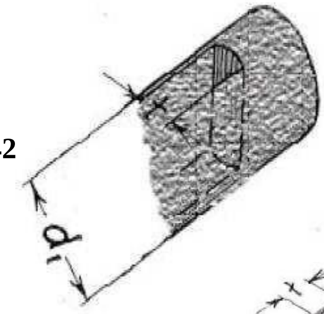


Fig. 2.42

8. Crushing failure of socket or rod (Fig. 2.43)

$$(d_3 - d_1)t\sigma_c = P$$

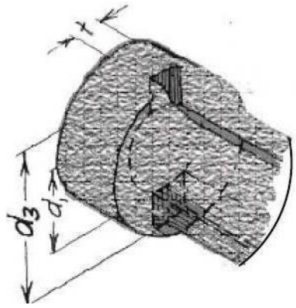


Fig. 2.43

9. Crushing failure of collar (Fig. 2.44)

$$\left(\frac{\pi}{4}\left(d_4^2 - d_1^2\right)\right)\sigma_c = P$$

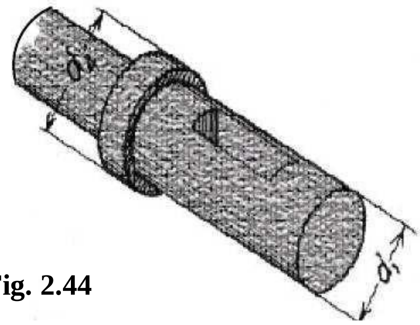


Fig. 2.44

10. Shear failure of collar (Fig. 2.45)

$$\pi d_1t_1\tau = P$$

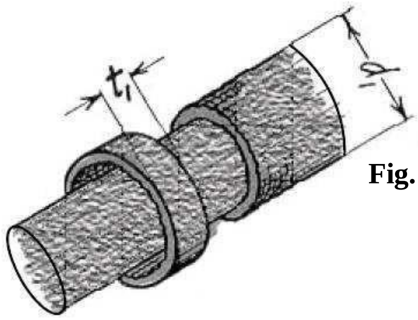
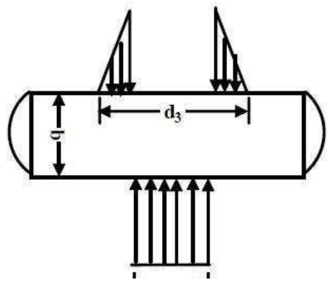


Fig. 2.45

Cotters may bend when driven into position. When this occurs, the bending moment cannot be correctly estimated since the pressure distribution is not known. However, if we assume a triangular pressure distribution over the rod, as shown in Fig. 2.46 (a), we may approximate the loading as shown in Fig. 2.46 (b)



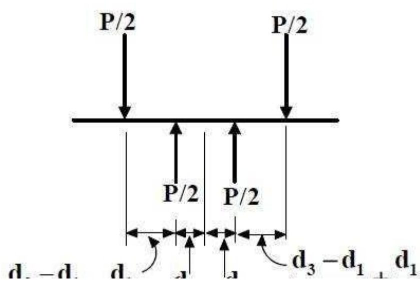


Fig. 2.46

This gives maximum bending moment = $\frac{P}{2}\left(\frac{d_3 - d_1}{6} + \frac{d_1}{4}\right)$ and the bending stress,

$$\sigma_b = \frac{\frac{P}{2}\left(\frac{d_3 - d_1}{6} + \frac{d_1}{4}\right)\frac{b}{2}}{\frac{tb^3}{12}} = \frac{3P\left(\frac{d_3 - d_1}{6} + \frac{d_1}{4}\right)}{tb^2}$$

Tightening of cotter introduces initial stresses which are again difficult to estimate. Sometimes therefore it is necessary to use empirical proportions to design the joint. Some typical proportions are given below:

$d_1 = 1.21d$; $d_2 = 1.75d$; $d_3 = 2.4d$; $d_4 = 1.5d$; $t = 0.31d$; $b = 1.6d$; $l = l_1 = 0.75d$;

$t_1 = 0.45d$; $s =$ clearance.

Design of a cotter joint:

Knuckle Joint

A knuckle joint (as shown in Fig. 2.47) is used to connect two rods under tensile load. This joint permits angular misalignment of the rods and may take compressive load if it is guided.

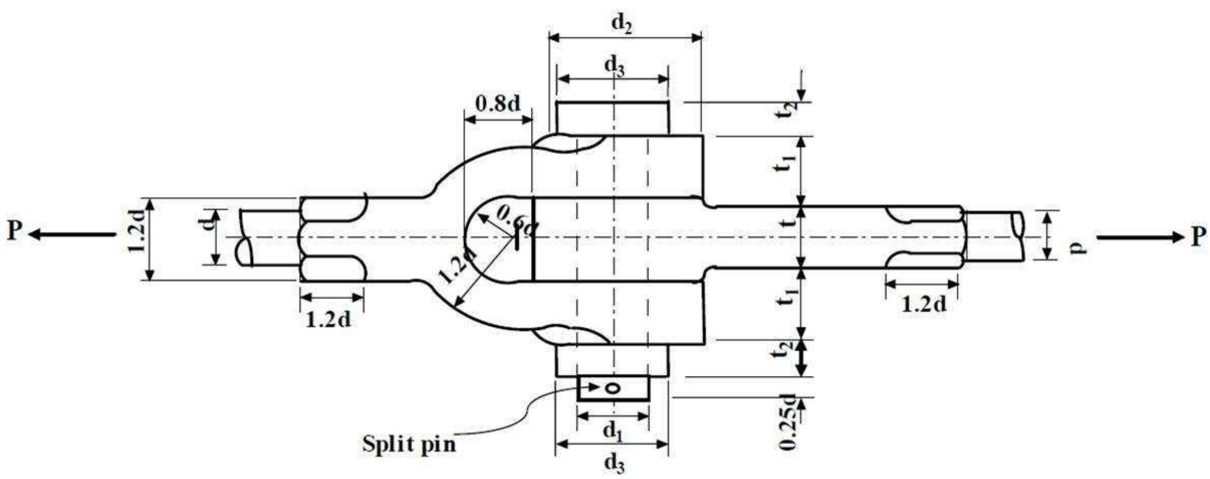


Fig. 2.47 – A typical knuckle joint

These joints are used for different types of connections e.g. tie rods, tension links in bridge structure. In this, one of the rods has an eye at the rod end and the other one is forked with eyes at both the legs. A pin (knuckle pin) is inserted through the rod-end eye and fork-end eyes and is secured by a collar and a split pin. Normally, empirical relations are available to find different dimensions of the joint and they are safe from design point of view. The proportions are given in the Fig. 2.47.

d = diameter of rod

$d_1 = d$ $\quad\quad$ $t = 1.25d$

$d_2 = 2d$ $\quad\quad$ $t_1 = 0.75d$

$d_3 = 1.5d$ $\quad\quad$ $t_2 = 0.5d$

Mean diameter of the split pin = 0.25 d

However, failures analysis may be carried out for checking. The analyses are shown below assuming the same materials for the rods and pins and the yield stresses in tension, compression and shear are given by σ_t, σ_c and τ.

1. Failure of rod in tension:

$$\frac{\pi}{4} d^2 \sigma_t = P$$

2. Failure of knuckle pin in double shear:

$$2\frac{\pi}{4} d_1^2 \tau = P$$

3. Failure of knuckle pin in bending (if the pin is loose in the fork):

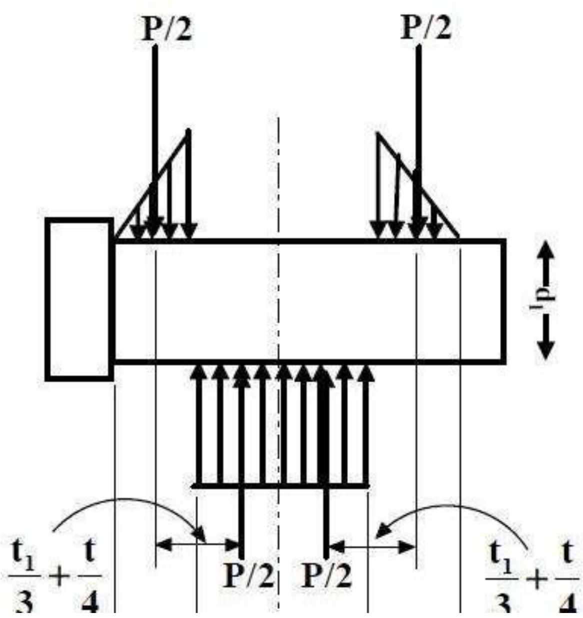
Assuming a triangular pressure distribution on the pin, the loading on the pin is shown in Fig. 2.48.

Equating the maximum bending stress to tensile or compressive yield stress we have,

$$\sigma_t = \frac{16P\left(\frac{t_1}{3} + \frac{t}{4}\right)}{\pi d_1^3}$$

4. Failure of rod eye in shear:

$$(d_2 - d_1)t\tau = P$$



5. Failure of rod eye in crushing:

$$d_1 t \sigma_c = P$$

6. Failure of rod eye in tension:

$$(d_2 - d_1) t \sigma_t = P$$

Fig. 2.48

7. Failure of forked end in shear:

$$2(d_2 - d_1) t_1 \tau = P$$

8. Failure of forked end in tension:

$$2(d_2 - d_1) t_1 \sigma_t = P$$

9. Failure of forked end in crushing:

$$2 d_1 t_1 \sigma_c = P$$

The design may be carried out using the empirical proportions and then the analytical relations may be used as checks. For example using the 2nd equation we have, $\tau = \frac{2P}{\pi d_1^2}$. We may now put value of d_1 from empirical relation and then find $F.S. = \frac{\tau_y}{\tau}$ which should be more than one.

<table>
<tr>
<td>Example Problem-1: Design a typical cotter joint to transmit a load of 50 kN in tension or compression. Consider that the rod, socket and cotter are all made of a material with the following allowable stresses: Allowable tensile stress σ_y = 150 MPa; Allowable crushing stress σ_c = 110 MPa; Allowable shear stress τ_y = 110 MPa.</td>
<td>Reference</td>
</tr>
<tr>
<td>Solution:
Axial load, $P = \frac{\pi}{4} d^2 \sigma_y$. On substitution this gives d=20 mm. In general standard shaft size in mm are:
6 mm to 22 mm diameter   2 mm in increment
25 mm to 60 mm diameter   5 mm in increment</td>
<td>Fig. 2.34 and 2.36</td>
</tr>
</table>

60 mm to 110 mm diameter — 10 mm in increment 110 mm to 140 mm diameter — 15 mm in increment 140 mm to 160 mm diameter — 20 mm in increment 500 mm to 600 mm diameter — 30 mm in increment We therefore choose a suitable rod size to be 25 mm.	
For tension failure across slot $\left(\frac{\pi}{4} d^2 - d_1 t\right)\sigma_t = P$. This gives $d_1 t = 1.58\times10^{-4}$ m². From empirical relations we may take $t = 0.4d$ i.e. 10 mm and this gives d_1 = 15.8 mm. Maintaining the proportion let d_1= 1.2 d = 30 mm.	**Fig. 2.37**
The tensile failure of socket across slot, $\left(\frac{\pi}{4}\left(d_2^2 - d_1^2\right) - \left(d_2 - d_1\right)t\right)\sigma_t = P$. This gives d2 = 37 mm. Let d2 = 40 mm.	**Fig. 2.38**
For shear failure of cotter $2bt\tau = P$. On substitution this gives b = 22.72 mm. Let b = 25 mm.	**Fig. 2.39**
For shear failure of rod end $2l_1 d_1 \tau = P$ and this gives l_1 = 7.57 mm. Let l_1 = 10 mm.	**Fig. 2.40**
For shear failure of socket end $2l\left(d_2 - d_1\right)\tau = P$. This gives l = 22.72 mm. Let L = 25 mm.	**Fig. 2.41**
For crushing failure of socket or rod $\left(d_3 - d_1\right)t\sigma_c = P$. This gives d3 = 75.5 mm. Let d3 = 77 mm.	**Fig. 2.43**
For crushing failure of collar $\left(\frac{\pi}{4}\left(d_4^2 - d_1^2\right)\right)\sigma_c = P$. On substitution this gives d_4 = 38.4 mm. Let d_4 = 40 mm.	**Fig. 2.44**
For shear failure of collar $\pi d_1 t_1 \tau = P$ which gives t_1= 4.8 mm. Let t_1 = 5 mm.	**Fig. 2.45**
Therefore the final chosen values of dimensions are: d = 25 mm; d_1= 30 mm; d_2 = 40 mm; d_3 = 77 mm; d_4 = 40 mm; t = 10 mm; t_1= 5 mm; l = 25 mm; l_1= 10 mm; b = 27 mm.	

DESIGN OF JOURNAL BEARING

1. Terms used in Hydrodynamic Journal Bearing

A hydrodynamic journal bearing is shown in Fig. 1, in which O is the centre of the journal and O′ is the centre of the bearing.

Let, D = Diameter of the bearing

d = diameter of journal

l= length of bearing

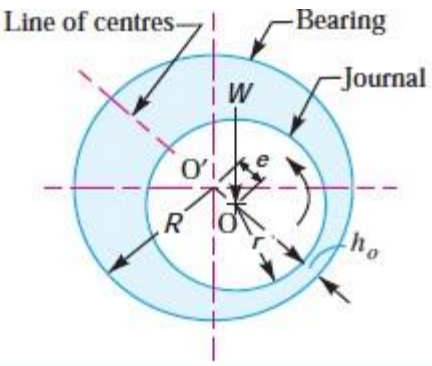


Fig. 1: Hydrodynamic Journal Bearing

The following terms used in hydrodynamic journal bearing are important from the subject point of view.

1. **Diametral clearance**. It the difference between the diameters of the bearing and the journal. Mathematically, diametral clearance, $\boxed{c = D - d}$ **.**

2. **Radial clearance**. It is the difference between the radii of the bearing and the journal. Mathematically, radial clearance, $\boxed{c_1 = R - r = \frac{D-d}{2} = \frac{c}{2}}$.

3. **Diametral clearance ratio**. It is the ratio of the diametral clearance to the diameter of the journal. Mathematically, diametral clearance ratio, $\boxed{\frac{c}{d} = \frac{D-d}{d}}$.

4. **Eccentricity**. It is the radial distance between the centre (O) of the bearing and the displaced centre (O′) of the bearing under load. It is denoted by e.

5. **Minimum oil film thickness**. It is the minimum distance between the bearing and the journal, under complete lubrication condition. It is denoted by h0 and occurs at the line of centres as shown in Fig. 1. Its value may be assumed as c / 4.

6. **Attitude or eccentricity ratio**. It is the ratio of the eccentricity to the radial clearance. Mathematically, attitude or eccentricity ratio, $\varepsilon = \frac{e}{c_1} = \frac{c_1 - h_o}{c_1} = 1 - \frac{h_o}{c_1} = 1 - \frac{2h_o}{c}$.

7. **Short and long bearing**. If the ratio of the length to the diameter of the journal (i.e. l / d) is less than 1, then the bearing is said to be short bearing. On the other hand, if l / d is greater than 1, then the bearing is known as long bearing.

Notes : 1. When the length of the journal (l) is equal to the diameter of the journal (d), then the bearing is called square bearing.

2. Bearing Characteristic Number and Bearing Modulus for Journal Bearings

The coefficient of friction in design of bearings is of great importance, because it affords a means for determining the loss of power due to bearing friction. It has been shown by experiments that the coefficient of friction for a full lubricated journal bearing is a function of three variables, i.e. $\frac{Zn}{p}, \frac{d}{c}, \frac{l}{d}$.

Therefore the coefficient of friction may be expressed as $\mu = \phi\left(\frac{Zn}{p}, \frac{d}{c}, \frac{l}{d}\right)$

Where ,

μ = Coefficient of friction,

ϕ = A functional relationship,

Z = Absolute viscosity of the lubricant, in kg / m-s,

n = Speed of the journal in r.p.m.,

p = Bearing pressure on the projected bearing area in N/mm^2, = Load on the journal ÷ l × d

d = Diameter of the journal,

l = Length of the bearing, and

c = Diametral clearance.

The factor ZN / p is termed as bearing characteristic number and is a dimensionless number.

The variation of coefficient of friction with the operating values of bearing characteristic number (ZN / p) as obtained by McKee brothers (S.A. McKee and T.R. McKee) in an actual test of friction is shown in Fig. 2. The factor ZN/p helps to predict the performance of a bearing.

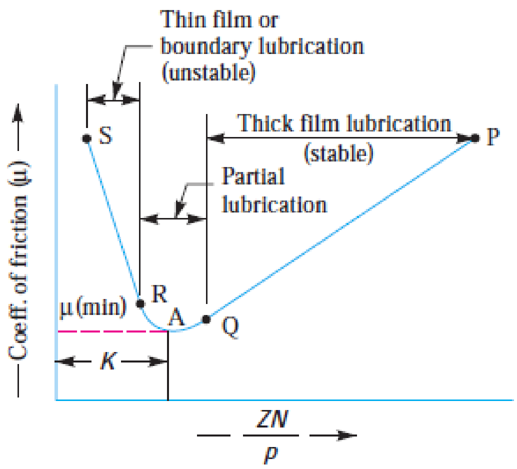


Fig. 2 Variation of coefficient of friction with ZN/p.

The part of the curve PQ represents the region of thick film lubrication. Between Q and R, the viscosity (Z) or the speed (N) are so low, or the pressure (p) is so great that their combination ZN / p will reduce the film thickness so that partial metal to metal contact will result. The thin film or boundary lubrication or imperfect lubrication exists between R and S on the curve. This is the region where the viscosity of the lubricant ceases to be a measure of friction characteristics but the oiliness of the lubricant is effective in preventing complete metal to metal contact and seizure of the parts. It may be noted that the part PQ of the curve represents stable operating conditions, since from any point of stability, a decrease in viscosity (Z) will reduce Zn / p. This will result in a decrease in coefficient of friction (μ) followed by a lowering of bearing temperature that will raise the viscosity (Z). From Fig. 2, we see that the minimum amount of friction occurs at A and at this point the value of Zn / p is known as bearing modulus which is denoted by K. The bearing should not be operated at this value of bearing modulus, because a slight decrease in speed or slight increase in pressure will break the oil film and make the journal to operate with metal to metal contact. This will result in high friction, wear and heating. In order to prevent such conditions, the bearing should be designed for a value of Zn / p at least three times the minimum value of bearing modulus (K). If the bearing is subjected to large fluctuations of load and heavy impacts, the value of Zn / p = 15 K may be used. From above, it is concluded that when the value of ZN / p is greater than K, then the bearing will operate with thick film lubrication or under hydrodynamic conditions. On the other hand, when the value of ZN / p is less than K, then the oil film will rupture and there is a metal to metal contact.

3. **Coefficient of Friction for Journal Bearings**

In order to determine the coefficient of friction for well lubricated full journal bearings, the following empirical relation established by McKee based on the experimental data, may be used.

Coefficient of friction $\mu = \left[\frac{33.25}{10^8} \times \frac{Zn}{p} \times \frac{d}{c}\right] + k$ (Eq. 19.5, pp. 19.3, Jalaludeen)

(When Z in N-s/m^2, or kg/m-s and p in N/mm^2)

$\mu = \left[\frac{33.25}{10^{10}} \times \frac{Zn}{p} \times \frac{d}{c}\right] + k$ (Eq. 19.6, pp. 19.3, Jalaludeen)

(When Z in centipoise, or kg/m-s and p in kgf/cm^2)

k = Factor to correct for end leakage. It depends upon the ratio of length to the diameter of the bearing (i.e. l / d). (Refer Fig. 19.2, pp.19.24, Jalaludeen) , and The design values can be taken from Table 19.5, pp. 19.13, Jalaludeen.

4. **Critical Pressure of the Journal Bearing**

The pressure at which the oil film breaks down so that metal to metal contact begins, is known as critical pressure or the minimum operating pressure of the bearing. It may be obtained by the following empirical relation, i.e. Critical pressure or minimum operating pressure, $p_c = \frac{Zn}{4.75 \times 10^6}\left(\frac{d}{c}\right)^2\left(\frac{l}{l+d}\right)$ N/mm^2 (Eq. 19.15, pp. 19.6, Jalaludeen)

When, Z in N-s/m^2

And $p_c = \frac{Zn}{475 \times 10^6}\left(\frac{d}{c}\right)^2\left(\frac{l}{l+d}\right)$ kgf.cm^2 (Eq. 19.16, pp. 19.6, Jalaludeen), When, Z centipoise

5. **Sommerfeld Number**

The Sommerfeld number is also a dimensionless parameter used extensively in the design of journal bearings. Mathematically, $S = \frac{Zn}{60 \times 10^6\, p}\left(\frac{d}{c}\right)^2$ (Eq. 19.8, pp. 19.4, Jalaludeen), Table 19.7 to Table 19.10.

6. **Heat Generated in a Journal Bearing**

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion. Mathematically, heat generated in a bearing, $H_g = \mu WV$ watts.

(when the load on bearing W in Newtons and V in m/s)

And $\boxed{H_g = \mu WV}$ kgf-m/min or $\dfrac{\mu WV}{J}$ kcal/min (Eq. 19.10, pp. 19.4, Jalaludeen)

(when W in Newtons and the rubbing velocity V in m/s)

And $\boxed{V = \dfrac{\pi dn}{60}}$ m/s

After the thermal equilibrium has been reached, heat will be dissipated at the outer surface of the bearing at the same rate at which it is generated in the oil film. The amount of heat dissipated will depend upon the temperature difference, size and mass of the radiating surface and on the amount of air flowing around the bearing. However, for the convenience in bearing design, the actual heat dissipating area may be expressed in terms of the projected area of the journal.

Heat dissipated by the bearing $\boxed{H_d = CA(t_b - t_a)}$ (Eq. 19.12, pp. 19.5, Jalaludeen)

Where C= Heat dissipation coefficient (values can be obtained from pp. 19.5, Jalaludeen)

A = projected area = $l \times d$ in m^2

t_b = temp. of bearing in ^{o}C

t_a = temp. of surroundings in ^{o}C

7. Design Procedure for Journal Bearing

The following procedure may be adopted in designing journal bearings, when the bearing load, the diameter and the speed of the shaft are known.

1 Determine the bearing length by choosing a ratio of l / d from Table 19.5, pp. 19.13, Jalaludeen.

2 Check the bearing pressure, p = W / l.d (Eq. 19.9, pp. 19.4, Jalaludeen), from Table 19.5, pp. pp. 19.13, Jalaludeen, for probable satisfactory value.

3 Assume a lubricant from Table 19.11, pp. 19.26, Jalaludeen, and its operating temperature (t0). This temperature should be between 26.5°C and 60°C with 82°C as a maximum for high temperature installations such as steam turbines.

4 Determine the operating value of Zn / p for the assumed bearing temperature and check this value with corresponding values in Table 19.5, pp. 19.13, Jalaludeen to determine the possibility of maintaining fluid film operation.

5 Assume a clearance ratio c / d from Table 19.5, pp. 19.13, Jalaludeen.

6 Determine the coefficient of friction (μ) by using the relation as discussed in Art. 3.

7 Determine the heat generated by using the relation as discussed in Art. 6.

8 Determine the heat dissipated by using the relation as discussed in Art. 6.

9 Determine the thermal equilibrium to see that the heat dissipated becomes at least equal to the heat generated. In case the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially cooled by water.

BALL AND ROLLER BEARING

Advantages and disadvantages of Roller bearing over sliding bearing

Advantages

1. Low starting and running friction except at very high speeds.
2. Ability to withstand momentary shock loads.
3. Accuracy of shaft alignment.
4. Low cost of maintenance, as no lubrication is required while in service.
5. Small overall dimensions.
6. Reliability of service.
7. Easy to mount and erect.
8. Cleanliness.

Disadvantages

1. More noisy at very high speeds.
2. Low resistance to shock loading.
3. More initial cost.
4. Design of bearing housing complicated.

Types of Rolling Contact Bearings

Following are the two types of rolling contact bearings:

1. Ball bearings; and
2. Roller bearings.

The ball and roller bearings consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, there are balls or rollers as shown in Fig. 3. A number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and is usually in two parts which are assembled after the balls have been properly spaced. The ball bearings are used for light loads and the roller bearings are used for heavier loads. The rolling contact bearings, depending upon the load to be carried, are classified as : (a) Radial bearings, and (b) Thrust bearings.

The radial and thrust ball bearings are shown in Fig. 4 (a) and (b) respectively. When a ball bearing supports only a radial load (WR), the plane of rotation of the ball is normal to the centre line of the bearing, as shown in Fig. 4 (a). The action of thrust load (WA) is to shift the plane of rotation of the balls, as shown in Fig. 4 (b). The radial and thrust loads both may be carried simultaneously.

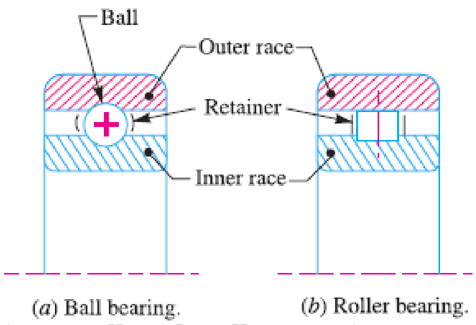


Fig. 3 Ball and Roller Bearing

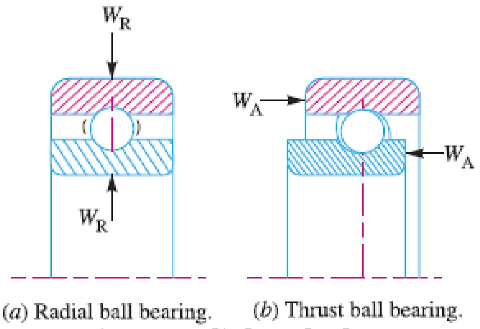


Fig. 4 Radial and Thrust Bearing

Types of Radial Ball Bearings

Following are the various types of radial ball bearings:

1. Single row deep groove bearing. A single row deep groove bearing as shown in Fig. 5 (a).

During assembly of this bearing, the races are offset and the maximum numbers of balls are placed between the races. The races are then centered and the balls are symmetrically located by the use of a retainer or cage. The deep groove ball bearings are used due to their high load carrying capacity and suitability for high running speeds. The load carrying capacity of a ball bearing is related to the size and number of the balls.

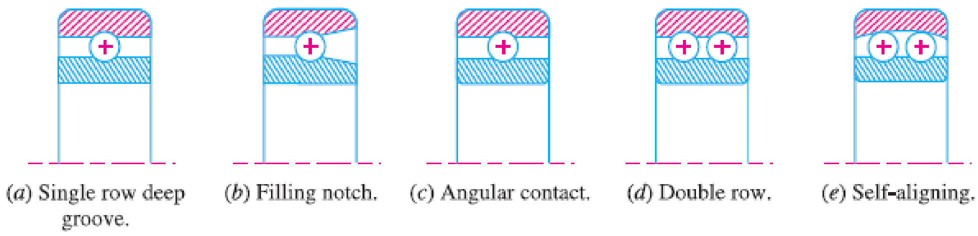


Fig. 5 Types of Radial Ball Bearing

2. Filling notch bearing.

A filling notch bearing is shown in Fig. 5 (b). These bearings have notches in the inner and outer races which permit more balls to be inserted than in a deep groove ball bearing. The notches do not extend to the bottom of the race way and therefore the balls inserted through the notches must be forced in position. Since this type of bearing contains larger number of balls than a corresponding un-notched one, therefore it has a larger bearing load capacity.

3. Angular contact bearing.

An angular contact bearing is shown in Fig. 5 (c). These bearings have one side of the outer race cut away to permit the insertion of more balls than in a deep groove bearing but without having a notch cut into both races. This permits the bearing to carry a relatively large axial load in one direction while also carrying a relatively large radial load. The angular contact bearings are usually used in pairs so that thrust loads may be carried in either direction.

4. Double row bearing.

A double row bearing is shown in Fig. 5 (d). These bearings may be made with radial or angular contact between the balls and races. The double row bearing is appreciably narrower than two single row bearings. The load capacity of such bearings is slightly less than twice that of a single row bearing.

5. Self-aligning bearing.

A self-aligning bearing is shown in Fig. 5 (e). These bearings permit shaft deflections within 2-3 degrees. It may be noted that normal clearance in a ball bearing are too small to accommodate any appreciable misalignment of the shaft relative to the housing. If the unit is assembled with shaft misalignment present, then the bearing will be subjected to a load that may be in excess of the design value and premature failure may occur. Following are the two types of self-aligning bearings:

(a) Externally self-aligning bearing, and (b) Internally self-aligning bearing.

In an externally self-aligning bearing, the outside diameter of the outer race is ground to a spherical surface which fits in a mating spherical surface in a housing, as shown in Fig. 5 (e). In case of internally self-aligning bearing, the inner surface of the outer race is ground to a spherical surface. Consequently, the outer race may be displaced through a small angle without interfering with the normal operation of the bearing. The internally self-aligning ball bearing is interchangeable with other ball bearings.

Types of Roller Bearings

Following are the principal types of roller bearings :

1. Cylindrical roller bearings. A cylindrical roller bearing is shown in Fig. 6(a). These bearings have short rollers guided in a cage. These bearings are relatively rigid against radial motion and have the lowest coefficient of friction of any form of heavy duty rolling-contact bearings. Such types of bearings are used in high speed service.

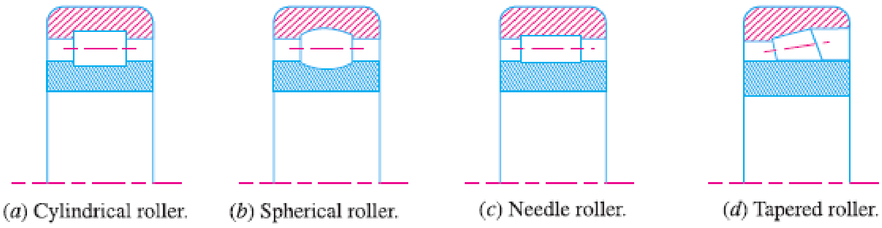


Fig. 6 Types of Roller Bearing

2. Spherical roller bearings. A spherical roller bearing is shown in Fig. 6 (b). These bearings are self-aligning bearings. The self-aligning feature is achieved by grinding one of the races in the form of sphere. These bearings can normally tolerate angular misalignment in the order of $\pm 1\frac{1}{2}^{\circ}$ and when used with a double row of rollers, these can carry thrust loads in either direction

3. Needle roller bearings. A needle roller bearing is shown in Fig. 6 (c). These bearings are relatively slender and completely fill the space so that neither a cage nor a retainer is needed. These bearings are used when heavy loads are to be carried with an oscillatory motion, e.g. piston pin bearings in heavy duty diesel engines, where the reversal of motion tends to keep the rollers in correct alignment.

4. Tapered roller bearings. A tapered roller bearing is shown in Fig. 6 (d). The rollers andrace ways of these bearings are truncated cones whose elements intersect at a common point. Such type of bearings can carry both radial and thrust loads. These bearings are available in various combinations as double row bearings and with different cone angles for use with different relative magnitudes of radial and thrust loads.