PROBLEMS IN ELECTRICAL ENGINEERING

(POWER ENGINEERING AND ELECTRONICS) WITH ANSWERS

BY

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PREFACE TO SEVENTH EDITION

THIRTY years have now passed since the first edition of this book appeared with a total of 792 problems. This number has steadily risen with successive editions and the addition of just over 100 new problems brings the present total to 1981.

Expansion has mainly been in the field of Electronic Engineering. Apart from the addition of new chapters on this subject, the scope of the more general chapters previously devoted to the field of Power Engineering has been widened to embrace the light current side. In particular, the increasing use of pulse techniques in the testing of lines and amplifiers has been covered in the chapter on Transients. The new chapter on Transistors has been restricted to the junction type which is rapidly replacing the thermionic valve for many applications and there can be little doubt that this device is now firmly established. Problems in this chapter include several on the general theory of four terminal networks necessary for a sound approach to the subject.

As in the sixth edition, the majority of problems are stated in M.K.S. and C.G.S. units in order to make the book equally suitable for those who wish to use either system. A note on the adoption of the rationalized M.K.S. system of units will be found in the Introductory, but the following points made in the preface to the sixth edition are worth repeating here. The great majority of problems in electrical engineering involve only the familiar practical units, the Ampere, Volt, Farad, etc. Possibly more than 90 per cent. of those in the present book fall into this class. The M.K.S. system is one that includes and extends these familiar units, thus giving a consistent unitary system for the whole field of electrical engineering science. To this extent, the adoption of the M.K.S. system will call for little readjustment of the student's outlook.

Acknowledgements are due to Mr. V. H. Attree, B.Sc., A.M.I.E.E., and Mr. M. G. Hartley, M.Sc., of The Manchester College of Science and Technology, for their help in the composition and checking of new problems.

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PREFACE TO FIRST EDITION

In engineering colleges and schools, the tuition class is now recognized as an important part of the course. Indeed, in electrical engineering, lectures and exercises should be developed together so far as possible, the ideal being one hour's tuition for each hour's lecture. This system should, whenever possible, be applicable to evening-class students as well as to those attending full-time day courses. Despite the difficulty of finding time for evening tuition classes, it may be of interest to mention that at The Royal Technical College, Glasgow, such classes were successfully established in the year 1926. In this Institution some of the group courses extend over three and even four evenings a week. Nevertheless, the institution of a voluntary tuition class, which all duly-enrolled evening students in electrical engineering might attend without additional fee, resulted in the regular attendance of well over 50 per cent. of all the students attending lectures. The system necessitates 600 separate and distinct questions each session.

Tuition may be given in one of two ways. The problems may be worked out on the blackboard by the teacher, or they may be worked out by the students in class, aided, where necessary, by the staff. It is the latter method that has been adopted at The Royal Technical College. An adequate number of teachers is available to render assistance when required, but "spoon-feeding" is discouraged. Work not completed in the class can be done at home, and marks—which count in the final result of the work of the session—are given for all exercises worked out in class or at home.

The present collection of problems was begun by the author in 1912 at the City and Guilds (Engineering) College, London. His activities as designer, teacher, and examiner for various universities and other public bodies, together with the active co-operation of his assistants, have made it possible to publish this book, and it is hoped that it may be useful to teachers and students in other colleges. And here the one difficulty from the author's point of view must be stated—namely, his inability to make individual acknowledgements. While it would be preposterous to claim originality for many of the problems, the author and his staff can conscientiously assert that they have not knowingly copied any of the examples from other publications. If, however, some have slipped in—as is almost inevitable—we hope to be pardoned. Possibly in some cases they have appeared in questions originated by the author for papers in which he has acted as examiner.

An endeavour has been made to exclude descriptive work on the one hand and purely arithmetic work on the other hand. The ideal throughout has been to set problems which illustrate or depend on definite principles or laws in electrical engineering. The data and the problems have been kept as practical as possible, so that the time spent in working out the examples will be of direct use to students in their future careers.

To facilitate reference and avoid useless repetition, general data, symbols, abbreviations, etc., have been collected in the "Introductory." Wherever possible, officially-adopted symbols and terms, even if distasteful, have been used. It is hoped that this information, together with a slide-rule and a book of trigonometrical and logarithmic (including natural logarithmic) tables, will give all that is required.

Without the generous aid of the members of his staff, this compilation would not have been possible, and the author's acknowledgements are especially due to Mr. F. E. Meade, M.I.E.E., Dr. M. G. Say, M.Sc., A.M.I.E.E., and Mr. G. Shearer, B.Sc., A.M.I.E.E. In order to make the book readily available to all students, whether members of a class or engaged in private study, the publishers have kindly co-operated by keeping the price as low as possible.

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May, 1929.

CONTENTS

The Contents include a General Index (pp. x–xi) and a Detailed Index (pp. xii–xviii).

The more elementary problems are usually at the start of each section.

An asterisk * before the answer to a problem denotes that, for various reasons—*e.g.* use of graphs, graphical solutions, mathematical approximations—some latitude is allowable in the figures. Other answers are given to slide-rule accuracy.

General data, symbols, abbreviations, etc., are collected on pp. 1–10.

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Ω	= ohms	kW	= kilowatts.
℧	= mhos	kVA	= kilovolt-amperes.
C	= coulombs	kVAr	= kilovars (reactive kVA).
J	= joules	kWh	= kilowatt-hours.
H	= henrys	kVAh	= kilovolt-ampere-hours.
F	= farads	MW	= megawatts.
W	= watts	MVA	= megavolt-amperes.

D. MISCELLANEOUS

Time :	sec or s, min, h.
Speed :	rev per sec, rev per min or r.p.s., r.p.m., m.p.h., etc.
Heat :	cal., k.cal. (= 1 000 cal.), B.Th.U.
Temperature :	°C, °F.
Power :	h.p. (British, = 746W), b.h.p.
Light :	c.p. = candle-power ; f.c. = foot-candles.
Frequency :	c/s = cycles per second.
	kc/s = kilocycles per second.
	Mc/s = megacycles per second.
Electromotive force :	e.m.f.
Potential difference :	p.d.
Alternating voltage :	a.v.
Alternating current :	a.c.
Direct voltage :	d.v.
Direct current :	d.c.
Root-mean-square :	r.m.s.
High voltage :	h.v. ; high tension : h.t.
Low voltage :	l.v. ; low tension : l.t.

3. Physical Constants

COPPER :—

Resistance of a standard annealed copper wire at 20° C is 1/58 Ω for 1 m length and 1 mm² cross-section, 0 0178 $\mu\Omega$-m (1·78 $\mu\Omega$-cm), or 0·68 $\mu\Omega$-inch.

Resistance of hard-drawn copper is taken as 102% of that of standard annealed copper.

Resistance-temperature coefficient : The resistance R_t of a wire at temperature t °C is given by

$$R_t = R_o (1 + \alpha t + \beta t^2)$$

where R_o is the resistance at 0° C and $\alpha = 0{\cdot}00426 = 1/284{\cdot}5$ per °C at 0° C ; $\beta = 0{\cdot}000\,001\,12$ and is neglected except where stated otherwise.

Specific gravity : 8·9.

Specific heat : 0 095.

Electrochemical equivalent : 0·000 328.

ALUMINIUM :—

Resistance of a standard aluminium wire, 1 m long and 1 mm² cross-section, is 0 0286 Ω at 20° C.

Resistance-temperature coefficient : $\alpha = 0{\cdot}00880$ per °C at 0° C.
Specific gravity : 2·7.

STEEL :—

Specific gravity of armature steel plates : 7·8 ; of transformer steel plates : 7·5.

Specific heat : 0·16.

To allow for insulation of stampings, assume : net iron length $= 0{\cdot}9 \times$ gross iron length.

4. **Unit Systems.**—The C.G.S. systems use as primary units the centimetre, the gram and the second. In the C.G.S. electromagnetic system the permeability of free space, μ_0, is unity by implication ; in the C.G.S. electrostatic system the permittivity of free space, κ_0, is unity. Because μ_0 and κ_0 in a consistent system must be such that $\mu_0\kappa_0 = 1/c^2$, where c is the velocity of free-space propagation, the C.G.S. electromagnetic and electrostatic systems are not mutually consistent in that derived electrical and magnetic quantities (e.g. voltage, current, magnetic flux) have different magnitudes in each. Thus 1 e.m.u. of current is $3 . 10^{12}$ e.s.u., and neither has the magnitude of 1 ampere.

The M.K.S. unit system incorporates the essential relation $\mu_0\kappa_0 = 1/c^2$; and, by employing as primary units the metre, the kilogram and the second, it gives derived quantities identical in magnitude with those of the engineer's "practical" electrical system (ampere, volt, ohm, etc.), which were originally obtained from the C.G.S., e.m.u. system by multiplying by powers of 10. Complete consistency has been obtained by invoking new "practical" units for force (newton) and magnetic flux (weber). With these additions the M.K.S. system is a single and completely self-consistent system, replacing the illogical non-consistent mixture of e.m.u., e.s.u. and practical units that has been in use for many years.

The rationalized M.K.S. system chooses μ_0 (and therefore κ_0) so as to base defining relations on the uniform-field concept instead of the spherical structure that was used originally in the definition of unit charges and unit magnetic poles. This results in the term 4π being omitted from uniform-field cases, in 2π appearing with cylindrical, and in 4π appearing in cases of spherical symmetry.

Rationalized M.K.S. System : Constants

Free-space velocity of propagation of electromagnetic waves :

$$c = 1/\sqrt{(\mu_0\kappa_0)} = 2{\cdot}998 . 10^8 \simeq 3 . 10^8 \text{ m/s.}$$

Free-space permeability (magnetic space-constant) :

$$\mu_0 = 4\pi/10^7 \text{ V-s/m}^2 \text{ per A/m, or H/m.}$$

Free-space permittivity (electric space-constant) :

$$\kappa_0 = 8{\cdot}854/10^{12} \simeq 1/(36\pi . 10^9) \text{ A-s/m}^2 \text{ per V/m, or F/m.}$$

Absolute permeability and permittivity :

$$\mu = \mu_r\mu_0 \text{ and } \kappa = \kappa_r\kappa_0$$

where μ_r and κ_r are respectively the relative permeability and permittivity, numerically equal to the values of permeability and permittivity in the C.G.S., e.m.u and e.s.u. systems respectively.

Relations between Unit Systems

Physical Quantity	Symbol	M K S Unit and Abbrev	C G S or Other Equivalent
Length	l, L	metre, m	10^2 cm
Mass	m, M	kilogram, kg	10^3 g
Time	t, T	second, s or sec	second
Force	F	newton, Nw	10^5 dynes
Energy	W	joule, J, Nw-m	10^7 ergs
Power	P	watt, W, Nw-m/s	10^7 ergs/s
Current	I	ampere, A	10^{-1} e m u.; $10c$ e.s.u.
Charge	Q	coulomb, C, A-s	10^{-1} e m u.; $10c$ e.s u.
Electric flux	Ψ	coulomb, C, A-s	$40\pi c$ e s.u
Potential	E	volt, V	10^8 e.m u.; $10^6/c$ e.s.u.
Potential difference	V	volt, V	10^8 e m u.; $10^6/c$ e s.u.
Electric field strength	$\mathcal{E}$	volt per metre, V/m	$10^4/c$ e s.u.
Electric flux density	D	coulomb per sq metre, C/m^2	$4\pi c. 10^{-3}$ e s.u
Resistance	R	ohm, Ω	10^9 e m.u.
Conductance	G	mho, $\mho$	10^{-9} e m.u.
Resistivity	ρ	ohm-metre, Ω-m	10^2 Ω-cm; 10^{11} e.m.u.
Conductivity	σ	mho per metre, $\mho$/m	10^{-2} $\mho$/cm; 10^{-11} e.m u.
Capacitance	C	farad, F	$10^{-5}c^2$ e s.u.
Permittivity	κ	farad per metre, F/m	$4\pi c^2. 10^{-7}$ e.s.u.
Magnetomotive force	F	ampere (-turn), A(T)	$4\pi. 10^{-1}$ e.m.u (gilbert)
Magnetic field strength	H	ampere (-turn) per metre, A(T)/m	10^{-2} AT/cm; $4\pi.10^{-3}$ e m.u (oersted)
Magnetic flux	Φ	weber, Wb; volt-second, V-s	10^8 e m.u (maxwell)
Magnetic flux density	B	weber per sq. metre, Wb/m^2	10^4 e.m u. (gauss)
Inductance	L	henry, H	10^9 e m.u
Permeability	μ	henry per metre, H/m	$10^7/4\pi$ e.m u.

c = free-space velocity of e.m. wave propagation $\simeq 3 \times 10^8$ m/s.

5. Miscellaneous Constants and Conversions

1 J = 1 Nw-m
= 10^7 ergs
= 0 24 cal

1 cal = 4 18 J

1 B.Th.U. = 778 ft-lb
= 1 058 J
= 253 cal

1 ft-lb = 1·36 J

1 lb-ft = 0·138 kg-m
= 1·35 Nw-m

1 Nw = 10^5 dynes
= 0 102 kg force
= 0·225 lb force

1 h.p. = 746 W

1 lb = 0 454 kg

1 in = 2·54 cm

1 m = 39·4 in

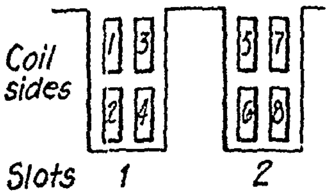
$e = 2\ 7183$; $g = 32{\cdot}2$ ft/s² $= 9{\cdot}81$ m/s²; $j = \sqrt{(-1)} = 1\underline{/90^\circ}$.

$\underline{/\pm\theta}$ = phase-angle $\pm\theta$ measured from reference axis.

arcsin x, arctan x . . . = angle of which the sine, tangent . . . is x.

electron charge, $1{\cdot}60 . 10^{-19}$ C, $1{\cdot}60 . 10^{-20}$ e.m.u., $4{\cdot}80 . 10^{-10}$ e.s.u.
electron charge/mass ratio, $1{\cdot}76 . 10^{11}$ C/kg, $1{\cdot}76 . 10^{7}$ e.m.u./g, $5{\cdot}27 . 10^{17}$ e.s.u./g.

$$1 \text{ Wb} = 10^8 \text{ maxwells (or lines)}$$
$$1 \text{ Wb/m}^2 = 10^4 \text{ gauss (or lines/cm}^2\text{)}.$$



6. **Commutator Windings.**—The system adopted for numbering coil-sides and slots in a 2-layer commutator winding is shown in the accompanying sketch. Upper coil-sides are numbered odd and lower coil-sides even.

SYMBOLS

p = number of pole-pairs.
a = number of pairs of parallel paths.
S = number of slots.
C = number of coils or commutator sectors.
u = number of coil-sides per slot.
y_b = back-pitch measured in coil-sides, or coil-span measured in slot-pitches.
y_f = front-pitch measured in coil-sides.
y_c = commutator pitch measured in sectors.
Z = number of conductors.
N_{ph} = number of phases.
y_{eq} = equipotential pitch measured in sectors.
y_{ph} = phase-pitch measured in sectors.

7. **Standard Curves.**—Reference is made in the text to the curves on pp. 6–10, included for use in problems involving magnetic circuits, hysteresis, and the characteristics of motors, generators and valves.

Where designs are needed for specific purposes, appropriate curves should be obtained.

8. **Sign Convention for Electronic Amplifiers.**—The voltage or current gain of an amplifier is given as positive when no phase reversal takes place and as negative when the output signal is in anti-phase with the input signal.

The input impedance of an amplifier is given as positive when an increase in input current is accompanied by an increase in the voltage across the input terminals.

The output impedance of an amplifier is given as positive when an increase in output current is accompanied by a decrease in the voltage across the output terminals.

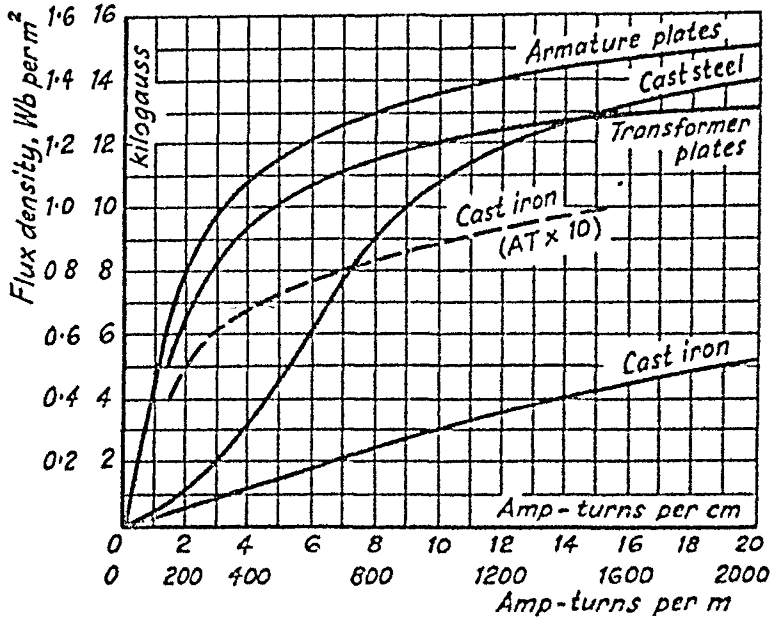


Fig. 1.—Magnetization curves, range 0 — 16,000 lines per cm² (0–1·6 Wb/m²).

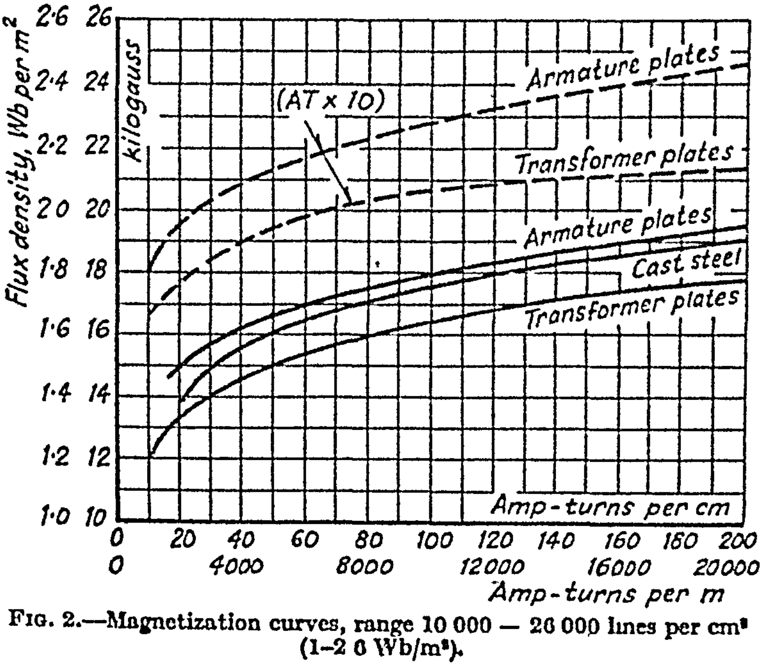


Fig. 2.—Magnetization curves, range 10 000 — 26 000 lines per cm² (1–2·6 Wb/m²).

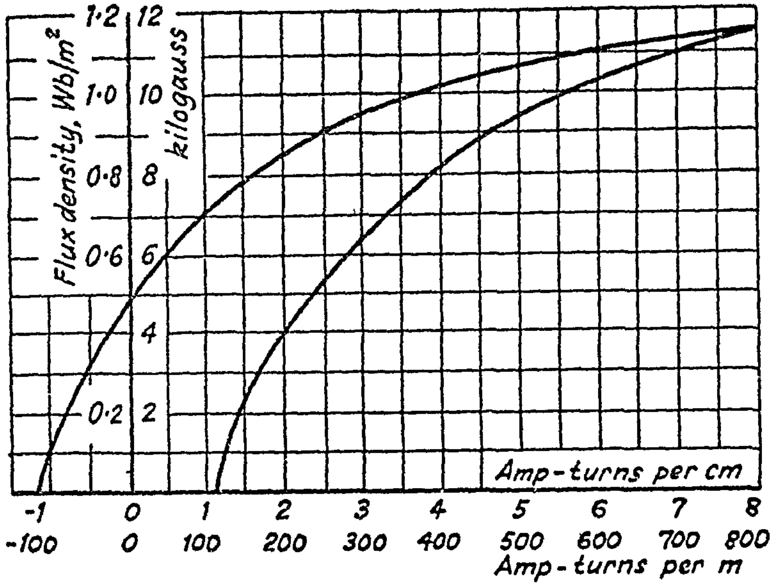


FIG. 3.—Hysteresis loop for transformer steel plates.

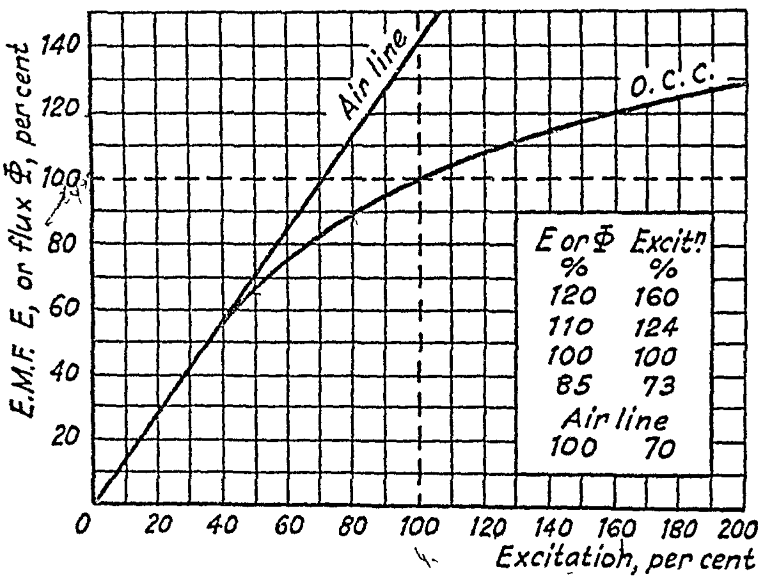


FIG. 4.—Saturation or magnetization curve or open-circuit characteristic for a generator or motor.

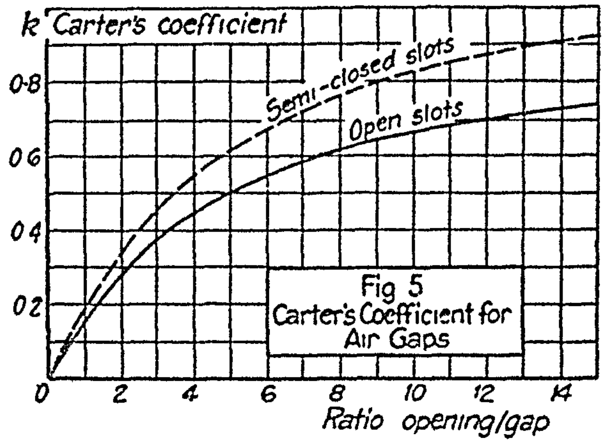


Fig. 5.—Carter's coefficient for air-gaps with slotted armatures.

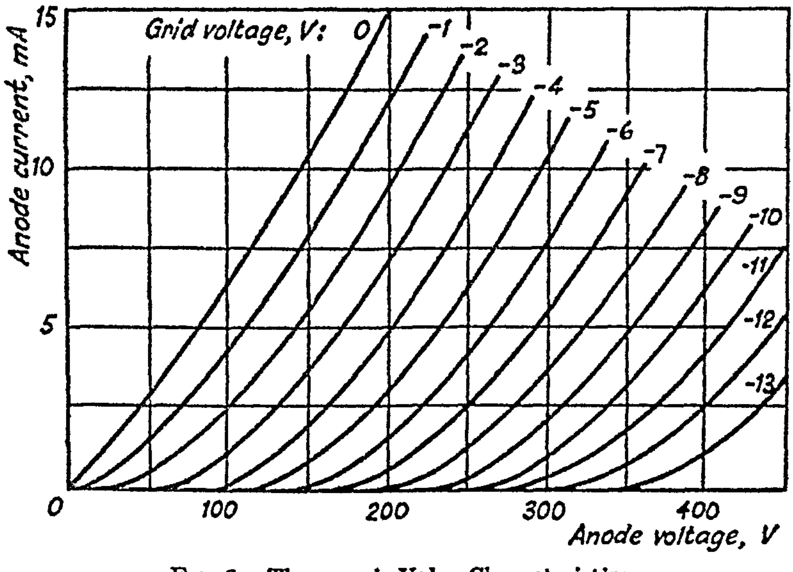


Fig. 6.—Thermionic Valve Characteristics.

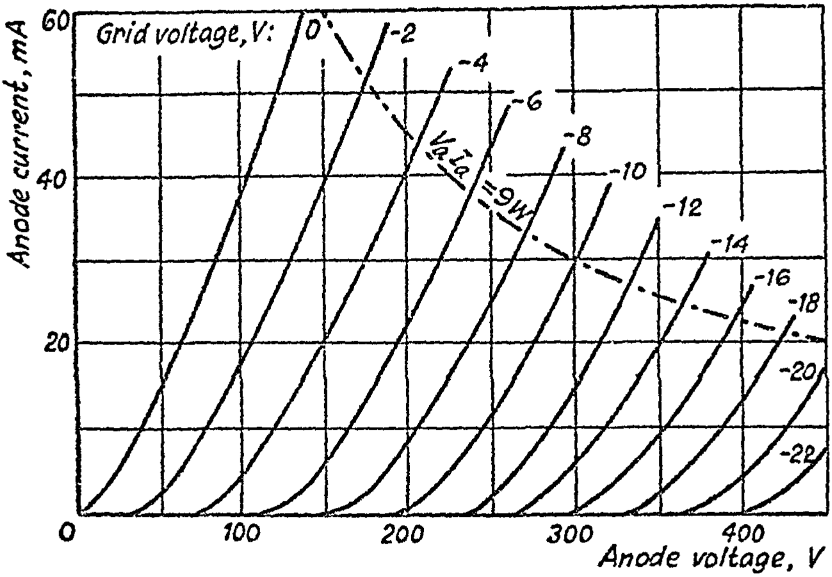


FIG. 7.—Thermionic Valve Characteristics.

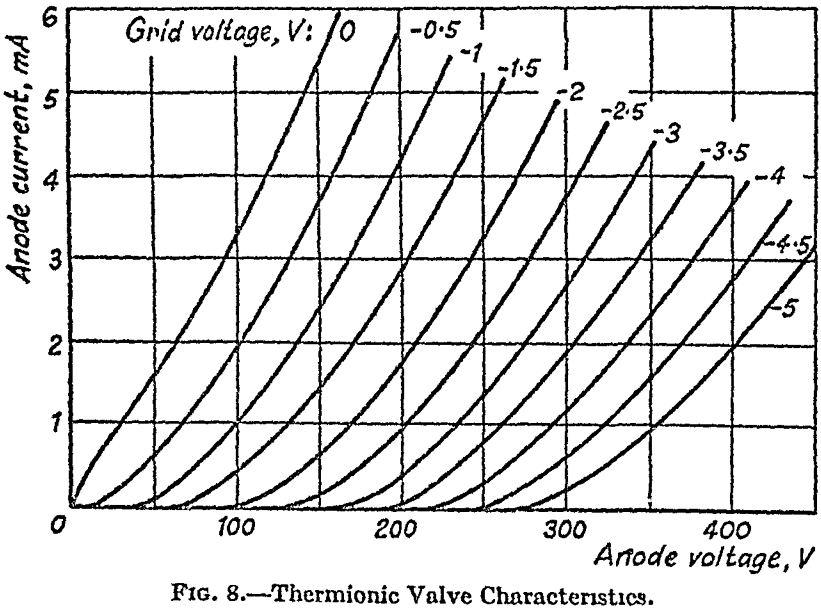


FIG. 8.—Thermionic Valve Characteristics.

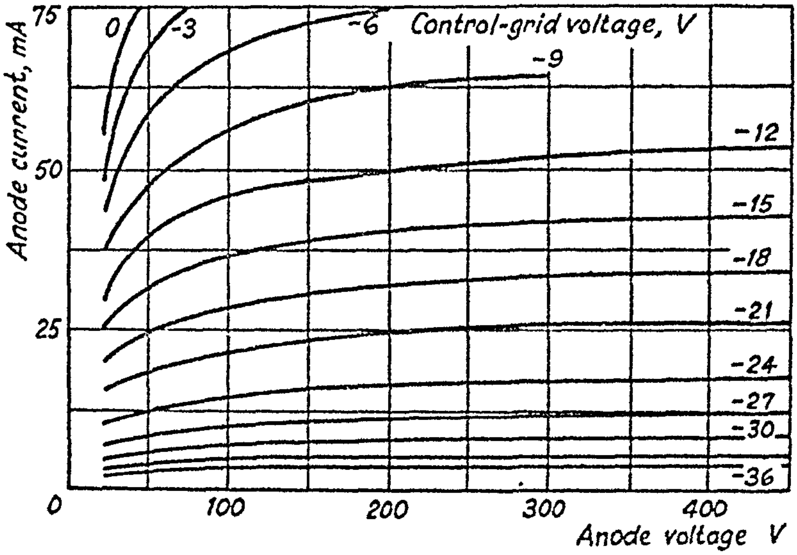


Fig. 9.—Thermionic Valve Characteristics.

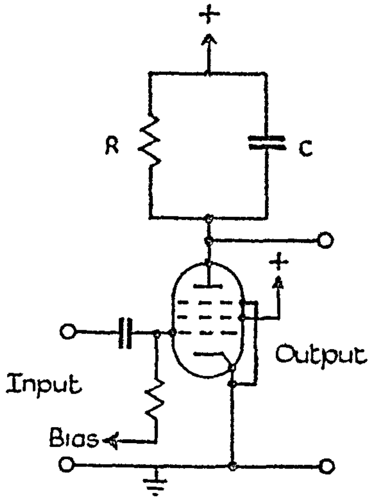


Fig 10 —Typical pentode amplifier with resistive load R and shunt capacitive load C. Mutual conductance of valve $= g_m$. Slope resistance high compared to R.

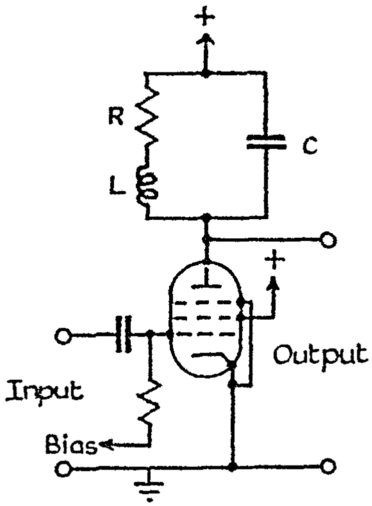


Fig. 11.—Typical pentode amplifier with resistive load R, shunt capacitive load C and shunt peaking inductance L. Mutual conductance of valve $= g_m$. Slope resistance high compared to R.

CHAPTER I

UNIT CONVERSIONS AND DIMENSIONS

Electrical-mechanical Conversions

1 What must be the horse-power of an engine to drive, by means of a belt, a generator supplying 7 000 lamps each taking 0·5 A at 250 V? The line drop is 5 V and the efficiency of the generator is 95%. There is a 2·5% loss in the belt drive. [1 292 h.p.]

2 Find the current and energy taken by a crane motor when raising 1 ton through 50 ft in 10 sec, when the supply is (*a*) 400-V, d.c.; (*b*) 400-V, 3-phase. Gear efficiency, 0 6; motor efficiency, 0 85; induction-motor power factor, 0 85. [(*a*) 74·5 A, 0·083 kWh; (*b*) 50·7 A, 0·083 kWh.]

3 An electric hoist makes 10 double journeys per hour. In each journey a load of 6 tons is raised to a height of 200 ft in 1¼ min and the hoist returns empty in 1¼ min. The hoist cage weighs ½ ton and has a balance weight of 3 tons. The efficiency of the hoist is 80% and of the driving motor 88%. Calculate (*a*) the electrical energy absorbed per journey, (*b*) the hourly consumption in kilowatt-hours, (*c*) the horse-power of the motor. [(*a*) 1·44 kWh; (*b*) 14·4 kWh; (*c*) 39·6 h.p.]

4 An output of 480 tons of material per 6-hour shift is required by an endless-rope haulage from a dip 1 000 yd long, gradient 1 in 8, the haulage speed being 3 m.p.h. The motor has to exert a constant pull of 1 200 lb on the rope to overcome all friction losses. Assuming an efficiency of 0·75, calculate the motor h.p. [58 h.p.]

5 A shaft (mean diameter, 6 in) is being turned in a lathe at 25 r.p.m. Taking the feed at ⅛ in, the tangential force of the tool against the job at 4 000 lb, and the overall efficiency of motor and gearing as 0·7, find the kWh consumed by the driving motor in a traverse of 3 ft. [0 97 kWh.]

6 Calculate the current taken by a 500-V, d.c. motor, efficiency 80%, driving a fan, efficiency 70%, which delivers 50 000 ft^3 of air per min at a water gauge of 6 in. [126 A.]

7 Find the current taken by a 480-V, d.c. motor driving a pump to raise 500 ft^3 of water per minute to a height of 90 ft. Motor efficiency 0·9; pump efficiency, 0·75. Allow a head of 10 ft for pipe friction. [220 A.]

8 Find the h.p. rating of a pump to supply 80×10^6 gallons of water per annum to a storage tank 16 ft in diameter. An automatic control is to start the pump at 45 lb/in² pressure and shut it off at 60 lb/in² pressure. Each operation takes 2 hours. Find also the annual cost with energy at ½d per kWh. Assume the efficiencies of pump and motor to be 0·6 and 0·9 respectively. [22 h.p., £141.]

9 A boiler feed pump delivers 800 000 lb of water per hour at 800 lb/in². The pump efficiency is 0·85. Calculate the line current when the pump is driven by (*a*) a 500-V, d.c. motor, efficiency 0·9; (*b*) a 500-V, 3-phase induction motor, efficiency 0·9, power factor 0·9. Assume hot water to weigh 59 lb/ft³. [(*a*) 215 A; (*b*) 138 A.]

10 A fully-charged car battery can give 100 Wh per charge under starting conditions. At each start, the engine has to be cranked at 60 r.p.m. for 10 sec against a resisting torque of 50 lb-ft. Allowing an overall efficiency for motor and gearing of 25%, estimate the number of starts per battery charge. [21*.]

11 Calculate the available continuous power of a hydro-electric station supplied from a catchment area A of 60 square miles with an annual rainfall F of 50 in and an effective head H of 1 000 ft. Assume yield factor k, to allow for run-off and loss by evaporation, to be 50%, and efficiency η of plant to be 80%. If the load factor be 33%, what should be the rating of generators installed? Show that average power $= 0{\cdot}00624\ \eta kAFH$ kW. [7 500 kW; 23 000 kW.]

12 Find the head in feet of a hydro-electric generating station in which the surface of a reservoir 1 acre in extent falls by 1 ft when 100 h.p.-hr is developed in the turbine. The efficiency of the turbine is 70%. [104 ft.]

13 A hydro-electric generating station is supplied from a reservoir of capacity 200 million ft³ at a head of 500 ft. (*a*) What is the total available energy in kilowatt-hours if the hydraulic efficiency be 0·8 and the electrical efficiency 0·9? (*b*) Find the fall in the reservoir level after a load of 12 000 kW has been supplied for 3 hr if the area of the reservoir is 1 sq. mile. (*c*) If the reservoir be supplied by a river at the rate of 40 ft³ per sec, what does this flow represent in kilowatts, in kilowatt-hours per day, and in kilowatt-hours per annum? [(*a*) $1{\cdot}69 \times 10^6$ kWh; (*b*) 1·83 in; (*c*) 1 220 kW; 29 300 kWh; $10{\cdot}7 \times 10^6$ kWh.]

14 A current of 80 A flows for 1 hr in a resistance across which there is a voltage of 2 V. Determine the velocity with which a weight of 1 ton must move in order that its kinetic energy shall be equal in amount to the energy dissipated in the resistance. [110·5 ft per sec.]

15 Find the amount of electrical energy expended in raising the temperature of 10 gall of water by 75° C. To what height could a weight of 5 tons be raised with the same expenditure of energy? Assume that the heater has an efficiency of 90% and the hoist an efficiency of 70%. [4·4 kWh ; 725 ft.]

16 An 80-V battery drives a 3-ton truck at 12 m.p.h. on the level against a frictional resistance of 120 lb. The overall efficiency is 0·6. Find the hourly discharge of the battery in ampere-hours. If the same power be taken from the battery on an up gradient of 1%, find the speed. [60 Ah ; 7·7 m.p.h.]

17 A 250-ton train is running at 30 m.p.h. ; the frictional resistance is 12 lb per ton and the gearing efficiency is 0·8. Find the kW input to the motors of efficiency 0·9 (*a*) on the level, (*b*) on a 1 in 100 gradient, (*c*) when on the level at this speed and accelerating at 0·5 ft/sec², (*d*) as (*c*) when on 1% gradient.
[(*a*) 249 ; (*b*) 715 ; (*c*) 975 ; (*d*) 1 440 kW.]

18 A 30-ton car has a frictional resistance of 600 lb. Find the energy needed to drive the car up a 5% gradient, 500 ft long, in 2 min. Gear efficiency 0·75, motor efficiency 0·9. Calculate the current for a line voltage of 550 V. [1·1 kWh ; 60 A.]

Electrical-thermal Conversions

19 A generating station has an overall thermal efficiency of 15·5%, and 1·80 lb of coal are burnt for every kilowatt-hour measured on the switchboard. Determine the calorific value of the fuel (*a*) in British thermal units per pound, (*b*) in kilo-calories per kilogram. [(*a*) 12 300 ; (*b*) 6 830.]

20 A small generating plant uses producer gas with a calorific value of 130 B.Th.U. per ft³. Find the volume of gas required per hour to generate 100 kW. The overall efficiency of the plant is 20%. [13 100 ft³.]

21 A gas fire consumes 50 ft³ of gas per hour, the calorific value of the gas being 500 B.Th.U. per ft³. If the efficiency of this fire is 50% calculate the loading of an electric fire to give the same heat output, and compare the hourly cost. Gas costs 9·5d per therm and electricity ¾d per unit.
[3·67 kW ; gas 2·375d ; electricity 2·75d.]

22 Calculate the thermal efficiencies of (*a*) a 3-pint, 1 000-W electric kettle, and (*b*) a 3-pint gas kettle, from the following data. The temperature of 1 quart of water is raised from 15° C to 95° C in 7¾ min in the electric kettle, and in 8½ min in the gas kettle, during which 2·5 ft³ of gas is used. Calculate also the cost in each case, with electricity at ¾d per unit and gas at 8d per therm. 1 therm = 100 000 B.Th.U. The gas has a calorific value of 500 B.Th.U. per ft³.
[(*a*) 81·8%, 0·097d ; (*b*) 28·8%, 0·1d.]

23 A well-lagged 100-gall tank with its heater weighs 200 lb and has a specific heat of 0·1. Find the kWh needed to raise the temperature of a full tank of (*a*) water, (*b*) oil, from 50° to 120° F. Take specific heat of oil as 0·5 and its specific gravity as 0·9. Ignore tank losses. [(*a*) 20·9 ; (*b*) 9·65 kWh.]

24 The water-cooled brake-pulley of an electric motor contains 2 pints of water at 100° C. If the input to the motor is 25 A at 250 V, find the time taken to boil off all the water. The efficiency of the motor is 80% and the latent heat of steam is 538 cal per g. [8 5 min]

25 An electric kettle having an efficiency of 85% is required to raise the temperature of 2 pints of water from 15° to 98° C in 10 min. Calculate the resistance of the heating elements for use on a 200-V circuit. If the thermal capacity of the kettle is 70 cal/°C, calculate the heat loss in radiation Express the thermal capacity in B.Th.U./°F.

[52 Ω ; 75 W ; 0 155 B.Th.U./°F.]

26 Estimate the loading necessary for an electric water heater to raise the temperature of 6 pints of water from 15° to 95° C in 10 min Take the heat loss in radiation during this period as 60 000 J, and the water equivalent of the heater as 125 g. Determine approximately the efficiency of the operation. How long would the heater take to raise the temperature of the water if its loading were halved ? *[2 070 W ; 91·5% ; 21 min.]

27 The heater element of an electric kettle has a constant resistance of 100 Ω, and the applied voltage is 250 V. Calculate the time taken to raise the temperature of 1 quart of water from 15° to 90° C, assuming that 85% of the power input to the kettle is usefully employed. If the water equivalent of the kettle is 100 g, find how long it will take to raise a second quart of water through the same temperature range immediately after the first. What is the efficiency of the second operation ?

[11·1 min. ; 10 2 min. ; 93%.]

28 The daily cycle of a thermal-storage electric heating installation begins at 6 a.m., when 200 000 lb of water is stored at 95° C. Until 5 p.m., 10 000 lb of water per hr is supplied for radiators at an average temperature of 75° C returning at 40° C. For washing, etc., a total of 50 000 lb at an average temperature of 72° C is supplied during the day and is replaced by water at 12° C. The electrical supply to the boiler, which has an efficiency of 97%, is available between 12 midnight and 4 a.m. Estimate the kW loading of the boiler, and the temperature of the water at 5 p.m. Neglect all losses [982 kW ; 60·8° C.]

29 The amount of hot water needed by a family of four in one week was found to be :

ablutions :	84 gall at 105° F
washing-up :	28 „ „ 140° F
cleaning :	7 „ „ 140° F
baths :	72 „ „ 100° F

The inlet temperature was 57° F and the energy consumption was 41·6 kWh. Find the overall efficiency. If the standby losses per week were 8·4 kWh, how much did the remaining losses amount to? [70·5% ; 3·9 kWh.]

30 A cubic tank of 1-yd side is filled to 90% capacity five times daily. The water is heated from 50° to 130° F. The loss per m² of tank surface per 1° F temperature difference is 8·05 W for unlagged tank and 3 22 W for lagged tank. Find the loading in each case and the corresponding efficiency. [10·65 kW, 69·5% ; 8·72 kW, 85%.]

31 A 3-phase, 440-V electrode boiler supplies 100 lb of superheated steam per hour. Under the conditions of operation, the total heat of steam from 32° F is 1 240 B.Th.U. per lb, the temperature of the feed water is 130° F and efficiency is 93%. Find the line current. [47 A.]

32 Two heaters A and B are in parallel across supply voltage V. Heater A produces 500 k.cal in 20 min and B produces 1 000 k.cal in 10 min. The resistance of A is 10 Ω. What is the resistance of B? If the same heaters are connected in series across the voltage V, how much heat will be produced in 5 min? [2·5 Ω ; 100 k.cal.]

33 A 5-lb electric iron has an initial rate of temperature rise of 33·5° C per min. (*a*) Estimate its loading in watts. (*b*) If the total heat dissipation be 3·4 W per °C for the whole surface, estimate the final steady temperature rise of the iron. Specific heat, 0·16. [(*a*) 850 W ; (*b*) 250° C.]

34 The star-connected elements of a 3-phase resistance oven are each wound with 220 in of nickel-chrome strip 0 01 in × 0 2 in. If the strip temperature is to be 1 000° C and the charge temperature 650° C, estimate the kW loading of the oven. Estimate also the strip temperature when the charge is 20° C. Take the radiating efficiency as 0 6 and the emissivity as 0 9. [10 kW ; 902° C.]

35 A 250-V, 1-kW single-element electric fire is to employ a nickel-chrome resistance wire operating at 1 000° C. Estimate a suitable diameter and length for the wire. Take the radiating efficiency = 1, the emissivity = 0·9, and the resistivity of the wire at 1 000° C = 42 4 × 10^{-6} Ω-cm. [0·272 mm ; 860 cm.]

36 What must be the useful rating of a tin-smelting furnace in order to smelt 50 kg of tin per hour? Smelting temperature of tin = 235° C ; specific heat = 0·055 ; latent heat of liquefaction = 13·31 k.cal per kg. Take the initial temperature of the metal as 15° C. [1·47 kW.]

37 A 3-phase arc furnace has the following characteristics : current, 7 500 A ; arc voltage, 90 V ; reactance and resistance of transformer and electrodes together (excluding arc), 0·003 Ω and 0 002 Ω respectively. Find (*a*) the power factor, (*b*) the electrical efficiency, (*c*) the kVA taken from the supply, and (*d*) the time taken to melt 5 tons of steel if the overall efficiency is 50%. Take the specific heat and latent heat of fusion of steel as 0·12 B.Th.U. per lb, and 16 B.Th.U. per lb respectively, and the melting point of steel as 2 500° F. Initial temperature, 60° F.

[(*a*) 0·978 ; (*b*) 85·7% ; (*c*) 2 415 kVA ; (*d*) 51·6 min.]

38 A room measures 12 ft × 12 ft × 14 ft. The air in it has to be renewed every 30 min and maintained at a temperature 10° C above that of the incoming air. Find the necessary rating of an electric heater, ignoring losses. Find the rating if the heat loss from walls, etc., is 400 B.Th.U./°F/hr. (For air : density, 0·08 lb/ft^3 ; specific heat, 0 24.) [0·41 kW ; 2·51 kW.]

Dimensions

39 Derive the dimensional equations for voltage and current in the LMTμ, LMTκ and LMTQ systems. Show that in each case the voltage-current product has the dimensions of power.

40 Derive the dimensional equations in the LMTμ and LTVI systems for (*a*) voltage gradient, (*b*) current density, (*c*) resistivity.

[(*a*) $M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2}\mu^{\frac{1}{2}}$, VL^{-1} ; (*b*) $M^{\frac{1}{2}}L^{-\frac{3}{2}}T^{-1}\mu^{-\frac{1}{2}}$, IL^{-2} ;
(*c*) $L^2T^{-1}\mu$, VLI^{-1}.]

41 Determine the physical dimensions of capacitance in the electromagnetic and electrostatic systems of units, and show that these units are related by the square of a velocity $c = 1/\sqrt{\kappa\mu}$, where κ is the permittivity and μ the permeability of the medium concerned. Find the magnitude of 1 F in terms of C.G.S. electrostatic units, given $c = 3 \times 10^{10}$ cm/s and 1 F = 10^{-9} e.m.u.

[$C_m = [L^{-1}T^2\mu^{-1}]$; $C_s = [L\kappa]$; 1 F = 9×10^{11} e.s.u.]

42 Show that torque can be expressed as work per unit angular displacement. Find (*a*) the relation between torque expressed in ton-ft and in kW per rev per min, (*b*) the relation between kg-m and joules per radian.

[(*a*) 1 ton-ft = 0·318 kW per rev per min ;
(*b*) 1 kg-m = 9·81 J per radian.]

43 A flywheel of moment of inertia 40 lb-ft^2 runs at 1 500 rev per min. Calculate the value of the stored energy in Wh.
[5·8 Wh.]

44 From the data given in the Introductory, find the relations between (*a*) British thermal unit and kilowatt-hour; (*b*) horse-power-hour and kilowatt-hour; (*c*) foot-pound and joule; (*d*) kilo-calorie and kilowatt-hour.
[(*a*) B.Th.U. $= 2{\cdot}93 \times 10^{-4}$ kWh or kWh $= 3\,418$ B.Th.U.; (*b*) h.p.-hr $= 0{\cdot}746$ kWh or kWh $= 1{\cdot}34$ h.p.-hr; (*c*) ft-lb $= 1{\cdot}36$ J or J $= 0{\cdot}735$ ft-lb; (*d*) k.cal $= 1{\cdot}16 \times 10^{-3}$ kWh or kWh $= 862$ k.cal.]

45 Using the LTVI system of dimensions, and given that 1 V = 1/300 C.G.S. e.s.u. of potential difference and 1 A $= 3 \times 10^9$ C.G.S. e.s.u. of current, find the number of C.G.S. e.s.u. of electric flux density in 1 M.K.S. rationalized unit. [$12\pi \times 10^5$.]

46 Given that 1 A $= 10^{-1}$ C.G.S. e.m.u. of current and that 1 V $= 10^8$ C.G.S. e.m.u. of potential difference, find the ratio of the size of the C.G.S. e.m.u. of inductance to the henry. Use the LTVI system of dimensions. [10^{-9}.]

47 The magnetic pull F of an electromagnet is given by $F \propto B^a A^b \mu^c$ where B = flux-density in the air-gap; A = cross-sectional area of the gap; and μ = the permeability of the air. Determine the values of a, b, and c.
[$a = 2$, $b = 1$, $c = -1$.]

48 Given that the energy stored per unit area in a parallel-plate condenser depends upon (i) the permittivity κ of the dielectric; (ii) the distance d between the plates; (iii) the applied potential difference V; determine how each factor influences the stored energy W.

$$\left[W \propto \frac{\kappa V^2}{d}.\right]$$

49 Is the following equation dimensionally correct for the current in a particular circuit containing mutual-inductance M, self-inductances L_1 and L_2, and resistances R_1 and R_2?

$$I = V\omega M \Big/ \{(\omega^2 M^2 + R_2)^2 + \omega^2 L_1 L_2 R_1^2\}^{\frac{1}{2}}.$$

If not, suggest any necessary corrections. [R_2 should read R_2^2.]

CHAPTER II

ELECTRIC FIELDS

Capacitance and Permittivity

1 Calculate the capacitance in microfarads of an air-insulated condenser with 18 plates each 10 cm square, the distance between plates being 2 mm. [530 $\mu\mu$F.]

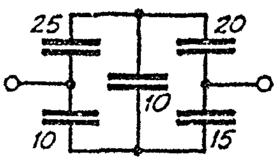
2 Find the equation of the voltage to be applied to a 50-μF condenser initially uncharged to produce a steady current of 10 mA. If charging ceases after 10 sec, calculate (*a*) condenser voltage, (*b*) charge, (*c*) stored energy.
[(*a*) 2 000 V; (*b*) 0·1 C; (*c*) 100 J.]

3 Find the capacitance of a spherical conductor 18 cm in diameter (*a*) when isolated in air, (*b*) when totally enclosed in a conducting spherical shell 20 cm in diameter and air dielectric.
[(*a*) 10 $\mu\mu$F; (*b*) 100 $\mu\mu$F.]

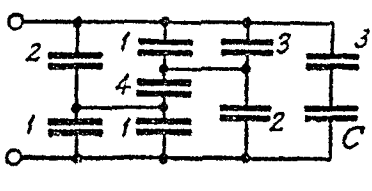
4 A bakelite-filled insulator, 1 m long, consists of coaxial cylinders, the inside and outside diameters of the outer and inner cylinders being 6 cm and 5·5 cm respectively. Calculate (*a*) the capacitance (relative permittivity = 4); (*b*) the charge in micro-coulombs at a potential difference of 25 kV.
[(*a*) 2 560 $\mu\mu$F; (*b*) 64·0 μC.]

5 Two capacitors A and B are placed in (*a*) series, (*b*) parallel. $C_A = 100\ \mu$F, $C_B = 50\ \mu$F. Find the current and the maximum stored energy in the circuit when a 240-V, 50-c/s voltage is applied. How is the voltage distributed in case (*a*)?
[(*a*) 2·5 A, 1·92 J, $V_A = 80$ V, $V_B = 160$ V; (*b*) 11·3 A, 8·63 J.]

6 Determine the capacitance of the condenser combination between terminals. The figures shown are capacitances in microfarads. [17·8 μF.]



7 The figures in the network refer to capacitance in microfarads. If the combined capacitance is 5 μF, find C. [21 μF.]



8 A plate condenser A is charged to 40 μC at 100 V. It is then connected to a similar type condenser B of 4 times the plate

area of A. Find the charge on each condenser and the loss in energy. [A, 8 μC ; B, 32 μC. 0·0016 J.]

9 An air condenser consisting of 2 parallel square plates of 50-cm side is charged to a p.d. of 250 V when the plates are 1 mm apart. Find the work done in separating the plates from 1 to 3 mm. Assume perfect insulation. [1 875 ergs.]

10 A plate condenser has three similar parallel plates, the outside two being metallically connected. Find the ratio of the capacitance when the inner plate is midway between the outers to the capacitance when the inner plate is three times as near one plate as the other. [$\frac{3}{4}$.]

11 A condenser is composed of two plates separated by a sheet of insulating material 3 mm thick and of relative permittivity 4. The distance between the plates is increased to allow of the insertion of a second sheet 5 mm thick and of relative permittivity κ_r. If the capacitance of the condenser so formed is one third of the original capacitance, find κ_r. [3·33.]

Electric Field

12 A parallel-plate condenser has plates 0·15 mm apart, a plate area of 1 000 cm^2, and a dielectric with a relative permittivity of 3. Find the electric flux density, the electric field intensity and the voltage between the plates if the condenser has a charge of 0·5 μC.
[18 85 e.s. units (5 μC/m^2) ; 6·28 e.s.u. (188·5 kV/m) ; 28·3 V.]

13 Calculate the dielectric flux (*a*) in electrostatic units, (*b*) in microcoulombs, between two parallel flat metal plates each 35 cm square with an air-gap of 1·5 mm between them, the potential difference being 3 000 V. A sheet of insulating material 1·5 mm thick is inserted between the plates, and the potential difference is raised to 7 400 V. What is the relative permittivity of this material if the charge is now 32 μC ?
[(*a*) 81 700 ; (*b*) 2·16 μC ; 6.]

14 The capacitance of the condenser formed by two parallel metal sheets, each 100 cm^2 in area, separated by a dielectric 2 mm thick, is 2×10^{-4} μF. A potential difference of 20 kV is applied. Find (*a*) the total dielectric flux in coulombs; (*b*) the potential gradient in kilovolts per cm ; (*c*) the relative permittivity of the material ; (*d*) the electric flux density.
[(*a*) 4 μC ; (*b*) 100 kV per cm ; (*c*) 4·5 ; (*d*) 1 510 e.s. units (400 μC/m^2).]

15 The electric field strength in a mass of porcelain (relative permittivity 6) in air is 1 000 V per cm. At the inner surface of the porcelain the field makes an angle 45° to the normal and emerges into the air. Find the angle of emergence of the external field, and its magnitude. [9·5° ; 4 300 V per cm.]

II. ELECTRIC FIELDS

Electric Force

16 A parallel plate condenser is immersed in alcohol of relative permittivity 26. The plates are charged to a potential difference of 20 kV, and the distance between them is 1·5 cm. Determine per cm² of plate area the force between the plates and the energy. [2 040 dynes ; 3 060 ergs.]

17 Find the greatest mechanical force, produced electrostatically, that can be exerted on 1 cm² of conducting material in air. Assume that air becomes conducting at a voltage gradient of 30 kV per cm. [397 dynes (0 00397 Nw).]

Voltage Gradient

18 Two series-connected, flat-plate condensers have plate areas of 300 in² and 60 in², plate separations of 0 02 in and 0·005 in, and permittivities of 1 and 6, respectively. Calculate the total voltage across the two condensers that will produce a gradient of 100 kV per cm between the plates of the first condenser. [6·13 kV.]

19 A steady voltage of 1 500 V is applied across two parallel metal discs of 10 cm radius, and 14 mm apart. Between the discs are three layers of dielectric : $t_1 = 2$ mm, $\kappa_1 = 3$; $t_2 = 5$ mm, $\kappa_2 = 4$; $t_2 = 7$ mm, $\kappa_3 = 6$. Calculate the voltage gradient and the energy stored in each dielectric.
[1 625, 1 220, 812 V/cm ; 21·9, 41·3, 38·4 μJ.]

20 A flat-plate condenser of 10 cm² superficial area has three dielectrics each 1 mm thick, and of relative permittivity 2, 3 and 5. Find the potential difference between the plates in volts for a charge of 10 000 $\mu\mu$C. Draw curves showing the potential across and the potential gradient through the dielectrics. Draw a line on each curve showing the potential and potential gradient for the same charge with air as the dielectric, and state the potential difference required. [1·17 kV ; 3·39 kV.]

21 Two concentric metal spheres, of radius r and R respectively, are maintained at a potential difference V. What ratio must the radii have for minimum voltage gradient in the intervening medium, for a given value of R ? [$r = \frac{1}{2}R$.]

22 Draw to scale equipotentials at every 5 kV, and a curve showing the variation of voltage gradient, between two concentric conductors maintained at a difference of 50 kV. The outer and inner radii of the inner and outer conductors are 1 cm and 10 cm respectively. What is the potential midway between the two conductors ? [13 0 kV.]

23 A 2-mm diameter wire is co-axial with a porcelain tube ($\kappa_r = 5$) of inside diameter 6 mm and outside diameter 12 mm,

A steady voltage of 1 000 V is applied between the wire and a metal sheath outside the porcelain. Find the voltage gradients in each of the dielectrics.
[Air : max. 8 150, min. 2 720 V/cm ; porcelain : max. 545, min. 273 V/cm ; air at outside surface of porcelain : 1 865 V/cm.]

24 A 50-$\mu\mu$F compressed-gas condenser for a 100-kV Schering bridge is of the concentric cylinder type. It is desired to limit the maximum stress in the inter-electrode space to 60 kV per cm. Estimate suitable values for the radius and active length of the inner, low-voltage electrode, if the inner radius of the high-voltage electrode is 7 cm. [3·7 cm ; 57·3 cm.]

25 The distance between the axes of two parallel conductors of radii 2 cm is 8 cm. If the conductors are maintained at a steady potential difference of 10 kV, draw a curve showing the variation of the voltage gradient along a line joining the axes of the conductors. Determine the ratio of the maximum and minimum values of gradient, and compare this with the similar ratio when the distance between axes of the conductors is reduced to 6 cm. [1·5 ; 1·25.]

26 If the breakdown dielectric strength of air be 38 kV per cm, estimate the greatest line voltage that can be used on a 3-phase transmission line with solid smooth conductors each 1 cm in diameter, spaced 2 m apart. [197 kV (peak).]

27 A parallel-plate condenser has a dielectric comprising a sheet of mica 1 mm thick (relative permittivity 5·5 ; resistivity 10^{16} Ω-cm) and a layer of oil 5 mm thick (2·2 ; 10^{10} Ω-cm). Find the root-mean-square voltage gradient in each dielectric material when a potential difference of 50 kV at 50 c/s is applied. [Oil : 91·9 kV per cm ; mica : 70·4 kV per cm.]

28 Two electrodes are so arranged that spark over occurs at a voltage of 3 000 V at normal temperature and pressure. What voltage will have to be applied under working conditions of (*a*) 110 lb per in^2 and 300° C, (*b*) 10 in mercury and − 30° C at a high altitude.
[(*a*) 12 800 V; (*b*) 1 185 V.]

CHAPTER III

MAGNETIC FIELDS

M.K.S. units are given in parenthesis

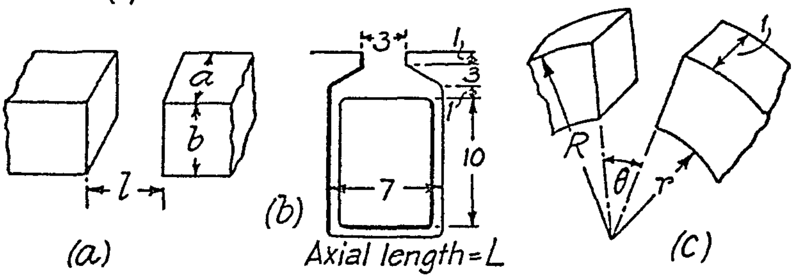
Permeance and Permeability

1 From the magnetization curves in Figs 1 and 2 plot curves of relative permeability to a base of flux density (*a*) for cast steel ; (*b*) for cast iron.

2 An iron ring with a mean circumference of 140 cm and cross-section 12 cm^2 is wound with 500 turns of wire. When the exciting current is 2 A, the flux is found to be 120 000 lines (1·2 mWb). What is the relative permeability of the iron ?

[1 113.]

3 Develop expressions for permeances of the air-paths in the following arrangements. Disregard fringing and assume that the lines of force are rectilinear in (*a*) and (*b*), and circular arcs in (*c*) :—



[(*a*) ab/l ; (*b*) 1·55 L ; (*c*) $(1/\theta)\log_e(R/r)$.]

Magnetic Field

4 Two concentrated coils each 20 cm in diameter and wound with 100 turns, are placed coaxially 10 cm apart with their planes parallel. A direct current of 1 A is passed in the same direction through each coil. Draw a curve showing the magnetic field strength at any point on the common axis up to a distance of 80 cm from one coil.

5 A glass tube 1 in diameter and 1 yd long is completely and uniformly wound with 900 turns of copper wire 0·9 mm diameter, insulated to 1·0 mm diameter. Find the strength of the magnetic field at the centre of the solenoid thus formed when a potential difference of 4 V is applied to its terminals. Temperature 20° C.

[24·4 oersteds (1 940 AT/m).]

6 Calculate the flux density (*a*) at the surface of, (*b*) 10 cm away from the centre of, a long straight isolated conductor 0·5 cm in diameter carrying a direct current of 100 A.
[(*a*) 80 lines per cm^2 (8 mWb/m^2) ; (*b*) 2 lines per cm^2 (0·2 mWb/m^2).]

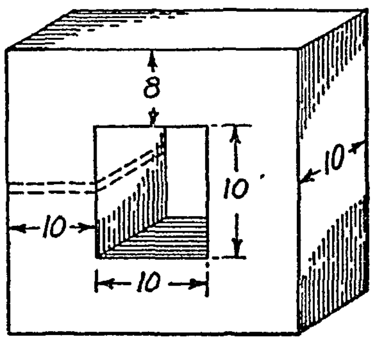
7 A long straight wire of diameter 1 cm carries a direct current of 1 000 A. Draw a curve showing magnetic field strength both inside and outside the wire on a radial line extending 10 cm from the axis of the wire.

8 A wire is bent into a plane square of 12 in side, and a current of 250 A is circulated round it. What is the magnetic flux density at the centre of the square?
[9·27 gauss (0·927 mWb/m^2).]

Magnetic Circuit

9 A cast-steel electromagnet has an air-gap of length 2 mm and an iron path of length 30 cm. Find the number of ampere-turns necessary to produce a flux density of 8 000 lines per cm^2 (0·8 Wb/m^2) in the gap. Neglect leakage and fringing.
*[1 500.]

10 Calculate the magnetization curve for (*a*) the complete iron circuit shown; (*b*) the same circuit with a 4-mm air-gap introduced as indicated by the dotted lines. The exciting coil has 200 turns. The material is transformer sheet steel. Neglect fringing and leakage, and plot total flux as a function of exciting current.



Dimensions in cm.

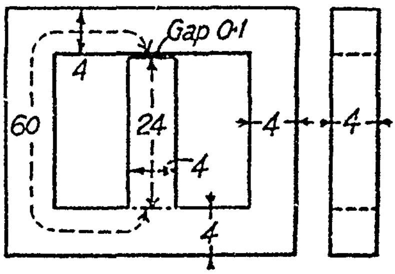
11 Explain the terms (*a*) magnetomotive force, (*b*) magnetic flux, (*c*) reluctance, (*d*) flux density. Estimate their respective approximate values in the following case:—A steel ring, 30 cm mean diameter and of circular section 2 cm in diameter, has an air-gap 1 mm long; it is wound uniformly with 600 turns of wire carrying a current of 2·5 A. Neglect magnetic leakage. (N.B.—The iron path takes about 40% of the total magnetomotive force.)
*[(*a*) 1 885 (1 500 AT) ; (*b*) 37 700 (0·377 mWb) ; (*c*) 0·05 (3·98 × 10^6 AT/Wb) ; (*d*) 12 000 (1·2 Wb/m^2).]

12 A cast steel ring has a circular cross-section 3 cm in diameter and a mean circumference of 80 cm. The ring is uniformly wound with a coil of 600 turns. (*a*) Estimate the current

required to produce a flux of 50 000 lines (0·5 mWb) in the ring. (*b*) If a saw-cut 2 mm wide is made in the ring, find approximately the flux produced by the current found in (*a*). (*c*) Find the current value which will give the same flux as in (*a*). Assume the gap density to be the same as in the iron and neglect fringing.
*[(*a*) 0·89 A ; (*b*) 18 400 maxwells (0·184 mWb) ; (*c*) 2·78 A.]

13 An iron ring of mean length 50 cm has an air gap of 1 mm, and a winding of 200 turns. If the permeability of the iron is 800 when a current of 1 A flows through the coil, find the flux density. [942 gauss (94·2 mWb/m²).]

14 A 680-turn coil is wound on the central limb of the cast steel frame shown. A total flux of 160 000 lines (1·6 mWb) is required in the gap. Find what current is required. Assume that the gap density is uniform and that all lines pass straight across the gap. Dimensions given are in centimetres.
*[1·95 A.]



15 It is required to build a cast iron magnet of rectangular cross-section with width equal to twice the breadth. Length of gap, ⅝ in ; length of iron path, 85 in ; total flux, 8 megalines (80 mWb) ; winding, 2 in deep ; core and gap density, 7 000 lines per cm² (0 7 Wb/m²). With wire at 1s 9d per lb and iron at 7d per lb find the cost of material. Find the saving or loss obtained by working the magnet at a density of 6 000 lines per cm² (0 6 Wb/m²). Use No. 14 S.W.G. with an exciting current of 10 A. (No. 14 S.W.G. wire weighs 58·1 lb per 1 000 yd ; cast iron 0 26 lb per in³.) *[£25 : 2 : 9 ; loss £1 : 16 : 6.]

16 A ring of cast steel has an external diameter of 24 cm and a square cross-section of 8 cm side. Inside and across the ring an ordinary steel bar 18 × 8 × 0 4 cm is fitted with negligible gap. Calculate the number of ampere-turns to be applied to one-half of the ring to produce a flux density of 10 000 lines per cm² (1 Wb/m²) in the other half. Neglect leakage. [700 AT.]

17 A ring of ordinary sheet steel stampings, of mean diameter 25 cm and net cross-sectional area 20 cm², has 2 000 turns. Find the mean rates of change of flux and of flux-density when the exciting current is raised from zero to (*a*) 0·25, (*b*) 1, (*c*) 3, (*d*) 5 A respectively, using the magnetization curves given in Figs. 1 and 2. Find also the instantaneous rate of change of the flux as the current passes through the value 1 A.
*[Megalines per A :—(*a*) 1·0 ; (*b*) 0·308 ; (*c*) 0·117 ; (*d*) 0·074
Kilolines per cm² per A :—(*a*) 50 ; (*b*) 15·4 ; (*c*) 5·8 ; (*d*) 3·7.
At 1 A, Megalines per A ≃ 0·087.]

Magnetic Force

18 A straight conductor 100 cm long and carrying a direct current of 50 A lies perpendicular to a uniform magnetic field of 10 000 lines per cm^2 (1 Wb/m^2). Find (*a*) the mechanical force acting on the conductor; (*b*) the mechanical power in watts to move the conductor against the force at a uniform speed of 5 m per sec; (*c*) the electromotive force generated in the conductor. Show that the electrical power produced is equal to the mechanical power used in producing motion.

[(*a*) 5×10^6 dynes (50 Nw); (*b*) 250 W; (*c*) 5 V.]

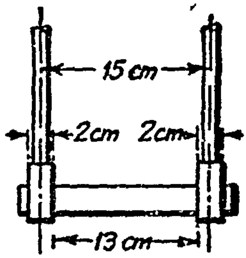
19 Find the maximum force in pounds per foot run between single-phase bus-bars (*a*) normally; (*b*) under steady short-circuit conditions. The normal load is 10 000 kVA at 15 kV, and the effective impedance 10%. The bus-bars are arranged 12 in apart. [(*a*) 0·04 lb; (*b*) 4 lb.]

20 Find the force in kilograms per metre length between two conductors 5 cm apart carrying 1 000 and 5 000 A respectively.

[2·04.]

21 The sketch represents the contacts of an oil switch. Assuming that the vertical conductors are long compared with the distance between them find approximately the force on the contact bar when the switch carries 10 000 A.

[5·38 kg (53 Nw).]



22 A long straight conductor carrying a current of 1 500 A is placed at the centre of a coil whose boundary is two-thirds of the circumference of a circle of radius 12 cm and the closing chord. The coil has 1 500 turns and carries a current of 4 A. Find the couple tending to rotate the coil.

[1 510 g-cm (0·15 Nw-m).]

23 A flexible wire 1 m long is formed into a square, the plane of which is perpendicular to a uniform field having a flux density of 10 gauss (1 mWb/m^2). Find the work done if the wire, under the electromagnetic forces set up when a current of 10 A is passed through it, assumes a circular shape. [$0·171 \times 10^{-3}$ J.]

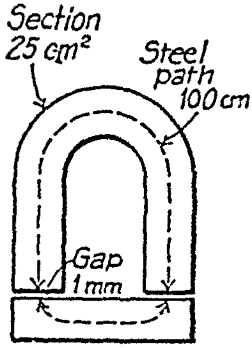
24 An alternator end-connector runs parallel to, and 5 cm distant from, the surface of the stator clamping ring, which can be considered to be an iron mass of permeability 500. Estimate the mechanical force per cm length on the conductor when carrying 10 000 A. [0·203 kg (2·0 Nw).]

25 A circular coil of T turns, radius r m, negligible inductance, and resistance R ohms, is rotated about a diameter in a uniform magnetic field of density B webers/m², at an angular velocity ω radians per sec. The axis of the rotation is perpendicular to the direction of the field. The ends of the coil are joined. Find the work done per revolution. [$B^2T^2\omega r^4\pi^3/R$ J.]

Lifting Power

26 Determine the force in kilograms necessary to separate two surfaces with 100 cm² of contact area when the flux density normal to the surfaces is 10 000 lines per cm² (1 Wb/m²). [406 kg.]

27 Calculate the force of attraction on the armature of the cast steel electromagnet shown when the flux density is 10 000 lines per cm² (1 Wb/m²). Find the ampere-turns required to produce this density.
*[447 lb (2 000 Nw) ; 2 500 ampere-turns.]



28 A horse-shoe type relay needs an excitation of 1 800 ampere-turns to raise the armature when the gap is 1·25 mm. Each pole shoe has an area of 2 cm² and the length of iron path is 40 cm. (*a*) Find the pull on the armature, assuming the iron to be unsaturated. (*b*) If the gap closes to 0·1 mm, find the force needed to pull the armature away, the excitation remaining constant. [(*a*) 29 lb (129 Nw) ; (*b*) 93 lb (410 Nw).]

29 A cast steel circular lifting magnet has an outside diameter of 50 in. The diameter of the inner circular pole is 16 in and the outer annular pole has the same area as the inner one. The mean length of magnetic path in the magnet is 60 in. The magnet lifts steel plates of negligible reluctance with an effective air gap of $\frac{1}{4}$ in at each pole. Find (*a*) the force of attraction in tons between magnet and plates, and (*b*) the flux density in the gaps, when the excitation is 12 000 ampere-turns.
*[(*a*) 11·2 tons ; (*b*) 10 400 gauss (1·04 Wb/m²).]

Induced E.M.F.

30 A wire of length 50 cm moves at right angles to its length at 40 m per sec in a uniform magnetic field of density 10 000 lines per cm² (1 Wb/m²). Calculate the electromotive force induced in the conductor when the direction of motion is (*a*) perpendicular to the field, (*b*) inclined at 30° to the direction of the field. [(*a*) 20 V ; (*b*) 10 V.]

31 A square coil of 10 cm side and with 100 turns is rotated at a uniform speed of 1 000 rev per min about an axis at right-angles to a uniform magnetic field having a flux density of 5 000 gauss (0·5 Wb/m^2). Calculate the instantaneous value of the induced electromotive force when the plane of the coil is (*a*) at right-angles to the field, (*b*) at 30° to the field, and (*c*) in the plane of the field. [(*a*) 0 ; (*b*) 45·3 V ; (*c*) 52·4 V.]

32 A circular coil of 100 turns with a mean diameter of 30 cm is rotated about a vertical axis in the earth's field at 16 rev per sec. Calculate the instantaneous value of the electromotive force in volts induced in the coil when its plane is (*a*) perpendicular to, (*b*) inclined at 30° to, (*c*) coincident with, the magnetic meridian. Take $H = 0{\cdot}18$ oersted (14·3 A(T)/m). [(*a*) 0 ; (*b*) 0·0111 V ; (*c*) 0 0128 V.]

33 A copper disc 40 cm diameter is rotated at 3 000 rev per min on a horizontal axis perpendicular to and through the centre of the disc, the axis lying in the magnetic meridian. Two brushes make contact with the disc, one at the edge and the other at the centre. If the horizontal component of the earth's field be 0·2 gauss (0·02 mWb/m^2), calculate the electromotive force induced between the brushes. [0·12 mV.]

34 A meter driving motor consists of a horizontal disc of aluminium, 20 cm in diameter, pivoted on a vertical spindle, and lying in a permanent magnetic field of density 3 000 lines per cm^2 (0·3 Wb/m^2). The current flow is radial from the spindle to the circumference of the disc. The circuit resistance is 0·225 Ω, and a potential difference of 2·30 V is required to pass a current of 10 A through the motor. Calculate (*a*) the rotational speed of the disc ; (*b*) the power lost in friction. [319 rev per min ; 0·5 W.]

35 A flat coil of 500 turns of fine wire, and of mean area 600 cm^2, revolves at a speed of 1 600 rev per min in a uniform magnetic field having a flux density of 800 gauss (80 mWb/m^2). Calculate the peak, average over a half cycle, and root-mean-square values of the electromotive force induced in the coil. [402 V. 256 V, 284 V.]

36 A coil of 100 turns is wound in two small diametrically opposite slots on a wooden cylinder : another coil of 50 turns is wound in two other diametrically opposite slots, the angle between the two coils being 30°. The cylinder is rotated at 1 000 rev per min in a uniform magnetic field having a flux density of 100 gauss (10 mWb/m^2) normal to the axis of the cylinder. The area of each coil is 400 cm^2. If the coils are connected in series, find the reading on an alternating-current voltmeter connected across the brushes. [4·3 V or 1·85 V.]

37 A wire, falling horizontally at right angles to a transverse field having a flux density of 10 000 gauss (1 Wb/m^2), has its ends joined by a flexible conductor, of which the parts lying within the field remain vertical under all conditions. If the length of the wire is 10 cm and the total weight and resistance of the circuit are 1 g and 1 Ω respectively, find (*a*) the distance traversed, (*b*) the velocity acquired, after 0·1 sec.
[(*a*) 8·6 cm; (*b*) 62 cm/sec.]

38 A metal ring of resistivity 2·62 $\mu\Omega$-cm, and density 2·7 g per cm^3, is allowed to fall from rest with the plane of the ring horizontal in such a manner that each element of the ring moves transversely through a radial field of 5 000 gauss (0·5 Wb/m^2). At what velocity is the ring falling after 0·001 sec and what is the ultimate velocity attained?
[0 269 cm per sec ; 0·277 cm per sec.]

39 The flux Φ linked by a coil of 100 turns varies during the period T of one complete cycle as follows: from time $t = 0$ to $t = \frac{1}{2}T$, $\Phi = \Phi_m\ (1 - 4t/T)$; and from $t = \frac{1}{2}T$ to $t = T$, $\Phi = \Phi_m\ (4t/T - 3)$. If T be $\frac{1}{50}$ sec, and Φ_m be 2 megalines (0·02 Wb), calculate the maximum value of the induced electromotive force. Plot to scale the flux and electromotive force waves.
[400 V.]

Self Inductance

40 Calculate the approximate resistance and inductance of a solenoid of mean diameter 1 cm, 1 m long, wound with 1 000 turns of copper wire 0·5 mm in diameter. What potential difference exists at the terminals of the solenoid at the instant when the current is 1 A, but increasing at the rate of 10 000 A per sec?
[2·76 Ω ; 0·0987 mH ; 3·75 V.]

41 Calculate the inductance of a toroid, 25 cm mean diameter and 6·25 cm^2 circular cross-section, wound uniformly with 1 000 turns of wire; also the e.m.f. induced when a current increasing at the rate of 200 A per sec flows in the winding. Develop any formula used.
[1 mH ; − 0·2 V.]

42 A circuit has 1 000 turns enclosing a magnetic circuit 20 cm^2 in section. With 4 A the flux density is 10 000 gauss (1 Wb/m^2) and with 9 A it is 14 000 gauss (1·4 Wb/m^2). Find the mean value of the inductance between these current limits and the induced electromotive force if the current fell uniformly from 9 A to 4 A in 0 05 sec.
[0·16 H ; 16 V.]

43 The mean diameter of a steel ring is 50 cm and a flux density of 10 000 gauss (1 Wb/m^2) is produced by 40 ampere-turns per cm. If the cross-section of the ring be 20 cm^2 and the number of turns 500, find (*a*) the inductance in henrys ; (*b*) the exciting current and the inductance when a gap 1 cm long is cut in the ring, the flux density being 10 000 gauss (1 Wb/m^2). Ignore leakage and fringing. *[(*a*) 0·08 H ; (*b*) 28·4 A, 0·035 H.]

44 A low-voltage release consists of a solenoid into which an iron plunger is drawn against a spring. The current taken from a 100-V, 50-c/s supply is 1·12 A with plunger out and 0·48 A with plunger drawn in. Assuming a constant resistance of 28·5 Ω, calculate the inductance and maximum flux linkages for both positions.

[0·27 H, 42·7 $\times$ 10^6 maxwell-turns (0·427 Wb-turns) ;
0·735 H, 44·7 $\times$ 10^6 maxwell-turns (0·447 Wb-turns).]

45 An iron ring with a mean diameter of 40 cm and a cross-section of 20 cm^2 has 100 turns of wire of negligible resistance. (*a*) How many volts and volt-amperes at 50 c/s must be supplied to obtain a maximum density of 10 000 lines per cm^2 (1 Wb/m^2), the excitation at this density being 4 ampere-turns per cm ? Find also the inductance. (*b*) What is the effect of introducing a 2-mm air-gap ?

[(*a*) 44·5 V ; 158 VA ; 0·04 H : (*b*) 44·5 V ; 662 VA ; 0·0095 H.]

46 A 1-phase, 50-c/s, 100-kVA transformer for 12 000/240-V ratio has a maximum flux-density of 12 000 lines per cm^2 (1·2 Wb/m^2) and an effective core section of 800 cm^2. The magnetizing current (root-mean-square value) is 0·2 A. Estimate the inductance of each winding on open-circuit.

[191 H ; 0·077 H.]

Mutual Inductance

47 Two identical 1 000-turn coils *X* and *Y* lie in parallel planes such that 60% of the lines of force produced by one coil link the other. A current of 5 A in *X* produces in it a flux of 5 000 lines (0·05 mWb). If the current in *X* changes from + 6 A to − 6 A in 0·01 sec, what will be the magnitude of the electromotive force induced in *Y* ? Calculate the self-inductance of each coil and the mutual-inductance.

[7·2 V ; each coil 10 mH ; 6 mH.]

48 Two coils of 1 250 and 140 turns respectively are wound on a common iron magnetic circuit of reluctance 0·002 (160 000 M.K.S.). Find the mutual inductance, neglecting leakage flux.

[1·1 H.]

49 A solenoid consists of 1 500 turns of wire wound on a length of 60 cm. A search coil of 500 turns enclosing a mean area of 20 cm^2 is placed centrally in the solenoid. Find (*a*) the mutual inductance of the arrangement, and (*b*) the electromotive force induced in the search coil when the current in the solenoid is changing uniformly at the rate of 250 A per sec.

[8·14 mH ; 0·786 V.]

50 A search coil having 50 turns enclosing a cross-sectional area of 2 cm^2 is placed at the middle of a solenoid having 500 turns of wire uniformly wound on a length of 12 cm and carrying an alternating current of 0 2 A at a frequency of 5 kc/s. Calculate the mutual inductance between the coils and the e m.f. induced in the search coil. [52 4 μH ; 0 329 V.]

51 A solenoid consisting of 1 000 turns of wire wound on a former of length 100 cm and diameter 3 cm is placed co-axially within another solenoid of the same length and number of turns, but of diameter 6 cm. Find approximately the mutual inductance and the coupling coefficient of the arrangement. [0·887 mH : 0·5.]

52 Two coils, X of 12 000 turns and Y of 15 000 turns, lie in parallel planes so that 45% of the flux produced by coil X links coil Y. A current of 5 A in X produces 5 000 maxwells (0 05 mWb) while the same current in Y produces 7 500 maxwells (0 075 mWb). Calculate (*a*) the mutual inductance, and (*b*) the coupling coefficient. [(*a*) 67·5 mH ; (*b*) 0 4.]

53 Two coils having 30 and 600 turns respectively are wound side-by-side on a closed iron circuit of section 100 cm^2 and mean length 150 cm. (*a*) Estimate the mutual inductance between the coils if the permeability of the iron is 2 000. (*b*) A current in the first coil grows steadily from zero to 10 A in 0 01 sec ; find the electromotive force induced in the other coil. [(*a*) 0 302 H ; (*b*) 302 V.]

54 The combined inductance of two coils connected in series is 0·6 H or 0 1 H, depending on the relative directions of the currents in the coils. If one of the coils when isolated has a self-inductance of 0 2 H, calculate (*a*) the mutual inductance, and (*b*) the coupling coefficient. [(*a*) 0·125 H ; (*b*) 0·72.]

55 A solenoid A of 500 turns, winding length 100 cm and diameter 2 cm is placed centrally within a solenoid B of 1 000 turns, winding length 100 cm and diameter 6 cm. Calculate the total inductance of the circuit when the coils are joined in series with (*a*) cumulative m.m.f., (*b*) opposing m.m.f. Calculate also the mutual inductance of the circuit. [(*a*) 4·04 ; (*b*) 3·25, 0·197 mH.]

Stored Energy

56 A solenoid 1 m in length and 10 cm in diameter has 5 000 turns. Calculate (*a*) the approximate inductance ; (*b*) the energy stored in the magnetic field when a current of 2 A flows in the solenoid. [0 246 H ; 0·492 J.]

57 A coil of inductance 0 25 H and negligible resistance is connected to a source of supply represented by $v = 4\ t$ volts. If the voltage is applied when $t = 0$ and switched off when $t = 5$

sec, find (*a*) the maximum value, (*b*) the r.m.s. value of the current, and (*c*) the energy stored during this period.
[(*a*) 200 A ; (*b*) 89·5 A ; (*c*) 5 000 J.]

58 An iron ring 15 cm in diameter and 10 cm^2 in cross-section is wound with 200 turns of wire. For a flux-density of 10 000 gauss (1 Wb/m^2) and a permeability of 500, find the exciting current, the inductance and the stored energy. Find the corresponding quantities when there is a 2-mm air-gap.
[3·75 A, 53·4 mH, 0·375 J ; 11·7 A, 17·1 mH, 1·17 J.]

59 A 250-V, 50-c/s contactor takes a current of 4 A at a power factor of 0·15 lagging when the armature is held in the open position, and a current of 1·5 A at a power factor of 0·07 lagging when the contactor is closed. Calculate the work done in closing the contactor. [0·72 ft-lb (0·98 J).]

Iron Losses

60 A laminated iron cylinder is rotated in a given magnetic field. The iron loss is 250 W at 600 rev per min and 312 W at 720 rev per min. Find approximately the loss if the laminations were twice as thick, the induction density increased by 25%, and the speed were 800 rev per min. [917 W.]

61 The hysteresis loop of a sample of sheet steel is carried to a density of 13 750 lines per cm^2 (1 375 Wb/m^2). The co-ordinates are such that 1 in = 10 ampere-turns per cm and 1 in = 10 000 lines per cm^2 (1 Wb/m^2), while the area of the loop is found to be 0·513 in^2. Find the loss in watts due to hysteresis if 1 000 cm^3 of this iron were subjected to an alternating density of 13 750 lines per cm^2 (1·375 Wb/m^2) at 25 c/s. [12·8 W.]

62 The hysteresis loss in a sample of iron was found to be 2·22 W per lb at a frequency of 50 c/s and a maximum density B of 10 000 lines per cm^2 (1 Wb/m^2). (*a*) Calculate therefrom the coefficient η in the expression :—Loss in ergs per cm^3 (or J per m^3) per cycle = $\eta B^{1.7}$: the specific gravity of the iron is 7·5. (*b*) Calculate the loss per lb and per kg at 25 c/s and 18 000 lines per cm^2 (1·8 Wb/m^2).
[(*a*) 0·00116 (735) ; (*b*) 3·02 W per lb, 6·67 W per kg.]

63 Calculate the hourly loss of energy in a specimen of iron the hysteresis loop of which is equivalent in area to 2 500 ergs per cm^3 (250 J/m^3). Frequency, 50 c/s ; density of iron 7·5 ; weight, 10 kg. [60 000 J.]

64 Calculate the loss per kg in a specimen of alloy steel for a maximum density of 10 000 gauss (1 Wb/m^2) and a frequency of 50 c/s, using 0·5 mm plates. Resistivity 25 $\mu\Omega$-cm ; density 7·8 g per cm^3, hysteresis loss 8 500 ergs per cm^3 (850 J/m^3) per cycle.
[Eddy-current loss = 0·52 W per kg ; hysteresis loss 2·24 W per kg ; total core loss 2·76 W per kg.]

CHAPTER IV

ELECTROCHEMISTRY AND BATTERIES

Electrochemistry

1 If a current of 10 A deposits 13·42 g of silver from a silver nitrate solution in 20 min, calculate the electro-chemical equivalent of silver. [0 001 118 g per C.]

2 Calculate the quantity of electricity and the steady current which will deposit 5 g of copper from a copper sulphate solution in 1 hr. [15 250 C ; 4·24 A.]

3 A leakage current of 0 2 A from a tram rail passes into an iron beam embedded in moist ground. If the electro-chemical equivalent of iron is 0 000 29 g/C, what weight of iron will be corroded away in one year ? [4 lb.]

4 A copper sulphate electrolytic cell, carrying 2 000 A, is used to produce refined copper. Estimate the annual output in tons if the cell is worked for an average of 100 hr per week. [12 tons.]

5 How much aluminium is produced from aluminium oxide (Al_2O_3) in 24 hr when the average current is 3 760 A and the current efficiency is 71% ? (Al is trivalent and its atomic weight is 27.) If the vat voltage is 7 V, how much energy is needed to produce 1 kg of aluminium ? [21·4 kg ; 29·5 kWh.]

6 Calculate (*a*) the energy used in producing chemical action, and (*b*) the number of kilo-calories developed in an electrolytic cell in 15 min, when a current of 100 A at a potential difference of 15 V is passed. The resistance of the solution is 0·05 Ω. [(*a*) 0·25 kWh ; (*b*) 107 k.cal.]

7 Calculate the time taken to build up a coating of nickel 1 mm thick on a cylinder of metal 10 cm in diameter and 10 cm long, with a current of 50 A. Data : specific gravity of nickel, 8·9 ; atomic weight, 58·7 ; divalent. Chemical equivalent of silver is 107·98, and 0·001 118 g is deposited per coulomb. How long would it take to deposit the same thickness of silver with the same current ? (Specific gravity of silver, 10·5.) [7·8 hr ; 2·5 hr.]

8 A 10-Ω resistor and a copper voltameter are connected in series and direct current is passed through them for 1 min. The current has a value of 2 A on switching-on and increases regularly at the rate of 3 A per min. Find the initial, average,

and final rates of (*a*) development of heat in the resistor, (*b*) deposition of copper.
[(*a*) 9·6, 31·2, 60 cal per sec ; (*b*) 0·656, 1·148, 1·64 mg per sec.]

9 A copper deposition cell has an electrolyte of cuprous and sodium cyanides. If, with an input of 200 W, copper is deposited at the rate of 1 lb per hr, find the energy-efficiency of the cell, given that the total electrode voltage is 0·5 V and the resistance of the electrolyte is 0·002 Ω. [84·5%.]

10 A silver and a copper voltameter are joined in parallel and a steady current is passed through them for 30 min. During this period the total quantity passed is 10 Ah and the amount of silver (electrochemical equivalent : 0·001 118) deposited is 12·5 g. Find the amount of copper deposited. Electrochemical equivalent of copper is 0·000 328. [8·15 g.]

11 How much caustic soda can be produced by the passage of 1 000 Ah of electricity through a solution of common salt in water ? Electrochemical equivalent of sodium, 0 000 238 6 g per C ; atomic weights Na, 23·05 ; O, 16 ; H, 1·008. [3·3 lb.]

12 Calculate the volume of water needed to compensate for gassing due to electrolysis in a fully charged acid battery caused by an overcharge of 5 A for 20 hr. Electrochemical equivalents in mg/C : hydrogen, 0·010 44 ; oxygen, 0 082 95. [33·6 cm^3.]

13 Given that the heat released is 34 k.cal when 1 g of hydrogen unites with oxygen to form water, calculate the minimum voltage needed to electrolyze water. Adding 15% to the voltage needed to decompose water, how much energy is needed at N.P.T. to release 1 m^3 of (i) H, (ii) O ? The electrochemical equivalent of hydrogen, $1{\cdot}044 \times 10^{-5}$ g per C.
[1·48 V ; (i) 4·1 kWh ; (ii) 8 2 kWh.]

14 A d.c. meter is tested by connecting it in series with a copper voltameter. If the current is held steady and the ammeter reads 5 A while the cell deposits 1 488 g in 15 min, calculate the correction for the instrument. [0·8% (low).]

Batteries

15 A storage battery consists of 55 cells in series. Each has an electromotive force of 2 V and an internal resistance of 0·001 Ω. The full load per cell is 0·04 A per in^2 of positive plate surface, and each cell has ten positive plates each 12 in square. Find the full-load terminal voltage and the power wasted in the battery if the connectors have a total resistance of 0 02 Ω.
[101·36 V ; 995 W.]

16 A secondary battery consists of 55 series-connected cells, each of internal resistance 0·001 Ω and electromotive force 2·05 V, and containing 10 positive and 11 negative plates each 1 ft square. If 0·04 A be taken from the cells per square inch of positive plate surface, what will be the potential difference at the distributing board, the leads to which have a total resistance of 0·017 Ω? [104·5 V.]

17 A 60-Ah cell rated on a 10-hr discharge is found to give 122% of its normal ampere-hour capacity on a 20-hr discharge, and 85% of its normal capacity on a 15-min discharge. Calculate the discharge current (*a*) at normal 10-hr rate; (*b*) at 20-hr rate; (*c*) at 15-min rate. [(*a*) 6 A; (*b*) 3·66 A; (*c*) 84 A.]

18 A discharged secondary battery is put on charge at 5 A for 3½ hr. At the end of this period it is disconnected and discharged through a resistor *R* ohms. If the duration of the discharge period is 6 hr and the terminal voltage remains constant at 12 V, determine the value of *R*. The Ah efficiency of the battery is 85%. [4·84 Ω.]

19 Calculate (*a*) the ampere-hour efficiency, and (*b*) the watt-hour efficiency of a secondary cell having a 20-hr charge rate of 10 A, and delivering 5 A for 36 hr on discharge with a mean terminal voltage of 1·96 V. The terminal voltage during charge has a mean value of 2·35 V. [(*a*) 90%; (*b*) 75%.]

20 The relation between the discharge current *I* and the time of discharge *t* of a battery is given by $I^n t = K$, where *n* and *K* are constants. If a battery gives 20 A for 8 hr and 40 A for 2·6 hr, find for how long it will give 30 A. [4·15 hr.]

21 Find the "trickle" current to be sent through an idle accumulator battery having a capacity of 50 Ah on 20-hr rating, in order to keep it fully charged. The discharge rate due to local action is 2% of the normal discharge rate. [50 mA.]

22 A shunt generator, the regulation curve of which is given by

Load current, A .	0	5	10	15	20
Terminal voltage .	221	218	214	208	200

charges an accumulator the open-circuit voltage/Ah characteristic of which is given by the following data:

Ah supplied	0	10	20	40	60	80	100
O.C. voltage	180	201	208	210	212	215	219

Charging commences when the open-circuit voltage of the accumulator is 180 V.

The internal resistance may be assumed constant at 1 Ω. Draw the charging current/time curve and estimate the time necessary to supply a charge of 50 Ah to the accumulator.

*[6 hr.]

23 A 250-V d.c. supply is available for charging 12-V automobile batteries. The voltage range per cell is 1·8 to 2·7 V from discharged to fully charged condition. If one 12-V battery has to be charged : (*a*) What resistance is needed for a charging current of 8 A ? (*b*) What will be the rating of the resistance in (*a*) ? (*c*) What percentage of the supplied energy is used in charging ? (*d*) What extra resistance will be needed to reduce the current to 1 A at the end of charge ? If fourteen 12-V batteries in series are charged : (*e*) Find resistances for initial charging currents of (i) 8 A, (ii) 100 A. (*f*) Find ratings of resistances in (i) and (ii). (*g*) Find percentage of the supplied power used in charging at beginning and at end of charge.

[(*a*) 29 9 Ω ; (*b*) 1913 6 W ; (*c*) 4·32% ; (*d*) 203·9 Ω rated at 1 A ; (*e*) (i) 12·35 Ω, (ii) 0·988 Ω ; (*f*) 9 880 W ; (*g*) at beginning of charge, 60·5% ; at end of charge, 90·7%].

CHAPTER V

RESISTANCE AND D.C. CIRCUITS

Resistance of Conductors and Resistivity

1 Find the resistance at 20° C of the following standard annealed copper wires :—(*a*) 1 mm^2 cross-section, 10 m long ; (*b*) 25 mm^2 cross-section, 200 m long ; (*c*) 0·2 in^2 cross-section, 8 miles long. [(*a*) 0·178 Ω ; (*b*) 0·138 Ω ; (*c*) 0·646 Ω.]

2 A rectangular metal strip has the dimensions $x = 10$ cm, $y = 0·5$ cm, $z = 0·2$ cm. Determine the ratio of the resistances R_x, R_y and R_z between the respective pairs of opposite faces. [2 500 : 6·25 : 1.]

3 Determine the resistance of a metal tube in terms of the external diameter D, the internal diameter d, the length l and the resistivity ρ. Calculate the resistance of a copper tube 0·5 cm thick and 2 m long. The external diameter is 10 cm.

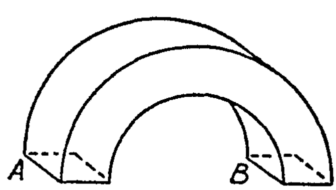
$$\left[R = \frac{4\rho}{\pi}\frac{l}{(D^2 - d^2)} = 23·1\ \mu\Omega.\right]$$

4 Calculate the loss in watts per kg of a current-carrying conductor in terms of the resistivity ρ $\mu\Omega$-cm, the current density σ A per mm^2, and the specific gravity S. [$10\ \sigma^2\ \rho/S$.]

5 Determine the length l and diameter d of a cylinder of copper in terms of the volume, x ; resistivity, ρ ; and the resistance between opposite ends, r.

$$\left[\left(\frac{r\,x}{\rho}\right)^{\frac{1}{2}} ; \left(\frac{16\,x\rho}{\pi^2 r}\right)^{\frac{1}{4}}\right]$$

6 Find the resistance of the semicircular copper conductor shown, between the equipotential faces A and B. Inner radius 5 cm, radial thickness 3 cm and axial thickness 3 cm. [3·84 $\mu\Omega$.]



7 A liquid resistor consists of two concentric metal cylinders of diameters $D = 30$ cm and $d = 15$ cm respectively with water of resistivity $\rho = 10\ 000$ Ω-cm between them. The length l of both cylinders is 50 cm. Derive an expression for the resistance of such an arrangement. If the actual resistance is only 70% of the theoretical value due to the presence of

water above and below the electrodes, determine the resistance for the values given. $\left[\frac{\rho}{2\pi l}\log_e\frac{D}{d};\ 15{\cdot}4\ \Omega.\right]$

8 A current of 15 A is conducted to earth by means of an embedded earthing electrode in the form of a hemisphere of radius 20 cm the flat side of which is in the plane of the surface of the ground. Estimate the resistance to earth and the potential between the electrode and a point on the surface of the ground distant 80 cm from the edge of the electrode. Take the soil resistivity as 4 000 Ω-cm. [31·9 Ω ; 382 V.]

9 A liquid resistance with parallel-plate electrodes has to absorb 40 kW at 400 V. The 5% soda solution used has a resistivity of 22 Ω-cm. Allowing a current density on the electrodes of 0·25 A per cm², find the distance between the electrodes. [73 cm.]

10 A cast-iron grid rheostat has to carry 230 A at 240 V for 15 sec with a temperature-rise of 50° C. Determine the cross-section of the grid in square inches and its length in feet. Take specific heat of iron = 0·115, density = 0·28 lb/in³ and resistivity = 34 μΩ-in. Ignore loss due to radiation, etc. [a = 0 094 in² ; l = 240 ft.]

11 A regulating resistance is wound with 150 ft of 0·048-in diameter wire of resistivity 12 × 10⁻⁶ Ω-cm. Calculate the length and diameter of a new resistance wire of resistivity 49 × 10⁻⁶ Ω-cm to replace the original. Assume the same conditions of emissivity and cooling. [93·6 ft ; 0 077 in.]

12 A porcelain cylinder, 5 cm in diameter, is wound with bare high-resistance wire having a resistance of 1 Ω per m length and mm² cross-section. The distance between consecutive turns equals the diameter d of the wire. If the external surface of the cylinder (excluding the ends) can dissipate 0·32 W/cm² at the permitted temperature rise, find the length of cylinder and the diameter and length of the wire for a loading of 100 W and a current of 1 A. [20 cm ; 0·58 mm ; 27 m.]

Insulation Resistance

13 The copper conductor resistance at 20° C of 500 yd of a certain cable is 0·58 Ω, and the insulation resistance is 975 MΩ. (a) Find the corresponding resistances per mile. (b) If the insulating material has its resistance halved by a temperature rise of 8° C, find the resistances per mile at a cable temperature of 38° C. [(a) 2·04 Ω, 277 MΩ ; (b) 2·14 Ω, 89·6 MΩ.]

14 The wire of a standard resistor is adjusted correctly to its marked value of 1 MΩ. What is the minimum permissible

value of the insulation resistance between the terminals to which the wire is attached so that the measured resistance shall be within 0·1% of the marked value? [999 MΩ.]

15 The insulation resistance of a 1-core cable is connected in series with a voltmeter across a 480-V supply. The voltmeter reads 5 V. When connected in series with a 50 000-Ω resistor across 240-V mains, the voltmeter reading is 90 V. Calculate the value of the insulation resistance. [2·85 MΩ.]

16 A test length of distributor cable 100 yd long has a conductor resistance of 0 13 Ω and an insulation resistance 10 000 MΩ. Find corresponding values for (*a*) 200 yd, (*b*) 1 mile, of similar cable at the same temperature.
[(*a*) 0·26 Ω; 5 000 MΩ; (*b*) 2·29 Ω; 570 MΩ.]

17 Two underground cables having conductor resistances of 0·7 Ω and 0 5 Ω and insulation resistances of 300 MΩ and 600 MΩ respectively are joined in series. Find the resultant conductor and insulation resistance. [1·2 Ω; 200 MΩ.]

18 A single-core cable of conductor diameter 1·5 cm and overall diameter 2·9 cm has an insulation resistance of 500 MΩ per mile. What is the resistivity of the insulation in MΩ-cm? The insulation resistance is to be increased to 900 MΩ per mile by an additional layer of insulation of resistivity $6{\cdot}5 \times 10^8$ MΩ-cm. What thickness of insulation is required?
[$7{\cdot}68 \times 10^8$ MΩ-cm, 1·25 cm.]

Resistance-temperature Coefficient

19 Taking the resistance-temperature coefficient to be 1/234·5 per 1° C at 0° C, prove that the resistance R_1 of a circuit of copper wire at temperature t_1 °C is related to the resistance R_2 at temperature t_2 °C by the expression

$$\frac{R_1}{R_2} = \frac{234{\cdot}5 + t_1}{234{\cdot}5 + t_2}$$

20 The resistance-temperature coefficient of phosphor bronze is $39\ 4 \times 10^{-4}$ per 1° C at 0° C. Find the coefficient for a temperature of (*a*) 20° C; (*b*) 100° C.
[(*a*) $36{\cdot}5 \times 10^{-4}$; (*b*) $28{\cdot}2 \times 10^{-4}$.]

21 It is found that the resistance of a coil of wire increases from 50Ω at 15° C to 58Ω at 55° C. Calculate the temperature coefficient at 0° C of the conductor material. [0 00425.]

22 A coil has a resistance of 18 Ω when its mean temperature is 20° C, and of 20 Ω when its mean temperature is 50° C. Find its mean temperature rise when its resistance is 21 Ω and the surrounding temperature is 15° C. [50° C.]

23 A strip of metal has a resistance of 100 Ω at 15° C. Find its mean resistance-temperature coefficient and the coefficient at 0° C if its resistance at 100° C is 126·2 Ω.

[$\alpha_{mean} = 0{\cdot}002\ 72$; $\alpha_0 = 0{\cdot}003\ 24$.]

24 A potential difference of 250 V is applied to a copper field coil at a temperature of 15° C, and the current is 5 A. What will be the mean temperature of the coil when the current has fallen to 3·91 A, the applied voltage being the same as before?

[85° C.]

25 Find the current flowing at the instant of switching a 25-W lamp on to a 200-V circuit, given that the incandescent filament temperature is 2 000° C and that the resistance-temperature coefficient at 15° C is 0·005. Neglect the term βt^2 in the temperature coefficient. [1·37 A.]

26 The B.S.I. Specification No. 225 : 1925 gives 105° C (by agreement) as the allowable temperature-rise in the rotor winding of a turbo-alternator, the measurement to be made by noting the increase of resistance. Estimate the allowable percentage increase of resistance (*a*) neglecting, (*b*) including the β-term in the formula involved when the ambient temperature is 40° C.

[(*a*) 38·4% ; (*b*) 40·1%.]

27 Show that if α_1 be the resistance-temperature coefficient of a conductor at t_1° C, the coefficient at t_2° C is given by

$$\alpha_2 = 1 \bigg/ \left\{\frac{1}{\alpha_1} + (t_2 - t_1)\right\}.$$

A specimen of copper wire of resistivity $1{\cdot}6 \times 10^{-6}$ Ω-cm at 0° C, and a temperature coefficient of 1/254·5 at 20° C. Find the resistivity and the temperature coefficient at 60° C.

[$2{\cdot}01 \times 10^{-6}$ Ω-cm ; 1/294·5.]

28 The resistance of a platinum-thermometer wire, measured at 50° C, 200° C and 500° C is 14·2032 Ω, 20·6064 Ω and 32·4900 Ω, respectively. Determine the equation connecting its resistance at t° C with its resistance at 0° C. To what temperature does a resistance of 50·4 Ω correspond?

[$R_t = 12(1 + 87 \times 10^{-4}t - 57 \times 10^{-8}t^2)\Omega$; 1 030° C.]

29 An insulating material has an insulation resistance of 100% at 0° C. For each rise in temperature of 5° C its resistance is reduced by 10%. Draw a curve showing insulation resistance to a base of temperature in °C. At what temperature is the insulation resistance halved? [33° C.]

30 Two materials, *A* and *B*, have resistance-temperature coefficients of 0·004 and 0·0004 respectively at a given temperature. In what proportion must *A* and *B* be joined in series to produce a circuit having a temperature coefficient of 0·001?

[*A* : *B* = 1 : 5.]

31 Two wires A and B are joined in series. At 0° C the resistance of B is 3·5 times the resistance of A. A has a temperature coefficient of 0 4% per °C, while that of the combination is 0·1% per °C, at 0° C. Find the temperature coefficient of B at that temperature. [0 0143%.]

32 Two coils, wound in opposition on the same core, have the same number of turns T and are connected in parallel to a 12-V battery. The resistance-temperature coefficients at 20° C of the respective coils are $\alpha_1 = 0\ 004$ and $\alpha_2 = 0\ 00228$. Find T so that the resultant ampere-turns at 20° C and at 150° C are 100 and the total loss in the two coils at 20° C is 15 W. [400.]

33 Given that the resistance of a series swamp-resistor is 60 times that of its voltmeter coil, and that the respective resistance-temperature coefficients per °C are $2{\cdot}5 \times 10^{-4}$ and 40×10^{-4}, find this coefficient for the whole circuit Find also the percentage rise in resistance in the voltmeter circuit for a temperature-rise of 40° C. [$3{\cdot}115 \times 10^{-4}$; 1·246%.]

Simple Circuits

34 Calculate the value of the resistors R in the following cases : (a) a voltmeter V of 20 000 Ω resistance connected across R reads 200 V, while the total current supplied to V and R is 0 05 A. (b) A voltage of 10 V is applied to R in series with an ammeter A of 0 1 Ω resistance, while A reads 50 A. (c) A voltmeter of 5 000 Ω resistance in series with R across 250 V mains reads 50 V. [(a) 5 000 Ω ; (b) 0·1 Ω ; (c) 20 000 Ω.]

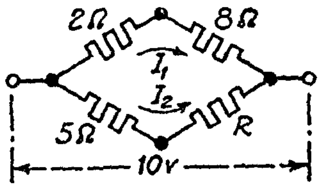
35 Two voltmeters, A and B, having resistances of 5 000 and 15 000 Ω respectively, are joined in series across 240 V. What is the reading of each voltmeter ? [A, 60 V ; B, 180 V.]

36 (a) A potential difference of 10 V is applied across a 2·5-Ω resistor. Calculate the current, the power dissipated and the energy transformed into heat in 5 min. (b) If a resistor is to dissipate energy at the rate of 250 W, find its resistance for a terminal voltage of 100 V.
[(a) 4 A, 40 W, 12 000 J ; (b) 40 Ω.]

37 A source of e.m.f. E volts and internal resistance r Ω supplies current to a heating coil. Calculate the resistance R of the coil so that (a) heating takes place most rapidly, (b) $\frac{3}{4}$ of the total energy developed by the generator is absorbed by the water.
[(a) $R = r$; (b) $R = 3\,r$.]

38 A direct-current arc has a voltage/current relation expressed by $V = 44 + 30/I$. It is connected in series with a resistor across a 100-V supply. If the voltages across the arc and the resistor are equal, find the ohmic value of the resistor. [10 Ω.]

39 For the given circuit calculate the value of the current in either branch and the value of the unknown resistance R when the total current taken by the network is 2·25 A. [$I_1 = 1$ A ; $I_2 = 1{\cdot}25$ A ; $R = 8\ \Omega$.]

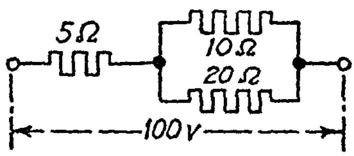


40 Two coils are connected in parallel and a voltage of 200 V is applied to the terminals. The total current taken is 25 A and the power dissipated in one of the coils is 1 500 W. What is the resistance of each coil? [26·67 Ω ; 11·43 Ω.]

41 The frame of an electric motor is connected to three earthing plates having resistances to earth of 10 Ω, 20 Ω, 30 Ω respectively. Due to a fault, the frame becomes live. What proportion of the total fault energy is dissipated at each earth connexion? [54·5% ; 27·3% ; 18·2%.]

42 A resistance R is connected in series with a parallel circuit comprising two resistances of 12 and 8 Ω respectively. The total power dissipated in the circuit is 70 W when the applied voltage is 20 V. Calculate R. [0·91 Ω.]

43 In the circuit shown, calculate the power dissipated in each resistor and the reading of a voltmeter connected across the 5-Ω resistor. [367 W in 5 Ω ; 826 W in 10 Ω ; 164 W in 20 Ω ; 42·9 V.]



44 A current of 20 A flows through two ammeters A and B joined in series. Across A the p.d. is 0·2 V and across B it is 0·3 V. Find how the same current will divide between A and B when they are joined in parallel. [A, 12 A ; B, 8 A.]

45 A resistance of 10 Ω is connected in series with two resistances each of 15 Ω arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken shall be 1·5 A with 20 V applied? [6 Ω.]

46 A voltage of 200 V is applied to a rheostat of 500 Ω. Find the resistance between two tapping points connected to a circuit needing 0·1 A at 25 V. Calculate the total power consumed. [79 Ω , 83·5 W.]

47 A 20-V battery with an internal resistance of 5 Ω is connected to a resistor of x ohms. If an additional 6-Ω resistor is connected across the battery, find the value of x so that the external power supplied by the battery remains the same. [7·5 Ω.]

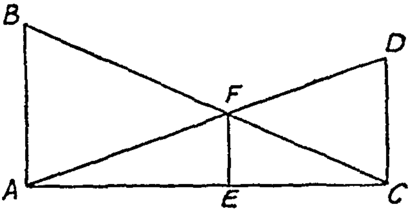
V. RESISTANCE AND D.C. CIRCUITS

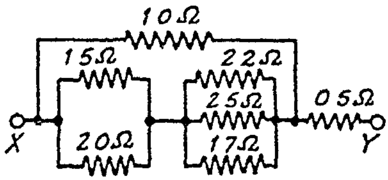
48 A constant voltage e is applied to n groups of rheostats in series, where each group has m identical rheostats in parallel. One rheostat burns out in one group. Find the percentage increase of current in each rheostat of the faulty group, and the percentage decrease of current in each rheostat of the sound groups.

$$\left[\frac{100(n-1)}{mn-n+1}\,;\;\frac{100}{mn-n+1}\right]$$

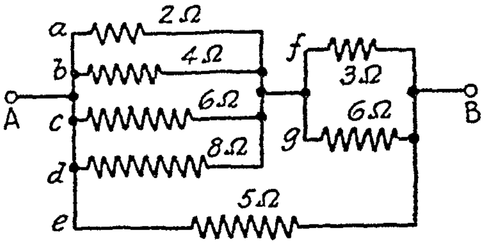
49 If AB, CD represent two resistances R_1, R_2, show that EF represents, to the same scale, the value of the resistance of R_1 and R_2 in parallel. Find graphically, and check by calculation, the resistance between points X, Y in the network shown.

[1·11 Ω.]





50 If 20 V be applied across AB, calculate the total current, the power dissipated in each resistor, and the value of a series resistance to halve the total current.



[10·75 A ; P_a = 21·1 W, P_b = 10·6 W, P_c = 7·05 W, P_d = 5·27 W, P_f = 60 7 W, P_g = 30·35 W, P_e = 80·0 W ; 1·86 Ω.]

51 A laboratory variable rheostat has five 10-Ω elements, which can be joined in series, parallel or series-parallel in any way desired. Find what combinations are possible when all five elements are used, and state the resistance in each case.

[2, 2 86, 3·75, 4·29, 5, 5·7, 6 25, 7·14, 8, 8·33, 8·57, 12, 12·5, 14, 16, 17·5, 20, 23 3, 26·7, 35, 50 Ω.]

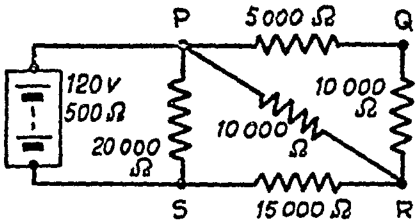
52 Three conductors A, B and C have conductances of 200 μ℧, 250 μ℧ and 300 μ℧. A and B in parallel are connected in series with C, and a direct voltage of 10 V is applied to the combination. Find the total conductance of the circuit and the current in each branch.

[180 μ℧ ; 800 μA ; 1 000 μA ; 1 800 μA.]

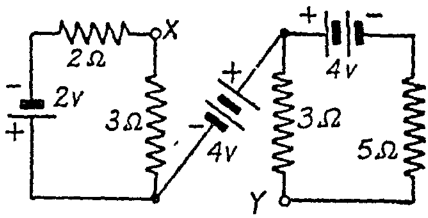
53 Two resistors, $R_1 = 2\,500\ \Omega$ and $R_2 = 4\,000\ \Omega$, are joined in series and connected to a 100-V supply. The voltage drops across R_1 and R_2 are measured successively by a voltmeter having a resistance of $50\,000\ \Omega$. Find the sum of the two readings. [97 V.]

54 A heater has two similar elements controlled by a 3-heat switch. Draw a connexion diagram for each position of the switch, and show diagrammatically the internal arrangement of the switch. What is the ratio of the heat developed for each position of the switch ? [4 : 2 : 1.]

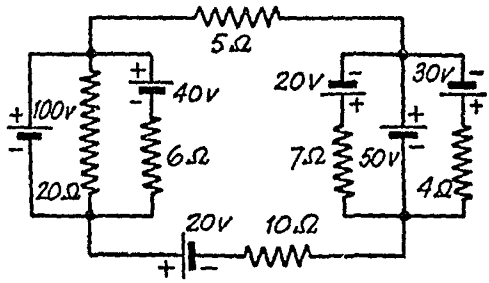
55 Determine (*a*) current given by the 120-V battery, (*b*) p.d. across *RS*, (*c*) magnitude and direction of current in *PR*. [11·17 mA ; 81·7 V ; 3·27 mA from *P* to *R*.]



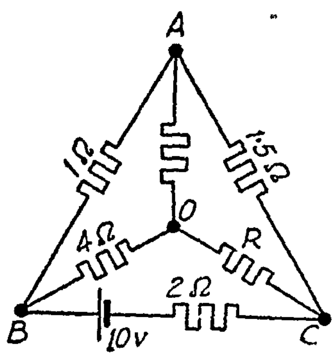
56 What is the difference of potential between the points *X* and *Y*, in the network shown ? [*Y* is 3·7 V positive with respect to *X*.]



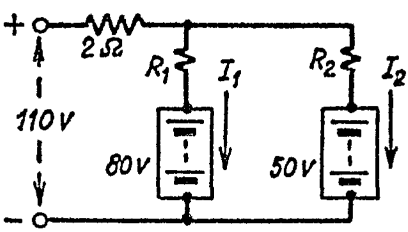
57 In the circuit shown, mark in the values and directions of all the branch currents.



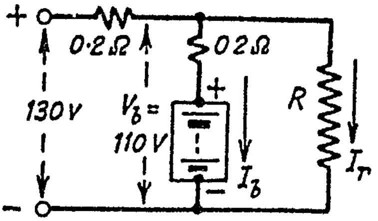
58 Find the value of *R* and the current flowing through it in the accompanying network when the current is zero in the branch *OA*. [6 Ω ; 0·5 A.]



59 What values must R_1 and R_2 have (a) when $I_1 = 4$ A and $I_2 = 6$ A, both charging; (b) when $I_1 = 2$ A discharging, and $I_2 = 20$A charging? (c) Under what condition is I_1 zero?
[(a) 2·5, 6·67 Ω; (b) 8·0, 1·2 Ω; (c) $R_2 = 2$ Ω.]



60 Find I_r and I_b when (a) $R = 2·3$ Ω; (b) $R = 0·5$ Ω; (c) For what value of R is $I_b = 0$?
[(a) $I_b = 25$ A charging, $I_r = 50$ A; (b) $I_b = 50$ A discharging, $I_r = 200$ A; (c) $R = 1·1$ Ω.]



61 A cell supplies 48·3 mA to a 25-Ω load and 25·9 mA to a 50-Ω load. Determine its e.m.f. and internal resistance, assuming both to be constant. Find also the terminal voltage of the cell when the connected load is 40 Ω.
[1·39 V; 3·9 Ω; 1 27 V.]

62 A battery comprises 10 cells each with an electromotive force of 2 V and an internal resistance of 0·02 Ω. Calculate the rate at which heat is developed in an external circuit of resistance 0 25 Ω when the cells are connected (a) all in series, (b) all in parallel.
[493 W; 15·7 W.]

63 A power of 36 W is to be expended in a resistor connected across the terminals of a battery having an electromotive-force of 20 V and an internal resistance of 1 Ω. Find (a) what values of resistance satisfy this condition, (b) the terminal potential difference of the battery for each value of resistance, and (c) the total power expenditure in each case.
[(a) 1/9th Ω and 9 Ω; (b) 2 V and 18 V; (c) 360 W and 40 W.]

64 Find the least number of cells, each of electromotive force 2 1 V and internal resistance 0·5 Ω, to produce a current of 6 A in an external resistance of 0 7 Ω. Sketch the arrangement. [12.]

65 If the maximum voltage per cell in a lead-acid battery is 2·5 V and the minimum 1·85 V, how many cells must there be in order to obtain a voltage of 220 V, and how many cells must be connected to the end-cell switch?
[119; 31.]

66 Two 12-V batteries with internal resistances 0·2 and 0·25 Ω respectively are joined in parallel and a resistor of 1 Ω is placed across the terminals. Find the current supplied by each battery.
[6·0 A; 4·8 A.]

67 The output of a lead-acid battery is to be kept constant at 2·5 kW over the range 2·2 to 1·8 V per cell during discharge. (*a*) What range of resistance is needed to maintain the load voltage at 125 V? (*b*) With a charging voltage of 200 V, a range of 1·8 to 2·5 V per cell during charge and R_{max} in (*a*) in circuit, find the starting and finishing charging currents. (*c*) For equal discharging and charging periods, find the ratio of the useful output in (*a*) to the total input in (*b*). (*d*) If the load voltage is to be kept at 125 V by varying the number of cells, how many end cells must there be? (*e*) If the charging currents in (*b*) are to be supplied by a variable-voltage generator of efficiency 0·9 and R_{min} in circuit, find the rating of the driving motor.
[(*a*) $R_{max} = 1{\cdot}45$, $R_{min} = 0{\cdot}05\ \Omega$; (*b*) 51 A, 17·2 A; (*c*) 0·37; (*d*) 13; (*e*) 10 h.p.]

68 A 250-V d.c. supply in series with a resistor of 0·5 Ω is used to charge 100 cells each of e.m.f. 2·2 V and internal resistance 0·01 Ω. Calculate the charging current; also the power dissipated in (*a*) heating the cells, (*b*) charging the cells, (*c*) heating the resistor. [20 A; (*a*) 400, (*b*) 4 400, (*c*) 200 W.]

69 A shunt generator with an armature resistance of 0·15 Ω and a field resistance of 30 Ω is used to charge a battery of 50 cells each with an internal resistance of 0·001 Ω. The e.m.f. induced in the generator is 130 V and the e.m.f. per cell is 2·2 V. Find the charging current. [97·2 A.]

CHAPTER VI

ALTERNATING QUANTITIES

Wave Values and Factors

1 A 50-c/s current has a peak amplitude of 100 A. Find the rate of change in amperes per second at time t where (a) $t = 0{\cdot}0025$, (b) $t = 0{\cdot}005$, (c) $t = 0{\cdot}01$ sec after $i = 0$ and is increasing.
[(a) 22 200 ; (b) 0 ; (c) — 31 400 A/sec.]

2 Find the root-mean-square value of the resultant current in a wire which carries simultaneously a direct current of 10 A, and a sinusoidal alternating current with a peak value of 10 A.
[12·2 A.]

3 Find the relative heating effects of two current waves of equal peak value, the one sinusoidal and the other rectangular in wave-form.
[1 : 2.]

4 Calculate the (i) form factor and (ii) peak factor of (a) a half-wave rectified sine wave, (b) a full-wave rectified sine wave.
[(a) (i) 1·57, (ii) 2·0 ; (b) (i) 1·11, (ii) 1·41.]

5 Calculate the root-mean-square value, the form and the peak factor of a periodic voltage having the following values for equal time-intervals, changing suddenly from one value to the next : 0, 5, 10, 20, 50, 60, 50, 20, 10, 5, 0, — 5, — 10 V, etc. What would be the root-mean-square value of a sine wave having the same peak value ?
[31 V ; 1·35, 1·93 ; 42·4 V.]

6 Calculate (i) the maximum value, (ii) the root-mean-square value, of the following quantities : (a) $40 \sin \omega t$, (b) $(A + B) \sin (\omega t - \pi/2)$, (c) $10 \sin \omega t - 17{\cdot}3 \cos \omega t$. Draw the vectors, showing the phase difference with respect to $A \sin (\omega t - \pi/6)$.

$$\left[\begin{array}{l}\text{(i) (a) } 40\ ;\ \text{(b) } (A + B)\ ;\ \text{(c) } 20. \\ \text{(ii) (a) } 28{\cdot}28\ ;\ \text{(b) } \dfrac{A + B}{\sqrt{2}}\ ;\ \text{(c) } 14{\cdot}14.\end{array}\right]$$

7 An alternating current varying sinusoidally with a frequency of 50 c/s has a r.m.s. value of 20 A. Write down the equation for the instantaneous value and find this value (a) 0·0025 sec, (b) 0·0125 sec, after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14·14 A ?
[(a) 20 A ; (b) — 20 A ; (c) 0·0033 sec.]

8 The voltages of a 50-c/s, 3-phase supply are : phase R, $v_R = 100 \sin \omega t$; phase Y, $v_Y = 100 \sin (\omega t - 2\pi/3)$; phase B,

$v_B = 100 \sin(\omega t - 4\pi/3)$. Find v_R, v_Y and v_B at the following instants after v_R passes through zero in a positive direction. (*a*) 1/600, (*b*) 1/500, (*c*) 1/400, (*d*) 1/300, (*e*) 1/200, (*f*) 1/100, (*g*) 1/50 sec. Show that $\Sigma v = 0$ in each case.
[(*a*) 50, – 100, 50 ; (*b*) 59, – 99, 40 ; (*c*) 71, – 97, 26 ; (*d*) 87, – 87, 0 ; (*e*) 100, – 50, – 50 ; (*f*) 0, 87, – 87 ; (*g*) 0, – 87, 87.]

9 An alternating quantity increases uniformly from 0 at 0° to F_m at α, remains constant from α to $(\pi - \alpha)$ and decreases uniformly from F_m at $(\pi - \alpha)$ to 0 at π. Calculate the r.m.s. value and average value of this wave for one half-cycle. Evaluate for $\alpha = 0$ $\pi/6$, $\pi/2$.

	$\alpha = 0$	$\pi/6$	$\pi/2$
r.m.s. :	F_m	$\sqrt{\frac{7}{9}}\,F_m$	$\sqrt{\frac{1}{3}}\,F_m$
average :	F_m	$\frac{5}{6}\,F_m$	$\frac{1}{2}\,F_m$

10 A 50-c/s voltage $v = 170\sqrt{2}\sin\omega t$ produces a current (*a*) $i = 10\sqrt{2}\sin\omega t$, (*b*) $i = 10\sqrt{2}\sin(\omega t - \pi/2)$, (*c*) $i = 10\sqrt{2}\sin(\omega t - \pi/6)$. In each case find (i) the equation of the power wave ; (ii) the maximum power ; (iii) the instantaneous power after t = 1/600, 1/500, 1/400, 1/300, 1/200 sec, reckoning $t = 0$ from the instant v crosses the abscissa axis in a positive direction ; (iv) the average power.
[(*a*) (i) $p = 1\,700(1 - \cos 200\,\pi t)$; (ii) 3 400 W ; (iii) 850, 1 175, 1 700, 2 550, 3 400 W ; (iv) 1 700 W. (*b*) $p = -1\,700 \sin 200\,\pi t$; (ii) 1 700 W ; (iii) – 1 470, – 1 615, – 1 700, – 1470, 0 W ; (iv) 0. (*c*) (i) $p = 1\,470 - 1\,470\cos 200\,\pi t - 850\sin 200\,\pi t$; (ii) 2 940 W ; (iii) 0, 208, 622, 1 470, 2 940 W ; (iv) 1 470 W.]

Graphical and Vectorial Representations

11 A current has the following constant values during the time intervals shown : 10 A, 0–0 01 sec. ; 5 A, 0·01–0·02 sec. ; zero, 0·02–0·03 sec ; – 5 A, 0·03–0·04 sec ; – 10 A, 0·04–0·05 sec. Plot the change of voltage when the circuit consists of (*a*) a 100-μF condenser, (*b*) a 100-μF condenser in series with a 50-Ω resistor. What is the maximum voltage across the circuit in each case ? [(*a*) 1 500 V ; (*b*) 1 750 V.]

12 The voltage across a condenser of 100 μF capacitance varies as follows :—(*a*) uniform increase from 0 V to 700 V in 10 sec ; (*b*) a uniform decrease from 700 V to 400 V in 2 sec ; (*c*) a steady value of 400 V ; (*d*) an instantaneous drop from 400 V to zero. Find the current flowing into or out of the condenser during each period. [(*a*) 7 mA ; (*b*) 15 mA ; (*c*) 0 ; (*d*) ∞.]

13 Find the sum of the five electromotive forces $e_1 = 20 \sin \omega t$; $e_2 = 10 \sin\left(\omega t + \frac{\pi}{6}\right)$; $e_3 = 15 \cos \omega t$; $e_4 = 10 \sin\left(\omega t - \frac{\pi}{3}\right)$; $e_5 = 25 \cos\left(\omega t + \frac{2\pi}{3}\right)$; by representing them vectorially. Express the resultant in the form $e \sin(\omega t + \psi)$.

[12·1 sin (ωt − 0 096).]

14 Two voltages $V_{AB} = 50$ V and $V_{BC} = 75$ V at the same frequency and with a phase difference between them of 60° act in series in a circuit. Find the voltage V_{AC} and its phase position relative to V_{BC}. [109 V ; 23·5°.]

15 A current of 10 A (r.m.s. value) alternating sinusoidally at 40 c/s is in series with an equal current alternating at 50 c/s. Write down an expression for the instantaneous current and find its value (*a*) 0·025 sec, (*b*) 0·05 sec, (*c*) 0·075 sec, (*d*) 0·1 sec, after the two currents have passed through their maximum positive values simultaneously. Draw the curves and show these values on the resultant.

[$i = 14{\cdot}14 (\cos 80\,\pi t + \cos 100\,\pi t)$;
(*a*) 14·14 A ; (*b*) 0 ; (*c*) 14·14 A ; (*d*) 28 28 A.]

16 Two sinusoidal sources of e.m.f. have r.m.s. values E_1 and E_2. When connected in series, with a phase displacement α, the resultant voltage read on an electrodynamic voltmeter is 41·1 V, and with one source reversed, 17·52 V. When the phase displacement is made zero a reading of 42·5 V is obtained. Calculate E_1, E_2 and α. [28 2 V ; 14·3 V ; 31°.]

17 Two 1-phase alternators supply 300 and 400 A respectively at a phase difference of 20° to a common load. Find the resultant current and its phase relation to its components.

[690 A ; 11·5° behind leading current.]

18 A voltage $e = 200 \sin 100\,\pi t$ is applied to a coil having $R = 200\ \Omega$ and $L = 638$ mH. Find the expression for the current, and the power taken by the coil.

[$i = 0{\cdot}707 \sin(100\,\pi t - \pi/4)$; 50 W.]

19 A current of 10 sin 314t A is passed through a series circuit consisting of a choke, resistance and condenser of 0·065 H, 10 Ω and 300 μF respectively. Plot in their proper phase relationships curves of (*a*) current, (*b*) induced voltage across the choke, (*c*) terminal voltage, (*d*) power in the circuit. State the values of the above quantities when $t = 1/100$ sec.

[(*a*) 0 ; (*b*) 204 V ; (*c*) 99 V ; (*d*) 0.]

20 A supply at 250 V r.m.s. and frequency 50 c/s is applied across terminals *AC* of a series circuit *ABC*. *AB* is an inductor of

0·05 H, and BC a 10-Ω resistor. At a given instant the potential of A is 20 V negative to C and is increasing positively. Find the potential of B relative to C at the same instant.

[154 V, B positive to C.]

21 In a series circuit $ABCD$ with 100 V r.m.s. at 50 c/s applied across AD, the components are: AB, an inductor of 0·07 H; BC, a capacitor of 60 μF; CD, a resistor of 20 Ω. Find the instantaneous value and polarity of the voltage across AB when C is 80 V negative to D and this p.d. is increasing.

[2·51 V, A positive to B.]

22 Four wires a, b, c, d are connected at a common point. The currents in lines a, b, and c are $i_a = 6 \sin$

$$\left(\omega t + \frac{\pi}{3}\right);\ i_b = 5 \cos\left(\omega t + \frac{\pi}{3}\right);\ i_c = 3 \cos\left(\omega t + \frac{2\pi}{3}\right);$$

the positive direction being towards the common point. Find the current in wire d. [7·34 sin $(\omega t - 1)$.]

23 The three phases of a generator are connected in star, one phase being inadvertently reversed. Determine the line voltages in terms of the phase voltage V_{ph}, and the phase angles. What would be the effect of the reversal of one phase if the phases were connected in mesh?

[Star: V_{ph}—150°—$\sqrt{3}V_{ph}$—150°—V_{ph}—60°.
Resultant voltage in mesh: $2V_{ph}$.]

24 Two sources of electromotive force, represented respectively by

200 sin ωt and 200 sin $\left(\omega t + \frac{\pi}{6}\right)$ volts, are in series. Express

the resultant in vector notation with reference to 200 sin ωt and calculate the r.m.s. current and the power supplied to a circuit of impedance $8 + j\,6\ \Omega$.

[$273\underline{/15°}$ V; 27·3 A, 5·96 kW.]

Effect on Instrument Readings

25 A moving-coil ammeter, a hot-wire ammeter, and a 100-Ω resistor are in series with a rectifier across a 200-V sine-wave voltage. The resistance of the rectifier is 100 Ω in one direction and 500 Ω in the other. Calculate the readings on the two ammeters, total power and power dissipated in rectifier.

[0·3 A, 0·746 A; 133·3 W, 77·7 W.]

26 A 50-c/s voltage of peak amplitude 100 V is applied to a 50-V battery in series with a half-wave rectifier which offers a resistance of 10 Ω to a charging current and an infinite resistance to a discharging current. Determine the energy dissipated in the rectifier in 1 hr.

[48 Wh.]

VI. ALTERNATING QUANTITIES

27 A symmetrical triangular voltage wave with a peak amplitude of 100 V and a frequency of 50 kc/s is applied across a 0·02 μF capacitor. Calculate the r.m.s. value of the current flowing in the capacitor. [0·4 A.]

28 A symmetrical triangular current wave of peak amplitude 10 A is passed through an inductor the reactance of which at 50 c/s is 40 Ω. Draw the wave-shape of the voltage across the terminals of the inductor, and calculate the peak and r.m.s. values. [254 A ; 254 A.]

29 A voltage derived from an impedanceless source, rises linearly from zero to 100 V in 1 ms, falls to zero in 100 μs, and repeats. Sketch the current that will flow in a 0·1-μF capacitor : state the r.m.s. and peak values. [31·6, 100 mA.]

30 Alternating currents of peak value 100 A have the following wave-forms :—(*a*) sinusoidal ; (*b*) full-wave rectified sinusoidal ; (*c*) rectangular ; (*d*) triangular. If these currents are passed successively through (i) a moving-coil ammeter and (ii) a hot-wire ammeter, connected in series, find the instrument readings in each case.

[(*a*) 0, 70·7 ; (*b*) 63·6, 70·7 ; (*c*) 0, 100 ; (*d*) 0, 57·8 A.]

CHAPTER VII

A.C. CIRCUITS

Single-phase Series Circuits

1 In a particular circuit a voltage of 10 V at 25 c/s produces 100 mA, while the same voltage at 75 c/s produces 60 mA. Draw the circuit diagram and insert values of the constants. At what frequency will the value of the impedance be twice that at 50 c/s ?

[$R = 88{\cdot}1\ \Omega$ and $L = 0{\cdot}3$ H in series ; 95 c/s.]

2 An inductance of 1 H is in series with a capacitance of 1 μF. Find the impedance of the circuit when the frequency is (*a*) 50 c/s, (*b*) 1 000 c/s.

[(*a*) 2 876 Ω capacitive ; (*b*) 6 121 Ω inductive.]

3 A coil of power factor 0·6 is in series with a 100-μF capacitor. When connected to a 50-c/s supply, the p.d. across the coil is equal to the p.d. across the capacitor. Find the resistance and inductance of the coil. [19·1 Ω ; 0·081 H.]

4 A resistor R in series with a capacitor C is connected to a 50-c/s, 240-V supply. Find the value of C so that R absorbs 300 W at 100 V. Find also the maximum charge and the maximum stored energy in C. [44 μF ; 0·0135 C ; 2·1 J.]

5 When 1 A is passed through three air-cored coils, A, B and C, in series, the voltage drops are respectively 6, 3, and 8 V on direct current, and 7, 5 and 10 V on alternating current. Find (*a*) the power-factor, (*b*) the power dissipated in each coil, and (*c*) the power-factor of the whole circuit when the alternating current flows. [(*a*) 0·86, 0·6, 0·8 ; (*b*) 6, 3, 8 W ; (*c*) 0·78.]

6 A current of 10 A flows in a circuit with a 30° angle of lag when the applied voltage is 100 V. Find (*a*) the resistance, reactance and impedance ; (*b*) the conductance, susceptance and admittance.

[(*a*) 8·66 Ω, 5 Ω, 10 Ω ; (*b*) 0·0866 ℧, 0·05 ℧, 0·1 ℧.]

7 A circuit has a fixed resistance of 2 Ω and a reactance of 10 Ω in series with a resistor R across 100-V constant-frequency mains. For what value of R is the power consumed in it a maximum ? [10·2 Ω.]

8 An alternating voltage $80 + j\,60$ V is applied to a circuit and the current flowing is $-4 + j\,10$ A. Find (*a*) the impedance

of the circuit, (*b*) the power consumed, and (*c*) the phase angle. [(*a*) 9·28 Ω; (*b*) 280 W; (*c*) 75° leading.]

9 Calculate (*a*) the admittance Y, (*b*) the conductance G, and (*c*) the susceptance B of a circuit consisting of a resistor of 10 Ω in series with an inductor of 0·1 H, when the frequency is 50 c/s.
[(*a*) $Y = 0{\cdot}0303$ ℧; (*b*) $G = 0{\cdot}0092$ ℧; (*c*) $B = -0{\cdot}029$ ℧.]

10 A resistor of 100 Ω is connected in series with a condenser of capacitance 50 μF to a supply at 200 V, 50 c/s. Find (*a*) the impedance, (*b*) the current, (*c*) the power factor, (*d*) the phase angle, (*e*) the voltage across the resistor and across the condenser.
[(*a*) 118·6 Ω; (*b*) 1·69 A; (*c*) 0 845; (*d*) 32·3°; (*e*) 169 V, 108 V.]

11 Find the impedance, current, power and power-factor of the following series circuits, and draw the corresponding vector diagrams. (*a*) R only, (*b*) L only, (*c*) C only, (*d*) R and L, (*e*) R and C, (*f*) R, L and C, (*g*) L and C. In each case the applied voltage is 200 V, the frequency is 50 c/s, $R = 10$ Ω, $L = 50$ mH, $C = 100$ μF.
[(*a*) 10 Ω, 20 A, 4 kW, 1·0; (*b*) 15 7 Ω, 12·72 A, 0, 0 lag; (*c*) 31·8 Ω, 6·29 A, 0, 0 lead; (*d*) 18·6 Ω, 10·75 A, 1·155 kW, 0 537 lag; (*e*) 33·3 Ω, 6 A, 400 W, 0·31 lead; (*f*) 18·9 Ω, 10·58 A, 2·01 kW, 0·58 lead; (*g*) 16·1 Ω, 12·4 A, 0, 0 lead.]

12 A voltage of 125 V at 60 c/s is applied across a non-inductive resistor connected in series with a condenser. The current is 2·2 A. The power loss in the resistor is 96·8 W, and that in the condenser is negligible. Calculate the resistance and the capacitance. [20 Ω; 50 μF.]

13 A low-voltage release consists of a solenoid of resistance 50 Ω into which an iron plunger is drawn against a spring. When supplied at 250 V, 50 c/s, a current of 2·5 A flows, which falls to 1 A when the plunger is drawn into the solenoid. Calculate, for both positions of the plunger, (*a*) the inductance of the solenoid, (*b*) the stored energy, and (*c*) the maximum flux-linkages.
[(*a*) 0 276 H, 0 78 H; (*b*) 1·72 J, 0·78 J; (*c*) 97·6 (0·976), 110·5 megamaxwell-turns (1·1 Wb-turns).]

14 An iron-cored coil takes 4 A at a power factor of 0·5 when connected to a 200-V, 50-c/s supply. When the iron core is removed and the voltage is reduced to 40 V the current rises to 5 A at a power factor of 0·8. Find the iron loss in the core, and the inductance in each case. [297 W; 0·138 H; 0 0158 H.]

15 A non-inductive load takes 10 A at 100 V. Calculate the inductance of a reactor to be connected in series in order that the same current be supplied from 220-V, 50-c/s mains. What

is the phase angle between the 220-V supply and the current? Neglect the resistance of the reactor. [0·062 H ; 68°.]

16 An inductive coil takes 10 A and dissipates 1 000 W when connected to a supply at 250 V, 25 c/s. Calculate (*a*) the impedance, (*b*) the effective resistance, (*c*) the reactance, (*d*) the inductance, (*e*) the power-factor, (*f*) the angle of lag.
[(*a*) 25 Ω ; (*b*) 10 Ω ; (*c*) 22·9 Ω ; (*d*) 0·146 H ; (*e*) 0·4 ; (*f*) 66·4°.]

17 An iron-cored choking coil of resistance 5 Ω takes 10 A when connected to 200-V, 50-c/s mains, and the power dissipated is 750 W. Assuming the coil to be equivalent to a series impedance, calculate (*a*) the iron-loss, (*b*) the inductance at the given value of the current, and (*c*) the power-factor.
[(*a*) 250 W ; (*b*) 0·059 H ; (*c*) 0·875.]

18 When a resistor and an inductor in series are connected to a 240-V supply, a current of 3 A flows lagging 37° behind the supply voltage, while the voltage across the inductor is 171 V. Find the resistance of the resistor, and the resistance and reactance of the inductor. [33·26 ; 30·74 ; 48 Ω.]

19 A current of 5 A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V, 50 c/s. If the voltage across the resistance is 125 V and across the coil 200 V, calculate (*a*) the impedance, reactance, and resistance, of the coil, (*b*) the power absorbed by the coil, and (*c*) the total power. Draw the vector diagram.
[(*a*) 40, 39 6, 5·5 Ω ; (*b*) 137·5 W ; (*c*) 762 5 W.]

20 When a voltage of 100 V at 50 c/s is applied to a choking coil *A*, the current taken is 8 A and the power 120 W. When applied to a coil *B* the current is 10 A and the power 500 W. What current and power will be taken when 100 V is applied to the two coils connected in series ? [4·52 A ; 140 W.]

21 A 200-V, 50-c/s supply is connected to a 20-Ω resistor in series with a choking coil. The reading of a voltmeter across the resistor is 120 V and across the coil is 144 V. Calculate the power and reactive volt-amperes in the coil and the power factor of the circuit. [121 W ; 855 VAr ; 0·7.]

22 An alternating current of frequency 100 c/s is passed through a non-inductive 10-Ω resistor in series with a choking coil of resistance 1·3 Ω and inductance 0·018 H. When the terminal voltage is at its maximum value, 100 V, what will be the voltage across the resistor ? [44·2 V.]

23 Find an expression for the current, and calculate the power, when a voltage represented by $v = 283 \sin 100\,\pi t$ is applied to a coil having $R = 50\ \Omega$ and $L = 0{\cdot}159$ H.
[$4 \sin (100\,\pi t - \pi/4)$, 400 W.]

VII. A.C. CIRCUITS

24 A coil having resistance R ohms and inductance L henries is connected across a variable-frequency alternating-current supply of 110 V. An ammeter in the circuit showed 15·6 A when the frequency was 80 c/s and 19·7 A when the frequency was 40 c/s. Find the values of R and L and calculate the time-constant of the coil. [4·95 Ω.; 0·01 H ; 0·00202 sec.]

25 Two coils have self-inductances of 0·1 and 0·2 H and a coupling coefficient of 0·35. What current will flow when the coils are joined in series across a 50-c/s, 125-V circuit ?

[1·0 A or 2·0 A.]

26 A reactor having negligible resistance and an inductance of 0 07 H is connected in series with a resistor of 20 Ω resistance across a 200-V, 50-c/s supply. Find (*a*) the current flowing in the circuit, (*b*) the cosine of the phase angle, (*c*) the voltage across (i) the reactor and (ii) the resistor, (*d*) the maximum value of the energy stored in the coil.

[(*a*) 6 73 A ; (*b*) 0·673 ; (*c*) (i) 148 V, (ii) 134·6 V ; (*d*) 3·17 J.]

27 A coil of resistance 10 Ω and inductance 0·1 H is connected in series with a condenser of capacitance 150 μF across a 200-V, 50-c/s supply. Calculate (*a*) the inductive reactance, (*b*) the capacitive reactance, (*c*) the impedance, (*d*) the current, (*e*) the power factor, (*f*) the voltage across the coil and the condenser respectively.

[(*a*) 31·4 Ω ; (*b*) 21·2 Ω ; (*c*) 14·3 Ω ; (*d*) 14 A ; (*e*) 0·7 lag ; (*f*) 460 V, 297 V.]

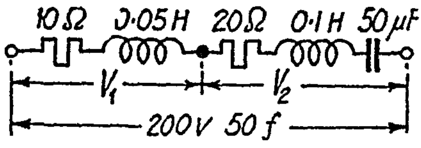
28 A voltage of 200 V is applied to a series circuit consisting of a resistor, an inductor and a capacitor. The respective voltages across these components are 170, 150 and 100 V and the current is 4 A. Find the power factor of the inductor and of the circuit.

[0·16 ; 0·97.]

29 A 230-V, 50-c/s voltage is applied to a coil of $L = 5$ H and $R = 2\ \Omega$ in series with a capacitance C. What value must C have in order that the p.d. across the coil shall be 250 V ?

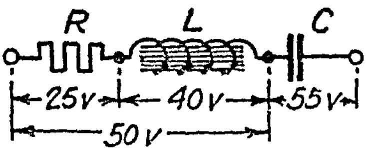
[26 μF.]

30 Draw a vector diagram for the circuit shown indicating the resistance and reactance drops, the terminal voltages V_1 and V_2, and the current. Find the value of (*a*) the current ; (*b*) V_1 and V_2 ; (*c*) the power factor.



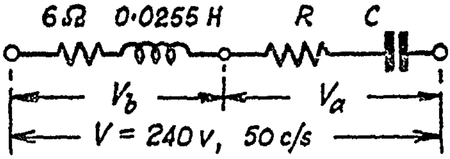
[(*a*) 5·84 A ; (*b*) 108·2 V, 221·5 V ; (*c*) 0·875 leading.]

31 In the arrangement shown, the condenser C has a capacitance of 20 μF, and the current flowing through the circuit is 0·345 A. If the voltages are as indicated, find the applied voltage, the frequency, and the loss in the iron-cored choking coil L.



[34·2 V ; 50 c/s ; 1·9 W.]

32 Find the values of R and C so that $V_b = 8\ V_a$ and V_b and V_a are in quadrature. Find also the phase relation between V and V_b, V_a and I.



[$R = 2{\cdot}66\ \Omega$; $C = 1\ 590\ \mu$F.]

33 A leaky condenser $Z_c = 74{\cdot}5\ \Omega$ is in series with a coil $Z_L = 40\ \Omega$ and a resistor $R = 56\ \Omega$. When a voltage $V = 200$ V is applied, $I = 2{\cdot}5$ A and the p.d. across R and Z_L combined is 194 V. Find the loss in the condenser. [24·3 W.]

34 Determine the magnitude of the pulsating energy in a series circuit of 10 Ω, 0 05 H, and 100 μF, when the terminals are connected to 200-V, 50-c/s mains. [5·76 J.]

35 A coil with $L = 2$ H and $R = 862\ \Omega$ is connected to a 230 V, 50 c/s supply. Calculate the energy stored when the instantaneous voltage is (*a*) zero, (*b*) a maximum.

[(*a*) 0·152 ; (*b*) 0·051 J.]

Single-phase Parallel Circuits

36 A circuit comprises a conductance G in parallel with a susceptance B. Calculate the admittance $G + j\ B$ if the impedance is $10 + j\ 5\ \Omega$. [0 08 $- j$ 0 04 ℧.]

37 Two circuits having the same numerical ohmic impedance are joined in parallel. The power factor of one circuit is 0·8 and the other 0·6. What is the power factor of the combination?

[0 707.]

38 When a 240-V, 50-c/s supply is applied to a resistor of 15 Ω in parallel with an inductor, the total current is 22·1 A. What value must the frequency have for the total current to be 34·0 A? [25·5 c/s.]

39 A coil of resistance 15 Ω and inductance 0·05 H is connected in parallel with a non-inductive resistor of 20 Ω. Find (*a*) the current in each branch circuit ; (*b*) the total current supplied ; and (*c*) the phase angle of the combination, when a voltage of 200 V at 50 c/s is applied.

[(*a*) 9·2 A, 10 A ; (*b*) 17·6 A ; (*c*) 22°.]

VII. A.C. CIRCUITS

40 Find the impedance, the current in each branch, the total current and power-factor of the following circuits :—

(*a*) resistance R in parallel with inductance L,
(*b*) resistance R in parallel with capacitance C,
(*c*) inductance L in parallel with capacitance C,
(*d*) R and L in series, in parallel with C.

In each case the applied voltage is 200 V at 50 c/s, $R = 10\ \Omega$, $L = 70$ mH and $C = 127{\cdot}2\ \mu$F. Draw in each case the circuit diagram and the vector diagram of the voltages and currents.

[(*a*) 9 1 Ω, 20 A, 9·1 A, 22 A, 0·91 lag,
(*b*) 9 8 Ω, 20 A, 8 A, 21·51 A, 0 93 lead ;
(*c*) 182 Ω, 9 1 A, 8 A, 1·1 A, 0 lag ;
(*d*) 57 9 Ω, 8·28 A, 8 A, 3·45 A, 0·99 lead.]

41 Two impedances, Z_1 and Z_2 are connected in parallel. The first branch takes a leading current of 16 A and has a resistance of 5 Ω, while the second branch takes a lagging current at power factor 0 8. The total power supplied is 5 kW, the applied voltage being $100 + j\ 200$ V. Determine the complex expressions for the branch and total currents, and for the circuit constants.

[$I_1 = -10\ 8 + j\ 11{\cdot}8$; $I_2 = 18\ 7 + j\ 9\ 3$; $I = 7{\cdot}9 + j\ 21{\cdot}1$ A.
$Z_1 = 5{\cdot}0 - j\ 13{\cdot}1$; $Z_2 = 8{\cdot}57 + j\ 6{\cdot}42$; $Z = 9\ 85 - j\ 1{\cdot}05\ \Omega$.]

42 Two circuits, the impedances of which are given by $Z_1 = 10 + j\ 15\ \Omega$ and $Z_2 = 6 - j\ 8\ \Omega$, are connected in parallel. If the total current supplied is 15 A, what is the power taken by each branch ? [737 W, 1 430 W.]

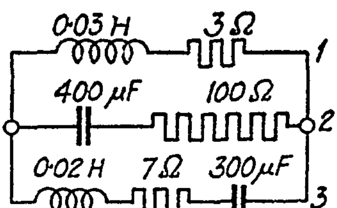
43 A circuit, with two branches $Y_1 = 0{\cdot}16 + j\ 0\ 12$ ℧ and $Y_2 = -j\ 0{\cdot}15$ ℧ in parallel, is connected to a 100-V supply. Find the total loss and the phase relationship between the branch currents and the supply current.

[1 600 W ; I_1 leads by 47·5°, I_2 lags by 79·5°.]

44 A small 1-phase, 240-V induction motor is tested in parallel with a 160-Ω resistor. The motor takes 2·0 A and the total current is 3 0 A. Find the power and power factor of (*a*) the whole circuit, (*b*) the motor. [(*a*) 580 W, 0 807 ; (*b*) 220 W, 0·46.]

45 A sinusoidal, 50-c/s voltage of 200 V supplies the three parallel circuits shown. Find the current in each circuit and the total current. Draw the vector diagram.

[(1) 20·2 A ; (2) 2 A ; (3) 24 8 A ; Total 29·4 A.]



46 A voltage of 240 V is applied to a pure resistor, a pure capacitor and an inductor, all in parallel. The resultant current is 2·3 A, while the component currents are 1·5, 2·0 and 1·1 A respectively. Find the resultant power factor and the power factor of the inductor. [0·88 ; 0·5.]

47 A resistor, an inductor and a condenser of 100 Ω, 0·1 H and 0·1 μF respectively, are connected in parallel and a voltage of 1 V is applied to the combination in turn at frequencies of (*a*) 50 c/s, (*b*) 5 000 c/s, (*c*) 500 000 c/s. Find the current in each branch and the main current and power factor for each condition.

(*a*)	(*b*)	(*c*)
10 0 mA	10·0 mA	10·0 mA
31·8 mA	0·318 mA	0·00318 mA
0·0314 mA	3·14 mA	314·0 mA
33·3 mA	10·4 mA	314·0 mA
0·8 lag	0·96 lead	0·032 lead

48 Two parallel circuits comprise respectively (i) a coil of resistance 20 Ω and inductance 0·07 H, and (ii) a condenser of capacitance 60 μF in series with a resistance of 50 Ω. Calculate the current in the mains and the power factor of the arrangement when connected across a 200-V, 50-c/s supply. A
[7·05 A ; 0·907 lag.]

49 Find what inductance must be placed in series with a lamp requiring 3·05 A, 410 W at unity power factor, when the supply is 230 V, 50 c/s. Find also the value of capacitance which must be placed across the supply terminals to raise supply power factor to unity. [0·19 H ; 34·1 μF.]

50 A coil having a resistance of 4 Ω and an inductance of 1 H is connected in parallel with a circuit comprising a similar coil in series with a condenser *C* and a non-inductive resistance *R*. Calculate the values of *C* and *R* so that the currents in either branch of the arrangement are equal but differ in phase by a quarter-period. Frequency 50 c/s.
[$C = 10{\cdot}3\ \mu$F ; $R = 310\ \Omega$.]

51 A reactor has a resistance of 5 Ω and an inductance of 0·04 H. Find a suitable shunt circuit such that the current taken by the combination will be 20 A at 100 V, at all frequencies.
[5 Ω ; 1 600 μF.]

52 Two coils *A* and *B* are connected in parallel and a voltage of 200 V at 50 c/s is applied to their common terminals. The A coils have resistances of 10 Ω and 5 Ω, and inductances of

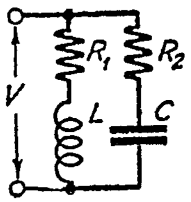
0·023 H and 0·035 H, respectively. Find (*a*) the current in each coil, (*b*) the total current, and (*c*) the power-factor of the combination. If a resistor of 20 Ω in series with a condenser of 100 μF capacitance be connected in parallel with coils *A* and *B*, find (*d*) the total current. The coupling between the coils is negligible.

[(*a*) 16·2 A and 16·5 A ; (*b*) 31·6 A ; (*c*) 0·63 ; (*d*) 30·3 A.]

53 An inductor *L* and a resistor *R* are connected in series. A capacitor *C* is shunted across *L* and *R*. At what frequency will the total current in the circuit be independent of the value of *R*, and what is the value of the current when the applied voltage is *V* ?

$$\left[f = \frac{1}{2\pi}\sqrt{\frac{1}{2LC}}\,;\ I = \omega CV.\right]$$

54 Find the condition that the currents in the two branches of the alternating-current circuit shown shall remain in quadrature when R_1 and R_2 are varied simultaneously. Determine (*a*) the frequency at which the total current remains constant in magnitude under this condition ; (*b*) the magnitude of this current.



$$\left[R_1R_2 = \frac{L}{C}\,;\ (a)\ \frac{1}{2\pi\sqrt{LC}}\,;\ (b)\ V\sqrt{\frac{C}{L}}.\right]$$

55 Two circuits of impedance $Z_A = 8 - j\,7\ \Omega$ and $Z_B = 5 + j\,6\ \Omega$ respectively are connected in parallel across a 100-V, 50-c/s supply. Calculate the current through each circuit and the total current supplied. What is the angle of phase difference of the branch currents I_A and I_B with respect to the applied voltage ? [9·41 A, 12·8 A, 15·7 A ; 41·2°, – 50·1°.]

56 A voltage 200 $/25°$ V is applied to a circuit composed of two parallel branches. If the branch currents are 10$/40°$ A and 20$/-30°$ A respectively, determine the kilovolt-amperes, kilovars, and kilowatts in each branch and in the whole circuit. What is the power factor of the combined load ?

[2·0 kVA, 0·518 kVAr, 1·93 kW ; 4·0 kVA, 3·28 kVAr, 2·29 kW ; 5·04 kVA, 2·76 kVAr, 4·22 kW ; 0·84 lagging.]

57 How is a current of 10 A shared by three circuits in parallel, the impedances of which are $2 - j\,5\ \Omega$, $6 + j\,3\ \Omega$ and $8 + j\,4\ \Omega$?

[5·68 A ; 4·57 A ; 6·12 A.]

58 Two impedances $14 + j\,5\ \Omega$ and $18 + j\,10\ \Omega$ are connected in parallel across a 200-V, 50 c/s supply. Determine (*a*) the admittance of each branch and of the entire circuit; (*b*) the total current, power, and power factor; and (*c*) the capacitance which when connected in parallel with the original circuit will make the resultant power factor unity.
[(*a*) $0{\cdot}0634 - j\,0{\cdot}0226$, $0{\cdot}0424 - j\,0{\cdot}0236$, $0{\cdot}1058 - j\,0\,0462$ ℧; (*b*) 23·1 A, 4·232 kW, 0·915; (*c*) 147 μF.]

59 A circuit A of resistance 8 Ω and inductive reactance 6 Ω is in parallel with a circuit B of resistance 3 Ω and inductive reactance 4 Ω. Find (*a*) the conductance, susceptance and admittance of the combined circuits; (*b*) symbolic expressions for the current in each circuit and for the total current; (*c*) the angle of lag of each circuit and the combined circuit.
[(*a*) 0·20, 0·22, 0·297 ℧; (*b*) $I_A = V\,(0{\cdot}08 - j\,0{\cdot}06)$, $I_B = V\,(0{\cdot}12 - j\,0{\cdot}16)$, $I = V\,(0{\cdot}20 - j\,0{\cdot}22)$; (*c*) arc cos 0·8, arc cos 0·6, arc cos 0·672.]

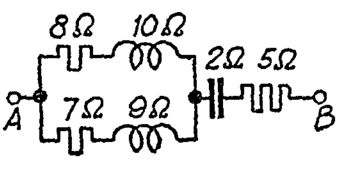
60 Two coils of resistances 10 Ω and 2 Ω, and inductances 0·02 H and 0·05 H respectively, are connected in parallel. Calculate (*a*) the conductance, susceptance, and admittance of each coil; (*b*) the total current taken by the circuit when connected across a 200-V, 50-c/s supply; (*c*) the characteristics of a single coil which will take the same current and power as taken by the original circuit.
[(*a*) 0·0717, 0·045, 0·0847 ℧; 0·008, 0·0628, 0·0634 ℧; (*b*) 26·8 A; (*c*) 4·43 Ω; 0·0191 H.]

61 Two impedances $Z_1 = R_1 + j\,X_{L1}$ and $Z_2 = R_2 + R + j\,(X_{L2} - X_{C2})$ are in parallel, where $R_1 = R_2 = 10\ \Omega$ and $X_{L1} = X_{L2} = 314 \times 0{\cdot}1\ \Omega$. Calculate the resistance of R and the capacitance C in branch 2 such that $I_1 = I_2$ and the phase angle between them is 90°. (Phase splitting circuit of phase-shifting transformer of a.c. potentiometer.) Find ϕ_1 and ϕ_2.
[21·4 Ω; 76 9 μF; $\phi_1 = 72°\ 20'$; $\phi_2 = -17°\ 40'$.]

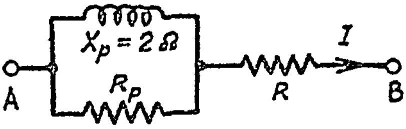
62 Two coils of resistances 20 Ω and 30 Ω, and self-inductances 0·1 and 0·05 H, respectively, are connected in parallel across a 200-V, 50-c/s supply. If the mutual inductance is 0·02 H determine (*a*) the current in each coil and (*b*) the total current.
[(*a*) 4·95 A, 5·08 A; (*b*) 9·6 A; or (*a*) 5·9 A, 6·75 A; (*b*) 12·3 A.]

Single-phase: Series-Parallel Circuits

63 In the arrangement shown, calculate the impedance AB and the phase angle between voltage and current.
[9·15 Ω; 17·2° lag.]



64 A voltage of 100 V is applied across AB to produce $I = 40$A. Find the value of R when (*a*) $R_p = 5\ \Omega$; (*b*) $R_p = 10\ \Omega$; also the power factor of the circuit in each case.



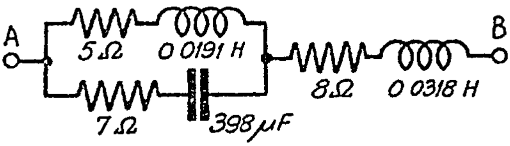
[(*a*) 1·12 Ω, 0·725 ; (*b*) 1·22 Ω, 0·642.]

65 A condenser of capacitance 50 μF, shunted by a non-inductive resistor of 100 Ω, is connected in series with a resistor of 50 Ω to a 200-V, 50-c/s supply. Calculate (*a*) the current in the condenser; (*b*) the current in the shunted resistance; (*c*) the total current.

[(*a*) 1·85 A ; (*b*) 1·18 A ; (*c*) 2·20 A.]

66 A 230-V, 1 000-c/s voltage is applied to a resistor in series with $C = 0{\cdot}05\ \mu$F. When C is shunted by a voltmeter of capacitance 0·06 μF, the reading is 100 V. Find the current when the voltmeter is disconnected. [0·0527 A.]

67 In the circuit shown determine what 50-c/s voltage must be applied across AB in order that a current of 10 A may flow in the condenser.

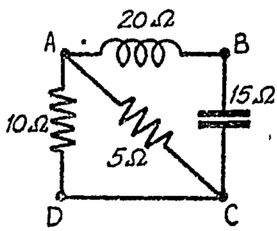


[288 V.]

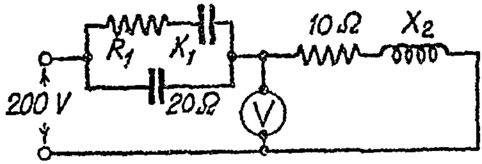
68 A 100-Ω resistor, shunted by a 0·4-H inductor, is in series with a condenser C. A voltage of 250 V at 50 c/s is applied to the circuit. Find (*a*) the value of C to give unity power-factor, (*b*) the total current, and (*c*) the current in the inductive branch. Draw the vector diagram. [64·9 μF, 4·07 A, 2·54 A.]

69 In a series-parallel circuit, the two parallel branches A and B are in series with C. The impedances are $Z_A = 10 + j\,8$, $Z_B = 9 - j\,6$, and $Z_C = 3 + j\,2$, and the voltage across C is $100 + j\,0$ V. Find the currents I_A and I_B and the phase angle between them. [$I_A = 4{\cdot}83 - j\,15{\cdot}1$, $I_B = 18{\cdot}6 - j\,0{\cdot}3$, phase difference = 73°.]

70 In the network shown determine the characteristics of the impedance between the points B and D.



[$50{\cdot}7\ \underline{/-65{\cdot}8^\circ}\ \Omega$.]

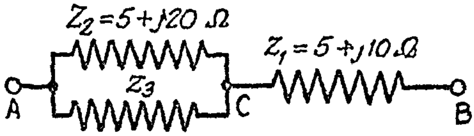


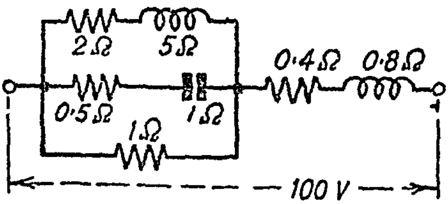
71 The circuit shown takes 12 A at a lagging power-factor and dissipates 1·8 kW when the voltmeter reading is 200 V. Calculate the values of R_1, X_1 and X_2. [$R_1 = 3{\cdot}14$; $X_1 = 2\ 17$; $X_2 = 13{\cdot}82\ \Omega$.]

72 An inductive impedance BC is connected in series with a parallel combination AB consisting of a condenser and a non-inductive resistance. The circuit constants are so adjusted that the currents in the parallel branches AB are equal, and that the voltage across AB is equal to and in quadrature with the voltage across BC. When a voltage of 200 V is applied to AC, the total power absorbed is 1 200 W. Calculate the circuit constants and draw a vector diagram.
[$R_{BC} = X_{BC} = 16\ 65\ \Omega$; $R_{AB} = X_{AB} = 33{\cdot}3\ \Omega$.]

73 Between points A and B are a non-inductive resistor R_1 and a condenser C in parallel, and between B and O is a coil of resistance R_2 and inductance L. The currents in R_1 and C are 5 and 12 A respectively. The voltage applied across AO is 200 V, 50 c/s, and its components AB and BO are each 140 V. Taking the vector representing the current in R_1 as the vector of reference, give the complex expressions for the currents in C and BO; also for the applied voltage and its components. Find the circuit constants, total power, and power factor.
[$I_C = 0 + j\ 12$; $I_{BO} = 5 + j\ 12$; $V_{AO} = 142{\cdot}8 + j\ 139{\cdot}9$ $V_{AB} = 140 + j\ 0$; $V_{BO} = 2{\cdot}8 + j\ 139{\cdot}9$; $R_1 = 28\ \Omega$, $C = 272\ \mu F$; $R_2 = 10\ \Omega$, $L = 0{\cdot}0127$ H ; 2·39 kW ; 0·92.]

74 When 220 V is applied across AB in the circuit shown, the total power input is 3·25 kW and the current is 20 A. Taking the voltage across AC as the vector of reference, give complex expressions for the currents in Z_2 and Z_3. [$I_2 = 3{\cdot}9\angle{-76°}$ A ; $I_3 = 22{\cdot}0\angle{48{\cdot}7°}$ A.]

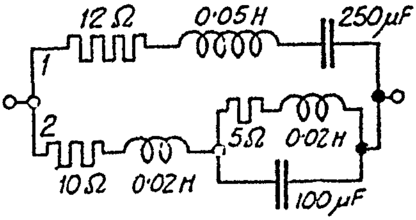




75 Determine the impedance of the circuit shown and the power consumed in each branch.

[$1{\cdot}12\angle{29{\cdot}5°}\ \Omega$, 0·216 kW, 1·24 kW, 3·10 kW, 3·16 kW.]

76 A potential difference of 100 V at 50 c/s is applied to the circuit shown. Calculate (*a*) the equivalent resistance and reactance; (*b*) the current in each branch circuit; (*c*) the power-factor of supply; (*d*) the current in the mains.



[(*a*) 7·4 Ω, 3·0 Ω; (*b*) $I_1 = 8\,1$ A, $I_2 = 4{\cdot}64$ A; (*c*) 0·926; (*d*) 12·5 A.]

Single-phase: Series (Acceptor) Resonance

77 A coil of resistance 2 Ω and inductance 0 01 H is connected in series with a condenser across 200-V mains. What capacitance must the condenser have in order that maximum current may occur at a frequency of (*a*) 25, (*b*) 50, (*c*) 100 c/s? (*d*) Find also the current, and the voltage across the condenser, in each case.
[(*a*) 4 050 μF; (*b*) 1 012 μF; (*c*) 254 μF; (*d*) 100 A; 157 V, 314 V, 628 V.]

78 Find the phase angle of the input impedance of a series circuit consisting of a 500-Ω resistor, a 60-mH inductor and a 0 053-μF capacitor at frequencies of (*a*) 2 000, (*b*) 2 820, (*c*) 4 000 c/s.
[(*a*) − 56·3°; (*b*) 0; (*c*) 56·3°.]

79 A resistor and a capacitor are in series with a variable inductor. When the circuit is connected to a 200-V, 50-c/s supply, the maximum current obtainable by varying the inductance is 0·314 A. The voltage across the capacitor is then 300 V. Find the circuit constants. [3·3 μF; 3·04 H; 637 Ω.]

80 A coil having a resistance of 5 Ω and an inductance of 0 1 H is connected in series with a condenser of capacitance 50 μF. A constant alternating voltage of 200 V is applied to the circuit. At what value of the frequency will the current be a maximum? Calculate this current, and the voltages across coil and across condenser for this frequency. Find also the voltage magnification at resonance. [71·2 c/s; 40 A, 1 790 V, 1 800 V; 8·94.]

81 A coil of inductance 9 H and resistance 50 Ω in series with a condenser is supplied at constant voltage from a variable frequency source. If the maximum current is 1 A at 75 c/s, find the frequency when the current is 0·5 A.
[75·77, 74·23 c/s.]

82 A constant voltage at a frequency of 1 Mc/s is applied to an inductor in series with a variable capacitor. When the capacitor is set to 500 μμF, the current has its maximum value; while it is reduced to one-half when the capacitance is 600 μμF.

Find (*a*) the resistance, (*b*) the inductance, (*c*) the *Q* factor of the inductor. [(*a*) 30·7 Ω; (*b*) 0·05 mH; (*c*) 10·8.]

83 A coil of resistance 40 Ω and inductance 0·75 H forms part of a series circuit for which the resonant frequency is 55 c/s. If the supply is 250 V, 50 c/s, find (*a*) the line current, (*b*) the power-factor, and (*c*) the voltage across the coil.
[(*a*) 3·88 A; (*b*) 0·62 leading; (*c*) 928 V.]

84 An inductor *L*, a capacitor *C* and a resistor *R* are connected in series. If $F = \omega/\omega_o - \omega_o/\omega$ and $n = R/\omega_o L$, where ω is the radial frequency and $\omega_o^2 = 1/LC$, find (*a*) an expression for the impedance of the combination in terms of *F*, *n* and *R*. If F_m is the value of *F* when the magnitude of the impedance has risen to 1·414 times its minimum value, find (*b*) the relationship between F_m and *n*. (*c*) Express the magnitude of the impedance in terms of *F*, F_m and *R*.
[(*a*) $Z = R(n + jF)/n$; (*b*) $F_m = n$;
(*c*) $|Z| = R\sqrt{1 + (F/F_m)^2}$.]

85 A circuit consists of a reactor having resistance and inductance of 5 Ω and 0·1015 H respectively, in series with a condenser of capacitance 100 μF. The circuit is connected to a 200-V alternating supply. Draw curves of current, inductive reactance, capacitive reactance, impedance, electromotive force induced in the reactor, and voltage across the condenser, to a base of frequency. Take values of frequency between 0 and 100 c/s.

Single-phase: Parallel (Rejector) Resonance

86 A coil of 20 Ω resistance has an inductance of 0·2 H and is connected in parallel with a condenser of 100 μF capacitance. Calculate the frequency at which the circuit will act as a non-inductive resistance of *R* ohms. Find also the value of *R*.
[31·8 c/s; 100 Ω.]

87 A constant voltage of 100 V at 50 c/s is applied to a coil of resistance 40 Ω and inductance 0 5 H, in parallel with a condenser of variable capacitance. Draw curves of (*a*) current in the coil, (*b*) current in the condenser, (*c*) total current, (*d*) total power factor, as the condenser capacitance is varied between 0 and 40 μF.

88 A coil of resistance 10 Ω and inductance 0·5 H is connected in series with a condenser. On applying a sinusoidal voltage, the current is a maximum when the frequency is 50 c/s. A second condenser is connected in parallel with this circuit; what capacitance must it have so that the combination acts like a non-inductive resistor at 100 c/s? Calculate the total current supplied in each case if the applied voltage is 220 V.
[6·7 μF; 22 A; 0·04 A.]

89 A circuit has an inductive reactance of 20 Ω at 50 c/s, its resistance being 15 Ω. For an applied voltage of 200 V at 50 c/s, calculate (*a*) the angle of phase difference between current and applied voltage; (*b*) the value of the current; (*c*) the value of the shunting capacitance to bring the resultant current into phase with the applied voltage; (*d*) the resultant current in case (*c*). [(*a*) 53°; (*b*) 8 A; (*c*) 102 μF; (*d*) 4·8 A.]

90 A coil of resistance R and inductance L is shunted by a capacitor C. Show that, for rejector (parallel) resonance, the effective resistance is L/CR. Show also that the circulating current is $V\sqrt{C/L}$, so long as the resistance is small. A circuit comprises an inductance of 10 μH associated with a resistance of 1 Ω, in parallel with a capacitance of 10^4 $\mu\mu$F. Find the current input to the circuit (*a*) for resonant frequency; (*b*) for 90% of resonant frequency. [(*a*) 0·001 V; (*b*) 0·0068 V.]

91 An inductor L, a capacitor C and a resistor R are connected in parallel. If $F = \omega/\omega_o - \omega_o/\omega$ and $n = \omega_o L/R$, where ω is the radial frequency and $\omega_o^2 = 1/LC$, find (*a*) an expression for the impedance of the combination in terms of F, n and R. If F_m is the value of F when the magnitude of the impedance has fallen to 0·707 times its maximum value, find (*b*) the relationship between F_m and n. (*c*) Express the magnitude of the impedance in terms of F, F_m and R.

[(*a*) $Z = nR/(n + jF)$; (*b*) $F_m = n$; (*c*) $|Z| = R/\sqrt{1 + (F/F_m)^2}$.]

92 A voltage consisting of a fundamental $V_1 = 100$ V (r.m.s.) at 50 c/s and a 10% fifteenth harmonic is applied to a coil having $R = 1\ \Omega$ and $L = 0{\cdot}001$ H. (*a*) Find I_1 and I_{15}. (*b*) Find what capacitance is needed to make a rejector circuit for the harmonic, and the resulting I_1 and I_{15}.
[(*a*) $I_1 = 95{\cdot}3$ A, $I_{15} = 2{\cdot}08$ A; (*b*) 43 3 μF, $I_1 = 95{\cdot}3$ A, $I_{15} = 0\ 433$ A.]

Single-phase: Locus Diagrams

93 A circuit has a constant reactor in series with a variable resistor R. Prove that the polar locus of the current is circular if a constant alternating voltage at constant frequency is applied to the circuit. Let the constant reactor be 1 Ω and the applied voltage be 100 V; draw a number of current vectors for various values of R, and note that the locus of the extremities of the current vectors is a circle.

94 Draw a circle representing the current locus of a circuit containing a reactance of 1 Ω and a variable resistance, across which a constant voltage of 100 V is applied. Find the maximum power input to the circuit in watts, and the corresponding current and power factor. What value of resistance does this condition require? [5 000 W; 70·7 A; cos $\phi = 0\ 707$; 1 Ω.]

95 A coil of resistance 2 Ω and reactance 5 Ω is connected in series with a non-reactive resistor which is continuously variable between zero and 8 Ω. Obtain by inversion the locus of the current when the circuit is connected to a 200-V supply; and hence find the maximum and minimum values of the current and the corresponding power factors.

*[87·2, 17·9 A; 0·37, 0·89.]

96 A resistor of 10 Ω is connected in series with an inductive reactor which is variable between 2 Ω and 20 Ω. Obtain by inversion the locus of the current vector when the circuit is connected to a 250-V supply. Determine the value of the current and the power factor when the reactance is (*a*) 5 Ω; (*b*) 10 Ω; and (*c*) 20 Ω.

*[(*a*) 22·8 A, 0·89; (*b*) 17·7 A, 0·71; (*c*) 11·1 A, 0·45.]

97 Use a circle diagram to find the current taken when five 50 W, 1·58 A lamps in series with a 40 μF condenser on a 125 V, 50 c/s supply are successively short-circuited. Calculate how many lamps are in circuit for maximum power consumption and the power consumed per lamp under this condition.

[1·53, 1·40, 1·26, 1·11, 0·98 A, 4 lamps, 98 W, 24·5 W.]

98 A circuit comprises a series arrangement of *R*, *L*, and *C*. Show that the current locus is circular for an applied voltage of constant peak value but varying frequency. Draw the circle for $R = 100$ Ω, $L = 0{\cdot}8$ mH, $C = 5\,000$ μμF, and $V = 10$ V. Derive a curve showing the current plotted to a base of frequency.

99 A coil of resistance 20 Ω and reactance 15 Ω is connected in parallel with a series circuit consisting of a fixed inductive reactor of 10 Ω and a non-reactive resistor *R* which is variable between 5 and 100 Ω. Draw the locus of the current vector with an alternating applied voltage of 400 V. Determine the value of the total current when the variable resistance is (*a*) 10 Ω, and (*b*) 50 Ω.

*[(*a*) 44·2 A; (*b*) 23·5.]

100 A 50-c/s, 200-V potential difference is applied to a coil of resistance 20 Ω and reactance 50 Ω, connected in parallel with an inductive reactor of 10 Ω in series with a non-reactive resistor *R* continuously variable between 0 and ∞. Obtain the admittance locus for the circuit and hence determine the value of the total current when it lags (*a*) 45°, (*b*) 60°, behind the applied voltage.

*[(*a*) 5·2 or 15·8 A; (*b*) 3·92 or 20·8 A.]

101 One branch of a parallel circuit contains a constant impedance $20\underline{/30^\circ}$ Ω: the other consists of a constant inductive reactor of 5 Ω in series with a non-reactive resistor which is variable between the limits of 2 Ω and 40 Ω. Obtain by inversion the locus of the variable voltage vector when a constant current of

10 A flows in the circuit. Determine therefrom the value of the voltage when the variable resistance is (*a*) 5 Ω, (*b*) 10 Ω. What will be the value of the voltage and of the variable resistance when the current in the circuit has a 45° angle of lag?

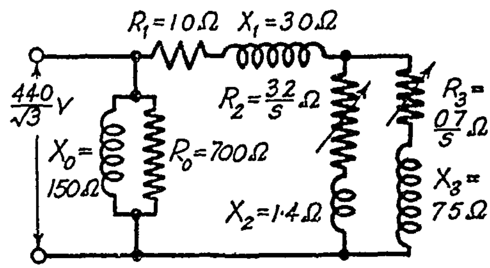
*[(*a*) 52·5 V; (*b*) 78 V; 49·5 V; 4·0 Ω.]

102 A variable load of unity power factor is shunted by an admittance (0 02 − *j* 0·2) ℧, and the parallel combination so formed is connected in series with an impedance (2 + *j* 5) Ω across a 400-V supply. Draw the locus diagram of input current, and find the value of the current when the load circuit is (*a*) open-circuited, (*b*) short-circuited, and (*c*) of equivalent resistance 5 Ω.

*[(*a*) 40 A; (*b*) 74 A; (*c*) 47 A.]

103 A circuit of impedance *r* + *j* 2 Ω is in parallel with an impedance 3 + *j* 4 Ω. In series with the combination there is an impedance 2 + *j* 3 Ω. Draw the locus of the current vector in the latter impedance when 200 V is applied across the whole circuit and *r* is varied from 1·5 to 6 Ω.

104 One phase of a double-cage, 440-V, 10-h.p., star-connected, 4-pole, 3-phase induction motor is represented by the equivalent circuit shown. The suffixes 0, 1, 2, and 3 refer to magnetizing impedance, stator, outer cage, and inner cage respectively. Draw the locus diagram for the input phase current.



Three-phase: Star Connexion

105 Calculate the active and reactive current components in each phase of a star-connected, 10 000-V, 3-phase alternator supplying 5 000 kW at a power-factor of 0·8. If the total current remains the same when the load power factor is raised to 0·9, find the new output.

[289 A, 216 A; 5 625 kW.]

106 A balanced star-connected load of 8 + *j* 6 Ω per phase is connected to a 3-phase, 230-V supply. Find the line current, power factor, power, reactive volt-amperes and total volt-amperes.

[13 3 A; 0 8; 4 250 W; 3 130 VAr; 5 280 VA.]

107 A balanced 3-phase, star-connected load of 150 kW takes a leading current of 100 A with a line voltage of 1 100 V, 50 c/s. Find the circuit constants of the load per phase.

[$R = 5\ \Omega$, $C = 810\ \mu F$.]

108 A balanced star-connected load is supplied from a symmetrical 3-phase, 400-V system. The current in each phase is 30 A and lags 30° behind the phase voltage. Find (*a*) the phase voltage, and (*b*) the total power. Draw the vector diagram showing the currents and voltages. [(*a*) 231 V ; (*b*) 18 kW.]

109 Three equal star-connected inductors take 8 kW at power factor 0 8 when connected to a 460-V, 3-phase, 3-wire supply. Find the line currents if one inductor is short circuited.

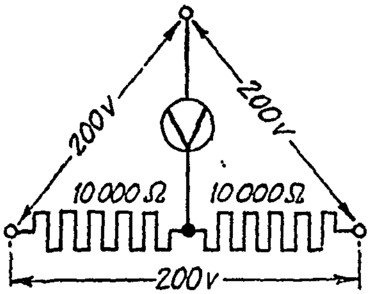
[21·7, 21·7, 37·6A.]

110 On a symmetrical 3-phase system, phase sequence *RYB*, a capacitive reactance of 8 Ω is across *YB*, and a coil $R + j X$ across *RY*. Find *R* and *X* such that $I_Y = 0$.

[$R = 6{\cdot}93\ \Omega$; $X = 4\ \Omega$.]

111 A voltmeter with a resistance of 10 000 Ω is connected as shown with two other 10 000-Ω non-inductive resistors across 200-V, 3-phase mains. Find the reading on the voltmeter (*a*) in the conditions shown ; (*b*) when the voltmeter is shunted by a 5 000-Ω resistor.

[(*a*) 115 V ; (*b*) 69·4 V.]



112 A 3-phase, 440-V supply is connected to three star-connected resistances such that the voltages across two of them are equal in magnitude and differ in phase by 100°. Find the ratio of the resistances and the voltage across each.

[1 : 1·88 : 1·88 ; 196 5 V, 287 V, 287 V.]

113 Non-reactive resistors of 10, 20, and 25 Ω are star-connected to the *R*, *Y*, and *B* phases of a 400-V, symmetrical system. Determine the current and power in each resistor and the voltage between star point and neutral. Phase sequence, *RYB*.

[16·5 A, 2·72 kW ; 13·1 A, 3 43 kW ; 11·2 A, 3·14 kW ; 68 V.]

114 A symmetrical 440-V, 3-phase system supplies a star-connected load with the following branch impedances : $Z_R = 100\ \Omega$, $Z_Y = j\,5\ \Omega$, $Z_B = -j\,5\ \Omega$. Calculate the voltage drop across each branch and the potential of the neutral point to earth. The phase sequence is *RYB*. Draw the vector diagram.

[$8\,800\ \underline{/-30^\circ}$, $8\,415\ \underline{/-31{\cdot}5^\circ}$, $8\,420\ \underline{/-28{\cdot}5^\circ}$; $8\,545\ \underline{/150^\circ}$.]

115 Determine the line currents in an unbalanced, star-connected load supplied from a symmetrical 3-phase, 440-V system. The branch impedances of the load are $Z_R = 5\underline{/30^\circ}\ \Omega$, $Z_Y = 10\underline{/45^\circ}\Omega$, and $Z_B = 10\underline{/60^\circ}\ \Omega$. The phase sequence is *RYB*.

[35·7 A, 32·8 A, 27·7 A.]

116 A star load, comprising two resistors and a pure inductor, is connected to a symmetrical 3-phase supply voltage. If the numerical impedance of all branches is the same, find the voltages across each branch as a percentage of the line voltage.
[23·2%, 86·4%, 77·5%.]

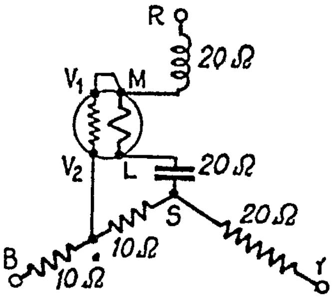
117 A symmetrical 3-phase, 440-V system supplies a star-connected load. The branch impedances are $Z_R = 10\underline{/30°}\ \Omega$, $Z_Y = 12\underline{/45°}\ \Omega$, and $Z_B = 15\underline{/40°}\ \Omega$. Assuming the neutral of the supply to be earthed, calculate the voltage to earth of the star point. Phase sequence, *RYB*. [19·4 V.]

118 Two lamps of equal resistance are connected across the lines *R* and *B* of a symmetrical 3-phase system. The junction of the two lamps is connected to the neutral *N* of the system through a condenser of reactance equal numerically to the resistance of either lamp. Show that the lamp connected to the leading phase takes 59 7% more current than the other.

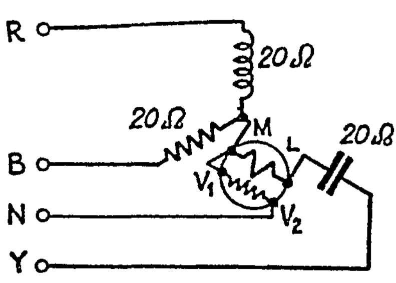
119 A resistor of 300 Ω and a condenser of 8 μF are connected in series across the lines A and B of a 50-c/s, 400-V, 3-phase system. Determine the voltage between the junction of the resistor and condenser and the line C. State the corresponding phase rotation. [ABC, 542 V ; ACB, 167 V.]

120 Two 500-ohm resistances and a condenser are joined in star to a 400-V, 3-phase, 50 c/s supply. State the phase rotation, and find the capacitance and equivalent series resistance of the condenser if the voltages across the resistances connected to the red and blue lines are 400 V and 300 V respectively.
[*RBY* ; 54·1 μF ; 21·05 Ω.]

121 Find the reading on the wattmeter when the network shown is connected to a balanced 400-V, 3-phase 3-wire supply. The phase sequence is *RYB*. Neglect instrument losses. [— 6·0 kW.]



122 Find the reading on the wattmeter when the network shown is connected to a balanced 400-V, 3-phase supply. The phase sequence is *RYB*. Neglect instrument losses. [— 1·69 kW.]



123 Three non-reactive resistors of 3, 4 and 5 Ω respectively are star-connected to a 3-phase, 400-V symmetrical system, phase sequence *RYB*. Find (*a*) the current in each resistor, (*b*) the power dissipated in each resistor, (*c*) the phase angles between the currents and the corresponding line voltages, (*d*) the star-point potential. Draw to scale the complete vector diagram.
[(*a*) 66·5, 59·5, 51·8 A ; (*b*) 13·2, 14·15, 13·4 kW ; (*c*) 26° 24′, 38° 10′, 25° 20′ ; (*d*) 34 V.]

124 Three identical coils, symmetrically arranged in space, are star-connected to a 400-V, 50-c/s, 3-phase supply. Each coil has resistance and inductance of 100 Ω and 0·8 H respectively, while the mutual inductance between each pair of coils is 0·3 H. Calculate the current taken by each coil and its power factor.
[1·24 A, 0·537.]

125 An unbalanced, star-connected load is fed from a symmetrical 3-phase system. The phase voltages across two of the arms of the load are $V_B = 295\underline{/97° \ 30'}$ and $V_R = 206\underline{/-25°}$. Calculate the voltage between the star-point of the load and the supply neutral.
[$52{\cdot}2 \ \underline{/- 49° \ 54'}$.]

126 Three impedances Z_R, Z_Y and Z_B are connected in star across a 440-V, 3-phase supply. If the voltage of the star-point relative to the supply neutral is $200 \ \underline{/150°}$ V and the *Y* and *B* line currents are $10 \ \underline{/- 90°}$ A and $20 \ \underline{/90°}$ A respectively, all with respect to the voltage between the supply neutral and the *R* line, calculate the values of Z_R, Z_Y and Z_B.
[$10 + j\,42{\cdot}72$, $32 + j\,4{\cdot}62$, $6 - j\,2{\cdot}31$ Ω.]

127 An unbalanced star-connected load is fed from a symmetrical 440-V, 3-phase system. The supply voltage of phase *R* is $254 \ \underline{/- 30°}$ and the voltage across the load in phase *R* is $206 \ \underline{/- 25°}$. Draw the vector diagram. Calculate (*a*) the voltage between the star-point of the load and the supply neutral, (*b*) the voltages across the loads in phases *Y* and *B*.
[(*a*) $52{\cdot}1\underline{/- 50{\cdot}2°}$ V ; (*b*) $268 \ \underline{/161{\cdot}1°}$; $295 \ \underline{/96{\cdot}5°}$ V.]

Three-phase : Mesh Connexion

128 A symmetrical 3-phase, 400-V system supplies a balanced mesh-connected load. The current in each branch circuit is 20 A and the phase angle is 40° (lagging). Find (*a*) the line current, and (*b*) the total power. Draw the vector diagram showing the voltages and currents in the lines and circuits for the three phases.
[(*a*) 34·64 A ; (*b*) 10·6 kW.]

129 A balanced mesh-connected load of $8 + j\,6$ Ω per phase is connected to a 3-phase, 230-V supply. Find the line current, power factor, power, reactive volt-amperes and total volt-amperes.
[40 A ; 0·8 ; 12·7 kW ; 9·53 kVAr ; 15·9 kVA.]

130 A 3-phase, 440-V mesh-connected system has the loads: branch RY, 20 kW at power factor 1 0; branch YB, 30 kVA at power factor 0 8 lagging; branch BR, 20 kVA at power factor 0 6 leading. Find the line currents and readings on wattmeters whose current coils are in phases R and B.

[$90{\cdot}5\ \angle 176{\cdot}5^\circ$; $111{\cdot}4\ \angle 14^\circ$; $36{\cdot}7\ \angle -119^\circ$; 39·8 kW; 16.1 kW.]

131 A 400-V, 50-c/s, 3-phase supply has 100 Ω between R and Y, 318 mH between Y and B, and 31·8 μF between B and R. Find (a) line currents for phase sequences (i) RYB, (ii) RBY. (b) Star-connected balanced resistors for the same power.

[(a) (i) $7\ 73\ \angle 15^\circ$, $7{\cdot}73\ \angle 165^\circ$, $4\ \angle -90^\circ$; (ii) $2\ 07\ \angle 20{\cdot}5^\circ$, $2{\cdot}07\ \angle 159\ 5^\circ$, $4\ \angle -90^\circ$, (b) 100 Ω.]

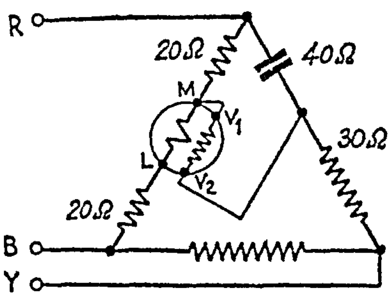
132 The currents in the RY, YB and BR branches of a mesh-connected system with symmetrical voltages are 25 A at power factor 0 8 lagging, 30 A at power factor 0·7 leading and 20 A at unity power factor respectively. Determine the current in each line. Phase sequence, RYB.

[$I_R = 44{\cdot}1$ A; $I_Y = 18{\cdot}3$ A; $I_B = 49{\cdot}5$ A.]

133 A 3-phase voltage is applied to a load consisting of two equal resistors R in series, phase B being connected to the junction. Find the ratio of the currents in the three lines and their relative phase positions.

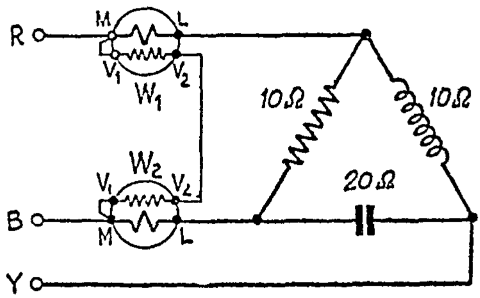
[$I_R : I_Y : I_B = 1 : 1 : \sqrt{3}$; $I_R = \angle 0^\circ$, $I_Y = \angle -60^\circ$, $I_B = \angle 150^\circ$.]

134 Find the reading on the wattmeter when the network shown is connected to a symmetrical 400-V, 3-phase supply of phase sequence RYB. Neglect instrument losses. Draw the vector diagram for the wattmeter current and voltage in relation to the line voltages.



[0·94 kW.]

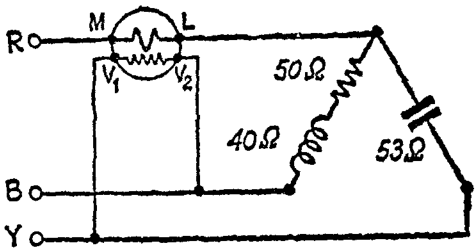
135 Find the readings on each of the two similar wattmeters when the network shown is connected to a symmetrical 400-V, 3-phase supply. The phase sequence is RYB. Draw the vector diagram.



[W_1: 14·93; W_2: 11·48 kW.]

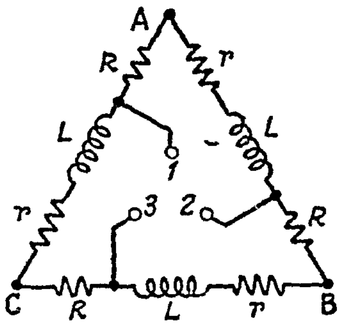
136 Find the reading on the wattmeter when the network shown is connected to a symmetrical 440-V, 3-phase supply. The phase sequence is RYB. Neglect electrostatic effects and instrument losses. Draw the vector diagram.

[$-$ 0·343 kW.]



137 The circuit shown in which $R = 200\ \Omega$, $r = 100\ \Omega$ and $L = 0{\cdot}552$ H is connected to a symmetrical 3-phase, 400-V, 50 c/s system at points 1, 2 and 3. Find the p.d. between A, B and C when the phase sequence of the supply is (a) 1, 2, 3 ; (b) 1, 3, 2.

[(a) 0, (b) 400 V.]



138 Three mesh-connected load impedances $Z_{RY} = 10 + j\ 17{\cdot}3$, $Z_{YB} = 10 + j\ 0$, $Z_{BR} = 15 - j\ 15$ are joined to the mesh-connected secondary winding of a transformer having a symmetrical voltage of 440 V per phase. Find (a) the line currents to the load, (b) the current in each secondary phase of the transformer. Neglect line and transformer impedances. Phase sequence RYB.

[(a) 39·5 $/-98° 30'$; 38·08 $/150°$; 43·6 $/27° 21'$.
(b) 21·4 $/-64° 54'$; 23·9 $/180° 45'$; 24·6 $/53° 24'$.]

Three-phase : Star-mesh Connexion

139 Three star-connected impedances, $Z_1 = 20 + j\ 37{\cdot}7\ \Omega$ per phase are in parallel with three mesh-connected impedances, $Z_2 = 30 - j\ 159{\cdot}3\ \Omega$ per phase. The line voltage is 398 V. Find the line current, power factor, power and reactive volt-amperes taken by the combination.

[3·37 $/-10{\cdot}4°$; 0·984 lag ; 2 295 W ; 420 VAr.]

140 Between any two terminals of a 3-phase balanced load the voltage is 415 V and the resistance is 3·0 Ω. The current in each of the three lines RYB is 100 A. Find the power factor of the load. Find also the resistance and reactance per phase of the load with (a) star connexion, (b) mesh connexion.

[0·625 (a) 1·5, 1·87 Ω ; (b) 4·5, 5·61 Ω.]

VII. A.C. CIRCUITS

141 A 3-phase, star-connected alternator feeds a 2 000-h.p. mesh-connected induction motor having a power factor of 0 85 and an efficiency of 0·93. Calculate the current and its active and reactive components in (*a*) each alternator phase; (*b*) each motor phase. The line voltage is 2 200 V.

[(*a*) 496, 421, 262 A; (*b*) 286, 243, 151 A.]

142 A 3 300-V, 200-kVA, star-connected alternator feeds a 200-h p., 400-V, mesh-connected induction motor through three 1-phase transformers with primaries connected in mesh and secondaries in star. Calculate (*a*) the phase-turn ratio of each transformer, (*b*) the primary and secondary phase current in each transformer, (*c*) the alternator phase current, (*d*) the motor phase current. The motor is developing full load at an efficiency and power factor of 0·9 and 0·85 respectively. Neglect transformer losses and magnetizing current.

[(*a*) 13/1; (*b*) 19·7, 256 A; (*c*) 34·1 A; (*d*) 148 A.]

143 A 440-V, 3-phase supply is connected through three 1-phase transformers to a load comprising three 10-Ω resistors connected in mesh. The transformers are connected in mesh on the primary side and in star on the secondary side. The turn ratio is 2 : 1. Neglecting magnetizing currents and losses, calculate (*a*) the load in kW, (*b*) the current in each resistor, (*c*) the line current to the load, and (*d*) the transformer secondary and primary phase currents.

[(*a*) 43·6 kW; (*b*) 38·1 A; (*c*) 66·0 A; (*d*) 66·0, 33·0 A.]

144 A 3-wire, 3-phase supply feeds a load consisting of 3 equal resistors. By how much is the load reduced if one of the resistors be removed, (*a*) when the load is star-connected, (*b*) when the load is mesh-connected?

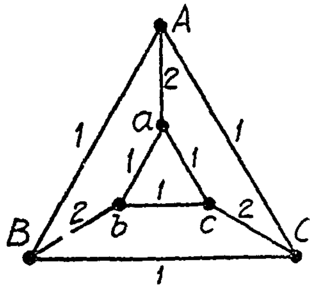
[(*a*) 50%; (*b*) 33·3%.]

CHAPTER VIII

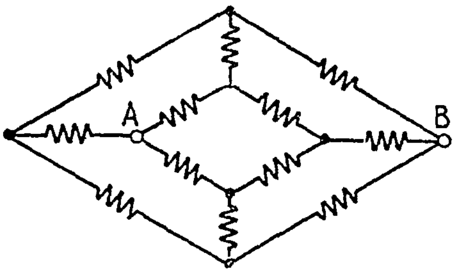
NETWORKS

Input Resistance and Impedance

1 A network of 9 conductors connects 6 points A, B, C, a, b, c, as shown. The figures denote resistances in ohms. Find (*a*) the resistance between A and a: (*b*) the resistance between C and a: (*c*) the resistance between c and a: (*d*) the resistance between C and A.
[(*a*) 1 Ω; (*b*) $1\frac{1}{12}$ Ω; (*c*) $\frac{7}{12}$ Ω; (*d*) $\frac{7}{12}\Omega$.]

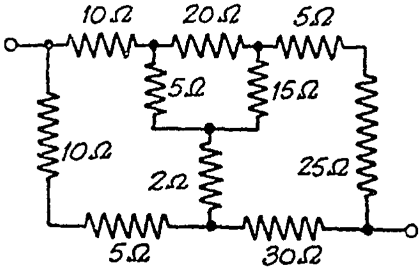


2 The diagram represents resistors each having the value R. Find the resistance between the junctions A and B. [$\frac{5}{6}$ R.]

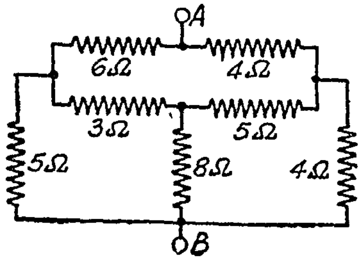


3 Twelve similar conductors each of 1 Ω resistance form a cubical framework. Calculate the resistance between (*a*) two adjacent corners, (*b*) two opposite corners of one face, (*c*) two opposite corners of the cube. [(*a*) $\frac{7}{12}$ Ω; (*b*) $\frac{3}{4}$ Ω; (*c*) $\frac{5}{6}$ Ω.]

4 Determine the resistance between terminals of the network shown. [23·6 Ω.]

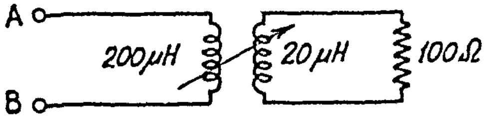


5 Determine the resistance between points A and B in the network shown. [4·23 Ω.]



alculate the impedance at 1 Mc/s between the input terminals of the circuit shown. The coupling coefficient is 0·1.

[6·1 + *j* 1 248 Ω.]



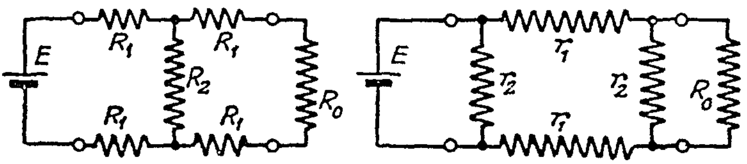
7 Two coupled coils have inductances of 0·01 H and 5 H and resistances of 5 Ω and 100 Ω respectively. If the coupling coefficient is 0 8, find the percentage change in the effective resistance of the primary at a frequency of 50 c/s when a resistance of 200 Ω is connected across the terminals of the secondary.

[4·74%.]

Equivalence of Networks

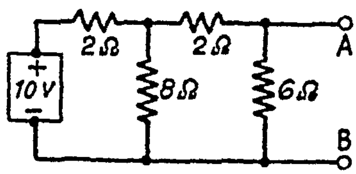
8 A network has two input terminals *AB* and two output terminals *CD*. The input impedance with *CD* open-circuited is 160 + *j* 354 Ω ; with *CD* short-circuited, 120 − *j* 546 Ω. The impedance across *CD* with *AB* open-circuited is 155 + *j* 248 Ω. Determine the equivalent T-section of the network.
[Series : 10 − *j* 136 Ω and 5 − *j* 242 Ω ; shunt, 152 + *j* 494 Ω.]

9 Find r_1 and r_2 in terms of R_1 and R_2 in the two networks, in order that the battery shall supply the same current, and



that there shall be the same current through R_0 in each case. Evaluate r_1, r_2, and E, if R_0 is 20 Ω, R_1 is 10 Ω, R_2 is 40 Ω and the current through R_0 is 5 A.
[$r_1 = 2R_1 + 2R_1^2/R_2$; $r_2 = 2R_1 + 2R_2$; 25 Ω, 100 Ω, 400 V.]

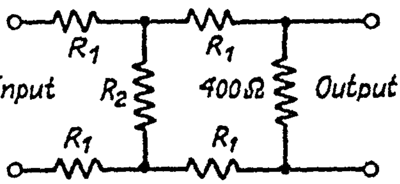
10 Determine (*a*) the equivalent voltage generator, and (*b*) the equivalent current generator, which may be used to represent the given network at the terminals *AB*.



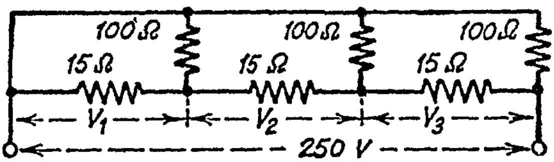
[(*a*) E.m.f., 5 V, series resistance 2·25 Ω ; (*b*) current, 2·22 A, shunt resistance 2·25 Ω.]

Transmission Characteristics

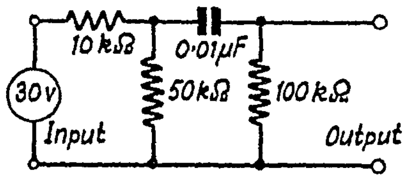
11 Calculate the values of the resistors R_1 and R_2 in order that the input resistance of the network shall be 500 Ω and the output voltage one-half of the input voltage. [77 Ω, 923 Ω.]



12 Determine the voltages across the 15-ohm resistors in the network shown. [69·2, 79·5, 101·3.]

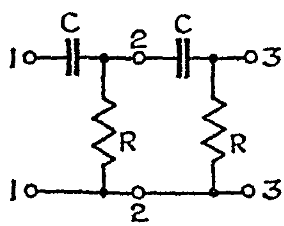


13 Determine the magnitude of the output voltage and its phase shift from the input voltage at a frequency of 1 kc/s in the network shown. [22·8 V ; 8·3°.]



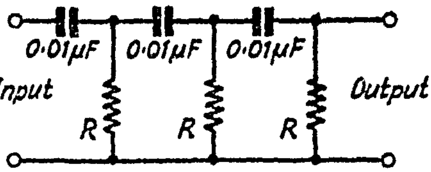
14 A network contains three T-sections. Each half of the series impedance is a 40-Ω resistor, and each shunt admittance comprises a condenser of 0·5 μF capacitance. Find the voltage required across one end to produce a current of $1 \underline{/0^\circ}$ A through a load resistance of 50 Ω at a frequency of 2 400 c/s. [$396 \underline{/68{\cdot}3^\circ}$ V.]

15 If a voltage $V \sin \omega t$ is applied across the terminals 11 of the network shown and $\omega CR = 1$, (*a*) evaluate β and ϕ if the resultant voltage across the terminals 33 is expressed in the form $\beta V \sin(\omega t + \phi)$. Find (*b*) the values of β and ϕ if the circuit is interrupted at the terminals 22 and a unity gain zero phase-shift buffer stage inserted.



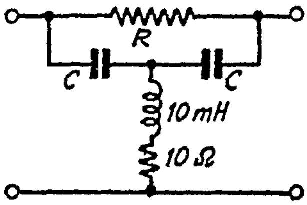
[(*a*) $\beta = 1/3$, $\phi = 90^\circ$; (*b*) $\beta = 1/2$, $\phi = 90^\circ$.]

16 Find the value of R to give the ladder network shown an output-to-input phase shift of 180° at a frequency of 1 kc/s, and calculate the attenuation.

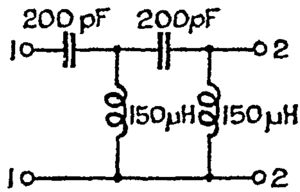


[6 490 Ω ; 29·2 db.]

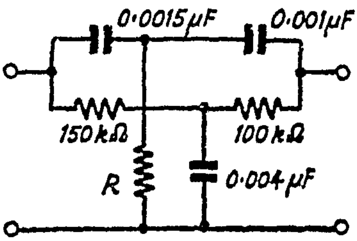
17 The bridge-T network shown is to have zero transmission at a frequency of 15·9 kc/s. Find C and R.
[0·005 μF ; 40 Ω.]



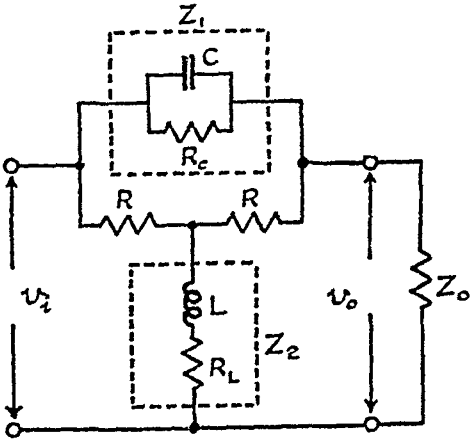
18 If a sinusoidal signal of amplitude 1 V r.m.s. is applied across the terminals 11 of the network shown, find the voltage appearing across the terminals 22 when the frequency is (*a*) 1 Mc/s and (*b*) 1 kc/s.
[(*a*) − 1·22 V ; (*b*) 1·41 μV.]



19 Determine the value of R to give zero transmission in the twin-T network shown, and the frequency at which this occurs.
[96·1 kΩ ; 840 c/s.]



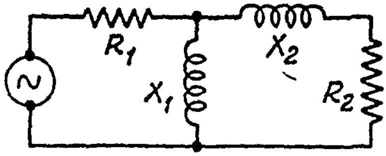
20 In the circuit shown, find (*a*) the relationships between R, Z_1, Z_2 and Z_o such that the input impedance is equal to Z_o. (*b*) What is the ratio v_o/v_i under these conditions? (*c*) If $Z_o = 75\Omega$, find values for C, R_c, L and R_L such that the ratio v_o/v_i is 0·75 at frequencies approaching zero and 0·95 at 1 Mc/s.



[(*a*) $R = Z_o$, $Z_1Z_2 = Z_o^2$; (*b*) $Z_o/(Z_1 + Z_o)$; (*c*) $C = 0{\cdot}016$ μF, $R_c = 25$ Ω, $L = 89$ μH, $R_L = 225$ Ω.]

Power Transfer and Matching

21 Find X_1 and X_2 in terms of R_1 and R_2 to give maximum power dissipation in R_2.



[$X_1 = \pm \sqrt{R_1^2 R_2/(R_1 - R_2)}$; $X_2 = \pm \sqrt{R_2(R_1 - R_2)}$.]

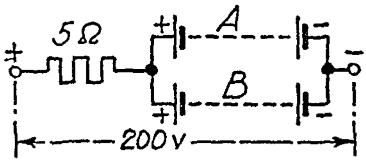
22 A load of $(20 - j\,X_c)\ \Omega$ is supplied from a source of e.m.f. 10 V r.m.s. and internal impedance $(10 + j\,20)\ \Omega$. Find the value of X_c to give maximum power dissipation in the load and calculate the value of this power. [20 Ω ; 2·22 W.]

23 Two reactors, each of 50-Ω reactance and negligible resistance, are connected in series across a 200-V, 50-c/s supply. Determine what value of resistance connected in parallel with one coil dissipates maximum power. [25 Ω.

24 A 1-kc/s generator has an internal impedance comprising a resistance of 50 Ω in series with an inductance of 0·01 H. It supplies a load of 1 000 Ω resistance. A capacitor C is connected in parallel with the load and a coil of inductance L and negligible resistance is put in series with the generator. Find the values of C and L which will enable maximum power to be dissipated in the load. [0·696 μF ; 24·6 mH.]

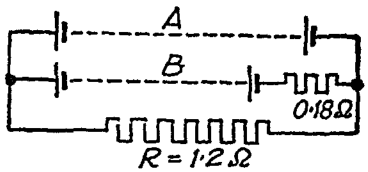
D.C. Networks—General Analysis

25 Two batteries are connected as shown to 200-V mains. Battery A has an electromotive force of 110 V and internal resistance 0·2 Ω; battery B has an electromotive force of 100 V and an internal resistance of 0·25 Ω. Determine the magnitude and direction of the current in each battery and the total current taken from the supply.



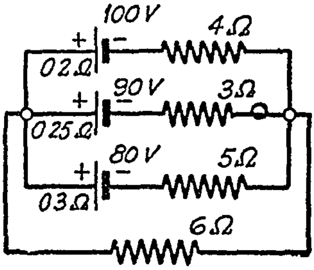
[I_A = 11·96 A, discharge ; I_B = 30·43 A, charge ; total, 18·47 A.]

26 In the circuit given, the batteries A and B consist of cells having on open-circuit an electromotive force of 2 V per cell and an internal resistance of 0·001 Ω per cell. There are 50 cells in battery A and 45 in battery B. Find the current flowing in each battery and in the resistor R.

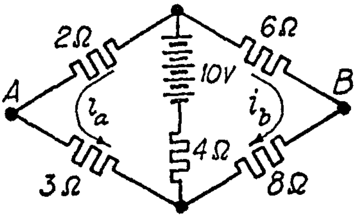


[I_A = 101 A, discharge ; I_B = 22 A, charge ; I_R = 79A.]

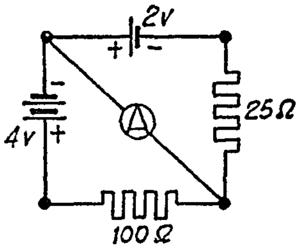
27 In the network shown determine the current in each battery and in the 6 Ω resistor.
[6·21 A ; 4·95 A ; 1·15 A ; 12·81 A.]



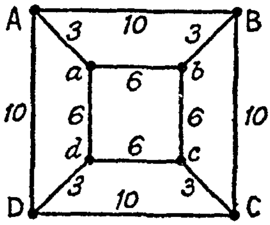
28 Calculate the current in the battery, the current in each branch, and the potential difference across AB in the network shown.
[$i_a + i_b = 1{\cdot}303$ A ; $i_a = 0{\cdot}96$ A ; $i_b = 0{\cdot}343$ A ; 0·14 V.]



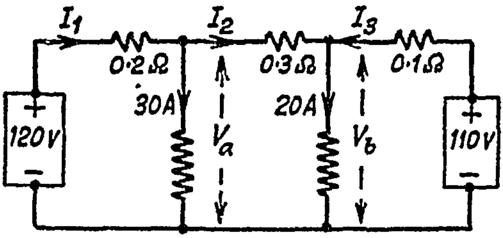
29 In the network shown determine the direction and magnitude of current flow in the milliammeter A having a resistance of 10 Ω. [26·7 mA.]



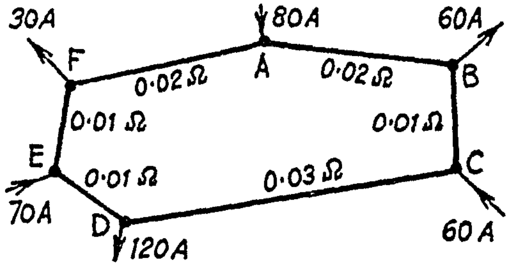
30 The figures give resistances in ohms. Find the currents in the ab, ad and aA branches when (a) 20 V acts in Aa from A to a, and 20 V in Cc from C to c, (b) as (a), but with the voltage in Cc reversed.
[(a) 0·72, 0·72, 1·48 A ; (b) 0·91, 0·91, 1·82 A.]



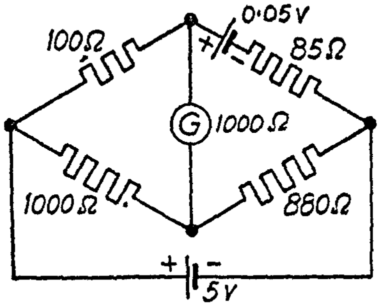
31 Find the currents I_1, I_2 and I_3 and the voltages V_a and V_b in the given network.
[$I_1 = 40$ A, $I_2 = I_3 = 10$ A ; $V_a = 112$ V, $V_b = 109$ V.]



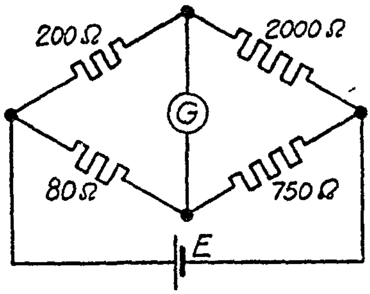
32 Find the currents in all branches of the network shown. [A to B, 89 A; C to B, 21 A; C to D, 89 A; E to D, 81 A; F to E, 11 A; A to F, 41 A.]



33 Determine the current in the galvanometer branch of the bridge network shown. [10·62 μA.]

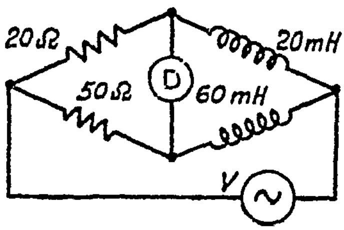


34 In the Wheatstone bridge network shown, the galvanometer G has a resistance of 1 000 Ω. A voltage of 4 V is applied by the battery E. Find the current through the galvanometer. Neglect the internal resistance of the battery. [17·4 μA.]

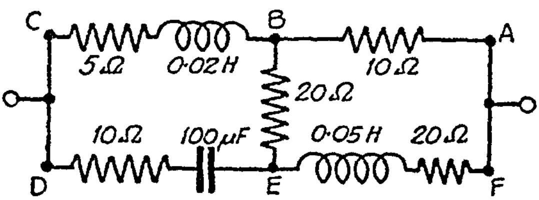


A.C. Networks—General Analysis

35 Find the current in the detector D of the given bridge network when the applied voltage is 4 V at a frequency of 1 000/2π c/s. The detector has a resistance of 30 Ω and the impedance of the supply is negligible. [4·67 mA.]



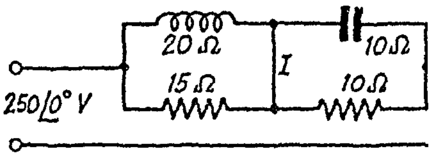
36 Calculate (*a*) the current in each branch, (*b*) the total current, and (*c*) the power factor, when the circuit shown is connected across a 200-V, 50-c/s supply.



[(*a*) $I_{AB} = 10{\cdot}5$ A, $I_{BC} = 13{\cdot}2$ A, $I_{DE} = 3{\cdot}84$ A, $I_{EF} = 3{\cdot}46$ A, $I_{BE} = 3{\cdot}93$ A; (*b*) 13·9 A; (*c*) 0·96.]

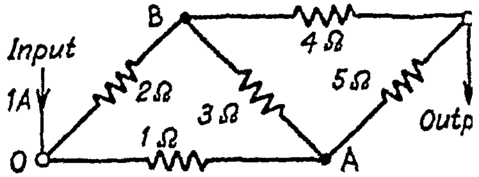
VIII. NETWORKS

37 Calculate the magnitude and phase, with reference to the applied voltage $250\underline{/0^\circ}$ V, of the current I in the central link.

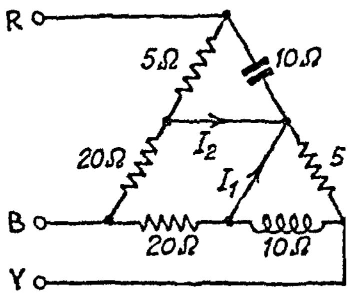


[$I = 16{\cdot}75\ \underline{/258^\circ}$ or 78°

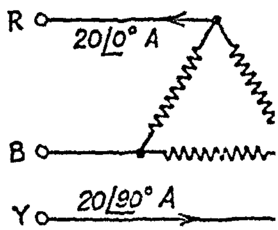
38 Find the current distribution in the network shown.
[OA, 0·572 ; OB, 0·428 ; AB, 0·095 ; AC, 0·477 ; BC, 0·528 A.]



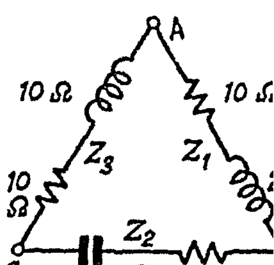
39 Find the magnitude of the current I_1 and its phase angle with reference to the current I_2 when the network shown is connected to a symmetrical 400 V, 3-phase supply. The phase sequence is RYB.
[33·5 $\underline{/148^\circ}$ A.]



40 Three 12-Ω resistors are mesh-connected to an unsymmetrical 3-phase supply of sequence RYB. If the currents in lines R and Y are as indicated, find the phase currents, and draw the vector diagram.
[14·9 $\underline{/-63{\cdot}5^\circ}$; 14·9 $\underline{/153{\cdot}5^\circ}$; 9·4 $\underline{/45^\circ}$ A.]



41 The unbalanced mesh network in the diagram is connected to a balanced 3-phase supply of line voltage 420 V and phase-sequence ABC. Calculate the loading of each branch of the equivalent star network.
[P_A, 0·59 kW at p.f. 0·04 lag ; P_B, 0 54 kW at p.f. 0·99 lag ; P_C, 16·9 kW at p.f. 0·98 lead.]



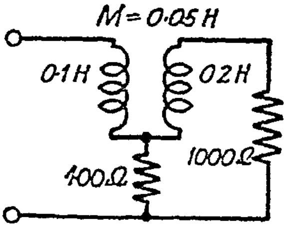
42 A 380-V, 3-wire, star-connected system has the following phase impedances: $Z_R = 4{\cdot}9 + j\ 2{\cdot}1\ \Omega$; $Z_Y = j\ 18\ \Omega$; $Z_B = 3{\cdot}8\ \Omega$. Find the loads on the equivalent mesh-connected system. Phase-sequence *RYB*.

[*RY*, 9·5 kW at p.f. 0·5 lag; *YB*, 24·2 kW at p.f. 0·8 lag; *BR*, 9·05 kW at p.f. 0·87 lead.]

43 Two similar circuits each of resistance 10 Ω and inductive reactance 8 Ω have a mutual inductive reactance of 4 Ω when placed in proximity. Calculate (*a*) the primary current when 0·5 ampere circulates in the closed secondary circuit, and (*b*) the primary voltage required to produce this current.

[(*a*) 1·6 A; (*b*) 37·4 V.]

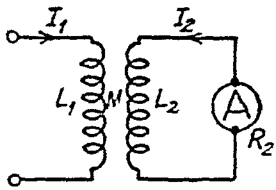
44 In the network shown the coils are wound on the same core, and have negligible resistance. Find the current in the 1 000-Ω resistor and its phase angle with respect to the applied voltage of 10 V at a frequency of 5 000/2π c/s when the coils (*a*) act in the same sense, (*b*) act against one another.



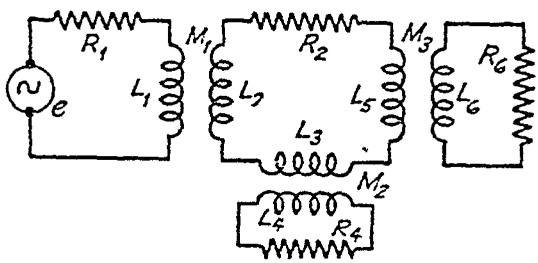
[(*a*) 3·63 $\underline{/-123{\cdot}7^\circ}$ mA; (*b*) 5·24 $\underline{/-60{\cdot}4^\circ}$ mA

45 In the equivalent transformer arrangement shown, prove that for high frequencies the current ratio I_1/I_2 is approximately

$$\frac{L_2}{M}\left[1 + \frac{R_2^2}{2\omega^2 L_2^2}\right].$$



46 In the multiple coupled circuits shown,
$L_1 = 100\ \mu$H,
$L_2 = 150\ \mu$H,
$L_3 = 120\ \mu$H,
$L_4 = 130\ \mu$H,
$L_5 = 160\ \mu$H,
$L_6 = 200\ \mu$H,

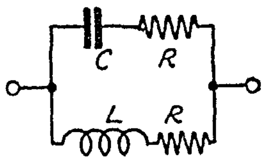


$M_1 = 50\ \mu$H, $M_2 = 80\ \mu$H, $M_3 = 140\ \mu$H, $R_1 = 40\ \Omega$, $R_2 = 60\ \Omega$, $R_4 = 80\ \Omega$, $R_6 = 30\ \Omega$, $e = 6\underline{/0^\circ}$ V, $\omega = 2 \times 10^6$. Find the current flowing in R_6.

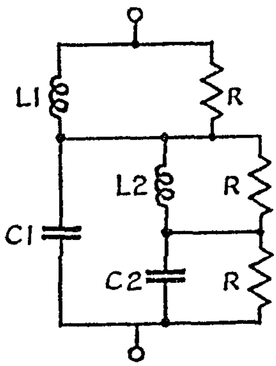
[3·69$\underline{/115{\cdot}8^\circ}$ mA.]

Constant Impedance Networks

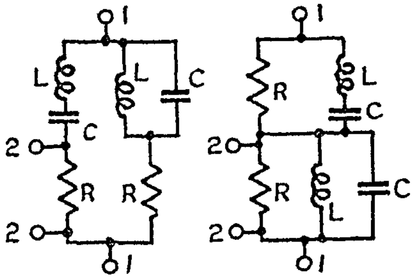
47 Prove that the impedance of the circuit shown is equal to R and is independent of the frequency if $R^2 = L/C$. Find its value when $L = 0{\cdot}02$ H, $C = 100\ \mu$F. Sketch a graph of the current in each branch to a logarithmic base of frequency. [14·14 Ω.]



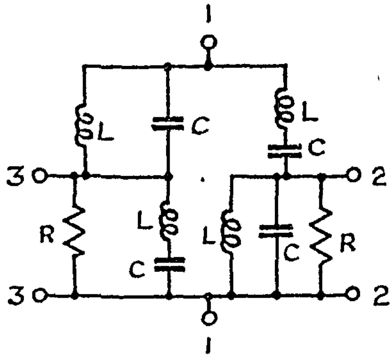
48 Find the relationships between L_1, L_2, C_1, C_2 and R such that the impedance of the circuit shown is independent of frequency. What is this impedance?
[$L_1/C_1 = L_2/C_2 = R^2$; R.]



49 In both the networks shown, prove that the impedance between the terminals 11 is equal to R and independent of frequency provided that $L/C = R^2$. Also show that the ratio of the voltage developed across the terminals 22 to that applied at the terminals 11 is as $1 : \sqrt{1 + F^2}$, where $F = \omega/\omega_o - \omega_o/\omega$ and $\omega_o^2 = 1/LC$.

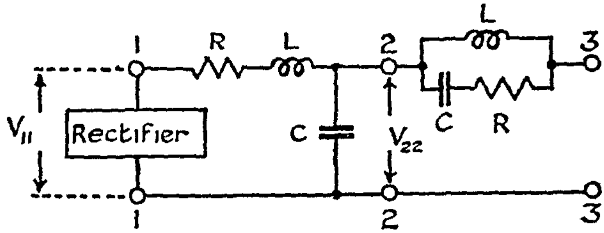


50 In the circuit shown find (*a*) the condition such that the impedance between the terminals 11 is independent of frequency and (*b*) state its value. Under these conditions find expressions for (*c*) the ratio v_{22}/v_{11} and (*d*) v_{33}/v_{11} in terms of F where $F = \omega/\omega_o - \omega_o/\omega$ and $\omega_o^2 = 1/LC$.



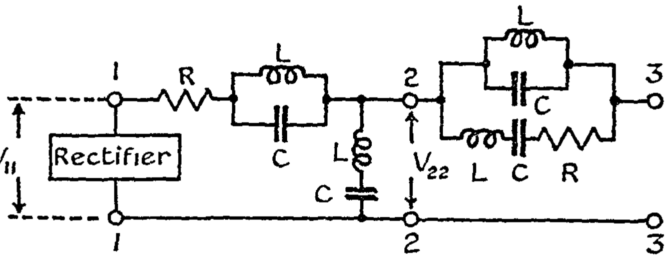
[(*a*) $L = 2CR^2$; (*b*) R; (*c*) $1/\sqrt{1 + F^4}$; (*d*) $1/\sqrt{1 + 1/F^4}$.]

51 A rectifier delivers d.c. power at the terminals 11 and has a smoothing filter comprising an inductor L and capacitor C between the terminals 11 and 22. The resistor R represents the source impedance of the rectifier together with the resistance of the inductor. The network included between the terminals 22 and 33 has components of the same value as those in the first section. Find (a) the relationship between L, C and R such that the source impedance looking into the terminals 33 is independent of frequency. (b) Find the ratio by which a ripple voltage, at radial frequency ω, across the terminals 11 is reduced at the terminals 22. (c) What is the source impedance under these conditions?



[(a) $2L/C = R^2$; (b) $v_{22}/v_{11} = 1/\sqrt{1+\omega^4L^2C^2}$; (c) R.]

52 A rectifier delivers d.c. power at the terminals 11 and has the smoothing filter shown between the terminals 11 and 22, R representing the impedance of the rectifier. The network included between the terminals 22 and 33 has components of the same value as those in the smoothing filter. Find (a) the

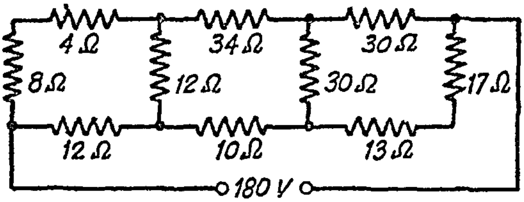


relationship between L, C and R such that the source impedance looking into the terminals 33 is independent of frequency. (b) What is this impedance? (c) If the rectifier has a ripple voltage v_{11}, express the ratio v_{22}/v_{11} in terms of F where $F = \omega/\omega_0 - \omega_0/\omega$, ω being the radial frequency and $\omega_0{}^2$ being equal to $1/LC$.

[(a) $2L/C = R^2$; (b) R ; (c) $v_{22}/v_{11} = F^2/\sqrt{F^4+1}$.]

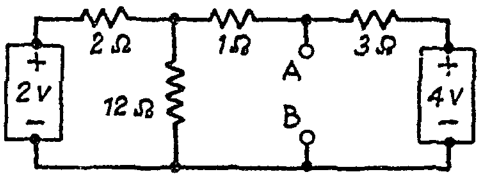
Network Theorems

53 Find the current in the 10 Ω resistor in the network shown by

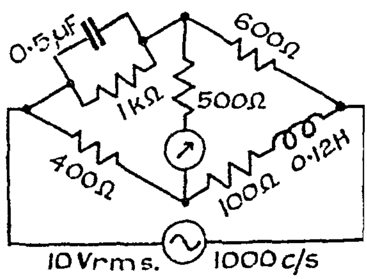


(*a*) star-mesh conversion, (*b*) Helmholtz's (Thevenin's) theorem
[4 A.]

54 Use (*a*) Thevenin's (Helmholtz's) theorem and (*b*) the Principle of Superposition to find the current in a 2-Ω resistor connected between *A* and *B* in the circuit shown.
[0·82 A.]



55 An alternating-current bridge is arranged as shown. Use Thevenin's theorem to find the current flowing in the detector.
[0·65/76° mA.]



CHAPTER IX

HARMONICS

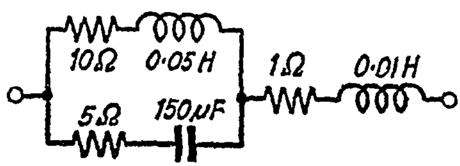
Single-phase Harmonics

1 An alternating voltage $v = 100 \sin(\omega t + 0\cdot5) + 20 \sin(3\omega t - 0\cdot2) + 10 \sin(5\omega t + 1\cdot5)$ is applied across a condenser of 30 μF capacitance. Derive an expression for the current and calculate its r.m.s. value. What is the percentage error in calculating the capacitance from the r.m.s. voltage and current? $\omega = 314$.
[$i = 100\,\omega C \sin(\omega t + 2\cdot07) + 60\,\omega C \sin(3\omega t + 1\cdot37) + 50\,\omega C \sin(5\omega t + 3\cdot07)$; 0·85 A ; 24% high.]

2 A voltage $v = 1\,000 \sin \omega t + 100 \sin 3\omega t$ is applied to a circuit of resistance 100 Ω and inductance 0·5 H. Find the power factor if $\omega = 2\pi 50$. [0·54.]

3 A condenser of capacitance 20 μF is connected to a supply which gives a voltage wave having two sinusoidal components :— $e_1 = 200 \sin 2\pi 50t$, and $e_{15} = 20 \sin 2\pi 750t$. Calculate the root-mean-square value of the current due to each component. Draw one complete cycle of the voltage and of the current wave. [0·89 A ; 1·33 A.]

4 A voltage of $150 + 440 \sin 314\, t$ is applied to the circuit shown. Determine the r.m.s. current in each branch, the total power, and the power factor of the circuit.



[$I_A = 21\cdot1$ A ; $I_B = 13\cdot7$ A ; $I_C = 18\cdot0$ A ; 5·7 kW ; 0·91.]

5 A potential difference of $100 + 200\sqrt{2} \sin 314\, t$ is applied to a circuit having a resistance of 10 Ω in series with a reactance of 15·7 Ω. Find the power expended and the impedance and power factor of the circuit. [2 155 W, 15·25 Ω, 0·656.]

6 A 50-c/s, 12 000-V alternator with reactance of 6·5 Ω and resistance 1 Ω is connected to a cable of gross capacity 2·9 μF. The wave form of the alternator is given by $e = 17\,000 \sin 2\pi 50t + 170 \sin 13\,(2\pi 50t)$. Calculate the maximum voltage impressed on the cable on open-circuit. [81 400 V.]

7 A voltage $e = 250 \sin \omega t + 50 \sin \left(3\omega t + \frac{\pi}{3}\right) + 20 \sin \left(5\omega t + \frac{5\pi}{6}\right)$ is applied to a circuit of resistance 20 Ω and inductance 0 05 H. Derive (*a*) an expression for the current, (*b*) the r.m.s. value of the current and of the voltage, (*c*) the total power supplied, and (*d*) the power factor : $\omega = 314$.
[(*a*) $i = 9{\cdot}85 \sin (\omega t - 0{\cdot}664) + 0{\cdot}978 \sin (3\omega t - 0{\cdot}122) + 0{\cdot}247 \sin (5\omega t + 1{\cdot}30)$; (*b*) 7 A, 181 V ; (*c*) 998 W ; (*d*) 0·79.]

8 A voltage wave containing a fundamental and a 30% third harmonic is applied to (*a*) an inductive circuit, (*b*) a capacitive circuit. Compare the reactance, calculated on a sinewave assumption, with that when no harmonic is present.
[(*a*) increased by 4% ; (*b*) reduced by 22%.]

9 A 0·225-H inductor is joined in series with a 5-μF capacitor. Find the supply frequency in order that the resonant frequency of the circuit is that of (*a*) the fundamental, (*b*) the third harmonic, and (*c*) the fifth harmonic, of the supply.
[(*a*) 150 c/s ; (*b*) 50 c/s ; (*c*) 30 c/s.]

10 A 12-kV, 50-c/s, 1-phase alternator is connected to an unloaded cable having a capacitance of 2·03 μF. If the total circuit inductance is 0 2 H, what harmonic in the supply voltage would produce resonance in the circuit ? [5th.]

11 A current $i = 200 \sin \omega t + 70 \sin 3\omega t + 30 \sin 5\omega t$ flows through a condenser having an effective series resistance of 5 Ω and a capacitance of 20 μF. Find the amplitude of the voltage of (*a*) the fundamental ; (*b*) each harmonic : $\omega = 314$.
[(*a*) 31 800 V ; (*b*) 3 730 V, 967 V.]

12 An alternating voltage represented by the expression $v = 200 \sin 2\pi 50t + 20 \sin 2\pi 150t$ is applied to a coil of 25 Ω resistance and 0·02 H inductance connected in series with a condenser of 40 μF capacitance. Derive an expression for the current and calculate the power factor of, and the power input to, the circuit.
[$i = 2{\cdot}58 \sin (2\pi 50t + 1{\cdot}25) + 0{\cdot}765 \sin (2\pi 150t + 0{\cdot}297)$; 0·34 ; 90·7 W.]

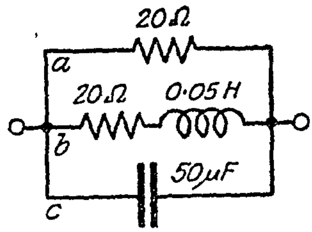
13 A current containing a fundamental, a 3rd and a 5th harmonic of amplitudes 50, 10, and 5 A respectively flows in a coil. If the power input to the coil is negligible and the r.m.s. voltage across it is 75 V, what is the inductance of the coil ? The frequency of the fundamental is 50 c/s. [0 00538 H.]

14 A voltage $v = 2\,000 \sin \omega t + 400 \sin 3\omega t + 100 \sin 5\omega t$ is applied to a series circuit comprising a resistance of 10 Ω, a capacitance of 80 μF, and a variable inductance. Find the

value of the inductance which will give resonance with the 3rd harmonic. What are the r.m.s. values of current and voltage when this value of inductance is in circuit? $\omega = 300$.

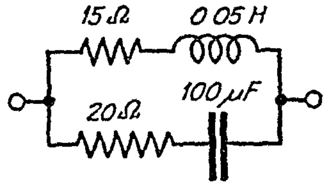
[0·041 H; 31·75 A; 1 445 V.]

15 A voltage $v = 200 \sin 314t + 50 \sin 942t$ is applied to the terminals of the circuit shown. Determine the r.m.s. value of the current in each branch, the total current, the power input, and the power factor.



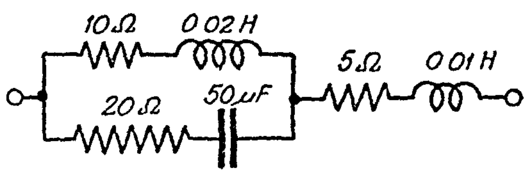
[$I_a = 7·3$ A, $I_b = 5·61$ A, $I_c = 2·78$ A; 11·72 A; 1 690 W; 0·98.]

16 A voltage $v = 300 \sin 314t + 60 \sin 942t$ is applied to the terminals of the circuit shown. Determine the r.m.s. value of the current in each branch and the total current.



[9 76 A, 5·94 A, 10·2 A.]

17 A voltage $v = 250 \sin 314t + 50 \sin 942t$ is applied to the terminals of the circuit shown. Determine the r.m.s. value of the total current, the power input, and the power factor.



[9·95 A, 1·605 kW, 0·89.]

18 An e.m.f. wave consists of a fundamental plus a higher harmonic the amplitude of which is 30% of the fundamental. It is applied to a circuit composed of R, L and C such that the phase angle between the fundamental of e.m.f. and current is 37°. Find the limits between which the power factor can vary, for different values of the higher harmonic frequency.

[0·819–0·766.]

19 An alternator voltage wave has one harmonic equal to 30% and another harmonic equal to 20% of the fundamental. Find the root-mean-square value of the voltage wave in terms of the fundamental.

[106·3%.]

20 A voltage wave consisting of a fundamental and a third harmonic has the following instantaneous values:

θ°	0	20	40	60	80	100	120	140	160
V	−17·32	16 88	64 28	103 92	115 80	94 48	69 28	46 96	30 20

Find the maximum value of the fundamental and of the third harmonic, and determine the phase-angle between them. [100 V ; 20 V ; 40°.]

21 A circuit with coil $L = 1$ H and $R = 20\ \Omega$ in series with $C = 20\ \mu$F is connected across a voltage consisting of positive halves only of a 50 c/s wave. If $\hat{v} = 500$ V, find (*a*) mean voltage, (*b*) second harmonic voltage, across condenser. [(*a*) 159·2 ; (*b*) 10·9 V.]

22 Draw one complete cycle of each of the following waves :

(a) $e = 100 \sin \omega t + 30 \sin 3\ \omega t$.
(b) $e = 100 \sin \omega t - 30 \sin 3\ \omega t$.
(c) $e = 100 \sin \omega t + 30 \cos 3\ \omega t$.
(d) $e = 100 \sin \omega t - 30 \cos 3\ \omega t$.
(e) $e = 100 \sin \omega t + 20 \sin 5\ \omega t$.
(f) $e = 100 \sin \omega t - 20 \cos 5\ \omega t$.
(g) $e = 100 \sin \omega t + 50 \sin 2\ \omega t$.
(h) $e = 100 \sin \omega t - 25 \cos 4\ \omega t$.

Three-phase Harmonics

23 If the phase voltage of a star-connected, 3-phase alternator contains a fundamental of 200-V amplitude, and 3rd, 5th, 7th, and 9th order harmonics of 40-, 25-, 20-, and 10-V amplitude respectively, calculate the ratio of line to phase voltage. [1·70.]

24 A 16-pole, 3-phase alternator has 144 slots. The winding is short-chorded by 1 slot-pitch and star-connected. Assuming a field form of $100 \sin \theta + 25 \sin 3\theta + 20 \sin 5\theta$, find the harmonics in (*a*) the phase voltage, (*b*) the line voltage, as a percentage of the fundamental. Determine also the phase and line r.m.s. values as a percentage of the fundamental.
[(*a*) $V_3 = 15{\cdot}25\%$, $V_5 = 2{\cdot}96\%$, $V_{phase} = 101{\cdot}2\%$;
(*b*) $V_5 = 2{\cdot}96\%$, $V_{line} = 100{\cdot}04\%$.]

25 Prove that no slot ripples can appear in the phase voltage wave of the following 3-phase alternators :—
(*a*) 16-pole machine with 147 slots.
(*b*) 16-pole machine with 150 slots.
At what slots will the phases start (or finish) in each case ?
[(*a*) 1, 50, 99 ; (*b*) 1, 26, 51 or 76, 101, 126.]

26 A 3-phase, star-connected alternator with a phase voltage $v = 800 \sin 314t + 200 \sin 942t + 100 \sin 1\,570t$ supplies three mesh-connected reactors each of resistance 20 Ω and inductance 0·1 H. Calculate the power factor and the power supplied to the load. [0·53, 41·6 kW.]

27 A 3-phase, 50-c/s alternator has a mesh-connected armature winding. The resistance and inductance per phase are 0·06 Ω

and 0·9 mH respectively, and the no-load phase voltage contains 3rd, 7th, 9th, and 15th harmonics having amplitudes of 4, 3, 2, and 1 per cent respectively of that of the fundamental. Calculate the circulating current on open-circuit when the excitation is such that the amplitude of the fundamental is 1 500 V. [50·6 A.]

28 The phase voltage of a 750-kW, 2 200-V, 3-phase, 50-c/s alternator has a 5% third-harmonic. What is the circulating current on normal voltage if the machine is mesh-connected? The resistance and inductance per phase are 0·25 Ω and 2·23 mH respectively. Express the loss due to the circulating current as a percentage of full-load output. [52 A ; 0 27%.]

29 The e.m.f. of one phase of a 50-c/s, 3-phase, mesh-connected alternator is $565 \sin \theta + 50 \sin 3\theta - 30 \sin 5\theta$ V. If the resistance and inductance per phase are 0 25 Ω and 5 mH, find the r.m.s. value of (*a*) the current circulating in the windings, and (*b*) the current in a 50-μF condenser connected across a pair of lines. [(*a*) 7·50 ; (*b*) 6·95 A.]

30 The phase-e.m.f. wave of a 3-phase alternator consists of a fundamental and a 30% third harmonic. If the amplitude of the fundamental is 3 100 V, calculate the r.m.s. value of the line voltage when the windings are connected (*a*) in star, (*b*) in mesh. If the leakage reactance of each phase is 10 0 Ω at 50 c/s, determine (*c*) the circulating current in case (*b*). Neglect resistance. [(*a*) 3 800 V ; (*b*) 2 190 V ; (*c*) 21·9 A.]

31 Three similar star-connected coils each of resistance 25 Ω and inductance 0 05 H are supplied from a 3-phase, star-connected alternator. The phase voltage is $v = 360 \sin \omega t + 60 \sin 3\omega t + 50 \sin 5\omega t$, where $\omega = 2\pi 50$. The star-points of the load and of the alternator are connected. Calculate the effective values of (*a*) the line current, (*b*) the line voltage, (*c*) the current in the neutral conductor, and (*d*) the total power absorbed by the load. What do these values become if the neutral wire is disconnected?
[(*a*) 8·67 A ; (*b*) 445 V ; (*c*) 2·88 A ; (*d*) 5 63 kW.
(*a*) 8·64 A ; (*b*) 445 V ; (*c*) 0 ; (*d*) 5·59 kW.]

32 Two 20 000-kVA, 10 000-V, 3-phase, star-connected alternators have each an armature reactance of 20% (each phase). The induced electromotive force has a third harmonic of amplitude 10% of the fundamental. The machines operate in parallel, and the earth connexion of each star point is made through a 10% reactance. Calculate (*a*) the circulating current in the earth connexions due to the third harmonic ; (*b*) the steady value of the fault current when one of the bus-bars develops a dead earth. [(*a*) 232 A ; (*b*) 7 695 A.]

IX. HARMONICS

33 The wave-form of the phase voltage of a 3-phase, star-con nected alternator is as follows :—

Time-angle, deg .	0	18	36	54	72	90	108	126
Voltage, V .	0	700	750	730	1 250	1 600	1 250	730

Draw the line-voltage wave and obtain the value of its 5t harmonic. *[875 V r.m.s.

CHAPTER X

TRANSIENTS

D.C. Transients : R and L

1 A circuit of resistance R ohms and inductance L henrys has a direct voltage applied to it. The current reaches 0·632 of its final steady value I in 1 sec. After the current has reached its final steady value the circuit is suddenly short-circuited. What will be the value of the current in terms of I 2 sec later ?
[0·135 I.]

2 Define the time-constant of a circuit. Draw the curve for the rise of current in a circuit consisting of a reactor of inductance 10 H and resistance 2 Ω, when a direct voltage of 100 V is applied. What is the value of the current 7·5 sec after switching on ?
[38·8 A.]

3 A direct voltage of 100 V is applied to a coil of resistance 10 Ω and inductance 10 H. What is the value of the current 0·1 sec after switching on, and the time taken for the current to reach one-half of its final value ? [0·95 A ; 0·69 sec.]

4 A coil of resistance 20 Ω and inductance 0·5 H is switched on to a direct-current, 200-V supply. Calculate the rate of change of the current (*a*) at the instant of closing the switch, and (*b*) when $t = L/R$. Find also (*c*) the final steady value of the current. [(*a*) 400 A/sec ; (*b*) 147 A/sec ; (*c*) 10 A.]

5 Calculate the voltage at the terminals of a coil of resistance 10 Ω and inductance 15 H at the instant when the current is 12 A and (*a*) increasing, (*b*) decreasing, at the rate of 6 A/sec. (*c*) Find the stored energy under both conditions.
[(*a*) 210 V ; (*b*) 30 V ; (*c*) 1 080 J.]

6 The resistance and inductance of a series circuit are 5 Ω and 20 H respectively. At the instant of closing the supply switch, the current increases at the rate of 4 A/sec. Calculate (*a*) the applied voltage, (*b*) the rate of growth of current when 5 A flows in the circuit, (*c*) the stored energy under both conditions.
[(*a*) 80 V ; (*b*) 2·75 A/sec ; (*c*) 0, 250 J.]

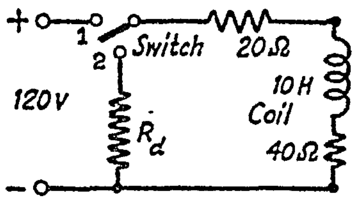
7 A coil of inductance 10 H and resistance 2 Ω is supplied at 20 V. Find (*a*) the time-constant of the circuit, (*b*) the maximum flux-linkages, (*c*) the maximum value of stored energy, (*d*) the

average e.m.f. induced when the circuit is opened if the current falls to zero in 0 5 sec.
[(*a*) 5 sec ; (*b*) 10^{10} flux-linkages or maxwell-turns (100 Wb-turns) ; (*c*) 500 J ; (*d*) 200 V.]

8 A coil has a resistance of 50 Ω and a time-constant of 1·25 sec. If the coil were rewound with the same weight of wire of half the diameter, calculate (*a*) the inductance; (*b*) the time-constant, and (*c*) the stored energy, when connected to a 600-V d.v. supply.
[(*a*) 1 000 H ; (*b*) 1·25 sec ; (*c*) 281 J.]

9 The shunt field winding of a 100-V generator has a resistance of 44 Ω and an inductance of 11 H. If 100 V be applied to the field terminals, how long will it take the current to reach (*a*) 0·95, (*b*) 0·99, of its final steady value ? Assume the inductance to be constant. (*c*) If the exciting current be reduced to zero in $\frac{1}{20}$ sec, calculate the average electromotive force induced.
[(*a*) 0·75 sec ; (*b*) 1·15 sec ; (*c*) 500 V.]

10 In the circuit shown, the switch is initially in position 1. (*a*) For $R_d = 500\ \Omega$, find the voltage across the field coil at the instant at which the switch is changed to position 2 ; (*b*) calculate the value of R_d for the voltage across the coil to be 120 V at the instant of switching.



(*c*) With R_d of the value found in (*b*), find the time taken to dissipate 95% of the stored energy.
[(*a*) 1 040 V ; (*b*) 40 Ω ; (*c*) 0·15 sec.]

11 A coil of resistance 10 Ω and inductance 15 H is connected across a 100-V, direct-current supply. Calculate the energy supplied to the coil from the mains in the first 5 secs. If the coil be disconnected from the mains after 5 secs, and immediately short-circuited, determine the energy recovered as heat.
[8 558 J, 697 J.]

12 A telegraph circuit consists of a battery with an electromotive force of 12 V and internal resistance 1 Ω, a non-inductive line of 45 Ω resistance, and a relay of 18 Ω resistance and 1·2 H inductance. If the armature of the relay is not attracted until the current rises to 0·05 A, what interval of time will elapse between the closing of the circuit and when the relay begins to operate ? [0·0058 sec.]

13 A six-cylinder engine has a 12-V coil-ignition system. The resistance in the primary circuit is 4 Ω and the inductance 0·01 H. (*a*) Find the time-constant of the primary circuit. (*b*) Assuming equal make and break periods, find the engine speed when the make period is equal to the time-constant, and

(c) the current at the end of the make period. Indicate the effect on the primary current of increasing and of decreasing the number of engine cylinders, and of lowering the battery voltage.
[0·0025 sec ; 4 000 r.p.m. ; 1·896 A.]

14 A 120-V, direct-current clock system is controlled from a master clock which sends successive impulses round the circuit. The original constants of the circuit are ; resistance, 20 Ω ; time of contact, 0·05 sec ; current at which cut-out operates, 0·5 A. Determine the additional resistance and inductance required if the system is to be supplied from 250-V mains.
[21·7 Ω ; 12·5 H.]

15 A battery of E volts is supplying a steady current to a series circuit of total resistance R ohms and inductance L henrys. A part R_1 of the total resistance is suddenly short-circuited. Derive (a) an expression for the current flowing in the battery subsequent to this operation. If $E = 100$ V ; $R = 20$ Ω ; $R_1 = 10$ Ω ; and $L = 2$ H, plot the current/time curve and determine the current (b) 0·1 sec, (c) 0·5 sec, after short-circuit.

$$\left[(a)\ i = \frac{E}{R - R_1}\left(1 - \frac{R_1}{R}e^{-\frac{(R-R_1)t}{L}}\right);\right.$$

$$\left.(b)\ 6{\cdot}97\ \text{A};\ (c)\ 9{\cdot}59\ \text{A.}\right]$$

16 A direct voltage applied to a coil of $L = 1$ H and $R = 10$ Ω is suddenly changed from V_1 to V_2. Derive an expression for the current during the transient. Plot the current/time curve for (a) $V_1 = 100$ V, $V_2 = 200$ V ; (b) $V_1 = 200$ V, $V_2 = 100$ V. Calculate its value in each case for (i) $t = 0\ 05$ sec, (ii) $t = 0{\cdot}5$ sec.

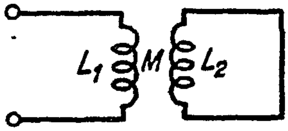
$$\left[i = \frac{V_2}{R} + \left(\frac{V_1}{R} - \frac{V_2}{R}\right)e^{-\frac{R}{L}t};\ (a)\ \text{(i) } 18{\cdot}94,\ \text{(ii) } 19{\cdot}93\ \text{A};\right.$$

$$\left.(b)\ \text{(i) } 16{\cdot}06,\ \text{(ii) } 10{\cdot}07\ \text{A.}\right]$$

17 A flux of 5 megalines (50 mWb) links all the turns of a 1 000-turn coil when the current is 4 A. A reversing voltage having a steady value of 100 V is suddenly applied when this current is flowing. Calculate the magnitude of the current (a) 0·25, (b) 0·5, (c) 2 sec after the application of the reversing voltage. The inductance of the coil is constant and its resistance is 50 Ω.
[(a) 0·22 A ; (b) 1·19 A reversed ; (c) 1·998 A reversed.]

18 A circuit of resistance 20 Ω and inductance 0·2 H in series has a direct voltage of 250 V suddenly applied to it. Find the voltage-drop across the inductance at the instant of switching on and at 0·01 sec later. What is the number of flux-linkages at these instants ?
[250 V, 92 V ; 0 ; 1·58 × 10^8 maxwell-turns (1·58 Wb-turns).]

19 In the circuit shown, $L_1 = 1$ H, $L_2 = 2$ H and $M = 1 \cdot 2$ H. Ignoring resistance, find the expression for the energy stored t sec after the circuit is connected to a direct voltage of 10 V. [$179t^2$ J.]



D.C. Transients : R and C

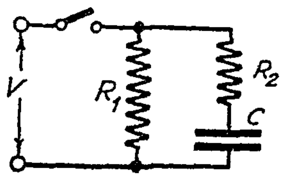
20 A condenser is charged through a large non-reactive resistance by a battery of constant voltage V. Derive an expression for the instantaneous charge on the condenser. If the condenser has a capacitance of 10 μF and the resistance is 1 MΩ, calculate the time taken for the condenser to receive 90% of its final charge. Draw the charge/time curve.

$$\left[q = Q\left(1 - e^{-\frac{t}{RC}}\right) ;\ 23 \text{ sec.}\right]$$

21 A condenser of 5 μF capacitance is connected to a constant-voltage source through a resistance of 2 MΩ. Calculate the time taken for the condenser to lose (*a*) 50%, (*b*) 63·2%, (*c*) 95% of its charge, when the voltage source is short-circuited.
[(*a*) 6·93 sec; (*b*) 10 sec; (*c*) 29·9 sec.]

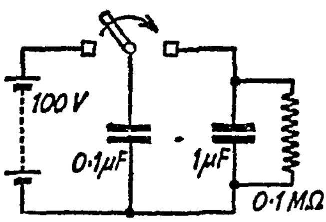
22 An 8-μF condenser is connected through a 1·5-MΩ resistance to a direct-current source. After being on charge for 24 secs the condenser is disconnected and discharged through a spark-gap. Determine what percentage of the energy input from the supply is liberated in the spark. [43·2%.]

23 Find how long it takes after the key is closed before the total current from the supply reaches 25 mA, when $V = 10$ V, $R_1 = 500\ \Omega$, $R_2 = 700\ \Omega$, and $C = 100\ \mu$F. [0·0735 sec.]



24 A resistance is connected across the terminals of a 20 μF condenser which has been previously charged to a potential difference of 500 V. If the potential difference falls to 300 V in 0·5 min, calculate the resistance in megohms. [2·94 MΩ.]

25 By means of the rotary switch operating at 6 000 rev per min, the 0·1-μF condenser is connected in turn to the battery and to the 1-μF condenser, each for a very short time. What is the final voltage attained by the 1-μF condenser? Calculate the voltage fluctuation on this condenser.



[46·5 V : 4·8 V.]

26 A 50-μF condenser and a 20 000-Ω resistor are connected in series across a battery of 100 V at instant $t = 0$. At instant $t = 0{\cdot}5$ sec, the voltage is suddenly increased to 150 V. Find the charge on the condenser at instant $t = 0{\cdot}75$ sec.
[$3{\cdot}19 \times 10^{-3}$ C.]

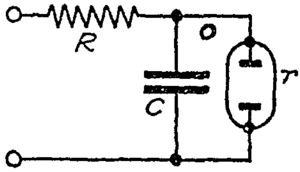
27 A cable 10 km long and of capacitance 2·5 μF discharges through its insulation resistance of 50 MΩ. By what percentages will the voltage have fallen 1, 2, and 5 min respectively after disconnection from its bus-bars? [38% ; 61·7% ; 90·9%.]

28 A 2-μF capacitor and an electrostatic voltmeter are connected to a 500-V supply. After disconnexion, the voltmeter reading falls to 250 V in 15 min. Find the leakage resistance. When the insulation of a cable is connected across the charged capacitor the voltage drops to 250 V in 10 min. Calculate the insulation resistance of the cable. [653 MΩ ; 1 300 MΩ.]

29 A condenser, across which is connected an electrostatic voltmeter of infinite resistance and of 10 $\mu\mu$F capacitance, is charged to a potential of 100 V. After disconnecting from the charging source the potential falls from 100 V to 50 V in 20·8 sec. When an additional 50 $\mu\mu$F air condenser is connected in parallel with the test condenser, the voltage falls from 100 V to 50 V in 55·5 sec. What is the insulation resistance of the test condenser? [10^{12} Ω.]

30 A resistance R and a 2-μF condenser are connected in series across a 200-V direct supply. Across the condenser is a neon lamp that strikes at 120 V. Calculate R to make the lamp strike 5 sec after the switch has been closed. If $R = 5$ MΩ, how long will it take the lamp to strike? [2·7 MΩ ; 9·2 sec.]

31 Determine the frequency with which the neon lamp glows when connected in the circuit shown. The lamp glows at V_2 and over, and is extinguished at V_1. The direct voltage V applied to the circuit is such that $V > V_2 > V_1$. The "equivalent resistance" r is negligible in comparison with R when the lamp is conducting and infinite when the lamp is non-conducting.



$$\left[1 \Big/ \left(RC\log_e \frac{V - V_1}{V - V_2}\right).\right]$$

D.C. Transients : R, L and C

32 A condenser of capacitance C charged to a potential V is connected to an inductance L in series with a resistance R. Evolve expressions for the current in the circuit after time t, when R^2 is (*i*) greater than $4L/C$, (*ii*) less than $4L/C$. If C

$= 1\ \mu F$, $L = 100\ \mu H$, find what time elapses before the instantaneous charge on the condenser is 10% of the original charge when R is equal to (a) 20 Ω, (b) 1 Ω.

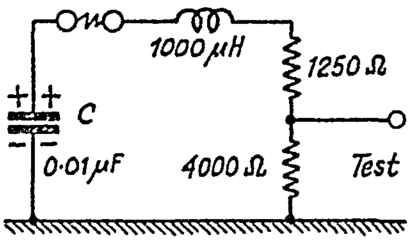
$$\left[(i)\ i = \frac{2V}{\sqrt{R^2 - 4L/C}} e^{-\frac{R}{2L}t} \sinh \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \cdot t\ ; \right.$$

$$(ii)\ i = \frac{2V}{\sqrt{4L/C - R^2}} e^{-\frac{R}{2L}t} \sin \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \cdot t\ ;$$

(a) 39 μsec ; (b) 15·1 μsec.]

33 A field circuit has an inductance $L = 9$ H and small resistance. When a steady current of 1 A is flowing in it, the coil is short-circuited across a 100-μF capacitor. Find the maximum voltage across the capacitor and the frequency of the discharge current. [300 V ; 5·3 c/s.]

34 At the instant of spark-gap breakdown a surge generator is represented by the circuit shown, in which the generator capacitance C is charged to 200 kV. Obtain an expression for the voltage across the test object (the surge impedance of which may be taken as infinite). What is the time to reach peak test voltage, and what is the initial rate of rise of the voltage ?



[153 ($e^{-0.017t} - e^{-5.23t}$) kV, t in μsec ; 1·1 μsec ; 800 kV/μsec.]

35 Calculate the capacitance to be shunted across a reactor of inductance 0·1 H and negligible resistance to prevent the induced voltage rising above 1 000 V when switching off a steady current of 10 A. [10 μF.]

A.C. Transients : R and L

36 A 50-c/s sinusoidal voltage of amplitude 400 V is applied to a series circuit of resistance 10 Ω and inductance 0·1 H. Find an expression for the value of the current at any instant after the voltage is applied, assuming that the voltage is zero at the instant of application. Draw curves of both transient and resultant current, and calculate the value of the transient current 0·02 sec. after switching on.

$$\left[i = 12{\cdot}1 \left\{ 0{\cdot}958 e^{-100t} + \sin(314t - 1{\cdot}26) \right\} ;\ 1{\cdot}56\ \text{A.} \right]$$

37 A 44-kV, 60-c/s voltage is applied to impedances $Z_1 = 15{\cdot}4 + j\ 12{\cdot}6\ \Omega$ and $Z_2 = 19{\cdot}3 + j\ 14{\cdot}8\ \Omega$ in series. If Z_2 is short-circuited at the instant of positive peak current, show that

the current $i = 3\,120\,(1 - e^{-460t})\sin(377t - 0{\cdot}68) + 1\,414\,e^{-460t}$. Draw graph from $t = 0$ to $t = 0{\cdot}01$ sec.

38 A voltage $v = \hat{v}\sin(500\pi t)$ is applied to a series circuit of resistance 25 Ω and inductance 0·2 H. Calculate the ratio of the maximum value to which the current rises to the steady-state maximum value when the voltage is applied at the instant $t = 0{\cdot}002$ sec. *[1·78 : 1.]

39 A 240-V, 50-c/s, sinusoidal voltage is applied to a coil having $R = 1\ \Omega$ and $X = 4\ \Omega$. Find the energy stored in the coil 1/400 sec after the voltage has passed through zero and is increasing. [11·46 J.]

A.C. Transients : R and C

40 A voltage $v = 400\sin(2\pi 50t)$ is applied to a resistance of 20 Ω and a capacitance of 100 μF in series. Plot curves of current i, and charge q, when this voltage is applied at the instant $t = 0{\cdot}00214$ sec. Calculate the value of the transient current, and of the resultant current, (*a*) 0·001 sec, and (*b*) 0·005 sec, after switching on.
[(*a*) 1·13 A ; 10·8 A ; (*b*) 0·152 A ; − 1·018 A.]

A.C. Transients : R, L and C

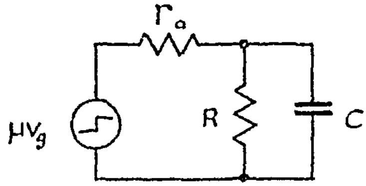
41 A 50-c/s, 3-phase generator with an earthed neutral has an inductance of 1·58 mH per phase, and is connected to bus-bars by means of an oil circuit-breaker. The capacitance to earth of the circuit between the generator and the switch is 0·0028 μF per phase. Due to a short-circuit on the bus-bars, the switch opens when the r.m.s. current is 7 000 A. Draw a curve showing the re-striking voltage which appears across the switch and determine the maximum rate of voltage rise.
[2 340 V per μsec.]

42 From the following data of a 50-c/s generator : e.m.f. to neutral, 7·5 kV r.m.s., reactance of generator and connected system, 4·0 Ω ; distributed capacitance to neutral, 0·01 μF ; resistance, negligible ; find (*a*) the maximum voltage across the contacts of a circuit-breaker when it breaks a short-circuit current at a zero-pause. Calculate also (*b*) the frequency of the transient oscillation, and (*c*) the average rate of rise of voltage up to the first peak of the oscillation.
[(*a*) 21·2 kV ; (*b*) 14·1 kc/s ; (*c*) 600 000 kV/s.]

43 In a s.c. test, with earthed neutral, on a 132-kV, 3-phase circuit-breaker, the power-factor of the fault was 0·3, the recovery voltage was 0·95 of full-line value, the breaking current was symmetrical, and the re-striking transient had a natural frequency of 16 000 c/s. Estimate the rate of rise of the re-striking voltage. [6·25 kV/μsec.]

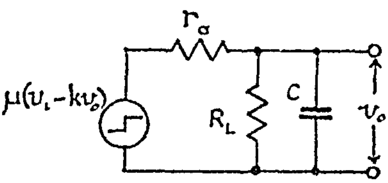
Transients in Valve Amplifiers

44 The equivalent circuit of a triode amplifier is shown. The valve has a slope resistance and amplification factor of r_a and μ respectively, and the anode load comprises a resistor R in parallel with a capacitor C. The input voltage between grid and cathode remains steady before time $t = 0$ and it then falls instantaneously by V volts after which it remains constant. Find (*a*) an expression for the resultant change of voltage across the anode load, and (*b*) modify this expression to cover the case when the triode is replaced by a pentode of very high slope resistance and of mutual conductance g_m.



$$\left[(a)\ V\frac{\mu R}{R+r_a}\left(1-e^{-\frac{R+r_a}{CRr_a}t}\right);\quad (b)\ Vg_mR\left(1-e^{-\frac{t}{CR}}\right).\right]$$

45 An amplifier employing negative feedback has a proportion of the output, kv_o, applied at the input terminals, the equivalent circuit being as shown (*a*) Find an expression for the current flowing in the capacitor C due to the rising edge of a pulse of amplitude v_i applied at the input. Assume the rise time of the incoming pulse to be infinitely short. (*b*) Find an expression for the time taken for the current in the capacitor to fall to one-tenth of its original value.



$$\left[(a)\ \frac{\mu v_i}{r_a}e^{-\frac{r_a+(1+\mu k)R_L}{Cr_aR_L}t};\quad (b)\ \frac{2\cdot 3\ Cr_aR_L}{r_a+(1+\mu k)R_L}.\right]$$

46 The input voltage to the pentode amplifier shown in figure 10 remains constant until time $t = 0$ and then rises exponentially according to the law $V(1 - e^{-at})$. If the CR value of the anode circuit is equal to $1/b$, find an expression for the output voltage when (*a*) $b = 2a$; (*b*) $b = a$; (*c*) $2b = a$. Evaluate the expressions obtained for values of $t = 1/a$, $2/a$, $3/a$ up to $10/a$ and plot curves of the output voltage as a fraction of the final value g_mVR. Plot also a curve showing the growth of the input voltage as a fraction of V. Take both input and output waveforms as rising positively to facilitate comparison. For each of the cases (*a*), (*b*), (*c*), give the percentage of the final value to which the output voltage has risen when the input has reached 90% of its final value.

[(*a*) $g_mVR(1 + e^{-2at} - 2e^{-at})$, 82% ;
(*b*) $g_mVR\{1 - e^{-at}(1 + at)\}$, 69% ;
(*c*) $g_mVR(1 - 2e^{-at/2} + e^{-at})$, 49%.]*

47 A number of amplifiers, each of the type shown in figure 10, are connected in cascade. In each stage the CR value of the anode circuit is $1/a$. If the input voltage to the first stage remains steady until time $t = 0$ and then rises exponentially according to the law $V(1 - e^{-at})$, find an expression for the output voltage at the anode of the first, second and third stages and hence (a) deduce an expression for the output of the nth stage. Draw a graph showing the rise of voltage at each succeeding anode of a ten-valve amplifier. Assume the low-frequency gain of each stage is equal to unity, and ignore the phase-reversal at alternate anodes introduced by the valves. Include a curve of the input voltage. Give the time, in terms of $1/a$, which elapses before each anode reaches (b) 50% and (c) 90%, of its final value.

[(a) $V\{1 - e^{-at}(1 + at + a^2t^2/\lfloor 2 + a^3t^3/\lfloor 3 \ldots + a^nt^n/\lfloor n)\}$; (b) $1{\cdot}7/a$, $2{\cdot}7/a$, $3{\cdot}7/a$, $4{\cdot}6/a$, $5{\cdot}6/a$, $6{\cdot}6/a$, $7{\cdot}6/a$, $8{\cdot}6/a$, $9{\cdot}5/a$, $10{\cdot}5/a$; (c) $3{\cdot}9/a$, $5{\cdot}3/a$, $6{\cdot}7/a$, $7{\cdot}9/a$, $9{\cdot}1/a$, $10{\cdot}3/a$, $11{\cdot}4/a$, $12{\cdot}5/a$, $13{\cdot}6/a$, $14{\cdot}7/a$.]*

48 The input voltage to the amplifier shown in figure 10 remains constant until time $t = 0$ and then rises linearly at a rate of V' volts per second until $t = t_1$ when the potential of the grid has risen by V volts. The input then remains constant. Taking the initial value as zero, the input may thus be specified as follows: $t < 0$, $v_g = 0$; $t = 0$ to $t = t_1$, $v_g = V't$; $t > t_1$, $v_g = V$. Find expressions for the voltage developed across the anode load, (a) for the period $t = 0$ to $t = t_1$, and (b) when $t > t_1$. (c) Draw a graph of the input voltage against time in terms of t_1 and then plot curves of the output voltage for the following cases: $CR = t_1/5$, $CR = t_1/2$, $CR = t_1$, $CR = 2t_1$. Ignore the phase-reversal introduced by the valve and show both input and output as fractions of their final values. Plot the curves up to time $t = 7t_1$. In each case give the rise time in terms of t_1 measured between the 10% and 90% amplitude points.

$$\left[(a)\ \frac{g_m VR}{t_1}\left\{t - CR(1 - e^{-t/CR})\right\};\right.$$
$$(b)\ \frac{g_m VR}{t_1}\left\{t_1 - CR(e^{t_1/CR} - 1)e^{-t/CR}\right\};$$
$$\left.(c)\ 0{\cdot}9t_1,\ 1{\cdot}4t_1,\ 2{\cdot}35t_1,\ 4{\cdot}85t_1.\right]^*$$

49 The input voltage to the amplifier shown in figure 11 falls instantaneously by V volts. Find expressions for the output voltage (a) in terms of b, γ, β and t when $1 > 4a/b$; (b) in terms of b, ϕ, ω and t when $1 < 4a/b$; and (c) in terms of b and t when $1 = 4a/b$. In all cases $a = 1/CR$, $b = R/L$ and the time t is measured from the instant when the input voltage falls. In case (a)

$\gamma = \sqrt{1 - 4a/b}$ and $\beta = b\gamma/2$; and in case (*b*) $\phi = \sqrt{4a/b - 1}$ and $\omega = b\phi/2$. Evaluate the appropriate expressions for the cases when $L = CR^2$, $L = 0{\cdot}5CR^2$, $L = 0{\cdot}25CR^2$ and $L = 0{\cdot}125CR^2$, taking values of $t = 1/a, 2/a, 3/a$ up to $t = 10/a$, and plot curves of the output voltage as a fraction of the final value. Include the simple exponential case when $L = 0$, and give (*d*) the percentage of the final value to which the output has risen in each of the first four cases at the instant when the last case, $L = 0$, has risen to the 90% point.

$$\left[\begin{array}{l}(a)\ g_m VR\left\{1 - e^{-bt/2}\left(\cosh \beta t + \frac{1+\gamma^2}{2\gamma}\sinh \beta t\right)\right\};\\ (b)\ g_m VR\left\{1 - e^{-bt/2}\left(\cos \omega t + \frac{1-\phi^2}{2\phi}\sin \omega t\right)\right\};\\ (c)\ g_m VR\left\{1 - e^{-bt/2}(1 + bt/4)\right\};\ (d)\ 129\%, 107\%, 97\%, 94\%.\end{array}\right]^*$$

50 The anode load circuit of the pentode amplifier of figure 11 is less than critically damped. If there is an instantaneous change of voltage at the input, find an expression for the percentage overshoot at the anode in terms of ϕ, where $\phi = \sqrt{4L/CR^2 - 1}$. Calculate the percentage overshoot in the following cases: (*a*) $L = CR^2$, (*b*) $L = 0{\cdot}7CR^2$ and (*c*) $L = 0{\cdot}35CR^2$. In case (*a*), state also the amount by which the voltage later falls below the final value.

$$\left[50\,\frac{1+\phi^2}{\phi}\,e^{-\frac{\arctan(-\phi)}{\phi}}\sin\left\{\arctan(-\phi)\right\};\ \begin{array}{l}(a)\ 30\%,\ -5\%;\\ (b)\ 16\%;\ (c)\ 1\%.\end{array}\right]$$

51 The input voltage to the amplifier of figure 11 remains constant until time $t = 0$ and then falls linearly at a rate of V' volts per second until $t = t_1$ when it remains steady at the new value. The total change in voltage at the grid is V volts. If the anode load is critically damped, develop an expression for the rise in voltage at the anode (*a*) for the period $0 < t < t_1$ and (*b*) for the period $t > t_1$. (*c*) To what fraction of its final value has the anode voltage risen at $t = 2t_1$ if $t_1 = CR$?

$$\left[\begin{array}{l}(a)\ \frac{g_m VR}{t_1}\left\{t - 3\frac{L}{R} + e^{-Rt/2L}\left(\frac{t}{2} + 3\frac{L}{R}\right)\right\};\\ (b)\ \frac{g_m VR}{t_1}\left[t_1 + e^{-Rt/2L}\left\{\frac{t}{2} + 3\frac{L}{R} - e^{Rt_1/2L}\left(\frac{t-t_1}{2} + 3\frac{L}{R}\right)\right\}\right];\\ (c)\ 0{\cdot}863.\end{array}\right]$$

52 The input voltage to the amplifier of figure 11 falls linearly at the rate of V' volts per second over a period of t_1 seconds before and after which it remains steady, the overall difference between the initial and final values being V volts If the anode circuit is less than critically damped, find expressions for the rise of out-

put voltage (*a*) for the period of the linear change on the grid, and (*b*) for the period after t_1. Taking $L = 0{\cdot}5CR^2$, plot curves of the rise of output voltage when the duration of the change at the input, t_1, has the following values: $0{\cdot}2CR$, $0{\cdot}5CR$, CR, $2CR$, $5CR$. Give (*c*) the maximum percentage overshoot in each case. Develop the expressions for the output voltage in terms of $a = 1/CR$, $b = R/L$, $\phi = \sqrt{4a/b - 1}$ and $\omega = b\phi/2$.

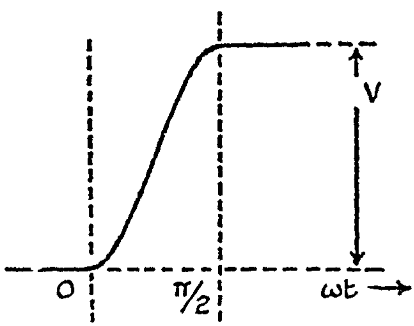
$$\left[(a)\,\frac{g_mVR}{t_1}\left[t + \frac{a-b}{ab} + \frac{e^{-bt/2}}{ab}\left\{(b-a)\cos\omega t + \frac{b-3a}{\phi}\sin\omega t\right\}\right];\right.$$

$$(b)\,\frac{g_mVR}{t_1}\left[t_1 + \frac{e^{-bt/2}}{ab}\left\{\left(b - a - e^{bt_1/2}(\overline{b-a}\cos\omega t_1 - \frac{b-3a}{\phi}\sin\omega t_1)\right)\cos\omega t\right.\right.$$

$$\left.\left. + \left(\frac{b-3a}{\phi} - e^{bt_1/2}(\overline{b-a}\sin\omega t_1 + \frac{b-3a}{\phi}\cos\omega t_1)\right)\sin\omega t\right\}\right];$$

$$\left.(c)\ 7\%,\ 7\%,\ 6\%,\ 5\%,\ 2\%.\right]^*$$

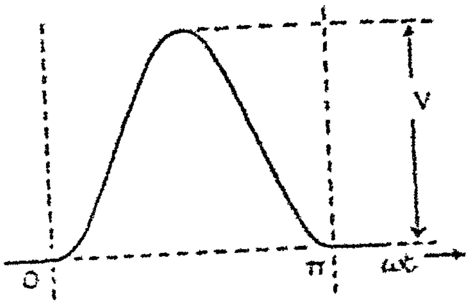
53 The input voltage to the amplifier of figure 10 is shown in the diagram. The voltage remains steady until time $t = 0$ and then rises to its final value, V, according to the law $V\sin^2\omega t$. If the relationship between the time constant of the anode load circuit and ω is given by $\gamma = 1/2\omega CR$, derive expressions for the output voltage (*a*) during the period of the sine-squared transition on the grid, and (*b*) during the ensuing period. Plot curves of the input and output voltages against ωt for values of $\omega t = 0$ up to $\omega t = \pi$. Take the cases when $\gamma = 1$, $\gamma = 2$, and $\gamma = 5$, and ignore the phase-reversal introduced by the valve. (*c*) Give in each case the fraction of the final value to which the output has risen when $\omega t = \pi/2$.



$$\left[(a)\ \frac{g_mVR}{2}\left\{1 - \frac{1}{1+\gamma^2}(e^{-t/CR} + \gamma\sin 2\omega t + \gamma^2\cos 2\omega t)\right\};\right.$$

$$\left.(b)\ g_mVR\left\{1 - \frac{1+e^{\pi\gamma}}{2(1+\gamma^2)}\,e^{-t/CR}\right\};\ (c)\ 0{\cdot}74,\ 0{\cdot}9,\ 0{\cdot}98.\right]^*$$

54 A sine-squared pulse of the form V $\sin^2\omega t$, as illustrated, is used to test the amplifier of figure 10. Derive expressions for the output voltage (*a*) during the period of the incoming pulse and (*b*) during the period immediately following, in terms of γ where $\gamma = 1/2\omega CR$. Draw a graph of the input voltage and plot curves of the output when $\gamma = 1$, $\gamma = 2$, $\gamma = 5$. Plot the curves against ωt, taking values up to $\omega t = 1{\cdot}5\pi$, and ignore the phase-reversal introduced by the valve. Give (*c*) the maximum value to which the output rises in each case.



$$\left[\begin{array}{l}(a)\ \dfrac{g_m VR}{2}\left\{1 - \dfrac{1}{1+\gamma^2}(e^{-t/CR} + \gamma \sin 2\omega t + \gamma^2 \cos 2\omega t)\right\}; \\ (b)\ \dfrac{g_m VR}{2(1+\gamma^2)}\, e^{-t/CR}(e^{2\pi\gamma} - 1); \\ (c)\ 0{\cdot}85\, g_m VR,\ 0{\cdot}95\, g_m VR,\ 0{\cdot}99\, g_m VR.\end{array}\right]^*$$

CHAPTER XI

INSTRUMENTS AND ACCESSORIES

Instruments : Electrostatic

1 An absolute electrometer has a movable circular plate 8 cm in diameter. If the distance between the plates during a measurement is 4 mm, find the potential difference when the force of attraction is 0·2 g weight. [1 188 V.]

2 The plates of an absolute electrometer, which are 10 in diameter and 0·8 in apart, are charged to a potential difference of 10 000 V. Calculate the force of attraction between the plates and the charge on each. [5 430 dynes (0·0543 Nw) ; 0·22 μC.]

3 An electrostatic voltmeter has two parallel plates. The movable plate is 10 cm in diameter. With 10kV between the plates the pull is 500 dynes (0·005 Nw). Find the change in capacitance for a movement of 1 mm of the movable plate. [0·103 $\mu\mu$F.]

4 The movable vane of a quadrant electrometer turned through 25 scale divisions when idiostatically connected to a potential difference of 60 V. When used heterostatically with the quadrants connected to a small voltage v and the needle to a 900-V supply, the deflexion was 10 scale divisions. Determine v. [0 8 V.]

5 An electrostatic voltmeter reading up to 1 000 V is controlled by a spring with a torsion constant of 0·001 g-cm per degree, and has a full-scale deflexion of 80°. The capacitance at zero is 10 $\mu\mu$F. What is the capacitance when the pointer indicates 1 000 V ? [20·9 $\mu\mu$F.]

6 An electrostatic voltmeter comprises three square parallel plates of 25 cm side. Two are fixed 10 cm apart in air and electrically connected. Between this pair and the third which can move into the space between, a potential difference is applied. With 12 kV, the movable plate is one-quarter-way in and with 32 kV it is half-way in. What potential difference is required to pull the plate three-quarter-way in ? Neglect fringing and assume that the displacement of the plate produces a proportional control force. [43 7 kV.]

7 A 120-V electrostatic voltmeter has a capacitance of 200 $\mu\mu$F and a torque of 3·8 dyne-cm ($3{\cdot}8 \times 10^{-7}$ Nw-m) at the full-scale deflexion of 90°. Draw curves showing how the capacit-

ance must vary with deflexion if the scale is to be (*a*) square-law, and (*b*) square-law below half-scale and linear above. Determine the efficiency figure, defined as the change of capacitance from full to zero scale reading divided by the capacitance at full scale, in each case. [(*a*) 0·415 ; (*b*) 0·7.]

Galvanometers

8 A moving-coil galvanometer deflects 10 cm on a scale 2 m away from its mirror with a current of 1 μA. When swinging freely on open-circuit, the time of one complete oscillation is 8 sec. The galvanometer is dead-beat when the total resistance of the circuit is 2 800 Ω. Find the moment of inertia of the moving system. [9·72 g-cm^2.]

9 In a particular moving-coil instrument, critical damping is to be provided only by eddy currents in the aluminium former carrying the moving coil. The moment of inertia of the moving system is 0 03 g-cm^2, the torsional constant of the control spring is 0 02 g-cm per radian and the loop resistance of the aluminium former is 0 006 Ω. Calculate the required coil flux linkages per radian deflexion.

[3 030 maxwells (3·03 × 10^{-5} Wb)/radian.]

10 The coil of a moving-coil galvanometer has 300 turns and is suspended in a uniform magnetic field of 1 000 gauss (0·1 Wb/m^2) by a strip of which the torsion constant is 2 dyne-cm (0·2 μNw-m) per radian. The coil is 2 × 2½ cm, with a moment of inertia of 1·5 g-cm^2, and the galvanometer resistance is 200 Ω. Calculate the value of resistance across the terminals to give critical damping. Assume damping to be entirely electromagnetic. [450 Ω.]

11 A moving-coil galvanometer gives a deflexion of 15 cm on a scale distant 250 cm for a current of 1 μA. When swinging freely, the time of one complete oscillation is 4 sec. The moment of inertia of the moving system is 10 g-cm^2. What total resistance in the galvanometer circuit is necessary for critical damping, all other damping being negligible? Draw a deflexion/time curve. [7 000 Ω.]

12 In an undamped galvanometer with a periodic time of 10 sec a current of 0·1 mA produced a steady deflexion of 150 divisions. Find the instantaneous quantity of electricity to produce a first swing of 100 divisions (*a*) when the instrument is undamped, (*b*) when the instrument is damped so that its decrement is 1·1. [(*a*) 106 μC ; (*b*) 111 μC.]

13 A (moving-coil) ballistic galvanometer gives a first swing of 60° for a discharge of 1 000 μC. Find the quantity of electricity to produce (*a*) a swing of 90° in the instrument, (*b*) a spot deflexion of 10 mm on a scale 1 m away. [1 500 μC, 9·55 μC.]

14 A tangent galvanometer has two deflecting coils X and Y of equal diameter and similarly situated with respect to the moving magnet. X has 50 turns, Y has 5 turns. (*a*) If 0·05 A flows in X, what current in Y will produce the same deflexion? (*b*) If 0·1 A in X produces a deflexion of 45°, what deflexion will 2 A in Y produce? (*c*) Find the deflexion when the currents in (*b*) (i) are additive, (ii) oppose one another.
[(*a*) 0·5 A; (*b*) 63·4°; (*c*) (i) 71·5°, (*c*) (ii) − 45°.]

15 A tangent galvanometer has a coil 20 cm in diameter. How far must the coil be moved along its own axis from the central position to reduce the sensitivity of the instrument to 1/10th of its former value? [19·1 cm.]

16 A tangent galvanometer has a coil with 20 turns of 30 cm diameter. In the earth's field ($B = 0{\cdot}18$ gauss (0·018 mWb/m^2)) the needle vibrates three times as slowly as when the controlling magnet is present. What is the current when the deflexion is 30°? [1·12 A.]

17 The coil of a tangent galvanometer has 50 turns and a mean diameter of 25 cm. It is to be replaced by a 30-turn coil which, when carrying twice the current of the original coil, would produce an equal magnetic force at its centre. Find the diameter of the new coil. [30 cm.]

Electrodynamic

18 A 10-A, electrodynamic ammeter is controlled by a spring having a torsion constant of 1 dyne-cm (10^{-7} Nw-m) per degree. The full-scale deflexion is 110°. Determine the inductance of the instrument when measuring a current of 10 A. Initial inductance is 2 μH and the change is linear. [2·21 μH.]

19 The inductance of a 50-V dynamometer voltmeter increases uniformly over the whole scale of 90°. The initial inductance is 0·25 H, and the torque for full scale deflexion is 0·4 g-cm, the current being 0·05 A. Determine the difference between d.c. and 50-c/s readings at (*a*) 50 V, (*b*) 25 V.
[(*a*) 0 2 V; (*b*) 0 08 V.]

20 The inductance of a 25-A electrodynamic ammeter changes uniformly at the rate of 0·0035 μH per degree. The torsion constant of the controlling spring is 10 dyne-cm (10^{-6} Nw-m) per degree. Determine the angular deflexion for full-scale.
[125·4°.]

21 A Siemens dynamometer has two coils of negligible resistance. When connected in series, a current of 6 0 A produces a torque which is balanced by a rotation of the torsion head through 100°. The instrument is adapted for use as a wattmeter by connecting one coil in series with a 200-Ω resistor across 250-V

mains, the other coil being in series with a load of 3·0 kW off the same mains. Find the angle through which the torsion head must be turned to balance the new torque. [41·7°.]

22 In a torsion-head type electrodynamometer a current of 25 A requires a deflexion of 90° in the head to give balance. (*a*) Find the range of the instrument (maximum angle = 360°). (*b*) For what current will the deflexion be 180° ? (*c*) What will be the deflexion for a current of 20 A ? (*d*) Find the constant of the instrument in ampere-degree units.
[(*a*) 50 A ; (*b*) 35·4 A ; (*c*) 57·6° ; (*d*) 2·64.]

23 The resistances of the two coils of a wattmeter are 0 01 and 1 000 Ω respectively and both are non-inductive. The load current is 20 A and the p.d. applied to the load is 30 V. Show the two ways in which the voltage-coil can be connected and find the error in the reading in each case.
[0·15% too high ; 0·67% too high.]

24 The shunt-coil of a torsion-head, null-reading type of electro-dynamometer wattmeter has a resistance of 6 600 Ω. When 120 V is applied to the shunt-coil and 20 A flows in the series coil, the head must be turned through 160° to bring the pointer to zero. What additional resistance must be inserted in the shunt circuit to make the constant of the instrument 20 W per degree ? [2 200 Ω.]

25 In a dynamometer wattmeter the moving coil has 500 turns of diameter 3 cm. Estimate the torque if the axes of the field and moving coils are at (*a*) 45°, (*b*) 90°, when the flux density in the field coils is 100 gauss (10 mWb/m²), the current in the moving coil is 0 05 A and the power being measured has a power factor of 0 7. [(*a*) 87·3 ; (*b*) 123·8 μNw-m.]

26 A dynamometer wattmeter measures power in a 50-c/s, 1-phase circuit without error, at all power factors. The resistance of the voltage coil and its series resistance are 400 Ω and 10 000 Ω respectively. The series resistance has a distributed self-capacitance equivalent to a shunt capacitance of 20 μμF. What is the self-inductance of the pressure coil ? [2 mH.]

Moving-coil

27 A millivoltmeter has two scales reading 0—20 mV and 0—100 mV, the resistances of the two ranges being 400 Ω and 2 000 Ω respectively. When used in conjunction with a thermopile, a certain reading changed from 19·5 to 23·4 mV when the range was altered from the lower to the higher scale. Calculate (*a*) the resistance of the thermopile, (*b*) its electromotive force, and (*c*) the correction multipliers.
[(*a*) 105·4 Ω ; (*b*) 24·6 mV ; (*c*) 1·26, 1·05.]

28 The moving coil of a galvanometer has 60 turns, a width of 2 cm and a depth of 3 cm. It hangs in a uniform radial field of 500 gauss (50 mWb/m^2). Find the turning moment of the coil when it is carrying a current of 1 mA.
[18 dyne-cm ($1{\cdot}8 \times 10^{-6}$ Nw-m).]

29 A moving-coil voltmeter with a resistance of 10 Ω gives full-scale deflexion with a potential difference of 45 mV. The coil has 100 turns, an effective depth of 3 cm and a width of 2 5 cm. The controlling torque exerted by the spring is 0·5 g-cm for full-scale deflexion. Calculate the flux-density in the air-gap.
[1 450 gauss (0·145 Wb/m^2).]

30 A voltmeter gives a full-scale reading of 300 mV. The moving coil is wound with 20 turns of copper wire each 0 274 mm. diameter. The mean length per turn is 13 cm and the mean depth of the coil is 4 cm. There is a resistance of 4 24 Ω in series with the coil, and the gap density is 1 500 gauss (0·15 Wb/m^2). Calculate the controlling couple in dyne-cm exerted by the springs when the reading is 300 mV.
[1 800 dyne-cm ($1{\cdot}8 \times 10^{-4}$ Nw-m).]

31 The following data refer to a moving-coil voltmeter:— resistance, 10 000 Ω; dimensions of coil, 3 cm $\times$ 3 cm; number of turns on coil, 100; flux-density in gap, 800 gauss (80 mWb/m^2), spring control, 30 dyne-cm per degree. Find the deflexion produced by 200 V. [48°.]

32 A moving-coil, permanent-magnet, indicating instrument for use with external shunts is to comply with the following conditions: full-scale voltage-drop across shunt, 75 mV, flux-density in the air-gap (unshunted), 1 500 gauss (0·15 Wb/m^2), resistance of instrument between ends of shunt leads, 5 Ω; resistance of moving-coil only (copper wire), 0 75 Ω; length and breadth of moving-coil, 2 5 $\times$ 2 2 cm; torque/weight ratio 147 dyne-cm/g; total weight of movement, 2·5 times weight of copper coil only. Calculate the air-gap flux-density to be diverted through the magnetic shunt to give the maximum (whole) number of turns, the coil section area, and the full-scale torque.
[50 gauss (5 mWb/m^2); 0·039 mm^2; 216 dyne-cm (21·6 μNw-m).]

Moving-iron

33 Find an expression for the deflecting torque of a moving-iron ammeter in terms of the current and of the rate of change of inductance with angle of deflexion. Given that the torque per degree is 4 dyne-cm (4×10^{-7} Nw-m), find the current to give a deflexion of (*a*) 30°, (*b*) 80°. The following figures give the relation between the deflexion and the inductance.

Deflexion, deg.	20	30	40	50	60	70	80	90
Inductance, μH	335	345	355 5	366·5	376 5	385	391 3	396

*$\left[\frac{1}{2}\frac{dL}{d\theta}I^2\right.$ Nw-m/deg. ; (*a*) 0·65 A ; (*b*) 1·43 A.]

34 The full-scale torque of a 5 A moving-iron ammeter is 0·1 g-cm. Estimate in μH per radian the rate of change of self-inductance of the instrument at full scale.

[0·785 μH/radian.]

35 The coil of a moving-iron voltmeter has a resistance of 500 Ω and an inductance of 1·0 H. The swamp resistor is 2 000 Ω. The meter reads 250 V when a direct voltage of 250 V is applied. What will it read when 250 V at 50 c/s is applied ? With what value of capacitance must the swamp be shunted to make the meter read correctly on 50 c/s ? [248 V ; 0·1 μF.]

36 If the deflecting torque of an instrument is directly proportional to the current to be measured, and the maximum current produces a deflexion of 90°, compare the deflexion in a spring-controlled instrument with a similar instrument having gravity control for a current equal to half the maximum value.

[45°, 30°.]

Hot-wire

37 The working wire of a single-sag, hot-wire instrument is 15 cm long, and is made of platinum-silver with a coefficient of linear expansion of 16×10^{-6}. The temperature rise of the wire is 85° C and the sag is taken up at the centre. Find the magnification (*a*) with no initial sag : (*b*) with an initial sag of 1 mm.

[(*a*) 19·2 ; (*b*) 14·9.]

Electrolytic

38 An unshunted, precision-grade, electrolytic, ampere-hour meter is rated at 1 A, 250 V, 25 kWh. If it deposits mercury into a uniform, circular tube of internal diameter 1·5 cm, what is the correct length of tube to correspond to a full-load run ? Take the electrochemical equivalent of mercury as 0·001039 and its density as 13·59. [15·6 cm.]

Induction Wattmeter

39 Find the value of the resistor which must be placed in shunt with the voltage coil of a 200-V, 50-c/s induction wattmeter to give phase compensation. The voltage coil has inductance and resistance of 4 78 H and 298 Ω respectively and is in series with an inductance of 3 H and a resistance of 190 Ω. [2 760 Ω.]

Rectifier

40 A sinusoidal alternating voltage of amplitude 100 V is applied to a circuit containing a rectifying device which entirely prevents current flowing in one direction, and offers a non-inductive resistance of 10 Ω to the flow of current in the other direction. Find the readings on (*a*) a hot-wire, (*b*) a moving-coil, ammeter in the circuit. [(*a*) 5 A ; (*b*) 3·185 A.]

41 A permanent-magnet, moving-coil ammeter indicates 20 mA when connected across two opposite corners of a bridge rectifier, the other two corners of which are connected in series with a condenser to a 100-kV, 50-c/s supply. Calculate the capacitance of the condenser. [707 $\mu\mu$F.]

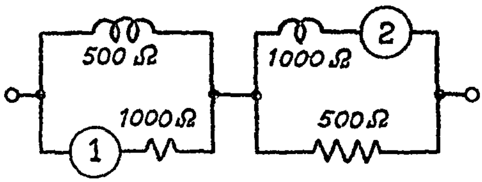
42 The characteristics of each of the 4 arms of a copper-oxide bridge rectifier are :—

Voltage V . .	0·1	0·15	0·2	0·24	0·28	0·34	0·38
Current mA . .	0·2	0·4	1	2	4	8	12

The bridge is connected across a supply = 1·0 sin θ + 0·1 sin 8θ. A milliammeter having $R = 20\ \Omega$ is joined across appropriate points of the bridge. Find ammeter reading (ignore " reverse " current). [5·33 A.]

Frequency Meter

43 Taking the accompanying diagram to represent the circuit in a Weston-type frequency indicator at 50 c/s, find the percentage change of current in the two coils 1 and 2, at right angles to each other, when the frequency is 5% below normal.



[Coil 1 : 1 4% increase ; Coil 2 : 6·8% decrease.]

44 An electrical-resonance frequency meter has two parallel circuits, each containing *R*, *L* and *C*. One circuit, tuned to 60 c/s, has $C = 1\ \mu$F and $R = 100\ \Omega$. The other circuit has $C = 1{\cdot}5\ \mu$F and $R = 100\ \Omega$. Find the inductance of the latter circuit and its resonant frequency in order that the currents in both circuits shall be equal at 50 c/s. [9·82 H ; 41·5 c/s.]

45 An electrical-resonance frequency meter has inductance and capacitance in each circuit. The capacitance in one circuit, $C_1 = 1\ \mu$F and $R_1 = R_2$. Both circuits are to carry equal currents at 50 c/s. Find L_2 and C_2 when the circuits are to be tuned at (*a*) f_1 = 60 c/s, f_2 = 40 c/s ; (*b*) f_1 = 52 c/s, f_2 = 48 c/s. [(*a*) 8·6 H, 1·85 μF ; (*b*) 11·5 H, 0·96 μF.]

XI. INSTRUMENTS AND ACCESSORIES

Instrument Errors : D.C. Instruments

46 An ampere-hour meter is calibrated to read energy on a 250-V, direct-current supply. On test a steady current of 15 A is passed through it for 300 min. If the meter readings before and after the test are 8 234·21 kWh and 8 253·18 kWh respectively, calculate the percentage error.

If the spindle turns through 291 rev during 5 min when a current of 20 A is passing through the meter, calculate the testing constant. [+ 0·9% ; 20 6 C/rev.]

47 Find the percentage error in the calculated value of the current in a tangent galvanometer due to an error of 1° in reading the angle of deflexion θ. Evaluate for $\theta = 30°$ and $\theta = 45°$. [4% ; 3 5%.]

48 A 20-A ampere-hour meter, the dial of which is marked in kWh, has an error of + 2% when used in a 250-V circuit. What will be the percentage error in the registration of the meter if it is connected for an hour in series with a load taking 4 kW at 200 V ? [27·5%.]

49 A d.c. Ah meter is rated at 5 A, 250 V. The declared constant is 5 A-sec/rev. Express the declared constant in revolutions/kWh. Calculate the full-load speed. In a test run at half load, the meter took 119 5 sec to complete 60·0 rev. Calculate the error of the meter.

[2 880 revs/kWh ; 60 r.p.m. ; + 0·42% (fast).]

50 Two ammeters are joined in series in a circuit carrying 10 A. Ammeter *A* has a resistance of 1 000 Ω and is shunted by 0 02 Ω. The corresponding values for ammeter *B* are 1 500 Ω and 0·01 Ω. What will the instruments read if the shunts are interchanged ? [*A*, 5 A ; *B*, 20 A.]

51 A voltmeter has a working coil of copper in series with an invariable swamp resistance. What must be the ratio of swamp resistance to coil resistance if the error introduced by a temperature rise between 15° C. and 25° C. is not to exceed 1% of the indication ? [3·0.]

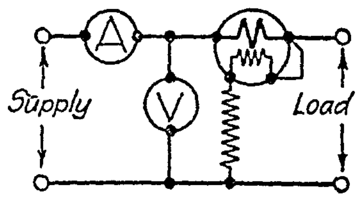
52 A voltmeter has a moving coil of copper wire, of resistance 800 Ω, and a swamp resistance of 19 700 Ω, made of platinoid. These values are for a temperature of 20° C, at which temperature the voltmeter reads correctly. Estimate the percentage error at 60° C. The resistance-temperature coefficient for copper is $42·8 \times 10^{-4}$ and for platinoid is $2·5 \times 10^{-4}$ at 0° C. [− 1·2%.]

Instrument Errors: A.C. Instruments

53 A dynamometer wattmeter with its voltage coil connected across the load side of the instrument reads 250 W. If the load

voltage be 200 V, what power is being taken by the load ? The voltage coil has a resistance of 2 000 Ω. [230 W.]

54 The instrument readings in the arrangement shown are :—ammeter, 2·4 A ; voltmeter, 200 V ; wattmeter, 22·0 W. The instrument resistances are :—ammeter, negligible ; voltmeter, 2 000 Ω ; wattmeter volt-coil 8 000 Ω ; wattmeter current-coil, 0·46 Ω. Find (*a*) the power factor of the load, (*b*) the power taken from the supply. Neglect instrument inductances.



*[(*a*) 0·0354 ; (*b*) 44·65 W.]

55 A dynamometer ammeter has a fixed coil with $R = 3\ \Omega$ and $L = 0{\cdot}12$ H, and a moving coil with $R = 30\ \Omega$ and $L = 0{\cdot}003$ H. It is calibrated on direct and used on 50-c/s current. Find the error when the moving coil is shunted across (*a*) the fixed coil ; (*b*) a 3-Ω non-inductive resistor in series with the fixed coil. [(*a*) 41% low ; (*b*) 0 05% low.]

56 The volt-circuit of an electrodynamic wattmeter has an inductance of 8 mH and a resistance of 2 000 Ω. What is the percentage error of the instrument when measuring an inductive load having a phase angle of 89° at 50 c/s ? Neglect the impedance of the current-coil and assume that the volt-circuit current is negligible compared with the load current. [7·2%.]

57 A dynamometer wattmeter reading correctly on direct current is used to measure the power in a circuit of resistance 2 Ω and inductance 0·25 H. The supply is 100 V at 50 c/s and the volt-circuit of the wattmeter has resistance 1 000 Ω and inductance 5 6 mH. What is the reading on the wattmeter ? Neglect the impedance of the current-coil. The voltage-coil is connected on the load side of the instrument. [13·35 W.]

58 The indication on a 110-V, 5-A wattmeter, used in conjunction with voltage and current transformers of nominal ratios of 100/1 and 20/1, is 350 W. If the resistance and inductance of the wattmeter volt-coil circuit are 362 Ω and 10 mH respectively, and the ratio-errors and phase angles of the voltage and current transformers at the particular working conditions are + 0·8%, + 45 min and − 0·2%, + 90 min, what is the true value of the power measured ? The load phase-angle is 50° lagging. Frequency, 50 c/s. [660 kW.]

59 The constant for a 3 phase, 2-element integrating wattmeter is 0 12 rev of disc per kWh. If the meter is normally used with a potential transformer of ratio 22 000/110 V and a current transformer of ratio 500/5 A, determine the error, expressed as a percentage of the correct reading, from the following test figures for the instrument only: line voltage, 110 V; current, 5·25 A; power factor, 1·0; time to complete 40 revolutions, 61 sec. [− 1·68%.]

60 In an induction watt-hour meter, the current and flux are in phase in the series system, but there is an angular deviation of 3° from quadrature between the voltage and flux in the voltage system. If the rotor speed at full load and unity power factor be correct, what is the percentage error at quarter load and 0 5 power factor? [9·2% slow.]

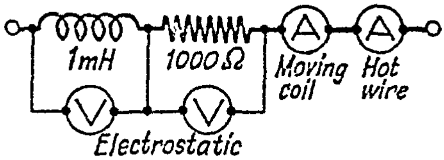
61 A 50-A, 230-V meter on full-load test makes 61 rev in 37 sec. If the normal disc speed is 520 rev per kWh, what is the percentage error? [0·75% slow.]

62 A 240-V, 5-A, single-phase energy meter has a registration constant of 1 200 rev per kWh. It is tested by means of a 240-V, 5-A wattmeter having 500 scale divisions which can be read to 0·1 division, and a stop-watch which can be read to 0 01 sec and which has negligible error. When tested at full load, the meter makes 40 rev in 99 8 sec. If the human error in timing be taken as $\pm$ 0 05 sec, estimate the limits within which the error of the meter may lie. The wattmeter is accurate to within 0 05% of its full-scale reading.

[0 07% fast; 0·33% fast.]

63 A rotating substandard energy meter is tested by a wattmeter of the same VA rating having a 500-division scale, which can be read to 0 1 division, and a stop-watch having a negligible error with a scale which can be read to 0 01 sec. The wattmeter is accurate to within 0 05% of its full-scale reading at all parts of the scale, and the total error in starting and stopping the stop-watch does not exceed 0·1 sec. Determine the duration necessary for a test of the energy meter at 100% load to enable the measured accuracy of the meter to be guaranteed to within 0 1%. For a test of the same duration at 10% load, calculate limits of accuracy of the determination. [370 sec; 0·73%.]

64 A current of $0{\cdot}5 + 0\ 3 \sin \omega t - 0\ 2 \sin 2\ \omega t$, where $\omega = 10^6$, is passed through the circuit shown. Determine the reading on each instrument.



[$V_L = 354$ V, $V_R = 561$ V, $I_{mc} = 0\ 5$ A, $I_{hw} = 0{\cdot}561$ A.]

65 What will be the reading when a voltage

$$e = 200 \sin \omega t + 40 \sin\left(3\omega t + \frac{\pi}{6}\right) + 30 \cos\left(5\omega t + \frac{\pi}{8}\right)$$ is

applied to an electrodynamic voltmeter? [145·8 V.]

66 A voltage $100 \sin \omega t + 40 \cos\left(3\omega t - \frac{\pi}{6}\right) + 50 \sin\left(5\omega t + \frac{\pi}{4}\right)$ is applied to the voltage coil of a wattmeter and through the current-coil is passed a current $8 \sin \omega t + 6 \cos\left(5\omega t - \frac{2\pi}{3}\right)$. What will be the reading on the wattmeter ? What percentage of this power is due to the fundamental ? [439 W ; 91·2%.]

Oscillographs : Electromagnetic

67 An electromagnetic oscillograph is required to reproduce the 5th harmonic of a 50-c/s wave correct to within 5% of its amplitude. What is the minimum frequency to which the instrument can be tuned ? Calculate the angle of phase-displacement for the corresponding harmonic.
[1 090 cycles per sec ; 26°.]

Shunts

68 Find the " multiplying power," (i.e. the ratio total current/galvanometer current), of a shunt of 200 Ω resistance used with a galvanometer of 1 000 Ω resistance. Determine the values of the shunt to obtain multiplying powers of 5, 10, 50, and 100.
[6 ; 250, 111, 20·4 and 10·11 Ω.]

69 A universal shunt has a total resistance of 10 000 Ω. Find the multiplying power of the shunt for a 2 500-Ω and a 5 000-Ω tapping if the galvanometer has a resistance (*a*) 50 Ω, (*b*) 5 000 Ω.
[(*a*) 4·02, 2·01 ; (*b*) 6, 3.]

70 A moving-coil instrument gives a full-scale reading of 24 mA when the p.d. across its terminals is 72 mV. Calculate (*a*) the shunt resistance for a full-scale deflexion corresponding to 120 A, (*b*) the series resistance for full-scale reading with 600 V. Find the power consumption in each case.
[(*a*) 0·60012 mΩ, 8·64 W ; (*b*) 24 997 Ω, 14·4 W.]

71 The coil of a measuring instrument has a resistance of 1 Ω, and the instrument reads up to 250 V when a resistance of 4 999 Ω is connected in series with it. Find (*a*) the current range of the instrument when used as an ammeter with the coil connected across a shunt of resistance 1/499 Ω, and (*b*) the exact value of the shunt resistance for the instrument to give a full-scale deflexion with a current of 50 A in the main circuit.
[(*a*) 25 A ; (*b*) 0·001001 Ω.]

72 A moving-coil ammeter has a fixed shunt of 0 02 Ω. With a coil-circuit resistance of R = 1 000 Ω and a p.d. of 0·5 V across it, full-scale deflexion is obtained. (*a*) To what shunted current does this correspond ? (*b*) Calculate the value of R to give full-scale deflexion when the shunted current I is (i) 10 A,

(ii) 75 A. (c) With what value of R is 40% deflexion obtained with $I = 100$ A?

[(a) 25 A; (b) (i) 400 Ω, (ii) 8 000 Ω; (c) 10 000 Ω.]

73 Two moving-coil ammeters have a resistance of 0·8 Ω and 1 Ω respectively, and each gives a full-scale reading with a current of 100 mA. Calculate the resistance of suitable shunts to give full-scale deflexions with 100 A. If the ammeters with shunts are placed in parallel in a circuit in which the total current flowing is 100 A, find the reading on each instrument.

[$S_1 = 8{\cdot}01 \times 10^{-4}$ Ω; $S_2 = 10{\cdot}01 \times 10^{-4}$ Ω; $I_1 = 55{\cdot}6$ A, $I_2 = 44{\cdot}4$ A.]

74 A moving-coil instrument giving a full-scale deflexion with 20 mA, and having a resistance of 1 Ω, is required to measure the current in a copper bus-bar of ·1 in^2 cross-section up to the limit of 1 000 A. Find the distance between two points on the bus-bar across which the instrument should be tapped when the bus-bar temperature is 20° C. With the same tap-points, if the bus-bar temperature rose to 40° C, how would the accuracy of the instrument readings be affected?

[29·5 in; 8% high.]

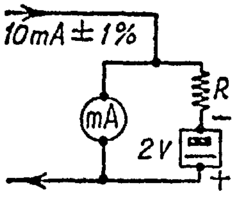
75 An ammeter, which reads correctly at 15° C, has a coil of copper wire with a resistance of 2 Ω at this temperature. The ammeter shunt has a constant resistance of 1/2 500 Ω. If a current of 100 A is flowing in the main circuit, what current will flow through the ammeter coil when its mean temperature is 60° C, and what will be the percentage error in the reading?

[0·017 A; − 15%.]

76 A moving-coil instrument has at normal temperature a resistance of 10 Ω and a current of 45 mA gives full-scale deflexion. If its resistance rises to 10·2 Ω due to temperature change, calculate the reading when a current of 2 000 A is measured by means of a 2 000-A shunt of constant resistance. What is the percentage error? [44·12 mA; − 1·95%.]

77 Find the length and breadth of the plates of a manganin shunt for a milliammeter to have a voltage drop of 75 mV at 1 000 A. The shunt is to have 5 plates each 2 mm thick and the radiation from the surface is at the rate of 0·2 W per cm^2: $\rho = 42$ $\mu\Omega$-cm. [Length 8·2 cm; breadth 4·6 cm.]

78 A current of approximately 10 mA is subject to fluctuations of ± 1%. It is desired to measure these variations. A 0·1—0—0·1 mA centre-zero moving-coil ammeter of resistance 50 Ω is available. Calculate the relative sensitivity of the measurement when the instrument is used (i) with a shunt to give a maximum deflexion with a current of 10·1 mA and (ii) as shown in the diagram with a value of



R chosen to give a zero reading on the ammeter when the main current is 10 0 mA. Determine the value of R.

[1/80·8 ; 200 Ω.]

79 . Calculate the constants of a shunt to extend the range of a 0–5 A moving-iron ammeter to 0–50 A. The instrument constants are $R = 0\ 09\ \Omega$ and $L = 90\ \mu$H. If the shunt is made non-inductive, and the combination is correct on direct current, find the full-scale error at 50 c/s. [0 01 Ω, 10 μH ; 3 8% low.]

80 A moving-coil ammeter is to give a full-scale torque of 0 5 g-cm when the voltage drop across the instrument is 0 075 V. The coil has 26½ turns of copper wire of diameter 0 2946 mm wound on a former 3 6 cm × 1 25 cm radius. Flux density in gap = 1 350. Find current for full-scale deflexion and swamp resistance for correct reading at 20° C. Calculate percentage error at 30° C if temperature coefficient of swamp resistance is negligible. [15·24 mA ; 4 1 Ω ; 0·65% low.]

Multipliers

81 Five lamps having resistances of 193, 203, 207, 197 and 200 Ω respectively are operated in series from the main supply on a tramcar. A voltmeter of 1 000 Ω resistance is connected across the 200-Ω lamp and reads 86 V. What is the voltage of supply? Find the minimum resistance the voltmeter should have in order that the normal voltage across the 200-Ω lamp shall not change by more than ½% when the voltmeter is connected.

[498·8 V ; 31 800 Ω.]

82 A moving-coil instrument has a resistance of 10 Ω and gives a full-scale deflexion when carrying 50 mA. Show how it can be adapted to measure voltages up to 750 V, and currents up to 100 A.

[14 990 Ω in series ; 0·005 Ω in shunt.]

83 A moving-iron voltmeter for a full-scale indication of 120 V has an inductance of 0 6 H and a resistance of 2 400 Ω. The instrument is designed for a 50-c/s circuit. Specify the series resistance required to increase the range 5-fold. [9 636 Ω.]

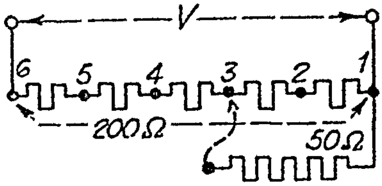
84 The operating coil of a moving-iron voltmeter has a resistance of 20 Ω and an inductance of 0 3 H. The swamp resistance is 2 000 Ω. With what capacitance must the swamp resistor be shunted in order that the meter shall read correctly on direct and on 50-c/s voltages? [0·0311 μF.]

85 The capacitance of a 0–2,000 V electrostatic voltmeter increases uniformly from 45 to 55 $\mu\mu$F from zero to full scale. Find the capacitance of a condenser to increase the range to 20 kV, and

the percentage error at half-scale reading. Show how the error can be reduced by using two condensers in series and connecting the instrument across one of them. [6·1 $\mu\mu$F ; 8·8% high.]

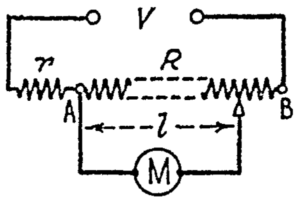
Voltage Dividers

86 A potential divider is arranged as shown. Find the position of 5 tappings on the 200-Ω coil, which is permanently connected across the mains, such that the current in the 50-Ω resistance rises from zero to that given by the full line voltage by equal increments. Specify the tappings by the resistances of the 5 steps formed by them.



[78, 66·5, 29·7, 15·8 and 10 Ω.]

87 For the circuit shown, determine the law of grading the resistor AB so that the current in the meter M is proportional to the distance of the tapping point from A. Draw a curve of resistance R_l to a base of length for the following values : $r = 5\ \Omega$, $R_{AB} = 20\ \Omega$, $M = 10\ \Omega$, $AB = 100$ cm. State also values of R_l for $l = 10$ cm, 50 cm, 90 cm. [1·65 Ω, 11·6 Ω, 18·8 Ω.]



88 A direct voltage of 240 V is applied to a uniform 200-Ω resistor abc. A 40-Ω resistor is tapped across ab. Find the resistance of ab in order that 1 A flows in the 40-Ω resistor.

[71·65 Ω.]

89 A voltage of 240 V is applied to a 10 000-Ω resistor. Across what part of it must a voltmeter of 4 000 Ω resistance be tapped to read 60 V ? What will the reading be if the voltmeter is tapped across 2 500 Ω ? [4 000 Ω ; 40·8 V.]

Instrument Transformers

90 A 50-c/s current transformer has a single-turn primary winding and a 200-turn secondary. The latter supplies a non-inductive ammeter of 1 Ω resistance with a normal current of 5 A. The flux requires 80 ampere-turns. Draw a vector diagram showing the currents, voltages and flux ; (a) calculate the maximum core-density for a core of 10 cm^2 net section ; (b) calculate the current ratio and phase angle. Neglect winding resistance and reactance.

[(a) 1 125 gauss (0·1125 Wb/m^2) ; (b) 200·64, 4° 34′.]

91 A 1 000/5 A, 50-c/s current transformer has a secondary burden comprising a non-reactive resistance of 1·6 Ω. The

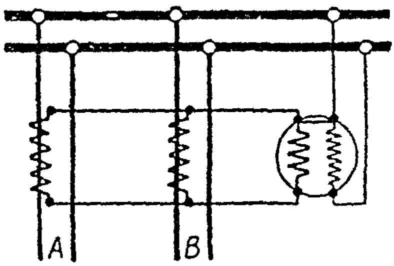
primary winding has 1 turn. Calculate the flux in the core and the current-ratio error on full-load. The iron loss in the core is 1·5 W at full-load. Neglect leakage reactance.

[18 000 maxwells ($1·8 \times 10^{-4}$ Wb) ; 3·75%.]

92 A relay current-transformer has a bar primary and 200 secondary turns. The secondary burden is an ammeter of resistance 1 2 Ω and reactance 0·5 Ω, and the secondary winding has a resistance of 0·2 Ω and reactance of 0·3 Ω. The core requires the equivalent of 100 *AT* for magnetization and 50 *AT* for core losses. (*a*) Find the primary current, the ratio error and the phase-angle when the secondary ammeter indicates 5·0 A. (*b*) By how many turns could the secondary winding be reduced to eliminate the ratio error?

[(*a*) 1 095 A, − 9·5%, 3·25° ; (*b*) 19.]

93 Two similar current transformers are connected as shown to read the total power in two similar feeders. The resistance and reactance of the wattmeter current coil are 0·09 Ω and 0 08 Ω respectively. The characteristics of each transformer are as follows :—



E.M.F., V	0·25	0 5	1 0	1·5	2·0
Magnetizing component, A .	0·9	1·5	2·5	3·2	4 0
Core-loss component, A . .	0·5	1·3	2·7	3·9	4 6

Turns-ratio, 500/5. The resistance and reactance of each transformer secondary winding are 0·02 Ω and 0·03 Ω respectively.

Estimate the phase-angle and ratio error of each transformer when 5 A is flowing through the wattmeter, (*a*) when the feeders are carrying currents equal in magnitude and phase, (*b*) when one feeder carries no current.

[(*a*) − 1·1%, − 1′ ; (*b*) − 0·6%, 0.]

94 The resistance and reactance of the secondary of a 500/5 current transformer are 0 02 Ω and 0·03 Ω respectively and the transformer characteristics are given by :—

E.M.F., V	0·25	0·5	1·0	1·5	2·0
Magnetizing component, A .	0 9	1·5	2·5	3·2	4·0
Core-loss component, A . .	[illegible]	1·3	2·7	3 9	4·6

An ammeter, a wattmeter current-coil and an induction relay are connected in series to the secondary winding. Their respective resistances are 0·08 Ω, 0 10 Ω, 0·14 Ω and their reactances 0·09 Ω, 0·07 Ω and 0·08 Ω. If the current in the instruments is 4 A, calculate the ratio and phase-angle errors (*a*) when all the instruments are in circuit, (*b*) when only the wattmeter is in circuit. Calculate the load VA in each case.

[(*a*) $-$ 1·4%, 1·25′, 6·4 VA ; (*b*) $-$ 0·001%, 2·5′, 1·96 VA.]

18 The arms of an alternating-current Maxwell bridge are arranged as follows: *AB* is a non-reactive resistor of 1 000 Ω in parallel with a condenser of capacitance 0·5 μF; *BC* is a non-reactive resistor of 600 Ω; *CD* is an unknown inductive resistor; and *DA* is a non-reactive resistor of 400 Ω. If balance is obtained under these conditions, find the values of the resistance and the inductance in *CD*. [240 Ω; 0 12 H.]

19 An Owen bridge is used to measure the properties of a sample of sheet steel at 2 kc/s. At balance, arm *AB* is R_2 = 834 Ω in series with C_2 = 0·124 μF, arm *BC* is the test specimen, arm *CD* is R_1 = 100 Ω, and arm *DA* is C_1 = 0·1 μF. Derive the balance conditions, and calculate the effective impedance of the specimen under the test conditions. Draw the vector diagram at balance.
[$R = R_1 C_1/C_2$; $L = R_1 R_2 C_1$; 132 $\underline{/52\ 8^\circ}$ Ω.]

20 An a c. bridge was made up as follows: arms *AB* and *BC* equal ratio arms, *CD* a variable condenser *C* in series with a variable resistor *R*, *DA* a standard air condenser of 0 001 μF in series with a fixed standard resistor of 500 Ω. The supply at 796 c/s was across *AC* and the detector across *BD*. Balance of the above bridge was obtained with *C* = 0 001 μF ± 10 μμF and *R* = 500 ± 5Ω. When an unknown condenser was connected across *CD*, the balance conditions changed to *C* = 360 ± 10 μμF and *R* = 0. Calculate the capacitance and power factor of the unknown condenser and the limits of accuracy.
[640 μμF ± 1 6%; 0·0039 ± 0·0002.]

21 The four arms of an alternating-current bridge are arranged as follows for the measurement of capacitance by the series-resistance method: *AB* is an imperfect condenser of capacitance C_1 and equivalent series resistance r_1; *BC* is a non-reactive resistor *P*; *CD* is a non-reactive resistor *Q*; and *DA* is a condenser of capacitance 0 5 μF with equivalent series resistance 0 4 Ω, in series with a non-reactive resistor *R*. Balance is obtained when *P* = 2 000 Ω, *Q* = 2 850 Ω, and *R* = 4·8 Ω, with a frequency of 450 cycles per sec. Calculate the values of C_1 and r_1, and determine the loss-angle of the condenser.
[0 713 μF; 3·65 Ω; arc tan 0 00734.]

22 The four arms of a bridge network are made up as follows: *AB*, a resistor of 50 Ω in parallel with an inductor of 0·1 H; *BC*, a resistor of 100 Ω; *CD*, an unknown resistor *R* in parallel with an unknown capacitor *C*; *DA*, a resistor of 1 000 Ω. A 50-c/s voltage is applied across *AC*. Find *R* and *C* when a vibration galvanometer across *BD* is undeflected.
[*R* = 7 060 Ω; *C* = 0·72 μF.]

23 In a four-arm, Wien bridge network arranged for measuring capacitance, the arm *AB* consists of an imperfect condenser C_1; *BC* and *CD* are non-reactive resistors of 1 000 Ω each; and *DA*

is a standard capacitor of 0·0115 μF in series with a resistor of 140 Ω. If $\omega = 7\,500$ and the bridge is balanced, find the shunt loss-resistance and the capacitance of C_1. [0·96 MΩ ; 0·0115 μF.]

24 A sheet of bakelite 0·18 in thick is tested at 50 c/s between electrodes 4·75 in diameter. The Schering bridge employed has a standard compressed-air condenser of 106 $\mu\mu$F capacitance ; a non-reactive resistor of $1\,000/\pi$ Ω in parallel with a variable condenser C ; and a non-reactive variable resistor R. Balance is obtained with $C = 0{\cdot}5$ μF and $R = 260$ Ω. Calculate the power factor and the permittivity of the sheet. [0·05 ; 5 88.]

25 A sample of bakelite was tested by the Schering bridge method at 25 kV, 50 c/s. Balance was obtained with a standard condenser of 106 $\mu\mu$F capacitance ; a condenser of capacitance 0·4 μF in parallel with a non-reactive resistor of 318 Ω ; and a non-reactive resistor of 120 Ω. Determine the capacitance, the equivalent series resistance, and the power factor of the specimen. Draw the vector diagram for the balanced bridge. [281 $\mu\mu$F ; 0·452 MΩ ; 0·04.]

26 In a low-voltage Schering bridge designed for the measurement of permittivity, the branch AB consists of two electrodes between which the specimen under test may be inserted ; BC is a standard air condenser of capacitance C_2 ; CD is a non-reactive resistor R_4 in parallel with a standard capacitor C_4 ; DA is a non-reactive resistor R_3 in parallel with a standard capacitor C_3. Without the specimen between the electrodes balance is obtained with the following values : $C_2 = 150$ $\mu\mu$F ; $C_3 = C_4 = 120$ $\mu\mu$F ; $R_3 = R_4 = 5\,000$ Ω. With the specimen inserted these values become $C_2 = 900$ $\mu\mu$F ; $C_3 = 200$ $\mu\mu$F ; $C_4 = 1\,000$ $\mu\mu$F ; and $R_3 = R_4 = 5\,000$ Ω In each test $\omega = 5\,000$. Find the relative permittivity and power factor of the specimen. [6 0 ; 0·02.]

27 An alternating-current Anderson bridge is arranged as follows : branch AB is an inductive resistor ; branches BC and ED are variable non-reactive resistors ; branches CD and DA are non-reactive resistors of 200 Ω each ; and branch CE is a condenser of 1 μF capacitance. The supply is connected to A and C, and the detector to B and E. Balance is obtained when the resistance of BC is 400 Ω and that of DE is 500 Ω. Calculate the resistance and inductance of AB, and draw a vector diagram for the balanced bridge. [400 Ω ; 0·48 H.]

28 The four arms of a Hay alternating-current bridge are arranged as follows : AB contains a coil of unknown impedance ; BC is a non-reactive resistor of 1 000 Ω ; CD is a non-reactive resistor of 883 Ω in series with a standard capacitor of 1·38 μF ; DA is a non-reactive resistor of 16 800 Ω. If the frequency of

the applied voltage is 50 cycles per sec and the bridge is balanced, determine the inductance and the resistance of the coil.

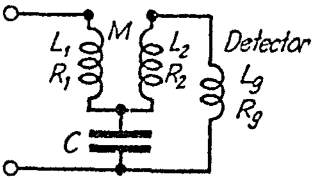
[20·4 H ; 2 440 Ω.]

29 The inductance of a coil is measured on the Heaviside-Campbell equal-ratio bridge. With the test coil short-circuited, balance is obtained when the adjustable non-reactive resistor is 12 63 Ω and the mutual inductometer is set at 0·1 mH. With the test coil in circuit, balance is obtained when the adjustable resistor is 25 90 Ω and the mutual inductometer is set at 15·9 mH. What is the resistance and inductance of the coil ?

[13·27 Ω ; 31·6 mH.]

30 In a modified Carey-Foster bridge the arm AB is of zero resistance ; BC is a non-reactive resistor of 100 Ω ; CD comprises a non-reactive resistor of 119 5 Ω in series with an unknown capacitor ; and DA contains a mutual inductance of 18 35 mH and a total non-reactive resistance of 200 Ω. The self-inductance of the coil in the arm DA is 40·6 mH. Determine the capacitance and the equivalent series resistance. [0 918 μF ; 1·5 Ω.]

31 In the Campbell frequency bridge shown, C has a fixed capacitance and M is a mutual inductance variable between 0 and $-$ 20 mH. If $R_1 = R_2 = 20$ Ω, $L_1 = L_2 = 30$ mH, $C = 2$ μF, $R_g = 180$ Ω, and $L_g =$ 10 mH, find (a) the lowest frequency that can be measured with the bridge, (b) the current in the detector at this frequency when the value of the mutual inductance is $-$ 10 mH and the applied voltage is 1 V. [796 cycles per sec ; 4 mA.]



D.C. Potentiometers

32 A standard cell of 1·0185 V used with a simple potentiometer balances at 50 cm. Calculate (a) the e.m.f. of a cell that balances at 72 cm ; (b) the percentage error in a voltmeter which balances at 64 5 cm when reading 1·33 V ; (c) the percentage error in an ammeter that reads 0 43 A when balance is obtained at 43·2 cm with the p d. across a 2-Ω resistor in the ammeter circuit.

[(a) 1·467 V ; (b) 1·2% high ; (c) 2 3% low.]

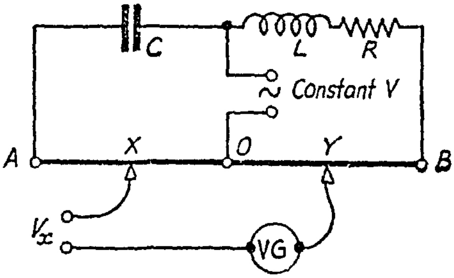
33 A simple potentiometer comprises a wire of resistance 21·0 Ω across which is connected a battery of 4·2 V. A test cell of voltage 1·43 V in series with a protective resistor of 15 Ω is tapped across the potentiometer and balance is obtained. Find the current flowing through the test cell when balance is disturbed by moving the tapping key a distance along the slide wire corresponding to 1 Ω.

[10·1 mA or 10·3 mA, according to direction.]

34 A potentiometer of total resistance 80 Ω and with a scale up to 2·00 V has a slide-wire 50 cm long on which there is a volt drop of 0·1 V. The test-circuit has an electromotive-force of 1·4300 V and a resistance (including galvanometer) of 850 Ω. If the galvanometer has a sensitivity of 17·5 mm per μA, and a minimum deflexion of 1 mm can be observed what is the limit of error in the reading of the test electromotive-force? What would the deflexion be if the position of the slider contact were changed by 2 mm? [$\pm$ 48 6 μV ; 8·2 mm.]

A.C. Potentiometers

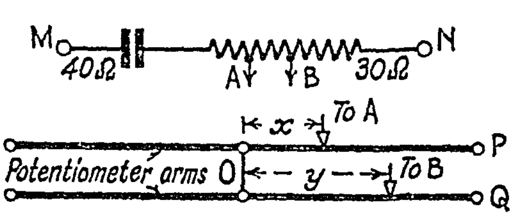
35 An a.c. potentiometer is arranged as shown, C being 6 μF. L and R are chosen such that the voltage-drops in each of the 100-Ω wires are in quadrature and equal to 1 V, when the supply frequency is 50 c/s. At 51 c/s balance with a voltage V_x is obtained by setting the contacts at X and Y where $OX/OA = 0{\cdot}8000$ and $OY/OB = 0{\cdot}6000$. Determine the change of phase of each of the potentiometer axes due to this change of frequency.



At 50 c/s, balance is effected with a voltage equal in magnitude and having the same phase angle with respect to the supply as V_x. Calculate the readings on each of the slide wires.

[OA, $-$ 12·45′ ; OB, $-$ 12·45′ ; 0·7816 ; 0·6037.]

36 Calculate the values of x and y when the voltage across AB is balanced on the a.c. potentiometer shown. Each semi-arm



of the potentiometer is 1 m long. The supply potential of M relative to N is $200/\underline{0°}$, while those of P and Q relative to zero are $1/\underline{70°}$ and $1/\underline{160°}$. The resistance between A and B is 0·25 Ω. [95·6 cm, $-$ 29 cm.]

37 A resistor of 100 Ω is connected in series with a 1 000-turn, iron-cored coil across a 50-c/s supply. The voltages across the resistor and across the coil, measured by an alternating-current potentiometer, are $0{\cdot}0587 - j0{\cdot}0650$ V and $0{\cdot}8595 + j0{\cdot}5010$ V respectively. The voltage measured across a 500-turn search-coil wound over the core is $-0{\cdot}4175 - j0{\cdot}2565$ V. Find the iron loss in the core, the copper loss in the coil and the leakage flux, considering the circuit as a transformer.

[0 155 mW ; 0 024 mW ; 7 lines (0·07 μWb).]

38 A non-reactive resistor of 1 000 Ω is connected in series with a coil and a condenser to a 50-c/s supply. If the voltages across each, measured by an alternating-current potentiometer, are $0{\cdot}6 - j\,0{\cdot}24$ V, $0{\cdot}6 + j\,0{\cdot}4$ V and $-0{\cdot}1 - j\,0{\cdot}4$ V respectively, find the power dissipated and the mean stored energy in each component.

[0 418, 0·264, 0 036 mW ; 0, 6·1, 4·21 ergs (0, 0 61, 0 421 μJ.).]

Power Measurement : Single-phase

3-ammeter Method

39 The readings on the ammeters connected for the three-ammeter method of power measurement are 2·5 A, 4 A, and 5·6 A in the non-inductive resistor, the load, and the main respectively. The terminal voltage is 300 V. Calculate the value of (*a*) the resistance : (*b*) the load impedance ; (*c*) the impedance of the combination ; (*d*) the power absorbed by the load ; (*e*) the power factor of the load ; (*f*) the total power supplied ; (*g*) the total power factor.

[(*a*) 120 Ω ; (*b*) 75 Ω ; (*c*) 53·6 Ω ; (*d*) 550 W ; (*e*) 0 458 ; (*f*) 1 300 W ; (*g*) 0·77.]

3-voltmeter Method

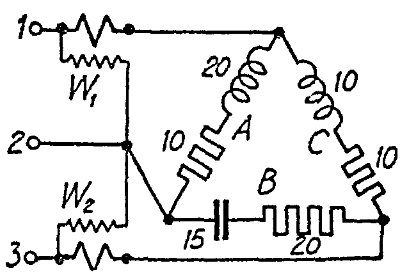
40 Three voltmeters are used to measure the power input to an inductive load in series with a resistance. The voltages across the non-inductive resistor, the load, and the combination respectively are 75 V, 80 V, and 140 V. The current is 4 A. Find (*a*) the impedance of the load and of the combination ; (*b*) the power absorbed by the load and by the non-inductive resistor ; (*c*) the power factor of the load and of the whole circuit.

[(*a*) 20 Ω, 35 Ω ; (*b*) 202 W, 300 W ; (*c*) 0·63, 0 9.]

Power Measurement—Three-phase

Wattmeter, Unbalanced Circuits

41 Determine for the 250-V, mesh-connected, unbalanced 3-phase system shown, (*a*) the current in each branch, (*b*) the current in each line, (*c*) the reading of each wattmeter. Find (*d*) the reading of each wattmeter when the current coils are in lines 1 and 2, and the voltage coils across 1–3 and 2–3 respectively. The resistances and reactances are in ohms. Draw



a vector diagram of voltages and currents. Phase sequence 1, 2, 3.

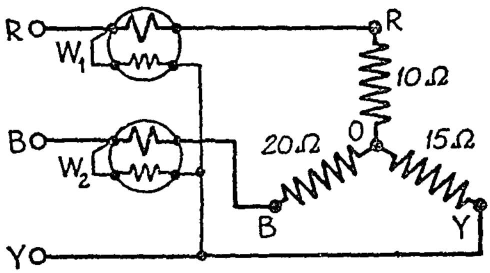
[(*a*) $I_A = 11{\cdot}15$ A, $I_B = 10$ A, $I_C = 17\ 7$ A; (*b*) $I_1 = 27{\cdot}1$ A, $I_2 = 3{\cdot}82$ A, $I_3 = 27{\cdot}2$ A; (*c*) $P_1 = 100$ W, $P_2 = 6\ 275$ W; (*d*) $P_1 = 5\ 920$ W, $P_2 = 455$ W.]

42 Three unequal non-reactive resistances are mesh-connected to a 440-V, symmetrical, 3-phase system. If the line currents are 50, 80, and 100 A, what is the total load? If two wattmeters are connected in circuit to measure the power input, determine the reading on each instrument. The current coils are connected in the lines carrying 50 and 100 A.

[56 5 kW; 15·25 kW and 41·25 kW.]

43 Three non-reactive resistances are connected across a symmetrical, 3-phase, 440-volt system as shown. Determine the current in each line, the total power input to the circuit, and the reading on each of the wattmeters. Phase sequence, *RYB*.

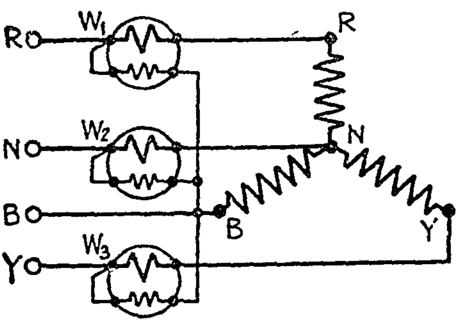
[20·6 A, 17·95 A, 14·7 A; 13·4 kW; $P_1 = 8{\cdot}2$ kW, $P_2 = 5{\cdot}2$ kW.]



44 Three loads are mesh-connected to a symmetrical, 3-phase, 440-V system. Load *a* takes 25 kW at unity power factor, load *b* takes 40 kVA at power factor 0·9 leading, and load *c* takes 45 kVA at power factor 0·7 lagging. Calculate the line currents and the readings of two wattmeters connected to measure the power input. The current coil of W_1 is connected in line *R* to the junction of *a* and *c*, the current coil of W_2 in the line *Y* to the junction of *a* and *b*. Phase sequence, *RYB*.

[102·5 A, 111 A, 192·5 A; 44 kW, 48·5 kW.]

45 A 440-V, 50-c/s, 3-phase, 4-wire system supplies a star-connected load, the branch impedances of which are $Z_R = 20 + j\ 10\ \Omega$, $Z_Y = 20 - j\ 25\ \Omega$, and $Z_B = 30\ \Omega$. Calculate the current in each line. If three wattmeters are connected as shown, determine the reading on each instrument. Phase sequence, *RYB*.



[$I_R = 11{\cdot}82 \angle -26° 42'$ A, $I_Y = 7{\cdot}98 \angle -68° 42'$ A, $I_B = 8{\cdot}47 \angle 120°$ A, $I_N = 10{\cdot}2 \angle -30° 12'$ A ; $W_1 = 4{\cdot}09$ kW, $W_2 = 2{\cdot}24$ kW, $W_3 = 3{\cdot}26$ kW.]

46 A 420-V, 3-phase, 3-wire system has a current of $10 \angle -30°$ A in phase R and $14 \angle -60°$ A in phase Y. Find the current in phase B and the readings on two wattmeters with current coils in phases R and Y and voltage coils connected to phase B. Phase sequence RYB ; reference vector V_R.

[$7{\cdot}32 \angle -77°$ A ; 4 200 W ; 0 W.]

Wattmeter, Balanced Circuits

47 Prove that the power in a balanced 3-phase circuit can be deduced from the readings of two wattmeters. Draw the relevant connexion and vector diagrams. If the readings on two wattmeters so connected are 5·0 kW and 0·5 kW, the latter reading being obtained after reversal of the current-coil connexions, calculate the power and power factor of the load. Show how two wattmeters are connected to measure the reactive volt-amperes in a circuit. [4·5 kW ; 0·426.]

48 A 3-phase, 3-wire, 415-V system supplies a balanced load of 20 A at power factor 0·8. The current coil of wattmeter 1 is in phase R, and of wattmeter 2 in phase B. Calculate (*a*) the power ; (*b*) the reactive kilovolt-amperes ; (*c*) the reading on 1 when its voltage coil is across R and Y ; (*d*) the reading on 2 when its voltage coil is across B and Y ; (*e*) the reading on 1 when its voltage coil is across B and Y ; (*f*) the reading on 2 when its voltage coil is across R and Y.

[(*a*) 11·5 kW ; (*b*) 8·625 kVAr ; (*c*) 3 26 kW ; (*d*) 8·24 kW ; (*e*) 4·98 kVAr ; (*f*) 4 98 kVAr.]

49 Two 3-phase alternators supply a balanced load. The readings of two wattmeters connected to measure the total power are 300 kW and 900 kW. Readings of ammeters in the line and in the two alternator circuits are in the ratio 5 : 4 : 3. Find the load on each alternator and the readings of the two wattmeters measuring it.

[265 kW, 935 kW ; 493 kW, − 228 kW ; 409 kW, 526 kW.]

50 The power input to a 2 000-V, 50-c/s, 3-phase motor, running on full load at an efficiency of 90%, is measured by two wattmeters which indicate 300 kW and 100 kW respectively. Calculate (*a*) the input ; (*b*) the power factor ; (*c*) the line current ; (*d*) the horse-power output.

[(*a*) 400 kW ; (*b*) 0·76 ; (*c*) 152 A ; (*d*) 482 h.p.]

51 A 500-V, 3-phase motor has an output of 50 h.p. and operates at a power factor of 0 85 with an efficiency of 90%. Calculate the reading on each of two wattmeters connected to measure the input. [28·15 kW, 13·35 kW.]

Wattmeter Connexions

52 A single-phase load is supplied from the R and B lines of a 440-V, symmetrical, 3-phase, 4-wire system. Two wattmeters are connected in the R and B phases with their voltage coils to N. If the instruments read 20 kW and − 10 kW respectively, determine the current taken by the load. [120 A.]

53 A wattmeter having a moving coil with $R = 100\ \Omega$ and $L = 0{\cdot}02$ H is used on a 300-c/s circuit. To measure reactive volt-amperes, the 3 000-Ω swamp resistor is replaced by a capacitor C. Calculate C; also the error in the reading of reactive volt-amperes when $\cos\phi = 0{\cdot}95$ lagging. [0·177 μF ; 9% high.]

54 A wattmeter reads 5·54 kW when its current coil is connected in the red phase, and its voltage coil is connected between the neutral and red phase of a symmetrical 3-phase system supplying a balanced load of 30 A at 400 V. What will be the reading on the instrument if the connexions to the current coil remain unchanged and the voltage-coil be connected between the blue and yellow phases? What does this figure represent?

[7 2 ; $\sqrt{3} \times$ kVAr per phase.]

55 A three-wire, 200-V, star-connected system has the following loads: $Z_R = 50 + j0$ ohms; $Z_Y = 40 + j20$ ohms; $Z_B = 25 - j60$ ohms. Find (a) the potential of the star-point, (b) readings on wattmeters 1 and 2 with current coils in phases R and Y and (i) voltage coils across RB and YB, (ii) voltage coils across BY and BR; (c) power and reactive volt-amperes from products of complex quantities (i) $V_{RB}\, I_R$ and $V_{YB}\, I_Y$, (ii) $V_{RO}\, I_R$, $V_{YO}\, I_Y$ and $V_{BO}\, I_B$; (d) $R_R\, I_R^2$, $R_Y\, I_Y^2$, $R_B\, I_B^2$, (e) $X_R\, I_R^2$, $X_Y\, I_Y^2$, $X_B\, I_B^2$. Rotation RYB.

[(a) $50\ \underline{/-169^\circ}$ V ; (b) (i) 465, 230 W, total 695 W, (ii) 130, 71 VAr, total 201 VAr ; (c) (i) 466 W, 419 VAr ; 230 W, 214 VAr, total 696 W, 205 VAr ; (ii) 491 W, 0 VAr ; 99 W, 50 VAr ; 106 W, 254 VAr ; total 696 W, 204 VAr ; (d) 490, 99, 106 W ; total 695 W ; (e) 0, 49 5, 254 VAr ; total 204·5 VAr.]

56 The upper and lower current coils of a two-element wattmeter are connected in the red and blue phases of a symmetrical 3-phase system supplying a 440-V star-connected balanced load. When the upper and lower voltage coils are connected between the yellow and red and the yellow and blue phases, the wattmeter reads 3 550 W ; when they are connected between the blue phase and star point and the star point and red phase respectively the wattmeter reads 806 W. What is the load current? [5 A.]

57 A mesh-connected load has impedances $Z_{RY} = 10 + j5\ \Omega$, $Z_{YB} = 15 + j15\ \Omega$, and $Z_{BR} = 5 + j8\ \Omega$ and is connected to a 400-V supply of phase sequence RYB. Calculate the readings of two wattmeters with (a) current coils in lines R and B and voltage coils connected to line Y, and (b) current coils in lines Y and B and voltage coils connected to line R.

[(a) $P_R = 4{\cdot}8$ kW, $P_B = 22{\cdot}3$ kW ; (b) $P_Y = 20{\cdot}08$ kW, $P_B = 7{\cdot}06$ kW.]

Harmonic Analysis

58 A 50-c/s current wave containing 3rd, 5th and 7th harmonics has an r.m.s. value of 10 A. By means of a shunt, 1/100th of this current is passed through the moving coil of a dynamometer instrument. Through the fixed coil are passed in turn sinusoidal currents of 150, 250, 350 c/s having maximum values 85 mA, 110 mA, 105 mA respectively, the phase of each current being adjusted to give a maximum reading on the instrument. The readings so obtained are 9·95, 6·24 and 9·51 divisions. Estimate the value of the 3rd, 5th and 7th harmonics as percentages of the fundamental. The instrument gives a reading of 100 divisions when a direct current of 100 mA is passed through the two coils in series. [16·97% ; 8·21% ; 13·13%.]

Resistance Thermometers

59 The temperature of a furnace is measured by means of a platinum resistance thermometer. The resistance of the thermometer is 5·00 Ω at 0° C, 6·85 Ω at 100° C, and 13·70 Ω at 496° C. Find the α and δ constants of the instrument and thence the true temperature when the resistance is 9·50 Ω. Use the formula $R_t = R_0 (1 + \alpha t)$ for the range 0°—100° C, and the correction $\delta t\ (t - 100)$ °C for the range above 100° C, to obtain true temperatures. [$\alpha = 0{\cdot}0037$; $\delta = 1{\cdot}495 \times 10^{-4}$; 248° C.]

Electronics Measurements

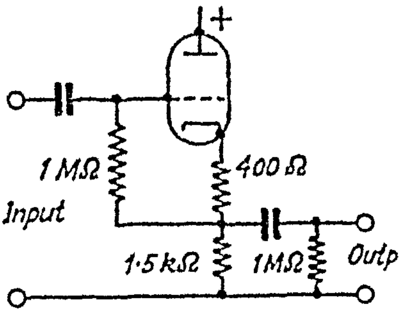
60 A multirange a.c. voltmeter having an impedance of 1 000 Ω/V is connected across the output of an oscillator. The meter reads 3·57 V on its 0–5-V range and 4·17 V on its 0–15-V range. Find the output impedance of the oscillator and its open-circuit output voltage. Assume the output impedance to be purely resistive. [1·38 kΩ ; 4·55 V.]

61 The following results were obtained from capacitive measurements on a triode valve :—

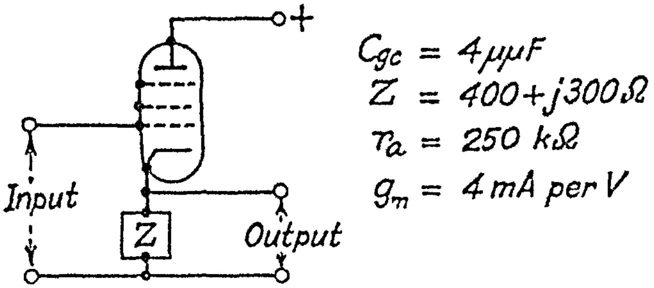
Grid to anode and cathode strapped	17 pF
Anode to grid and cathode strapped .	7 pF
Cathode to grid and anode strapped .	20 pF

If the valve has an amplification factor of 5 and a slope resistance of 3 000 Ω, what is the effective value of the input capacitance with a 2 000-Ω anode load? [21 pF.]

50 Calculate the output impedance of the cathode-follower circuit shown. The valve has a slope resistance of 20 kΩ and an amplification factor of 40 and the reactance of the coupling capacitors may be assumed negligible at the operating frequency. [557 Ω.]

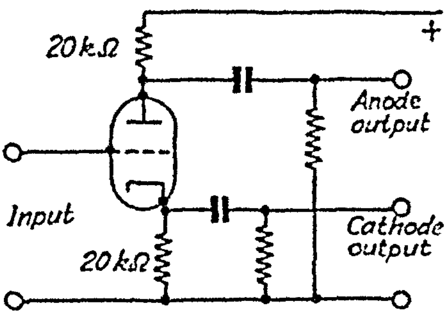


51 In the cathode-follower circuit shown, the screen grid of the pentode valve is decoupled to the cathode by a condenser of



negligible reactance. Find (*a*) stage gain, (*b*) output impedance, (*c*) input impedance at 5 Mc/s. Neglect all other capacitances. [(*a*) 0·7 ; (*b*) 250 Ω ; (*c*) 22·6 $\angle -64°$ kΩ.]

52 The triode valve used in the phase-splitter shown has a slope resistance of 20 kΩ and an amplification factor of 50. Assuming the coupling capacitors and grid resistors to be effectively infinite, find (*a*) the anode output impedance and (*b*) the cathode output impedance.
[(*a*) 19·7 kΩ ; (*b*) 755 Ω.]



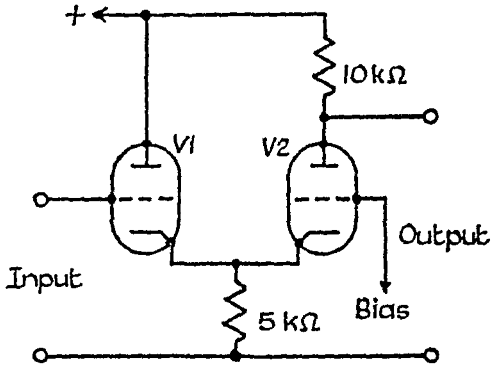
Multi-valve Stages

53 Using a single section of a double triode with an amplification factor of 100, a stage gain of 45 may be obtained using an anode load of 100 kΩ. Find the stage gain obtained with both sections of the valve connected in parallel and using an anode load of 47 kΩ. Assume the two sections to be identical and the static operating conditions to be unchanged. [48·5.]

54 Three valves, each having a mutual conductance of 10 mA/V and an amplification factor of 5, are connected in parallel. A resistive load of 12 Ω is fed from the anodes by a transformer designed for maximum power transfer. The input signal to the grids is 20 sin ωt volts. Find (*a*) the required turns-ratio of the transformer, (*b*) the peak-to-peak current swing in each valve, (*c*) the r.m.s. voltage across the transformer primary, and (*d*) the power developed in the load.
[(*a*) 3·73 : 1 ; (*b*) 200 mA ; (*c*) 35·4 V ; (*d*) 7·5 W.]

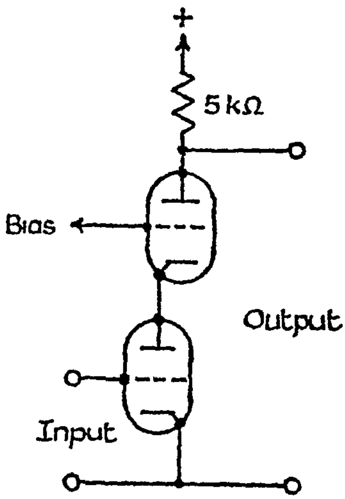
55 Two triodes V1 and V2 have their anodes connected together and feed a 15-Ω load through a 25 : 1 step down transformer. V1 and V2 have slope resistances of 100 kΩ and 20 kΩ, and amplification factors of 100 and 20, respectively. Find the power in the load if a signal of 1 V r.m.s. is fed to (*a*) V1 grid with V2 grid earthed, (*b*) V2 grid with V1 grid earthed, and (*c*) V1 and V2 grids together. [(*a*) 3 84 mW ; (*b*) 3 84 mW ; (*c*) 15·4 mW.]

56 Find (*a*) the gain and (*b*) the output impedance of the cathode coupled amplifier shown. Each valve has a slope resistance of 3 kΩ and an amplification factor of 30 and the grid of V2 is effectively earthed at the operating frequency. (*c*) By what percentage will the gain be reduced if a 2-kΩ resistor is inserted in the anode circuit of V1, and (*d*) what will then be the output impedance?

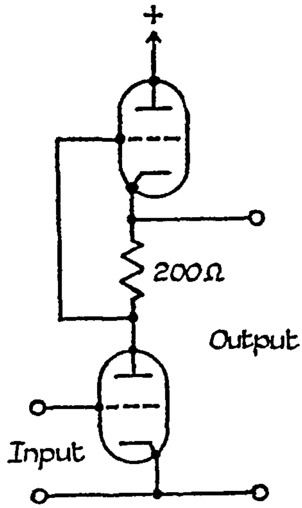


[(*a*) 18·5 ; (*b*) 3·73 kΩ ; (*c*) 12% ; (*d*) 4·4 kΩ.]

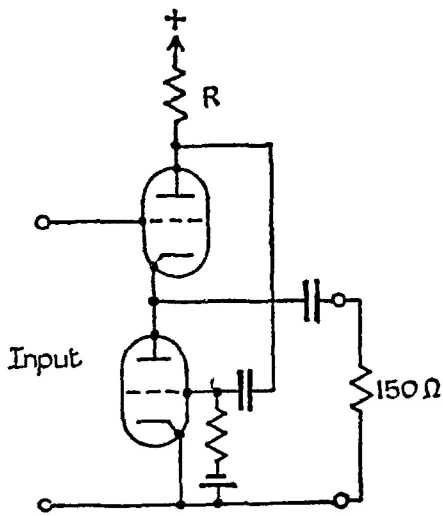
57 The cascade amplifier shown has two similar valves of slope resistance 3 kΩ and amplification factor 30. The grid of the upper valve is effectively earthed at all frequencies at which the circuit is to operate. Find (*a*) the gain between the input and output terminals, (*b*) the output impedance and (*c*) the gain between the input terminals and the anode of the lower valve.
[(*a*) − 46 ; (*b*) 4·75 kΩ ; (*c*) − 2·38.]



58 The shunt-regulated amplifier shown employs two valves of slope resistance 3 kΩ and amplification factor 30. Find (*a*) the gain, (*b*) the output impedance, and (*c*) the gain between the input terminals and the anode of the lower valve.
[(*a*) – 22 1 ; (*b*) 790 Ω ; (*c*) – 22·6.]



59 In the circuit shown, find the value of *R* such that a current change in the upper valve is accompanied by an equal and opposite change in the lower valve. Find the gain and output impedance under these conditions Assume no loss or phase change in the inter-valve or load coupling circuits. Both valves have a slope resistance of 3 kΩ and an amplification factor of 30.
[*R* = 110 Ω ; 0 72, 47 Ω.]



Power Output Stages and Power Amplifiers

60 The maximum permissible anode dissipation of a certain triode is 120 W. Find its power output when used as (*a*) a class A amplifier with an efficiency of 20%, (*b*) a class C amplifier with an efficiency of 70%. [(*a*) 30 W ; (*b*) 280 W.]

61 Plot a curve showing (*a*) the anode rating, and (*b*) the overall efficiency, of a power amplifier valve to give a radio-frequency output of 1 kW for anode efficiencies of 40% to 90%, assuming that the cathode heating power is 20% of the anode rating. State the overall efficiency when the anode efficiency is 70%.
[66%.]

62 One point on the I_a/V_a characteristic of a triode with an amplification factor of 5 and a slope resistance of 2 000 Ω is given by 50m A and 180 V at $V_g = -10$ V. The valve is operated with an anode resistance load of 3 660 Ω, a grid bias of − 26 V, and a supply voltage of 330 V. The safe dissipation of the valve and the resistor are 6 5 W and 3·5 W respectively. Find the permissible range of the continuously applied alternating grid voltage if the ratings of the valve and resistor are not to be exceeded. [5·9 V − 8·33 V r.m.s.]

63 Using the triode curves given in Fig. 7, estimate the anode voltage and the grid bias required to give a no-signal anode dissipation of 6 W, when the anode supply voltage is 300 V and the d.c. resistance of the anode load is (*a*) 2 500 Ω, (*b*) 1 000 Ω, (*c*) zero.
*[(*a*) 240, − 7·6 V ; (*b*) 280, − 10 V ; (*c*) 300, − 11·4 V.]

64 A triode has the following characteristics :

V_a	40	80	120	160	200	240	280	320
I_a for V_g = 0	3·2	8	14					
− 4			3·4	7 6	13·1			
− 8				0·8	3·4	7 6		
− 12						1·1	3·7	7·7

Plot the characteristics and add an anode dissipation curve for 2 W. The valve is operated with a resistance load of 22 300 Ω. The no-signal values of anode current and voltage are 8·6 mA and 168 V respectively and the signal voltage applied to the grid is ± 4 V. Find (*a*) the supply voltage, (*b*) the fundamental-frequency output power, and (*c*) the percentage second-harmonic distortion.
*[(*a*) 360 V ; (*b*) 0 08 W ; (*c*) 3 0%.]

65 The characteristics of a triode are as follows :

V_a	V_gV	0	−10	−20	−30	−40	−50
200 V	I_a mA	150	100	50	10	2	0
300 V	I_a mA	—	—	150	100	50	10

The load consists of a series circuit of inductive reactance 3 000 Ω and a resistance 2 000 Ω. The supply voltage is 500 V and the grid is biased to − 40 V. For a grid swing of ± 30 V peak, plot the path of operation on the I_a/V_a and I_a/V_g characteristics and find the efficiency of the arrangement as a generator of a.c. power. *[3 %.]

66 Find the maximum output power of a triode operating under class A conditions with amplification factor 5 and slope resistance 5 000 Ω for a grid swing of ± 10 V (max.). [62·5 mW.]

67 Find the ratio of a transformer required to couple a loudspeaker of impedance 15 Ω to an output valve having a slope resistance of 2 500 Ω, the optimum load being three times this figure. [22 3 : 1.]

68 An output valve having a slope resistance of 2 000 Ω, an optimum load of 4 000 Ω and an amplification factor of 5 is required to deliver an output of 1 W to a loudspeaker, the impedance of which may be regarded as a resistance of 2 Ω. Find the anode voltage fluctuation, the minimum permissible value of grid bias, and the ratio of the output transformer. [± 89 5 V ; 27 V ; 44·7.]

69 A triode of slope resistance 1 000 Ω and amplification factor 5 is transformer-coupled to a 4-Ω resistive load. Find (*a*) the turns ratio of the transformer for maximum power output. If the input signal to the grid is 7 07 sin ωt volts, find (*b*) the current in the load, and (*c*) the a.c. voltage at the anode. [(*a*) 15·8 : 1 ; (*b*) 197 mA r.m.s. ; (*c*) 12·5 V r.m.s.]

70 A triode amplifier has a 5-kΩ resistive anode load and draws a mean anode current of 50 mA, the anode supply voltage being 450 V. The valve has an amplification factor of 10 and a mutual conductance of 5 mA/V. If the input signal is 20 sin ωt volts find (*a*) the useful power in the load, (*b*) the total power in the load, and (*c*) the valve anode dissipation. [(*a*) 2 W ; (*b*) 14·5 W ; (*c*) 8 W.]

71 The anode characteristics of a triode used in a power-output stage are given in Fig. 7. The output transformer ratio is 16/1, the d.c. resistance of the primary is 500 Ω, the load on the secondary is a resistance of 15 Ω and the cathode-bias resistor is 250 Ω, by-passed by a capacitor of negligible reactance at all operating frequencies. For an anode supply voltage of 275 V, find graphically (*a*) the no-signal anode voltage and current, and (*b*) the anode dissipation. If a signal of 5 V r.m.s. is applied to the stage, find also (*c*) the power output, and (*d*) the anode efficiency. *[(*a*) 255 V, 30 mA ; (*b*) 7·65 W ; (*c*) 1·02 W ; (*d*) 13·3 %.]

72 Fig. 9 gives the anode characteristics of a pentode valve used in a power-output stage which has an h.v. supply of 250 V.

The load is a resistance of 15 Ω, supplied through a transformer with a turns ratio of 20/1. Find (*a*) the grid bias required to give an anode dissipation of 9 W, (*b*) the power output for an input signal of 15 V peak, and (*c*) the anode efficiency for this output. Construct a curve of output voltage to a base of input voltage. *[(*a*) − 16·7 V ; (*b*) 3 W ; (*c*) 33%.]

73 The valves of a class A push-pull triode power-output stage have anode characteristics given by Fig. 7. The anode supply is 300 V and the anode dissipation of each valve is 6 W. If the load is a 12·5-Ω resistor and the ratio of the total primary to secondary turns of the output transformer is 40/1, find (*a*) the output power, and (*b*) the anode efficiency, for an input signal of 7·75 V r.m.s. *[(*a*) 3 W ; (*b*) 25%.]

74 A class B radio-frequency push-pull amplifier delivers a total output of 6 kW. The dynamic resistance of the anode-to-anode tank circuit is 3 000 Ω. Calculate the useful load across each valve if the amplitude of the fundamental of the anode current is one-half the peak-value. [1 500 Ω.]

75 A 1 Mc/s class C amplifier drives a tank circuit having an inductance of 200 μH, and a capacitance adjusted to give unity-power-factor loading. The grid bias is − 240 V, the h.v. supply 1 000 V, and a sinusoidal grid swing of ± 300 V peak is applied. The anode current in milliamperes is given by $I_a = 0{\cdot}6V_a + 3{\cdot}0V_g$. If the tank circuit load is adjusted to the optimum value, such that the lowest anode and highest grid voltages are equal, estimate (*a*) the power given to the tank circuit, (*b*) the efficiency, (*c*) the *Q* value of the tank circuit. Ignore grid loss and cathode heater power.
[(*a*) 44 W ; (*b*) 81% ; (*c*) 8.]

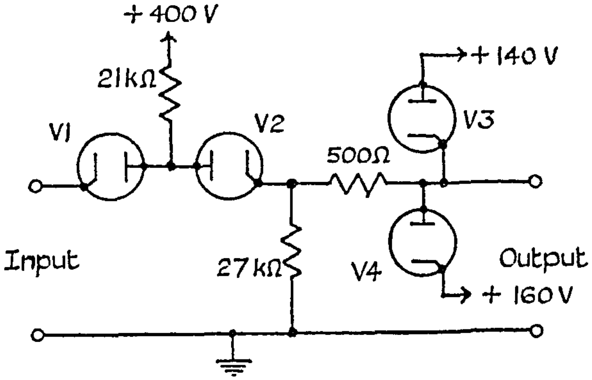
76 The approximate characteristics of a small transmitting valve are given by $I_a = 0{\cdot}6V_a + 3{\cdot}0V_g$, where I_a, V_a and V_g denote the anode current in milliamperes, the anode and grid voltages respectively. The valve is used as a class C power amplifier. The root-mean-square value of the drive and tank-circuit voltages are 127 and 212 V respectively. The input frequency is 0·796 Mc/s, the supply voltage is 400 and the grid bias is − 100 V. Determine analytically or graphically the efficiency of the amplifier and the *Q* value of the tank circuit. The inductance of the tank circuit is 100 μH. [0·62 ; 4·2.]

77 The characteristics of a small transmitting valve may be represented by $I_a = 0{\cdot}6V_a + 3{\cdot}0V_g$, where I_a is the anode current in milliamperes, and V_a and V_g are the anode and grid voltages respectively. The anode supply voltage is 1 000 V and the grid bias is − 240 V. Calculate the power output when the angle of flow is 140°. The amplitude of the drive voltage is 300 V.
[45·9 W.]

Diode Limiters

78 A square-wave generator of output impedance 10 kΩ gives a 20-V signal when the output is connected to earth through a 5-kΩ load resistor, the voltage excursion being from + 20 V to + 40 V with respect to earth. If the anode of a diode is also connected to the output of the generator, the cathode being connected to a d.c. supply of 35 V and of source impedance 500 Ω, find the maximum voltage to which the signal then rises. Assume the forward impedance of the diode to be constant at 200 Ω. [35·9 V.]

79 Four diodes are arranged as shown; all voltages are with respect to earth. If the voltage across the input terminals has the following values: (*A*) 240 V, (*B*) 162 V, (*C*) 158 V, (*D*) 150 V, (*E*) 142 V, (*F*) 138 V, (*G*) 110 V, all positive with respect to earth, find the corresponding voltages at the output terminals



and the current flowing in V1 in each condition. If the input voltage v varies with time t as follows: $v = 110$ V from $t = 0$ to $t = 9$ μs, v rises linearly from 110 V at $t = 9$ μs to 150 V at $t = 10$ μs, $v = 150$ V from $t = 10$ μs to $t = 15$ μs; v rises linearly from 150 V at $t = 15$ μs to 240 V at $t = 99$ μs, v falls linearly from 240 V at $t = 99$ μs to 110 V at $t = 100$ μs, use the seven points calculated above to plot a graph of output voltage against time and show the input voltage dotted. The four diodes each have a forward impedance of 200 Ω and zero conductance in the reverse direction.

[(*A*) 161 V, (*B*) 160·3 V, (*C*) 158 V, (*D*) 150·1 V, (*E*) 142·3 V, (*F*) 139·7 V, (*G*) 139 V, (*A*) 0, (*B*) 3·6 mA, (*C*) 5·6 mA, (*D*) 6·3 mA, (*E*) 7 mA, (*F*) 8·7 mA, (*G*) 13·7 mA.]

Oscillators and Waveform Generators

80 A parallel-tuned circuit has a capacitance of 500 μμF, a coil *Q*-factor of 40, and a resonance frequency of 500/2π kc/s. It is used in a tuned-anode oscillator with a valve of slope re-

sistance 10 kΩ and amplification factor 40. Find the frequency of oscillation and the minimum value of the mutual coupling between anode and grid coils for sustained oscillation.
[80 kc/s ; 212 μH.]

81 The minimum value of mutual inductance between grid and anode circuits, required for the maintenance of oscillations in a tuned anode oscillator, is 0·74 mH when the load on the oscillator is equivalent to a resistance of 20 Ω in the tuned circuit, and 0 98 mH when the load is equivalent to a resistance of 40 Ω. Find the frequency of oscillation if the valve has a slope resistance of 20 000 Ω and an amplification factor of 10. [29 kc/s.]

82 Find approximately the condition for sustained oscillations in a triode with tuned grid circuit, given that the tuning capacitance is 0 01 μF, the loss resistance 100 Ω, the amplification factor 10, and the slope resistance 10 000 Ω. [M = 1 mH.]

83 The output circuit of a tuned-anode oscillator has a decrement of $\pi/10$. It is (*a*) directly coupled to an amplifier valve, (*b*) inductively coupled to an amplifier valve, (*c*) connected to an aerial which forms the capacitive branch of the output circuit. The anode current contains a second and fourth harmonic of 30% and 10% of the fundamental amplitude respectively. Determine the percentage harmonic content of the input voltage to the amplifier in (*a*) and (*b*) and of the aerial current in (*c*).
[(*a*) 2%, 0·267% ; (*b*) 2%, 0 267% ; (*c*) 4%, 1·07%]

84 The anode current of a triode oscillator has an alternating-current component of 10 mA peak at a certain frequency. Find the current flowing in the tuned anode load which comprises a coil with 2% decrement in parallel with a condenser.
[1·57 A peak.]

85 The anode current of an oscillator is given by $i_a = 2\ (\sin \omega t - 0{\cdot}2 \sin 3\ \omega t)$ amp during one half-cycle and is zero during the other half. The anode voltage is given by $v_a = 5\ 000\ (1 - 0\ 7 \sin \omega t)$ volts. Find the power dissipated at the anode and the overall efficiency if the filament heating absorbs 300 W.
[1 220 W ; 53·5%.]

86 A high-frequency class C power oscillator has an anode supply voltage of 1 000 V and uses a triode valve with grid-cathode and grid-anode spacings of 1 mm and 7 mm respectively. The mean anode current is 30 mA, the oscillatory component of the anode voltage is 900 V peak and the angle of flow is 120°. Assuming that the anode current consists of rectangular pulses and that the maximum value of the grid voltage is equal to the minimum value of the anode voltage, estimate the frequency at which the anode efficiency of the oscillator is 20% less than its low frequency value, and the efficiency at this frequency.
[70 5 Mc/s ; 54·5%.]

87 A simple gas-filled triode timebase-waveform generator consists of a 0 05-μF capacitor, charged from a 300-V supply through a resistor which can be varied from 5 to 50 kΩ, and discharged through the valve when its anode-cathode voltage exceeds 200 V. Assuming the flyback time to be negligible, find the repetition-frequency range of the timebase if the valve ceases to conduct when the capacitor voltage drops to 15 V.

[380–3 800 c/s.]

88 The anode load of a screen grid valve consists of a coil of inductance 159 μH and resistance 25 Ω tuned by a variable capacitor. If the slope resistance of the valve is $-$ 35 kΩ find the lowest frequency at which the valve will oscillate.

[940 kc/s.]

CHAPTER XVII

MODULATION AND DEMODULATION

Modulated Waves

1 A transmitting aerial is radiating an unmodulated carrier, the radiated power being 1 200 W. If the carrier is now modulated simultaneously by two pure notes to depths of 20% and 40% respectively, calculate the total power radiated. [1 320 W.]

2 The reading on an ammeter in an aerial circuit is 10·86 A when the aerial current is sinusoidally modulated to a depth of N%. The ammeter reading becomes 11·49 A when the depth of modulation is increased to $(N + 20)$%. Calculate the r.m.s. value of the unmodulated carrier. [10 A.]

3 A circuit consisting of a coil of 25 μH and a condenser of 0·0005 μF has an effective resistance of 0·15 Ω. The e.m.f. induced in it from a neighbouring circuit is 1 V r.m.s. at its resonant frequency. Calculate this frequency and the current induced in the circuit. If the e.m.f. is modulated to a depth of 100% at 1 000 c/s, calculate the r.m.s. value of the new current and its depth of modulation. [1·42 Mc/s, 6·66 A ; 7·0 A, 43·2%.]

4 Find the current produced in a circuit of inductance 400 μH, capacitance 300 $\mu\mu$F and resistance 5 Ω, when an e.m.f. of 1 V r.m.s. is induced in it at the resonant frequency. If the carrier is simultaneously modulated to 40% at 5 000 c/s and to 50% at 8 000 c/s, find the new current in the circuit. [0·2 A, 0·2005 A.]

5 An 800-kc/s carrier wave is modulated equally at 50 and 5 000 c/s. In what ratio will these be reproduced by a receiver tuned to 800 kc/s, the input circuit of which has a logarithmic decrement of 0·0314 ? Assume that the remainder of the receiver has a flat response. [1·6 : 1.]

6 The frequency of an oscillator is varied slowly over a range ± 1·5 kc/s about its mean frequency of 1 Mc/s. The oscillator output is applied to a tuned amplifier which has a Q-factor of 100 and is de-tuned by 5 kc/s. Find the depth of modulation of the resultant amplitude-modulated signal. [13·5%.]

Modulators and Modulated Amplifiers

7 In an anode-choke modulator, the amplification factor of the control triode is 10, its slope resistance is 10 000 Ω and the grid has an applied sinusoidal signal voltage of ± 3 V (peak)

at 1 000 cycles per sec. The slope resistance of the oscillator triode is 50 000 Ω, and the choke inductance is 10 H. Estimate the variation of the anode voltage of the oscillator.

[$\pm$ 24·7 V (peak).]

8 The output from a class C amplifier is modulated to a depth of 50%. Calculate the turns-ratio of the transformer required for transformer-coupled anode modulation if the modulator slope-resistance is 2 000 Ω. The amplifier is supplied at 3 000 V and operates with an efficiency of 65%. The valve is rated at 1 000 W. [1 : 0·75.]

9 The anode dissipation of a class C, anode-modulated power amplifier is 472 W when the modulation depth is 60%. The efficiency of the power amplifier is 60%, and of the modulator valve is 25%. Find the carrier power and the valve dissipation at 100% modulation. Find also the A.F. output and the rating of the modulator valve to give this degree of modulation. Calculate the overall efficiency at 60% modulation.

[600 W ; 600 W ; 500 W ; 2 000 W ; 23 6%.]

10 An anode-modulated, class C amplifier is required to supply a total power of 200 W when modulated to 80% by a class B modulator. Find the necessary rating of the power amplifier valve and each of the modulator valves, if the power amplifier and modulator operate at efficiencies of 75% and 55% respectively. [66 6 W ; 26 5 W.]

11 A grid-modulated class C amplifier attains an instantaneous efficiency of 65% at the modulation peaks. Estimate its average efficiency for a sinusoidal modulation of depth 80%. What must be the rating of the valve to deliver a modulated output of 2 kW ? If the same valve were used in an anode-modulated class C amplifier, what output would be obtained for an average efficiency of 65%, and what would require to be the rating of the modulator valve working at an efficiency of 20% ?

[48% ; 4 1 kW ; 1·5 kW ; 7·6 kW.]

12 A push-pull modulator stage comprising two valves, each rated at 750 W and operating at an efficiency of 55%, is used to anode-modulate a class C power amplifier operating at an efficiency of 60%. The power amplifier is rated at 3 000 W and is operating at its fully-loaded condition Find the carrier power and the maximum depth of modulation obtainable.

[3 400 W ; 80 2%.]

13 A valve of rating 500 W is to be used as (*a*) a grid-modulated class C power amplifier, (*b*) a class C power amplifier anode-modulated by a class A modulator having an efficiency of 25%. It is required to modulate to a depth of 70%. Find the carrier power and overall efficiency in each case. Find also the instan-

taneous efficiencies of the power amplifiers at the peaks and troughs of grid modulation. Assume that the unmodulated power amplifier efficiencies are (*a*) 35% and (*b*) 70% respectively.

[(*a*) 269 W, 43·6% ; (*b*) 938 W, 43·7%. 59·5%, 10·5%.]

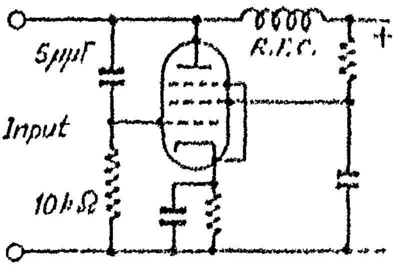
14 A grid-modulated, class C amplifier utilizing a valve having a rating of 2 100 W is modulated to a depth of 60%. Calculate the R.F. power output, the percentage of power contained in the sidebands, and the percentage increase of power due to modulation. If the depth of modulation is changed so that the amplifier efficiency increases by 0·75%, calculate the new depth of modulation, and the power dissipated in the valve. Take the unmodulated efficiency as 30%.

[1 062 W ; 15·2% ; 18% ; 90% ; 1 784 W.]

15 A 500-kc/s oscillator has a tuned-circuit inductance of 1 mH and its output is frequency-modulated by a reactance-valve modulator. The modulator phase-shift network consists of a 1 400-Ω resistor and a 6-$\mu\mu$F capacitor, the latter being connected between grid and anode of the pentode modulator valve. If the modulating voltage varies the valve mutual conductance by $\pm$ 1 mA/V, find the limits of the frequency deviation of the oscillator.

[479 kc/s – 521 kc/s.]

16 The pentode valve in the frequency modulator circuit shown has a mutual conductance of 2 mA/V. Find the stage input impedance at a frequency of 5 Mc/s in terms of a parallel combination of resistance and the appropriate reactance parameter. It may be assumed that the decoupling capacitors have negligible reactance and that the impedance of the R.F. choke is effectively infinite.



[670 Ω in parallel with 30·8 $\mu\mu$F.]

Detectors and Frequency Changers

17 A diode detector circuit with an efficiency of 90% represents an effective load of 0·25 MΩ across the source of modulated R.F. to which it is connected. Estimate suitable values for the resistance and capacitance to be used with the diode for satisfactory operation at modulation frequencies and depths up to 10 kc/s and 60% respectively.

[0·45 MΩ ; 265 $\mu\mu$F.]

18 A diode in a dem[illegible] [illegible]uit has an inter-electrode capacitance of 10 $\mu\mu$F. [illegible] [illegible] RC combination with C = 100 $\mu\mu$F. [illegible] [illegible] of R required when [illegible] audio-frequency co[illegible] [illegible] [illegible]centage modulation de[illegible] are respectively (*a*) 400 c/s and 30%, (*b*) 400 c/s and 80%, (*c*) 5 000 c/s and 30%. [(*a*) 12 6 MΩ ; (*b*) 3 MΩ ; (*c*) 1 MΩ.]

19 A simple diode detector has a load of 500 kΩ in parallel with a capacitance of 100 μμF. If the maximum modulation depth of the input signal is 80%, find the highest modulation frequency that can be detected without harmonic distortion. [2 390 c/s.]

20 The static characteristic of an anode-bend detector is parabolic, $I_a = p(q + V_g)^2$, and two points on the curve are given by $V_g = 0$, $I_a = 8$ mA ; $V_g = -5\ 0$ V, $I_a = 2$ mA. Find the increase in the mean anode current produced by an alternating grid voltage of 1·5 V r.m.s. The grid bias equals − 5 V. [0 18 mA.]

21 Plot the anode current due to a sinusoidal signal voltage of 12 V peak applied to the grid of a triode detector, the characteristic of which at a constant anode voltage of 80 V is as follows :

Grid voltage, V . .	+10	5	0	−2	−4	−6	−8
Anode current, mA . .	5 6	3 75	1·9	1·2	0 6	0 2	0

Estimate the mean increase in the rectified current due to the signal. The valve operates with a bias of − 2 V. *[0·9 mA.]

22 The transfer characteristic of a valve is given by $I_a = 16 + 4V_g + 0\ 1V_g^2$ mA, where I_a is the instantaneous value of the anode current and V_g the corresponding grid voltage. If two input voltages, of 0·1 and 0 4 V r.m.s. and of different frequencies, are applied to the valve, find (*a*) the steady component, and (*b*) the r.m s. value of the difference-frequency component of the anode current. The grid bias is − 3 V. [(*a*) 4 92 mA ; (*b*) 5·66 μA.]

23 The dynamic characteristic of a frequency-changer circuit is known to be parabolic and is defined by the following pairs of values :

Anode current, mA . .	5	1 5	0
Grid-cathode voltage, V .	0	− 5	− 10

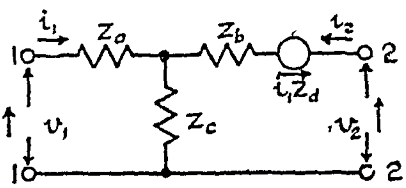
Determine the amplitude of the difference-frequency component of the output voltage when the following three voltages in series are applied between grid and cathode : 1 mV at 1 Mc/s, 3 V at 1·1 Mc/s and a direct voltage of − 2 V. The anode load impedance is 10 kΩ. [2·4 mV.]

CHAPTER XVIII

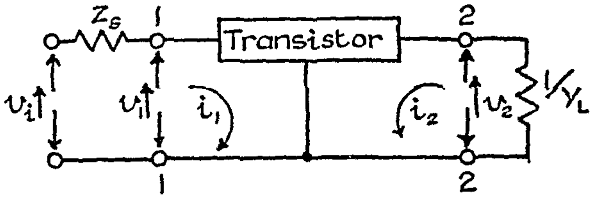
TRANSISTORS AND TRANSISTOR AMPLIFIERS

"z" Parameters

1 Prove the mesh equations, $v_1 = i_1z_{11} + i_2z_{12}$ and $v_2 = i_1z_{21} + i_2z_{22}$, from the network shown. The "z" parameters are defined as follows :—
$z_{11} = v_1/i_1$ measured with 22 open-circuited ;
$z_{22} = v_2/i_2$ measured with 11 open-circuited ;
$z_{12} = v_1/i_2$ measured with 11 open-circuited ;
$z_{21} = v_2/i_1$ measured with 22 open-circuited.



2 The diagram shows a transistor, represented by a four-terminal network, operating from a source of impedance Z_S and connected to a load Z_L. Use the mesh equations to find



expressions for (a) the voltage gain v_2/v_1, (b) the current gain i_2/i_1, (c) the input impedance v_1/i_1, and (d) the output impedance v_2/i_2, in terms of the "z" parameters z_{11}, z_{22}, z_{12}, z_{21}.

[(a) $Z_Lz_{21}/\{(z_L + z_{22})(Z_S + z_{11}) - z_{12}z_{21}\}$;
(b) $-z_{21}/(Z_L + z_{22})$; (c) $z_{11} - z_{12}z_{21}/(Z_L + z_{22})$;
(d) $z_{22} - z_{12}z_{21}/(Z_S + z_{11})$.]

3 A transistor circuit is matched so that the impedance of the driving source is equal to the input impedance of the transistor, and the load impedance is equal to the output impedance. Express in terms of the "z" parameters (a) the input impedance, (b) the output impedance, and (c) the power gain defined as the ratio of the power developed in the load to the power available from the source.

[(a) $z_{11}\sqrt{1 - z_{12}z_{21}/z_{11}z_{22}}$; (b) $z_{22}\sqrt{1 - z_{12}z_{21}/z_{11}z_{22}}$;
(c) $z_{21}^2/z_{11}z_{22}(1 + \sqrt{1 - z_{12}z_{21}/z_{11}z_{22}})^2$.]

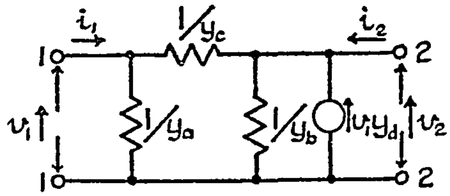
4 A transistor has a load of impedance Z_L and is driven from a source of impedance Z_S. From the expressions for (*a*) voltage gain, (*b*) current gain, (*c*) input impedance, and (*d*) output impedance, in terms of the "z" parameters z_{11}, z_{22}, z_{12}, z_{21}, develop expressions for the same quantities (*a*), (*b*), (*c*) and (*d*) in terms of the parameters z_{11}, z_{22}, z_{in}, z_{out} and α, where z_{11} and z_{22} are unchanged, z_{in} is defined as the input impedance with the output short-circuited, z_{out} is the output impedance with the input short-circuited, and α is the current gain with the output short-circuited. In case (*a*), give alternative expressions.

[(*a*) $-\alpha z_{22}Z_L/\{Z_L(Z_S+z_{11})+z_{22}(Z_S+z_{in})\}$ or $-\alpha z_{22}Z_L/\{Z_S(Z_L+z_{22})+z_{11}(Z_L+z_{out})\}$; (b) $\alpha z_{22}/(Z_L+z_{22})$; (*c*) $z_{11}(Z_L+z_{out})/(Z_L+z_{22})$; (*d*) $z_{22}(Z_S+z_{in})/(Z_S+z_{11})$.]

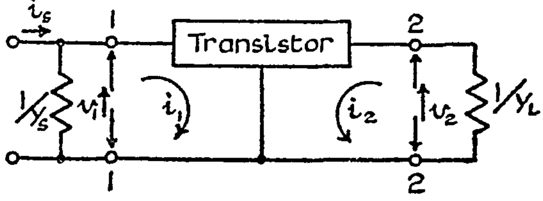
"y" Parameters

5 Prove the mesh equations $i_1 = v_1y_{11} + v_2y_{12}$ and $i_2 = v_1y_{21} + v_2y_{22}$ from the network shown. The "y" parameters are defined as follows :—

$y_{11} = i_1/v_1$ measured with 22 short-circuited ;
$y_{22} = i_2/v_2$ measured with 11 short-circuited ;
$y_{12} = i_1/v_2$ measured with 11 short-circuited ;
$y_{21} = i_2/v_1$ measured with 22 short-circuited.



6 A transistor has a load of admittance Y_L and is driven from a current source of admittance Y_S. Using the mesh equations for the four-terminal network representing the transistor, find



expressions for (*a*) the voltage gain v_2/v_1, (*b*) the current gain i_2/i_S, (*c*) the input admittance i_1/v_1, and (*d*) the output admittance i_2/v_2, in terms of the "y" parameters y_{11}, y_{22}, y_{12}, y_{21}.

[(*a*) $-y_{21}/(Y_L+y_{22})$; (*b*) $y_{21}Y_L/\{(Y_L+y_{22})(Y_S+y_{11})-y_{12}y_{21}\}$; (*c*) $y_{11}-y_{12}y_{21}/(Y_L+y_{22})$; (*d*) $y_{22}-y_{12}y_{21}/(Y_S+y_{11})$.]

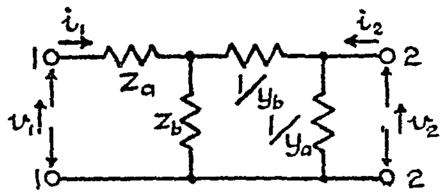
7 A transistor is connected to a load of admittance Y_L and is driven from a current source of admittance Y_S. From the expressions for (*a*) voltage gain, (*b*) current gain, (*c*) input admittance, and (*d*) output admittance, in terms of the "*y*" parameters $y_{11}, y_{22}, y_{12}, y_{21}$, develop expressions for the quantities (*a*), (*b*), (*c*) and (*d*) in terms of the parameters $y_{11}, y_{22}, y_{in}, y_{out}$ and u, where y_{11} and y_{22} are unchanged, y_{in} is defined as the input admittance with open-circuited output, y_{out} is the output admittance with open-circuited input, and u is the voltage gain with the output open-circuited. In case (*b*) give alternative expressions.

[(*a*) $uy_{22}/(Y_L + y_{22})$;
(*b*) $-uy_{22}Y_L/\{Y_L(Y_S + y_{11}) + y_{22}(Y_S + y_{in})\}$
or $-uy_{22}Y_L/\{Y_S(Y_L + y_{22}) + y_{11}(Y_L + y_{out})\}$;
(*c*) $y_{11}(Y_L + y_{out})/(Y_L + y_{22})$; (*d*) $y_{22}(Y_S + y_{in})/(Y_S + y_{11})$.]

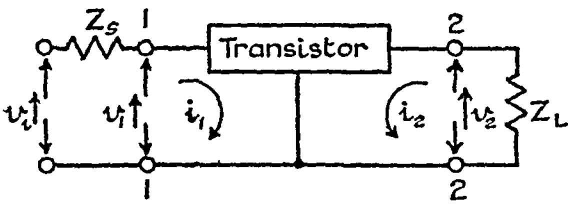
"*h*" Parameters

8 Prove the mesh equations $v_1 = i_1h_{11} + v_2h_{12}$ and $i_2 = i_1h_{21} + v_2h_{22}$ from the network shown. The "*h*" parameters are defined as follows :—

$h_{11} = v_1/i_1$ measured with 22 short-circuited ;
$h_{22} = i_2/v_2$ measured with 11 open-circuited ;
$h_{12} = v_1/v_2$ measured with 11 open-circuited ;
$h_{21} = i_2/i_1$ measured with 22 short-circuited.



9 A transistor is connected to a load of admittance Y_L and is driven from a source of impedance Z_S. Using the mesh equations for the four-terminal network representing the transistor,



find expressions for (*a*) the voltage gain v_2/v_i, (*b*) the current gain i_2/i_1, (*c*) the input impedance v_1/i_1, and (*d*) the output admittance i_2/v_2 in terms of the "*h*" parameters $h_{11}, h_{22}, h_{12}, h_{21}$.

[(*a*) $-h_{21}/\{(Z_S + h_{11})(Y_L + h_{22}) - h_{12}h_{21}\}$;
(*b*) $h_{21}Y_L/(Y_L + h_{22})$; (*c*) $h_{11} - h_{12}h_{21}/(Y_L + h_{22})$;
(*d*) $h_{22} - h_{12}h_{21}/(Z_S + h_{11})$.]

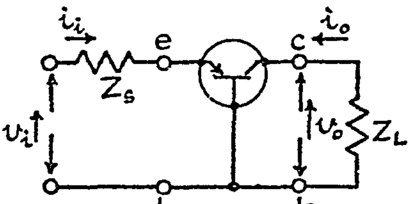
Relationships between "z", "y", and "h" Parameters

10 Using the three pairs of mesh equations in terms of the "z", "y" and "h" parameters respectively, find expressions for the "z" parameters (a) in terms of the "y" parameters, and (b) in terms of the "h" parameters.

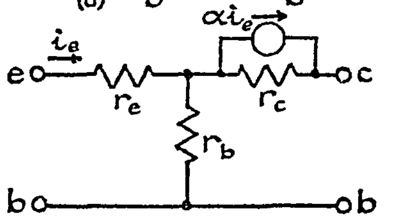
[(a) $z_{11} = y_{22}/(y_{11}y_{22} - y_{12}y_{21})$, $z_{22} = y_{11}/(y_{11}y_{22} - y_{12}y_{21})$, $z_{12} = - y_{12}/(y_{11}y_{22} - y_{12}y_{21})$, $z_{21} = - y_{21}/(y_{11}y_{22} - y_{12}y_{21})$; ($b$) $z_{11} = h_{11} - h_{12}h_{21}/h_{22}$, $z_{22} = 1/h_{22}$, $z_{12} = h_{12}/h_{22}$, $z_{21} = - h_{21}/h_{22}$.]

Low-frequency T Equivalent Circuit

11 A common base amplifier is represented for the purpose of analysis by figure (a). If the transistor be represented by the low-frequency T equivalent circuit shown at (b), find the "z" parameters and evaluate these for the case when $r_e = 20 \Omega$, $r_b = 700\ \Omega$, $r_c = 1\ M\Omega$, and $\alpha = 0\ 97$. Find also the voltage gain, current gain, input impedance and output impedance if $Z_S = 500\ \Omega$ and $Z_L = 3\ k\Omega$.



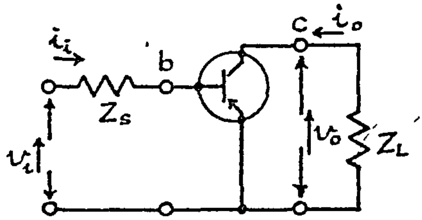
(a)



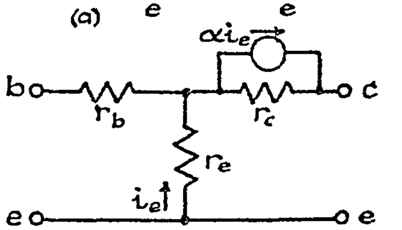
(b)

[$z_{11} = 720\ \Omega$, $z_{22} = 1\ 001\ k\Omega$, $z_{12} = 700\ \Omega$, $z_{21} = 971\ k\Omega$; $v_o/v_i = 5{\cdot}34$, $i_o/i_i = - 0{\cdot}967$, $Z_i = 43\ \Omega$, $Z_o = 445\ k\Omega$.]

12 The basic circuit of a common emitter amplifier is shown at (a). If the transistor is represented by the equivalent circuit at (b), find the "z" parameters and evaluate these when $r_e = 20\ \Omega$, $r_b = 700\ \Omega$, $r_c = 1\ M\Omega$, and $\alpha = 0\ 97$. Find also the voltage gain, current gain, input impedance and output impedance if $Z_S = 500\ \Omega$ and $Z_L = 3\ k\Omega$



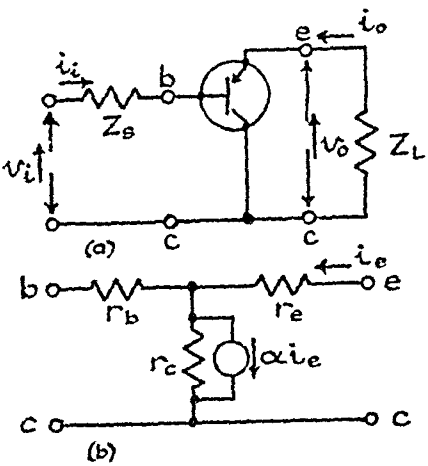
(a)



(b)

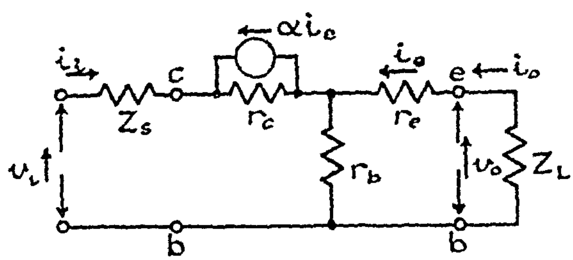
[$z_{11} = 720\ \Omega$, $z_{22} = 30{\cdot}02\ k\Omega$, $z_{12} = 20\ \Omega$, $z_{21} = - 970\ k\Omega$; $v_o/v_i = - 48{\cdot}7$, $i_o/i_i = 29{\cdot}4$, $Z_i = 1{\cdot}31\ k\Omega$, $Z_o = 45{\cdot}9\ k\Omega$.]

13 The basic circuit of a common collector amplifier is shown at (*a*). If the transistor be represented by the equivalent circuit at (*b*), find the "*z*" parameters and evaluate these when $r_e = 20\ \Omega$, $r_b = 700\ \Omega$, $r_c = 1\ \mathrm{M}\Omega$, and $\alpha = 0{\cdot}97$. Find also the voltage gain, current gain, input impedance and output impedance if $Z_S = 500\ \Omega$ and $Z_L = 3\ \mathrm{k}\Omega$.



[$z_{11} = 1\ 001\ \mathrm{k}\Omega$, $z_{22} = 30{\cdot}02\ \mathrm{k}\Omega$, $z_{12} = 30\ \mathrm{k}\Omega$, $z_{21} = 1\ \mathrm{M}\Omega$; $v_o/v_i = 0{\cdot}984$, $i_o/i_i = -\ 30{\cdot}3$, $Z_i = 92\ \mathrm{k}\Omega$, $Z_o = 50\ \Omega$.]

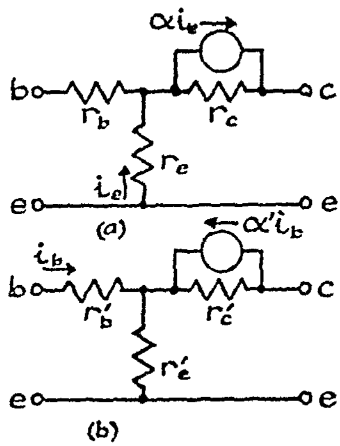
14 A voltage generator of source impedance Z_S is connected across the output terminals of a common base amplifier, that is between the collector and base. Representing the transistor by the low-frequency T equivalent circuit shown, find (*a*) the fraction of this voltage which appears between emitter and base if the external impedance across these terminals is Z_L. Find also



the ratio of the current in Z_L to the current in Z_S. Draw diagrams showing the same conditions but with the transistor connected (*b*) as a common emitter stage, and (*c*) as a common collector stage, and find the same two quantities, v_o/v_i and i_o/i_i, in each case. $Z_S = 500\ \Omega$, $Z_L = 3\ \mathrm{k}\Omega$, $r_e = 20\ \Omega$, $r_b = 700\ \Omega$, $r_c = 1\ \mathrm{M}\Omega$, $\alpha = 0{\cdot}97$.

[(*a*) $v_o/v_i = 0\ 000\ 69$, $i_o/i_i = -\ 0{\cdot}19$; (*b*) $v_o/v_i = 0{\cdot}000\ 45$, $i_o/i_i = -\ 0{\cdot}005\ 4$; (*c*) $v_o/v_i = 0{\cdot}14$, $i_o/i_i = -\ 0{\cdot}03$.]

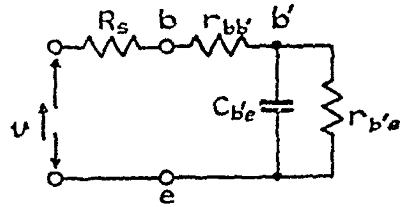
15 The low-frequency T equivalent circuit for a common emitter stage is shown at (*a*). The circuit is redrawn at (*b*) showing the current generator as a function of the base current. Equate the "*z*" parameters of the two circuits and thus find the relationship between r_b, r_e, r_c, α and r'_b, r'_e, r'_c, α'.



$[r'_b = r_b,\ r'_e = r_e,\ r'_c = (1 - \alpha)r_c,\ \alpha' = \alpha/(1 - \alpha).]$

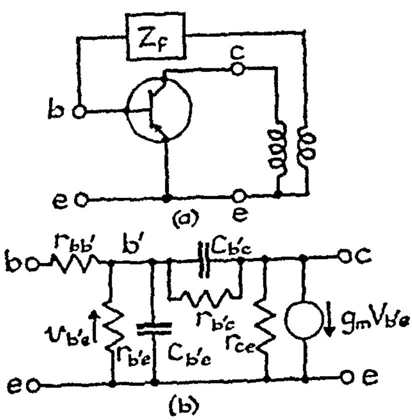
Hybrid π and Single Frequency Equivalent Circuits

16 A common emitter amplifier has a load of low impedance so that the frequency response is effectively determined by the input characteristics. Using the input section of the hybrid π common emitter equivalent circuit to represent the input conditions as shown, determine the frequency at which the gain will have fallen by 3 db when the stage is operated from a source of impedance R_S.



$[\{1/(R_S + r_{bb'}) + 1/r_{b'e}\}/2\pi C_{b'e}.]$

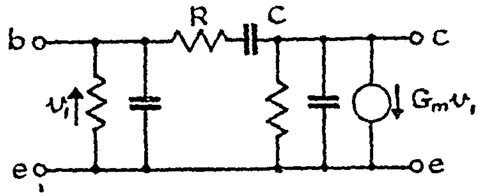
17 The basic circuit of a 500 kc/s neutralized common emitter amplifier is shown at (*a*), the transistor being represented by the hybrid π equivalent circuit shown at (*b*). The neutralizing impedance, Z_f, has such a value that the inherent feedback due to the elements $r_{b'c}$, $C_{b'c}$ is exactly neutralized, and the input impedance is thus the value obtained by considering the output terminals of the transistor to be short-circuited, while the output impedance is the value obtained with the input short-circuited. If $r_{ce} = 50$ kΩ, $r_{b'c} = 2{\cdot}5$ MΩ, $r_{b'e} = 1\,250$ Ω, $r_{bb'} = 100$ Ω, $g_m = 40$ mA/V, $C_{b'c} = 10$ pF and $C_{b'e} = 1\,000$ pF, find the input resistance



and capacitance R_i and C_i, and the output resistance and capacitance R_o and C_o, where R_i, C_i and R_o, C_o are parallel components. Make approximations where justified.

[$R_i = 680\ \Omega$, $C_i = 800$ pF, $R_o = 19$ kΩ, $C_o = 44$ pF.]

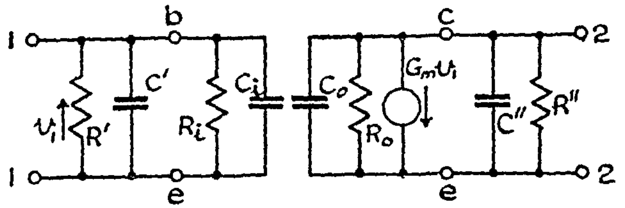
18 Convert the hybrid π equivalent circuit of Problem 17 to the single frequency circuit shown by taking the y_{12} and y_{21} parameters of each. Using the data of Problem 17, find R and C.



If the feedback transformer has a step-down ratio of 5 : 1, find the values of the external neutralizing components R_f and C_f, these being the series components of Z_f. Find also the modulus value of G_m.

[$R = 10{\cdot}5$ kΩ, $C = 9{\cdot}3$ pF, $R_f = 2{\cdot}1$ kΩ, $C_f = 46{\cdot}5$ pF, $G_m = 37$ mA/V.]

19 Representing the neutralized amplifier of Problems 17 and 18 by the circuit shown, find the equivalent parallel components R', C' and R'', C'' of the external neutralizing impedance Z_f, and hence find (*a*) the total effective input resistance and

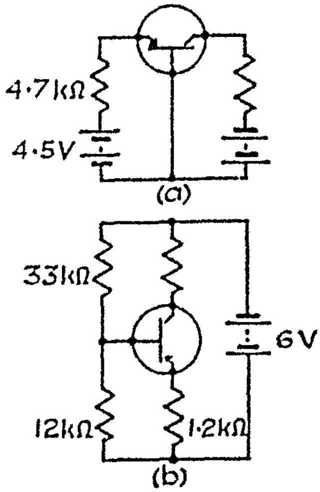


capacitance, and (*b*) the total effective output resistance and capacitance. If a load resistor R_L in parallel with a lossless inductor L is connected across the output terminals 22, find (*c*) the power gain when the values of R_L and L are chosen to make this a maximum. Give (*d*) the value of L.

[(*a*) 660 Ω, 843 pF ; (*b*) 18 400 Ω, 46 pF ; (*c*) 36 db ; (*d*) 2·2 mH.]

20 Using the hybrid π equivalent circuit shown at (*b*) in Problem 17, show that the short-circuit current gain $\alpha' = \alpha'_o/(1 + jf/f'_{co})$, where α'_o is the short-circuit current gain at zero frequency and f'_{co} is the cut-off frequency defined as the frequency at which α' has fallen by 3 db. Using also the relevant data of Problem 17, make approximations where justified.

D.C. Bias Conditions

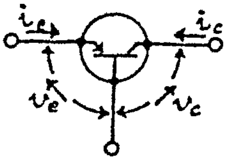


21 The illustrations at (*a*) and (*b*) give basic bias circuits for a common base and common emitter amplifier respectively. In each case calculate the value of the collector current In case (*a*), $I_{co} = 10\ \mu A$ and $\alpha = 0{\cdot}97$. In case (*b*), $I'_{co} = 150\ \mu A$ and $\alpha' = 30$ In each case take a voltage drop between base and emitter of 0·15 volts, and assume the values for α and α' hold for the d.c. condition.

[(*a*) 0·91 mA ; (*b*) 0·98 mA.]

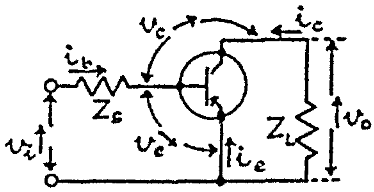
Transistor Equations

22 The performance of a transistor at low frequencies is defined by the two equations, $v_e = i_e r_1 + i_c \gamma r_1$ and $v_c = i_c r_2 + i_e \alpha r_2$, where v_e, v_c i_e, i_c are specified in the diagram and $r_1 = v_e/i_e$ with $i_c = 0$, $r_2 = v_c/i_c$ with $i_e = 0$, $\alpha = -i_c/i_e$ with $v_c = 0$, and $\gamma = -i_e/i_c$ with $v_e = 0$. Express r_1, r_2, α, γ in terms of the parameters of the low-frequency T equivalent circuit, r_e, r_b, r_c, α. Neglect r_b compared to r_c in the case of α.



[$r_1 = r \; + r_b$, $r_2 = r_c + r_b$, $\alpha = \alpha$, $\gamma = r_b/(r_e + r_b)$.]

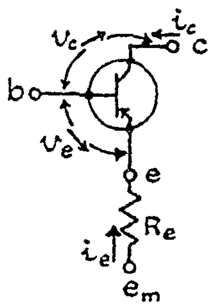
23 A transistor the performance of which is defined by the equations $v_e = r_1(i_e + \gamma i_c)$ and $v_c = r_2(i_c + \alpha i_e)$, is connected as shown in the basic diagram. If $Z_s = 500\ \Omega$, $Z_L = 3\ k\Omega$, $r_1 = 720\ \Omega$, $r_2 = 1\ M\Omega$, $\gamma = 0{\cdot}97$ and $\alpha = 0\ 97$, find the voltage gain v_o/v_i and the current gain i_c/i_b.



[$v_o/v_i = -47{\cdot}5$, $i_c/i_b = 29{\cdot}8$.]

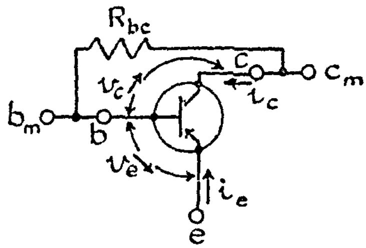
24 A common emitter amplifier has a feedback resistor R_e=100 Ω connected as shown. If the transistor performance is defined

by the equations $v_e = r_1(i_e + \gamma i_c)$ and $v_c = r_2(i_c + \alpha i_e)$, find (*a*) modified values, r_{1m}, r_{2m}, γ_m, α_m, of the four constants r_1, r_2, γ, α, which will define the performance of the transistor and feedback resistor combined. If the amplifier is operated from a source of impedance 500 Ω, and is connected to a load of impedance 3 kΩ, find (*b*) the voltage and (*c*) the current gains. The values of the four constants are $r_1 = 720\ \Omega$, $r_2 = 1\ \text{M}\Omega$, $\gamma = 0{\cdot}97$, $\alpha = 0{\cdot}97$.



[(*a*) $r_{1m} = R_e + r_1$, $r_{2m} = r_2$, $\gamma_m = \gamma r_1/(R_e + r_1)$, $\alpha_m = \alpha$; (*b*) $-$ 17·9 ; (*c*) 29 2.]

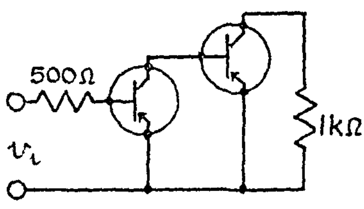
25 A common emitter amplifier has a feedback resistor $R_{bc} = 10$ kΩ connected as shown. The transistor performance is defined by the equations $v_e = r_1(i_e + \gamma i_c)$ and $v_c = r_2(i_c + \alpha i_e)$. Find (*a*), modified values r_{1m}, r_{2m}, γ_m, α_m, of the constants r_1, r_2, γ, α, which will define the performance of the transistor and feedback resistor combined. If the transistor amplifier is driven from a source of impedance 500 Ω and connected to a load of impedance 3 kΩ, find (*b*) the voltage gain and (*c*) the current gain. The transistor constants are $r_1 = 720\ \Omega$, $r_2 = 1\ \text{M}\Omega$, $\gamma = 0{\cdot}97$, $\alpha = 0{\cdot}97$.



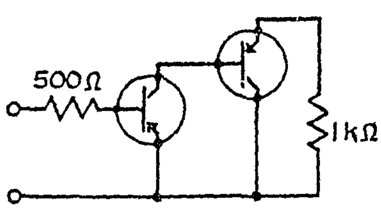
[$r_{1m} = r_1\{1 + r_2(1 - \alpha\gamma)/R_{bc}\}/(1 + r_2/R_{bc})$, $r_{2m} = r_2/(1 + r_2/R_{bc})$, $\gamma_m = \gamma/\{1 + r_2(1 - \alpha\gamma)/R_{bc}\}$, $\alpha_m = \alpha$; (*b*) $-$ 12 8 ; (*c*) 2·86.]

Two-stage Amplifiers

26 The basic circuit of a two-stage low-frequency amplifier is shown. If the transistor constants are $r_e = 20\ \Omega$, $r_b = 700\ \Omega$, $r_c = 1\ \text{M}\Omega$, $\alpha = 0{\cdot}97$, find (*a*) the overall voltage gain and (*b*) the overall current gain. [(*a*) 530 ; (*b*) 970.]



27 The basic circuit of a two-stage low-frequency amplifier is shown. The transistor constants are $r_e = 20\ \Omega$, $r_b = 700\ \Omega$, $r_c = 1\ \text{M}\Omega$, $\alpha = 0{\cdot}97$. Find (*a*) the overall voltage gain and (*b*) the overall current gain.



[(*a*) 800 ; (*b*) 500.]

CHAPTER XIX

PROPERTIES OF LINES, CABLES AND INSULATORS

Overhead Lines : Sag and Span

1 A single span of a transmission line is 500 ft in length, the supporting structures being level. The conductors have a cross-section of 0 4 in^2 hard-drawn copper. Find the sag which must be allowed if the tension is not to exceed one-fifth of the ultimate strength (60 000 lb per in^2), (*a*) in still air; and (*b*) with a wind pressure of 0·9 lb per ft and a $\frac{1}{2}$-in ice coating. In the latter case find also the vertical sag. [(*a*) 10 ft; (*b*) 16 ft, 14·9 ft.]

2 An overhead conductor consists of 7 strands of silicon-bronze having an ultimate strength of 8 000 kg per cm^2 and an area of 2 2 cm^2. When erected between supports 600 m apart and having a 15-m difference in level, find the vertical sag which must be allowed so that the factor of safety shall be 5 with the wire loaded with 1 kg of ice per m and a wind pressure of 1·75 kg per m. The wire weighs 2·03 kg per m. [30 6 m.]

Overhead Lines : Inductance

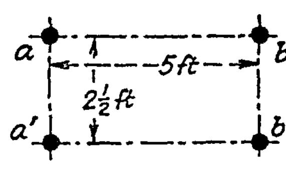
3 Calculate the loop inductance per mile of a 1-phase circuit comprising 2 parallel conductors 1 m apart and 1 cm in diameter. [3·57 mH.]

4 A 1-phase circuit comprises two parallel conductors 0 25 in in diameter spaced 3 ft apart. Calculate the inductance per mile of the loop if the material of the conductor is (*a*) copper, (*b*) steel, relative permeability 50. [(*a*) 3·81; (*b*) 11·69 mH.]

5 A 10-mile transmission line consists of a pair of conductors, $\frac{1}{2}$ in in diameter, spaced (*a*) 4 ft, (*b*) 2 ft, apart. Find the impedance of the loop formed by joining the conductors at one end, for a frequency of 25 c/s. [(*a*) 7·08Ω; (*b*) 6 56Ω.]

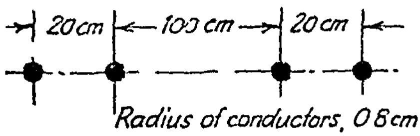
6 A wire 6 mm in diameter is suspended at a constant height 10 m above sea water, which constitutes the return conductor. Calculate the inductance of the system per km. [1 81 mH.]

7 In the single-phase line shown, the conductors have a diameter of $\frac{1}{2}$ in: *a* and *a*′ in parallel form one connexion, and *b* and *b*′ in parallel the return connexion. Calculate the total inductance of the line per mile assuming the current to be equally divided.



[1 88 mH.]

8 A 1-phase supply is effected by conductors arranged as shown, the current being equally divided between both conductors forming a pair. Find the inductance per km of the system. [1·31 mH per km.]

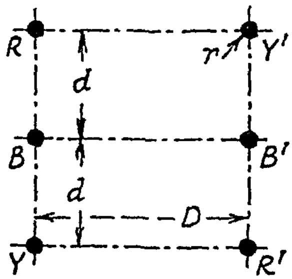


9 Find the inductance per mile per conductor (line to neutral) of a 3-phase system in which 0 5-in diameter conductors are placed at the corners of an equilateral triangle, the distance between conductors being 5 ft. [1·85 mH.]

10 The 0·77-in diameter conductors of a 3-phase, 3-wire system are situated at the corners of a triangle. giving conductor spacings of 12, 18 and 27 ft. The conductors are transposed at regular intervals and the load is balanced Calculate the inductance per mile per conductor (line to neutral). [2·115 mH.]

11 A 3-phase, 3-wire system consisting of $\frac{1}{2}$-in diameter conductors spaced 10 ft apart in a horizontal plane supplies a balanced load. Calculate the inductance per mile of each conductor (line to neutral). Explain the significance of the complex number in the expression.
[$L_R = 1\ 95 - j\ 0\ 193$; $L_Y = 1{\cdot}845$; $L_B = 1\ 95 + j\ 0\ 193$ mH/mile.]

12 A symmetrical, double-circuit, 3-phase line is to have its six similar conductors arranged as shown. Calculate the effective inductance in millihenries per mile of each phase (line to neutral), assuming the supply currents to be balanced and equally divided. The phase sequence is RYB. Take I_R as datum. D=10 ft, d = 5 ft, and r = 0·5 in.
[$L_R = 0{\cdot}818 + j\ 0\ 0811$; $L_Y = 0\ 818 - j\ 0{\cdot}0811$; $L_B = 0{\cdot}827$.]



Overhead Lines : Capacitance

13 Two parallel overhead railway conductors, 25 ft apart, are at a height of 17 ft above earth. One line is supplied at 11 000 V, $16\frac{2}{3}$ cycles per sec. If the other line is open-circuited at both ends, calculate the voltage induced between it and earth when the insulation resistance between the two lines is (a) infinite, (b) 5 MΩ per mile. The conductor diameter is 0·4 in.
[(a) 3 770 V, (b) 3 920 V.]

14 A wire 6 mm in diameter and 1 km in length is suspended at constant height 10 m above sea water. Calculate the capacitance between conductor and earth. [0 0027 μF.]

15 Calculate the capacitance per kilometre of a pair of parallel wires 5 mm in diameter, spaced uniformly 20 cm apart in air. [0 0064 μF.]

16 A 1-phase overhead line 20 miles long consists of 2 parallel wires each 0 2 in diameter, 5 ft apart. If the line voltage be 50 kV at 50 c/s, calculate the charging current with the line open-circuited. [2·2 A.]

Overhead Lines : Corona

17 Determine the disruptive critical voltage and the critical voltages for local and general corona on a 3-phase overhead transmission line, consisting of three stranded copper conductors, spaced 8 ft apart, at the corners of an equilateral triangle. Air temperature and pressure are 70° F and 29 in of mercury respectively. Conductor diameter, 0·408 in ; irregularity factor, 0 85 ; and surface factors 0 7 and 0 8. [97 kV ; 116, 132 kV.]

Cables : Capacitance, One-core

18 A submarine cable is 3 500 km long and has a conductor 0 5 cm in diameter with a gutta-percha covering 0 5 cm thick. Calculate the total capacitance of the cable, taking the relative permittivity of gutta-percha as 4. [708 μF.]

19 A 10-mile length of 1-phase concentric cable takes a charging current of 8 A when connected to 11 000-V, 50-c/s bus-bars. The inner conductor has a diameter of 0·5 in, and the insulation is 0 4 in in radial thickness. Find the value of the relative permittivity. [2·48.]

20 A 1-core cable 1 km in length has a core diameter of 1 0 cm and a diameter under the sheath of 2 5 cm. The relative permittivity is 3 5. The power factor on open-circuit is 0 03. Calculate (*a*) the capacitance of the cable ; (*b*) its equivalent insulation resistance ; (*c*) the charging current ; (*d*) the dielectric loss, when the cable is connected to 6 600-V, 50-c/s bus-bars. [(*a*) 0·211 μF ; (*b*) 5 × 10^5 Ω ; (*c*) 0 44 A ; (*d*) 87 W.]

Cables : Capacitance, Three-core

21 Show how the "equivalent star capacitance" of a 3-phase, 3-core, metal-sheathed cable is obtained from a single measurement of capacitance between any two cores, and draw a vector diagram showing the charging currents due to the various

capacitances present in such a cable. A 3-core cable gives on test a capacitance measurement of 3 μF between two cores. Find the line charging current of the cable when connected to 11 000-V, 50-c/s bus-bars. [12 A.]

22 In a 3-phase, 3-core, metal-sheathed cable the measured capacitance between any two cores in 2 μF. Calculate the kilovolt-amperes taken by the cable when it is connected to 50-c/s, 11 000-V bus-bars. [152 kVA.]

23 A mile of 3-core, 3-phase, metal-sheathed cable gave the following results on test for capacitance:—(i) capacitance between bunched conductors and sheath, 1·0 μF, (ii) capacitance between two conductors bunched with the sheath and the third conductor, 0·6 μF. With the sheath insulated, find the capacitance (*a*) between any two conductors, and (*b*) between any two bunched conductors and the third conductor. Calculate (*c*) the capacitance current per conductor per mile when connexion is made to 10 000-V, 50-c/s bus-bars.

[(*a*) 0 367 μF ; (*b*) 0·489 μF ; (*c*) 1 33 A.]

24 Calculate the charging current taken by 10 miles of a 3-core, metal-sheathed cable when connected to 10-kV, 25-c/s bus-bars. The capacitance measured between any two cores is 0·3 μF per mile. If a current of 30 A be taken by a load at the receiving end with a power factor of 0 9 lagging, calculate the current and power factor at the sending-end, neglecting line drop and leakage. [5·44 A ; 28·1 A, 0·96.]

Cables : Stress and Grading

25 Find the diametral dimensions for the 1-core, metal-sheathed cable giving the greatest economy of insulating material for a working voltage of 85 kV, if a dielectric stress of 60 kV per cm can be allowed. Prove the formulæ used.

[Conductor, 2·83 cm dia. ; sheath, 7·70 cm inside dia.]

26 Show that for a given voltage V and maximum stress ϵ_{max} in the dielectric of a 1-core cable, the sheath diameter D is a minimum when $D : d = e$, where d = conductor diameter and e = base of natural logarithms. Determine D and d for V = 10 kV and ϵ_{max} = 23 kV/cm. [d = 0·87 cm ; D = 2·36 cm]

27 An 85-kV, 1-core, metal-sheathed cable is to be graded by means of a metallic intersheath. (*a*) Find the diameter Δ of the intersheath and the voltage at which it must be maintained in order to obtain the minimum overall cable diameter D. The insulating material can be worked at 60 kV per cm. (*b*) Prove the formulæ used, and compare the conductor and outside diameters (d and D) with those of an ungraded cable of the same material under the same conditions.

[(*a*) d = 1·04 cm, Δ = 2·84 cm, D = 5·34 cm, 53·8 kV to earth ; (*b*) d = 2 84 cm, D = 7·70 cm.]

28 A 1-core, lead-sheathed cable joint has a conductor of 10 mm diameter and two layers of different insulating material, each 10 mm thick. The relative permittivities are 8 (inner) and 2·5 (outer). Calculate the potential gradient at the surface of the conductor when the potential difference between the conductor and the lead sheathing is 60 kV. [70 kV per cm.]

29 Find the maximum working voltage of a 1-core, lead-sheathed cable joint with conductor 1 cm diameter and sheath 5 cm inside diameter. Two insulating materials are used :—*A*, maximum working potential gradient, 60 kV per cm ; permittivity, 4. *B*, maximum working potential gradient, 50 kV per cm ; relative permittivity, 2·5. [65 kV.]

30 A single-core, lead-covered cable is to be designed for 66 kV to earth. Its conductor radius is 0 5 cm and its three insulating materials, *A*, *B* and *C* have relative permittivities of 4, 4 and 2 5 with maximum permissible stresses of 50, 40 and 30 kV/cm respectively. Find the minimum internal diameter of the lead sheath. [7·44 cm.]

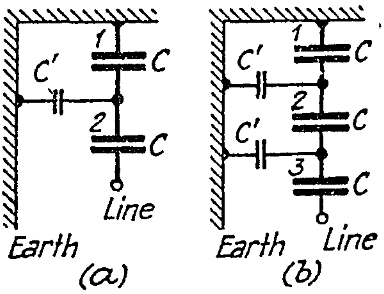
31 A single-core cable has a conductor diameter *d* and an inside sheath diameter *D*. The dielectric will withstand a stress of 60 kV per cm. Find the maximum working voltage when $d = 1$ cm and $D = 7\ 5$ cm. Draw a curve showing the maximum working voltage as a function of the conductor diameter as *d* varies from 1 cm upwards, *D* remaining constant at 7·5 cm. What is the best conductor diameter and the corresponding working voltage ? [60·5 kV ; 2·76 cm ; 83 kV.]

Cables : Insulation Resistance

32 Find the insulation resistance per mile of a 2-core, concentric cable having an inner conductor of diameter 1 33 cm and an outer conductor of inside diameter 3 62 cm. The insulating material is paper with a specific resistance of $8 \times 10^{12}\ \Omega$–m at the temperature considered. Prove the formula used. [790 MΩ.]

Insulators and Bushings

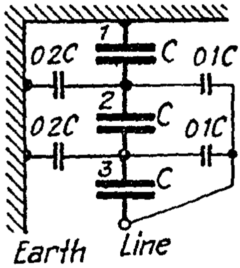
33 Calculate the maximum voltage that the strings of suspension insulators (*a*) and (*b*) can withstand if the maximum voltage per unit is not to exceed 17·5 kV. The equivalent condenser arrangement is shown in sketch : $C' = \frac{1}{8} C$. [(*a*) 33 kV ; (*b*) 44 kV.]



34 Each conductor of a 33-kV, 3-phase system is suspended by a string of three similar insulators, the mutual capacitance of which across units is 9 times the shunt capacitance between unit and earthed framework. Calculate the voltage across each insulator. [Bottom 7·41 ; middle 6·13 ; top 5 51 kV.]

35 A string of six insulator units has self-capacitances equal to ten times the pin-to-earth capacitances. Find (*a*) the voltage distribution from top to bottom insulator as a percentage of the total voltage ; (*b*) the string efficiency.
[(*a*) 9 9, 10·9, 13·0, 16·4, 21·3, 28 6% ; (*b*) 58·3%.]

36 Investigate the voltage distribution over a string of three suspension insulators. The equivalent capacitance arrangement is shown in the sketch. The self-capacitance of each unit is *C*. (*a*) Express the voltage across each insulator as a percentage of the line voltage to earth. (*b*) If the capacitance to the line of the lower link-pin were increased to 0·3 *C* by means of a guard ring, investigate the redistribution of voltage. (*c*) Find for each case the "string efficiency."



[(*a*) 31%, 30%, 39% ; (*b*) 32·6%, 32·4%, 35% ; (*c*) 85% ; 95%.]

37 A string of 6 suspension insulators is to be graded to obtain uniform distribution of voltage across the string. If the pin-to-earth capacitances are all equal to *C* and the mutual capacitance of the top insulator is 10 *C*, find the mutual capacitance of each unit in terms of *C*.
[From top to bottom insulator : 10, 11, 13, 16, 20, 25 *C*.]

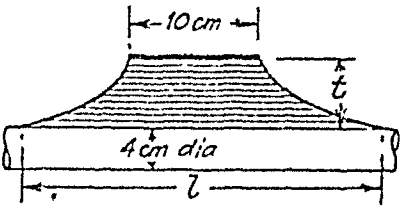
38 A string of eight suspension insulators is to be fitted with a grading-ring. If the pin-to-earth capacitances are all equal to *C*, find the values of line-to-pin capacitances that would give a uniform voltage distribution over the string.
[$\frac{1}{7}$, $\frac{1}{3}$, $\frac{3}{5}$, 1, $1\frac{2}{3}$, 3 and 7 *C*.]

39 A transformer bushing for 50 kV consists of the following :—

		Permittivity.
Copper rod :	outside diameter 4 cm	—
Treated paper	,, ,, 5 ,, ...	3
Compound	,, ,, 10 ,, ...	2·5
Porcelain	,, ,, 15 ,, ...	5

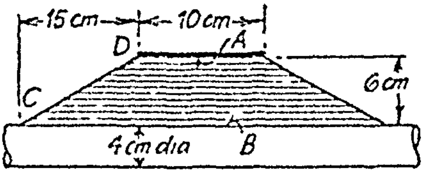
Find the maximum and minimum voltage gradients.
[Paper : 19·3 and 15·4 kV/cm ; compound : 18·5 and 9·3 kV/cm porcelain : 4·65 and 3·1 kV/cm.]

40 A condenser bushing for a r.m.s. voltage of 80 kV to earth is designed to have a uniform radial voltage gradient. The insulating material used has a maximum permissible working voltage stress of 10 kV (peak) per cm. Assuming a uniform and very small thickness of insulation between each successive foil, determine the values of t and l, and draw the locus of the ends of the tinfoil layers.



Estimate the value of t for a bushing without foils having the same maximum radial stress. [4·24 cm ; 31·2 cm ; 14·7 cm.]

41 The sketch shows the dimensions of a condenser bushing for 30 kV to earth. The successive layers of insulation are of uniform
for 30 kV to earth. The successive layers of insulation are of
uniform and very small thickness. Plot the variation of voltage gradient radially between A and B, and axially between C and D. At what points do the minimum radial and axial gradients occur?



What is the ratio of maximum stress to minimum stress, radially and axially?
[Midway between A and B ; midway between C and D ; 1·56 ; 1·56.]

42 A conductor 2·8 cm diameter is passed centrally through a porcelain bushing ($\kappa_r = 4$) having internal and external diameters of 8 cm and 9 cm respectively. The voltage between the conductor and an earthed clamp surrounding the porcelain is 15 kV r.m.s. Determine whether or not corona will be present in the air-space round the conductor. [Yes.]

Impedance and Propagation Constants of Lines and Cables

43 A 3-phase, 132-kV, 50-c/s, overhead transmission line has steel-cored aluminium conductors of equivalent copper area of 0·175 in^2 and effective diameter 19 9 mm, spaced equilaterally 4 m apart. The insulation resistance is 500 MΩ per km. Calculate the line constants (*i.e.*, resistance, inductance, capacitance, leakance, reactance, susceptance) per km at 20° C.
[0 153 Ω ; 1·25 mH ; 0·0093 μF ; 0 002 μ℧ ; 0 392 Ω ; 2·9 μ℧.]

44 A 2-core cable has, per loop-kilometre, a resistance of 10 Ω, an inductance of 0·01 H, a capacitance of 0·1 μF, and an insulation resistance of 0 01 MΩ. Calculate its characteristic impedance if of infinite length. Frequency 5 000/2π c/s. [816 Ω]

45 A 100-km, 50-c/s, 3-phase transmission line has a resistance of 0·5 Ω per km per line, an inductive reactance of 1 Ω and capacitance $1{\cdot}59 \times 10^{-2}$ μF per km, line-to-neutral. The leakance is negligible. Calculate the characteristic impedance Z_0, the propagation constant γ and the quantities $A = C = \cosh \gamma l$, $B = Z_0 \sinh \gamma l$, and $D = \frac{1}{Z_0} \sinh \gamma l$ in the equations

$$V_s = V_r A + I_r B \text{ and } I_s = I_r C + V_r D$$

where V_s, V_r, I_s, I_r represent the sending- and receiving-end voltage and current respectively.
*[$Z_0 = 474 \angle -13{\cdot}3° \ \Omega$; $\gamma = 2{\cdot}36 \times 10^{-3} \angle 76{\cdot}7°$; $A = 0{\cdot}975 \angle 0{\cdot}73°$; $B = 110{\cdot}9 \angle 63{\cdot}7°$; $D = 0{\cdot}0005 \angle 90{\cdot}22°$.]

46 Calculate the velocity of propagation of electric waves (*a*) in an overhead line of capacitance $0{\cdot}114 \times 10^{-12}$ F per cm and inductance $0{\cdot}978 \times 10^{-8}$ H per cm ; (*b*) in a cable of inductance $0{\cdot}75 \times 10^{-8}$ H per cm and capacitance $1{\cdot}33 \times 10^{-12}$ F per cm. Estimate the relative permittivity of the insulating material in case (*b*). [(*a*) 3×10^{10} cm per sec ; (*b*) 10^{10} cm per sec, $\kappa_r = 9$.]

47 An overhead transmission line has per mile a resistance of 0·5 Ω, a capacitance of 0 014 μF, an inductive reactance of 0·67 Ω and negligible leakance. Calculate the characteristic impedance and the propagation constant of the line. Frequency 50 c/s. [$435 \angle -18{\cdot}35° \ \Omega$; $1{\cdot}92 \times 10^{-3} \angle 71{\cdot}65°$.]

Travelling Waves

48 A 3-phase transmission line has conductors 1·5 cm in diameter spaced 1 m apart in equilateral formation. The resistance and leakance are negligible. (*a*) Calculate the natural impedance of the line. (*b*) If an electric wave be propagated on the line at a voltage of 11 000 V, find the line current associated with this wave. (*c*) If the load end of the line have resistance 1 000 Ω per phase, star-connected, find the rate of energy absorption and the rate of reflection, and state the form of reflection. (*d*) What must be the value of the end resistance per phase in order that the energy of the wave shall be completely dissipated? (*e*) If the line be connected to a cable extension with inductance $0{\cdot}5 \times 10^{-8}$ H and capacitance 1×10^{-6} μF per cm per phase, find the amounts of transmitted and reflected power.
[(*a*) 300 Ω ; (*b*) 21·2 A ; (*c*) 287 kW, 117 kW, current reversed ; (*d*) 300 Ω ; (*e*) 250 kW, 155 kW.]

49 A surge of 10 kV magnitude travels along a cable towards its junction with an overhead line. The inductance and capacitance of the cable and overhead line are respectively 0·3 mH, 0·4 μF, and 1·5 mH, 0·012 μF per mile. Find the voltage rise at the junction due to the surge. [18·55 kV.]

50 A surge of 25 kV travelling in a line of natural impedance 500 Ω arrives at a junction with two lines of impedances 700 Ω and 200 Ω respectively. Find the surge voltages and currents transmitted into each branch line.
[11·83 kV, 16·9 A ; 11·83 kV, 59·2 A.]

51 The ends of two long transmission lines, A and C, are connected by a cable B, 1 km long. The surge impedances of A, B, and C are 500, 70, and 600 Ω respectively. A rectangular voltage wave of 10 kV magnitude and of infinite length is initiated in A and travels to C. Determine the first and second voltages impressed on C, and the voltage at a point on A, 0·5 km from the junction of A and B, 30 μs after the initial wave has reached the junction of B and C. The velocity of the wave in B is 10^{10} cm per sec. [4·4 kV ; 7·04 kV ; 5·88 kV.]

52 A travelling wave of $V_{max} = 100$ kV enters an overhead transmission line of surge impedance 500 Ω and conductor resistance 7 Ω per km. What will the value of V_{max} be 100 km further on, and what is the power loss and the heat loss of the wave during the time required to traverse this distance? Neglect losses in the insulation and assume a wave velocity of 300 000 km per sec. Determine also the corresponding values for a cable of surge impedance 50 Ω and wave velocity 150 000 km per sec.
[50 kV, 15 000 kW, 860 cal. ; 0·092 kV, 200 000 kW, 2 300 cal.]

53 A 500-kV, 2-μs, rectangular surge on a line having a surge impedance of 350 Ω approaches a station at which the concentrated earth capacitance is 3 000 $\mu\mu$F. Determine the maximum value of the transmitted wave. [488 kV.]

54 An inductance of 800 μH connects two sections of a transmission line each having a surge impedance of 350 Ω. A 500-kV, 2-μs rectangular surge travels along the line towards the inductance. Draw the voltage wave transmitted beyond the junction if the inductance has negligible resistance. Determine the maximum value of the transmitted wave. [416 kV.]

55 A surge whose value is given by

$$v = 300\,(e^{-0\cdot 04 \times 10^{6}t} - e^{-10^{6}t})\,\text{kV}$$

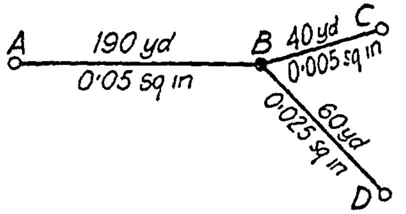
reaches a point on a 50-c/s, 132-kV line where a piece of apparatus is connected between line and earth. Compare the ratio of the maximum rate of change of voltage at the terminal of the apparatus due to the surge with that due to the normal working voltage. Determine also the maximum voltage V_m of the surge, and the times taken for the surge voltage to rise from zero to V_m, and to fall from V_m to $V_m/2$.
[8 520 : 1 ; $V_m = 252$ kV ; 3·35 μs ; 18·1 μs.]

CHAPTER XX

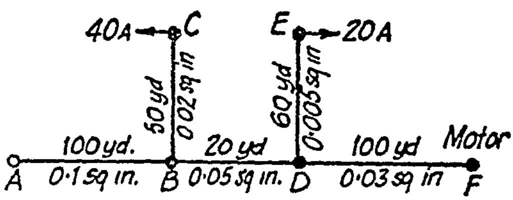
DISTRIBUTORS AND DISTRIBUTION NETWORKS

2-wire Distributors : D.C.

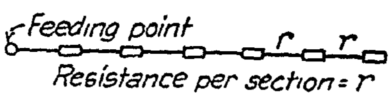
1 The copper cable system shown is loaded at C with 25 A, and at D with a motor A constant voltage of 275 V is maintained by a generator at A. When the motor is on full load, the potential difference at C is 255 V. Find the full-load current of the motor and its terminal voltage. [30 A ; 261·3 V.]



2 In the 2-wire, d.c. distributor given, the voltage at A is maintained at 266 V. To what value will the voltage at E fall if the voltage at C drops to 254 V due to the starting of the motor at F? Calculate also the voltage at F when the motor takes a current of 50 A. The cross-sections given are those of each core. Resistivity 0·7 $\mu\Omega$-in. [245 V ; 250·6 V.]



3 Six tram-cars running equally spaced along a single track, as in sketch, each return 50 A into the rails. What must be the total resistance of the rails in order to keep the voltage drop within the statutory limit of 7 V, assuming all the current to be carried by the rails. What power loss will there be with this resistance? [$6r = 0{\cdot}04\Omega$; 1 517 W.]



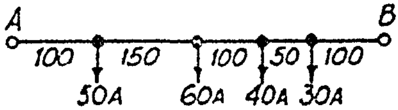
4 Power is supplied from a 550-V substation to two tram-cars 1 mile and 3 miles distant respectively. The former takes 40 A and the latter 20 A. The resistance of the trolley wire is 0·4 Ω per mile and of the track 0 03 Ω per mile. Find the voltage across each car and the total transmission loss. [524·2 V ; 507 V ; 1·89 kW.]

5 An electric train moving on a section of line between two substations takes a constant current of 500 A. The substations are

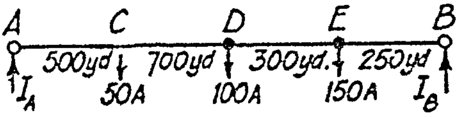
5 miles apart and are maintained at 590 V and 580 V respectively. The track resistance is 0·06 Ω per mile go and return. Show graphically the variation of current received from either substation, and find the point of minimum potential along the track. [2·67 miles from 590-V station.]

6 A trolley wire supplied from one end has the following loads :— 150 A at 100 yd ; 100 A at 300 yd ; 80 A at 1 000 yd ; and 120 A at 1 200 yd from the feeding end. Find the voltage at the various load points if a voltage of 550 V be maintained at the supply end. The trolley wire resistance is 200 $\mu\Omega$ per yd, and the rail return has a resistance of 150 $\mu\Omega$ per yd.
[534·3 V ; 513·3 V ; 464·3 V ; 455·9 V.]

7 A 2-conductor street main has a total length of 500 m and is loaded as shown, distances being given in metres. Both ends A and B are supplied at 250 V. If the minimum allowable voltage at the consumers' terminals is to be 245 V, find the necessary cross-section of each conductor of the main. Resistivity, 1·7 $\mu\Omega$-cm. [1·0 cm².]

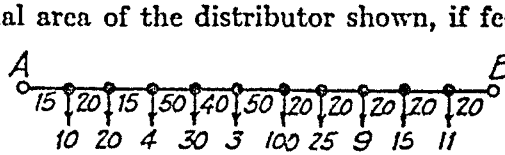


8 The resistance of the two conductors of the cable loaded as shown is 0·1 Ω per 1 000 yd for both conductors. Find the current supplied at A and at B, the current in each section, and the voltages at C, D and E. Both A and B are maintained at 200 V.



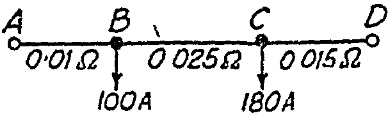
[$I_A = 88{\cdot}5$ A ; $I_B = 211{\cdot}5$ A ; $I_{CD} = 38{\cdot}5$ A ; $I_{ED} = 61{\cdot}5$ A ; $V_C = 195{\cdot}6$ V ; $V_D = 192{\cdot}9$ V ; $V_E = 194{\cdot}7$ V.]

9 Find the cross-sectional area of the distributor shown, if fed from both ends at the same voltage, and if the maximum voltage drop is not to exceed 4 V. The distances are given in yards and the current loadings in amperes. Take the resistivity as 0·7 $\mu\Omega$-in. [0·135 in².]



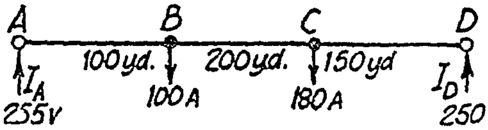
10 Three 50-A loads, A, B, C, are connected to a 250-V supply point through separate cables having resistances 0·05, 0·1 and 0·02 Ω respectively. A is joined to B by a 0·1-Ω connector, and B is joined to C by a 0·15-Ω connector. Find the voltages at A, B and C. [V_A, 247·26 V ; V_B, 246·78 V ; V_C, 248·74 V.]

11 A 2-conductor distributor AD is fed at A and D at 255 V and 250 V respectively, and is loaded as shown. The resistances given are those of each conductor. Find the value of the current in each section of the cable and the voltage at each load point.



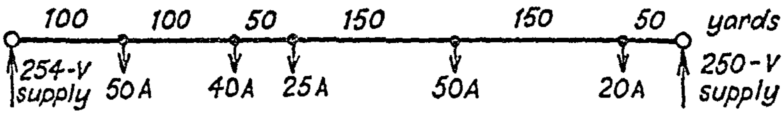
[$I_{AB} = 184$ A ; $I_{BC} = 84$ A ; $I_{DC} = 96$ A ; $V_B = 251{\cdot}8$ V ; $V_C = 247{\cdot}1$ V.]

12 A 2-core distributor cable is fed from both ends and is loaded as shown. The cross-section of each conductor is 0·25 in^2 and the distances are given in yards. Find the current flowing into the cable at A and at D, and the magnitude of the current in section BC. Find also the voltage at B and at C. Resistivity, 0·7 $\mu\Omega$-in.



[$I_A = 193$ A ; $I_D = 87$ A ; $I_{BC} = 93$ A ; $V_B = 251{\cdot}1$ V ; $V_C = 247{\cdot}4$ V.]

13 A 2-wire distributor is loaded as shown, the two ends being fed at 254 and 250 V respectively.



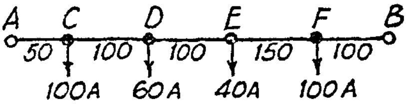
Find the cross section of the conductor for a minimum consumers' voltage of 240 V. [0 063 in^2.]

14 A distributor 1 200 yd long carries a uniformly-distributed load of 0·5 A per yd. It is supplied at both ends from a substation by feeders, one 900 yd, the other 600 yd, long. The feeders have a cross sectional area 50% greater than that of the distributor. Find the point where the consumer's voltage is lowest, and the current supplied by each feeder.
[545 yd from 900-yd feeder point ; 272·5 A in 900-yd feeder ; 327·5 A in 600-yd feeder.]

15 A 2-wire distributor AB is 500 yd long and has a resistance of 0·1 mΩ/yd (go and return). At C, 100 yd from A, is a load of 50 A : at D, 200 yd from A, there is a load of 100 A. From C to E there is a uniform load of $\frac{1}{2}$A/yd. (a) When supplied from A, find the position of E for a maximum drop to any consumer of 3 V. (b) When loaded as in (a) and fed also at B at a voltage 2 V higher than at A, find the voltage drop at D and at E.
[(a) $CE = 78$ yd, (b) D and E both 1 V below A.]

16 A 250-yd, 2-wire main fed from one end is loaded uniformly at the rate of 1·5 A per yd, the resistance of each conductor being 0·2 Ω per 1 000 yd. Find the voltage necessary at the feeding end to maintain 250 V (*a*) at the middle, (*b*) at the distant end of the cable. [(*a*) 264 V; (*b*) 268·75 V.]

17 The two conductors of a distributor cable 500 yd long have a total resistance of 0·075 Ω. A voltage of 250 V is maintained at feeding points *A* and *B*. The cable is loaded with a uniform lighting load of 1 A per yd and with the additional concentrated loads shown in the diagram. The distances are in yards. Find (*a*) the lowest voltage, (*b*) the point at which it occurs, and (*c*) the current fed into the cable at *A* and at *B*.



[(*a*) 242·8 V; (*b*) *E*; (*c*) $I_A = 422$ A, $I_B = 378$ A.]

18 A 250-yd, 2-wire main fed from both ends *A* and *B* is loaded uniformly at the rate of 1·5 A per yd, the resistance of each conductor being 0·2 Ω per 1 000 yd. Find the voltage at *A* and at *B* for a minimum of 240 V at consumers' terminals (*a*) when $V_A = V_B$, (*b*) when $V_A = V_B + 6$. In case (*b*), find also supply currents I_A and I_B.
[(*a*) 244·7 V; (*b*) $V_A = 248·2$ V, $V_B = 242·2$ V: $I_A = 248$ A, $I_B = 128$ A.]

19 A direct-current, 2-wire distributor 575 yd long is fed at both ends *A* and *B* at 440 V. The load consists of 100 A at 100 yd from *A*, 150 A at 150 yd from *A*, and a uniform loading of 1 A per yd for the last 350 yd. The resistance of each conductor is 0·05 Ω per 1 000 yd. At what point is the load voltage a minimum and what is its value? [435·5 V at 275 yd from *A*.]

20 A direct-current, 2-wire distributor *XY* is 350 yd long and is fed at both ends *X* and *Y* at 250 V and 253 V respectively. It is loaded as follows at distances from the end *X*; 50 A at 80 yd, 100 A at 140 yd, a uniform loading of 2 A per yd from 180 yd to 210 yd and from 250 yd to 300 yd, and 40 A at 270 yd. The resistance of each conductor is 0·005 Ω per 100 yd. Show graphically the current and voltage distribution along the distributor and find the point of minimum voltage.
[140 yd from *X*.]

21 A 2-wire distributor *AD* 250 yd long is fed at *A* at 250 V and loaded as follows:—75 A at *B* 100 yd from *A*, 50 A at *C* 200 yd from *A*, and 25 A at *D* 250 yd from *A*. If the voltage at *D* is to be 245 V, find the cross-section of the conductors for minimum volume of copper. Resistance of 1 000 yd of copper of 1 in² cross-section is 0·028. [0·264, 0·186, 0·108 in².]

22 A cable AB, of length 150m, feeds two cables BC and BD of respective lengths 100m and 80m. Loads of 80 A, 60 A and 40 A are taken at points B, C and D respectively. Find the cross-sections in mm² of the cables for minimum total weight of copper if the voltage drop between supply point A and load points C and D is not to exceed 10 V.

[AB, 137 ; BC, 65·8 ; BD, 35·1 mm².]

23 A load of 100 A at 550 V is taken at the end of a line 2 000 ft long which is supplied at 600 V. An additional load of 100 A is to be taken at A, the midway point, while 550 V is to be maintained at the far end. (a) Find the cross-section of an auxiliary parallel line from the supply end to A. (b) If the same loads were supplied from a two-part conductor using minimum material, what would be the percentage saving in material? Calculate the load at A in (a) and in (b).

[(a) 0·064 in² ; (b) 3·125%. (a) 57·5 kW ; (b) 57·06 kW.]

24 A bus running at 30 m.p.h. takes 100 A from a system which has a loop resistance of 0·25 Ω per mile and is fed at equipotential points $\frac{1}{2}$ mile apart. Find the rate of change of voltage with respect to time at the bus when it is (a) 220 yd, (b) 440 yd, from a feeding point. [(a) 0 104 V/sec ; (b) 0.]

25 A 6 000-yd rail system has a loop resistance of 0·015 Ω per 1 000 yd. End A is earthed. A negative feeder joined to B, 4 000 yd from A, makes the potential of B 2 V below that of A when the current loading is uniformly distributed. Find (a) the current in the negative feeder when the maximum p.d. between any two points of the rail is 7 V, and (b) the potential at the middle and distant end of the return rail with respect to A.

[(a) 830 A ; (b) 2·96 and 3 98 V positive to A.]

26 A third-rail system, length 5 000 yd, has a total resistance of 0 015 Ω per 1 000 yd and is uniformly loaded at 300 A per 1 000 yd. End A is earthed. A negative feeder at B, 3 000 yd from A, keeps the potential at B 3 V below that at A. Sketch graphs showing the current and potential distribution along the rail. Find (a) the current in the negative feeder, (b) the maximum p.d. between any two points on rail, and (c) the potential above earth at distant end of rail.

[(a) 1 116 A ; (b) 9 V ; (c) 6 V.]

27 A section of railway 3 miles long is supplied from substations at each end of the section. Combined resistance of positive and track rails is 0·05 Ω per mile of track. At substation A, the supply voltage falls uniformly from 650 to 600 V as the current rises from 0 to 1 000 A. At substation B the voltage falls from

680 to 600 V as the current rises from 0 to 1 000 A. If the train takes a constant current of 1 250 A, what will be the current supplied by the substation *A* and the voltage at the train when the train is (*a*) at *A*, (*b*) at *B*, and (*c*) at middle of section.
[(*a*) 1 065 ; (*b*) 250 ; (*c*) 658 A ; (*a*) 597 ; (*b*) 600 ; (*c*) 568 V.]

28 A tramway section 2 miles long has a track resistance of 0·03 Ω per mile and has an average load of 200 A per mile. If the supply end of track is at earth potential, find potential at the remote end. If a cable joins the remote end to a track booster to reduce the potential to 7 V, find what current it carries.
[12 V ; 83·35 A.]

3-wire Distributors: D.C.

29 A load supplied on the 3-wire, direct-current system takes a current of 200 A on the positive side and 120 A on the negative side. The outer conductors of the supply cable have each a resistance of 0·05 Ω ; the middle wire has one-half the cross-section of either outer. Find the voltage to be maintained at the feeding point between the respective outers and the middle wire in order that the consumers' voltage shall be 200 V between either outer and the middle wire. [218 V ; 198 V.]

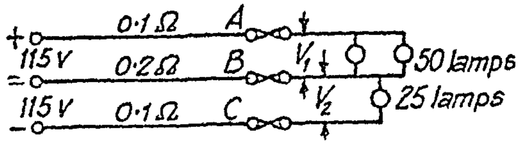
30 A 3-wire feeder 1 200 yd long carries 110 A on one side and 90 A on the other. The supply voltage is 225 V. If the load voltages differ by 5 V, find the section of conductor (neutral = $\frac{1}{2}$ of outer) and the load voltages. [0·58 in^2, 217·5, 222·5 V.]

31 In a 3-wire system, the supply voltage is 250 V on each side. The load on one side is 3 Ω, and on the other 4 Ω. The resistance of each of the three conductors is 0·05 Ω. Find the load voltages.
[3-Ω side, 244·9 V ; 4-Ω side, 247·9 V.]

32 In a 3-wire distributor, each conductor has a resistance of 0·13 Ω. The supply voltage between outers is 230 V. At the load end are connected 15 Ω between positive and neutral, 20 Ω between negative and neutral, and 10 Ω between positive and negative. Find the power absorbed in each of these loads.
[821 W, 627 W, 5 476 W.]

33 A direct-current, 3-wire system with 460 V between outers supplies 250 kW on the positive and 400 kW on the negative side, the voltages being balanced. Calculate the voltage on the positive and negative sides respectively if the neutral wire becomes disconnected from the balancer set. Assume the loads to consist of constant resistances. [284 V, 176 V.]

34 The 3-wire system with line resistances as shown supplies a lighting load, and a fuse is incorrectly inserted in the middle wire at B. The normal current per lamp at 110 V is 1 A, and the resistance of the lamps may be assumed constant. If the supply is 115/230 V, find the voltage across each side of the system. (a) with all lamps switched on; (b) if the fuse at B melts while those at A and C hold; (c) if the fuse at C melts while those at A and B hold.



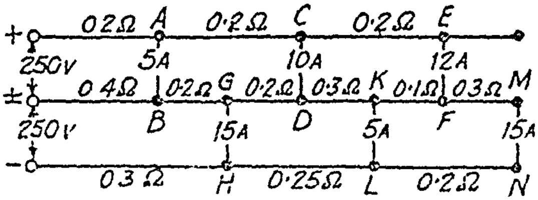
[(a) $V_1 = 105·9$. $V_2 = 116·7$; (b) $V_1 = 74·3$, $V_2 = 148·7$; (c) $V_1 = 101$, $V_2 = 0$.]

35 A 3-wire distributor is loaded at A, B and C as follows;—

	A	B	C
Current, positive to neutral, A	25	65	15
„ neutral to negative, A	35	40	25

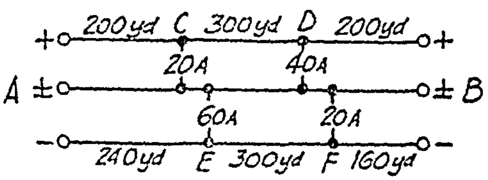
Show on a sketch the value of the current in each part of the circuit.

36 A 3-wire direct-current system is supplied at one end at 250 V between outers and middle wire, the loadings and conductor resistances being as shown. (a) Calculate the voltage between middle wire and outer at each load point. (b) If loads EF, KL and MN are switched off, what will be the voltage across AB, CD, and GH?



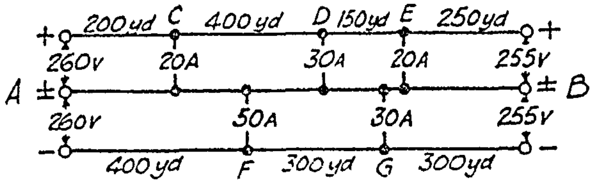
[(a) AB, 247·8 V; CD, 245·6 V; EF, 245·9 V; GH, 233·7 V; KL, 220·7 V; MN, 218·9 V. (b) AB, 247 V; CD, 244 V; GH, 244·5 V.]

37 A 3-wire distributor, 700 yd in length, is fed from both ends A and B, where the voltage between either outer and the middle wire is 200 V. The diagram shows the loading in amperes and the distances in yards. The resistance of each outer is 0·05 Ω per 100 yd, and of the neutral 0·1 Ω per 100 yd. Calculate the potential difference at each load point.



[$V_C = 201·09$; $V_D = 196·11$ V; $V_E = 189·52$ V; $V_F = 196·85$ V.]

38 A 3-wire distributor, loaded as shown, is fed at A at 260 V between either outer and the middle wire, and at B at 255 V.



The resistance of each outer is 0·4 Ω per 1 000 yd and the middle wire is half the cross-section of either outer. Calculate the potential difference at each load point, and the power supplied at each end.

[$V_C = 257{\cdot}32$ V; $V_D = 253{\cdot}56$ V; $V_E = 253{\cdot}35$ V; $V_F = 246{\cdot}64$ V; $V_G = 249$ 82 V. $P_A = 25{\cdot}2$ kW; $P_B = 13{\cdot}5$ kW.]

39 A 3-wire distributor, 300 yd long, and fed from one end, is loaded uniformly at the rate of 0·25 A per yd on the positive side, and 0 15 A per yd on the negative side. The resistance of each outer is 0 04 Ω, and of the middle wire 0 08 Ω. Calculate the voltage at the distant end of the distributor if 200 V is maintained between outers and middle wire at the feeding point. [197·3 V; 200·3 V.]

40 A 3-wire distributor, 300 yd long, is supplied from one end at 506/253 V and is uniformly loaded at the rate of 0 3 A per yd on the positive side and of 0·2 A per yd on the negative side. There are in addition concentrated loads of 20 A, 40 A and 30 A across outers at points distant 100 yd, 150 yd, and 300 yd respectively from the feeding point. Find the cross-section of the distributor in order that the voltage between outers at the distant end shall be 500 V. The middle conductor is to be one-half the cross-section of either outer. Find for this condition the respective voltages between outers and middle at the distant end of the distributor. Resistivity, 0·7 $\mu\Omega$-in.

[0·237 in^2; 248·8 V, 251·2 V.]

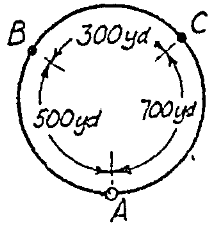
Ring-mains: D.C.

41 A ring-main is supplied at point S and loaded at point A with 100 A, and at point B with 70 A. The sectional resistances are: SA, 0 2 Ω; AB, 0·3 Ω; AS, 0 5 Ω. (a) Find the current in AB. (b) Find the voltage drop between S and each load point. (c) Find the voltage drop between S and A when load at B is removed.

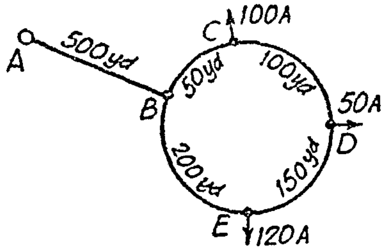
[(a) 15 A, A to B; (b) S to A, 23 V; S to B, 27·5 V; (c) 16 V.]

42 In the direct-current ring main shown, a voltage of 500 V is maintained at A. At B a load of 150 A is taken and at C a load of 200 A is taken. Find the voltages at B and C. The resistance of each conductor of the main is 0 03 Ω per 1 000 yd.

[$V_B = 494\ 2$ V , $V_C = 493\ 4$ V.]



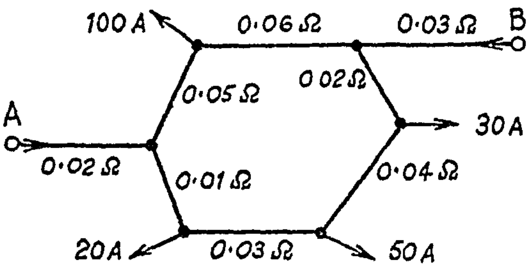
43 A 2-wire ring main is loaded as shown and fed from point A. The feeder cable AB has a cross-section of a in² and the ring main a cross-section of b in² per conductor. The distances are given in yards. Find the cross-sections a and b which will give the least weight of copper in the system if the maximum drop from A is not to exceed 15 V. Resistivity, 0·7 $\mu\Omega$-in.



[$a = 0{\cdot}625$ in² ; $b = 0\ 24$ in².]

44 Find the currents supplied to the ring main from A and B (a) for equal voltages at A and B, (b) for the voltage at B higher than that at A by 5 V.

[(a) $I_A = 114$ A, $I_B = 86$ A ; (b) $I_A = 65$ A, $I_B = 135$ A.]



Balancers : D.C.

45 On a 3-wire, direct-current generating station with 500 V between outers there is a load of 1 500 kW on the positive side and 2 000 kW on the negative side. Calculate the current in the middle wire, the current in each of the balancer armatures (neglecting losses) and the total current supplied by the generators.

[2 000 A ; 1 000 A ; 7 000 A.]

46 The load on a direct-current, 3-wire system with 500 V between outers consists of a lighting current of 1 500 A on the positive and 1 300 A on the negative side, while motors connected across the outers absorb 500 kW. Assuming that at this loading the balancer machines have each a loss of 5 kW, calculate the load on the main generators and on each of the balancer machines.

[1 210 kW ; 20 and 30 kW]

47 In a 500/250-V, direct-current, 3-wire system there is an out-of-balance load of 200 kW on the positive side. The loss in each balancer is 10 kW and the current in the negative main is 2 800 A. Calculate the current in each armature of the balancer set and the total load on the generators.

[+ ve, 360 A ; — ve, 440 A ; 1 620 kW.]

48 A direct current, 3-wire system with 500 V across the outers supplies 1 000 A on the positive, 750 A on the negative side, and 2 000 A across the outers. The motor-balancers have each an armature resistance of 0·2 Ω and take 8 A on no-load. Find (*a*) the voltage across each balancer, (*b*) the total load on the generators and the current loading of each balancer unit. The balancer field windings are in series across 500 V.

[(*a*) 225 V, 275 V ; (*b*) 1 442 kW, 116 A, 184 A.]

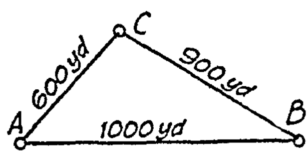
49 A 125/250-V, 3-wire system has an out-of-balance current of 50 A, the larger load being 500 A. The balancer set has an overall efficiency of 0·76 and each armature has a resistance of 0·1 Ω. Calculate the current in each of the balancer machines and the output of the main generator. Show the currents in a diagram.

[22 A, 28 A ; 119·5 kW.]

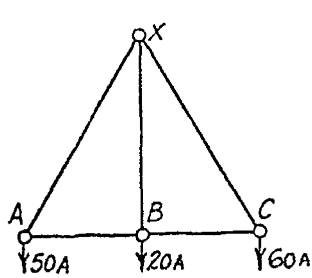
Distribution Networks : D.C.

50 Three substations *A*, *B*, *C*, are situated as shown, the normal loads at *A*, *B* and *C* being 500 kW, 750 kW and 450 kW respectively. Find the point that provides the most economical centre of distribution.

[514 yd along, and 141 yd perpendicular to, *AB*.]



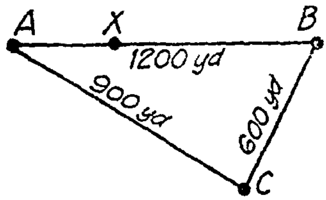
51 Loads of 50 A, 20 A and 60 A are supplied at points *A*, *B*, *C* which are connected by feeders of total resistance 0·05 Ω, 0·1 Ω, 0·04 Ω respectively to the substation *X*, the voltage at which is 250 V. *A* and *B*, and *B* and *C* are linked by interconnectors of total resistance 0·1 Ω and 0·15 Ω respectively. What are the voltages at the points *A*, *B* and *C*?



[$V_A = 247{\cdot}58$ V ; $V_B = 247{\cdot}74$ V ; $V_C = 247{\cdot}64$ V.]

52 A 2-conductor distributor *AB*, 1 200 yd long, is loaded uniformly with *a* amperes per yd, and is supplied from a genera-

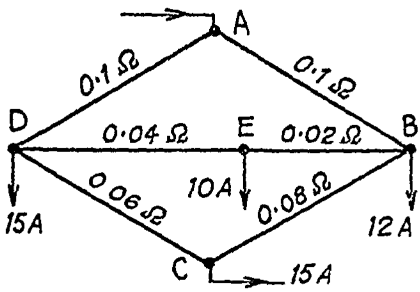
ting station at C by two feeders CA, 900 yd long and CB, 600 yd long, as shown. The section of each feeder conductor is 50% greater than that of each distributor conductor. Find (a) the position X of minimum voltage, and (b) the current in each feeder.



[(a) 545 yd from A ; (b) $I_{CA} = 545$ A, $I_{CB} = 655$ A.]

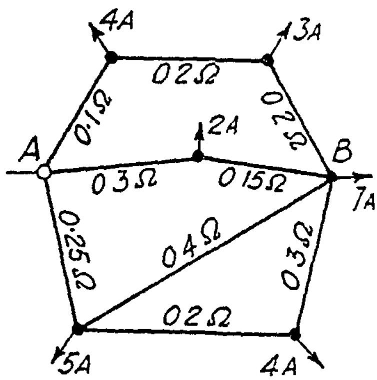
53 A direct-current network is loaded as shown. Find the voltage drop between the supply point A and the point of lowest potential. Resistances are for go and return.

[3·1 V at C.]



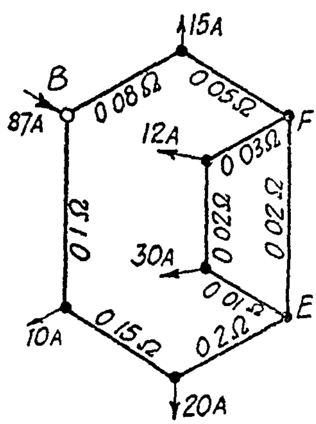
54 In the direct-current network shown, A is the feeding point. The resistances of the various sections (both conductors together) are indicated. Determine the resistance of an additional cable between A and B to reduce the potential difference between these points to 2 V. Show the resulting current distribution when the equalizer is in use.

[0·53 Ω.]

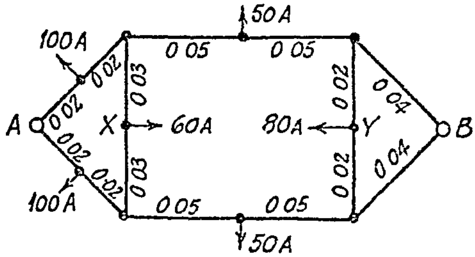


55 In the direct-current network shown, the feeding point is at B. The resistance of each section (both conductors) is givens in ohms. Find the current in the section E F, its direction, and the voltage between points E and F. Show the current distribution.

[21 8 A, F to E ; 0·436 V.]



56 A and B are at the same potential, being fed from the same source. Calculate the currents entering the network at A and at B, also the voltage drops at X and Y. All resistances, for both conductors, are given in ohms.

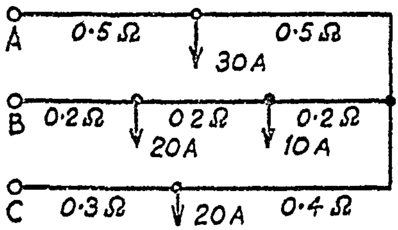


[292 A at A, 148 A at B ; 4·74 V, 3·76 V.]

57 A 2-wire direct current distributor network consists of a square of sides 100 yd. At the corners B, C, D, E the loads are 20, 15, 60 and 30 A respectively. The points B and E are fed from a station A distant 50 and 100 yd respectively. Find the maximum permissible resistance per 1 000 yd of distributor conductor, if the minimum supply voltage is to be 230 V and all conductors have the same cross section. [0·54 Ω.]

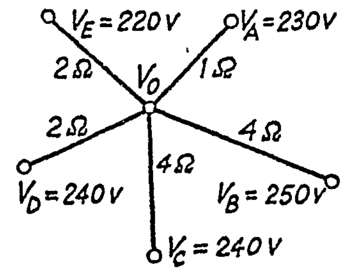
58 Three distributors A, B, C, joined at one end, are loaded as shown. Find the point of lowest load voltage (a) when $V_A = V_B = V_C$; (b) when $V_A = 240$ V, $V_B = 250$ V, $V_C = 230$ V

[(a) 12 V drop at 30 A load in A; (b) $V = 227·2$ V at 20 A load in C.]



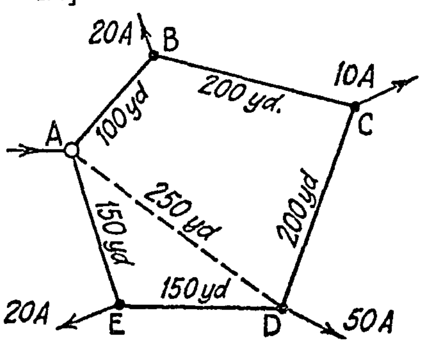
59 Five lines A, B, C, D, E, run to a common point. With the supply voltages and resistances as shown and no load taken at V_O, find the value of V_O and the current leaving each supply point.

[$V_O = 238$ V; $I_A = -3·0$, $I_B = 4·25$, $I_C = 1·75$, $I_D = 3·5$, $I_E = -6·5$ A.]



60 For the same voltage-drop between the supply point A and the load point D in the network shown, compare the weights of copper required with and without an inter-connector between A and D. Conductors have a common cross-section in each scheme.

[1 : 1·88.]

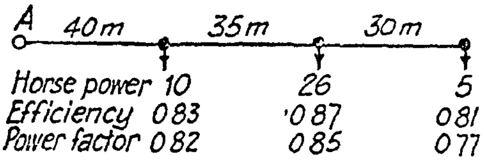


61 A substation S supplies loads of 15 kW and 40 kW at 500 V at the load points A and B respectively which are distant 1 mile from each other and from S. Compare the minimum amounts of copper which can be used when A and B are supplied (*a*) by separate cables from S, and (*b*) by separate cables from a feeding point P, midway between A and B, which is connected to S by a single cable. In each case the substation voltage must not exceed the load voltage by more than 5% of the latter.

[(*a*) : (*b*) 1 : 1·86.]

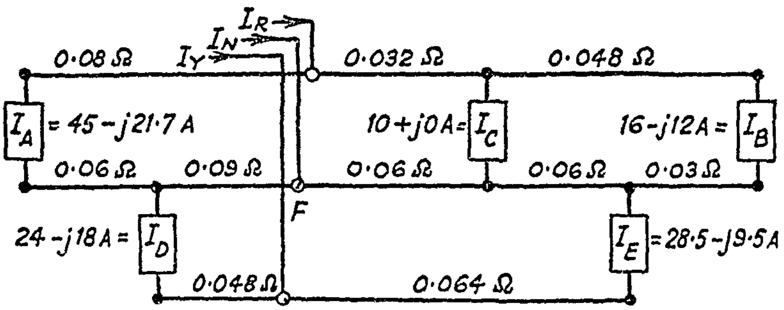
Distributors : A.C.

62 A 1-phase distributor cable has motor loads as shown. The mean motor voltage is 220 V. Find the cross-section of each core of the cable for a transmission loss of 5%. The feeding point is at A.



[45 mm².]

63 A 1-phase, 3-wire system is fed at F from a transformer



which maintains 250 V on each side. Find the transformer currents I_R, I_Y and I_N ; also the voltage at each load.
[$I_R = 71 - j\,33\,7$; $I_Y = 52\,5 - j\,27\,5$, $I_N = 18{\cdot}5 - j\,6\,2$ A. $V_A = 241{\cdot}8$ V ; $V_B = 248\,95$ V ; $V_C = 249\,3$ V ; $V_D = 250\,75$ V ; $V_E = 247\,3$ V.]

64 A 1-phase distributor has a resistance of 0·2 Ω and a reactance of 0·3 Ω. At the far end the voltage $V_b = 240$ V, the current is 100 A and the power factor 0 8. At the mid point a, a current of 100 A is supplied at power factor 0·6 with reference to the voltage V_a at a. Find the supply voltage V_s and the phase angle between V_s and V_b.

[292 V ; 4°]

65 In a 4-wire, 3-phase distribution system with 240 V between lines and neutral, there is a balanced motor load of 500 kW at power factor 0 8. Lamp loads connected between respective lines and neutral absorb 50 kW, 150 kW and 200 kW. Calculate the current in each line and in the neutral wire of the feeder cable supplying this load.

[1 042 A ; 1 420 A ; 1 615 A ; 551 A.]

66 A 3-phase, 4-wire, 416/240-V, 50-c/s system has a balanced 3-phase motor load of 20 kW at power factor 0·8 lagging and 1-phase loads of 25 A at unity power factor, 45·7 A at power factor 0 9 leading, and 30 A at power factor 0·8 lagging, the phase sequence being in the order given. Find the current in each line and neutral, and the capacitance required across each phase to obtain unity power-factor in each line.

[56·6, 69, 64·7, 29·6 A ; 271, 13·5, 515 μF.]

67 A 3-phase, 4-wire distributor supplies a balanced voltage of 400/230-V to a load consisting of 80 A at power factor 0·8 lagging, 70 A at power factor 0·9 lagging and 50 A at unity power factor, on phases *RYB* respectively. Calculate the voltage at the supply end of the *R* phase relative to the load voltage of the same phase. The resistance of each core is 0 1 Ω.

[$235\underline{/-2{\cdot}2^\circ}$ V.]

68 The following loads are connected to a 3-phase, 4-wire 400/230 V system :—(i) a 3-phase, 15 h.p. motor with efficiency 0·85 and power factor 0·9 lagging, (ii) a 3-phase oven taking 5 kW at unity power factor, (iii) a 1-phase, 400-V, 3 h p motor with efficiency 0 8 and power-factor 0·8 lagging connected across *R* and *Y*, (iv) 1-phase loads of 1 kW at power factor 0·9 lagging connected across *R-N*, 3 kW at power factor 0·9 leading connected across *Y-N*, 4 kW at unity power factor connected across *B-N*. Calculate the current in each line and neutral.

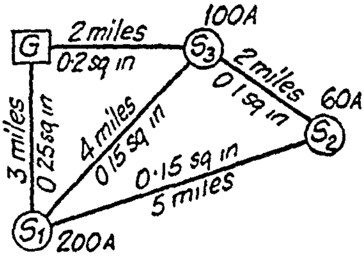
[41·2, 44, 44·6, 5 6 A.]

69 A factory is supplied from 3-phase, 380-V, 50-c/s mains. The load comprises (i) ten 3-phase, 10-h.p. induction motors each having a full-load efficiency of 84% and a power factor of 0 8 ; (ii) a 3-phase, 100-h.p. synchronous motor having an efficiency of 92% and running with a leading power factor of 0·75 ; (iii) 1 200 lamps, each rated at 50 W, 220 V. Assuming all loads to be balanced, calculate (*a*) the line current for (i), (ii) and (iii) respectively, (*b*) the line current for full-load conditions when all loads are switched on, and (*c*) the power factor of the total load.

[(*a*) (i) 291 A, (ii) 284 A, (iii) 157 A ; (*b*) 603 A ; (*c*) 1·0.]

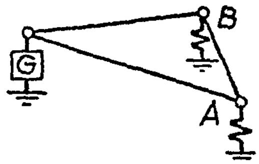
70 Three substations S_1, S_2 and S_3 are supplied with 3-phase energy from a distributing centre *G* as shown. Distances are

given in miles, the cross-sections of each conductor of the 3-core cables in square inches, and the loads on the substations in amperes per line. A voltage of 10 000 V is maintained at G. Calculate (a) the current in each section of the network, (b) the bus-bar voltage at each sub-station. Assume unity power factor and neglect voltage drop due to reactance. Resistivity, 0·7 $\mu\Omega$-in.



[(a) currents : $GS_1 = 190$ A, $GS_3 = 170$ A, $S_3S_2 = 48$ A ;
(b) voltages : $S_1 = 9\,825$ V, $S_2 = 9\,795$ V, $S_3 = 9\,871$ V.]

71 An alternating-current network is arranged as shown. The impedances of the star-connected loads at A and B are $50\underline{/37°}\ \Omega$ and $40\underline{/26°}\ \Omega$ respectively. The line voltage at G is 13 200 V. All lines have negligible leakance and capacitance. The resistances and reactances (line-to-neutral) values are :—



Line GA,	Resistance	2·5 Ω ;	Reactance	2·3 Ω
AB,	,,	1·4 Ω ;	,,	1·0 Ω.
BG,	,,	1·5 Ω ;	,,	1 2 Ω.

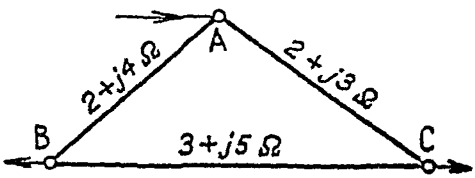
Find the voltages at A and B, and the current in the line AB.

[$V_A = 12\,400$ V ; $V_B = 12\,500$ V ; $I_{AB} = 25$ A.]

72 A single-phase, ring distributor ABC is fed at A. The loads at B and C are 40 A at power factor 0·8 lagging and 60 A at power factor 0·6 lagging respectively, both expressed relative to the voltage at A. The total impedances of the three sections AB, BC and CA are $2 + j\,1$, $2 + j\,3$, and $1 + j\,2\ \Omega$. Determine the current in each section with respect to the supply voltage at A.

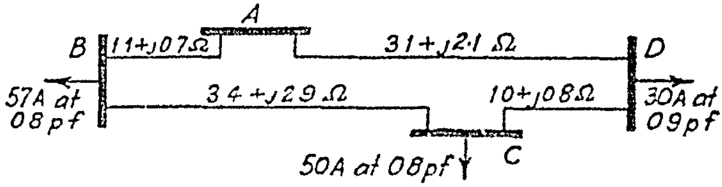
[AB, $39\,5 - j\,25$; BC, $7·5 - j1$; CA, $-\,28·5 + j\,47$ A.]

73 A 3-phase, 6 350-V/phase station at A supplies balanced loads of 50 A at power factor 0·8 lag to B and 40 A at power factor 0·9 to C. (Power factors refer to voltage at A.) (a) Find current in each feeder and the voltage at B and at C. (b) With the same loadings ; find the corresponding values when a boost of 480 V, in phase with the phase voltage, is injected into the section AC.



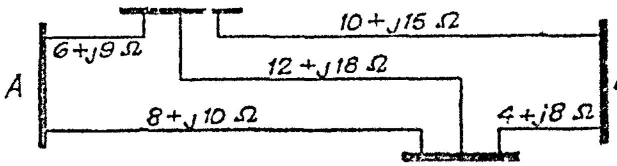
[(*a*) *A* to *B*, 35·6 − *j* 26·8 A ; *A* to *C*, 40·4 − *j* 21·2 A ; *C* to *B*, 4 4 − *j* 8·7 A. V_B = 6 174 V, V_C = 6 205 V. (*b*) *A* to *B*, 18 + *j* 8·5 ; *A* to *C*, 58 − *j* 51 ; *C* to *B*, 22 − *j* 88·5. V_B = 6 328, V_C = 6 561.]

74 The diagram represents a 3-phase system supplied with 11 kV at *A*. The load currents are balanced and the power factors,



all of which are lagging, refer to the supply voltage at *A*. The impedances per phase are shown in ohms. Calculate the voltage at *C* and its phase angle relative to the supply voltage.
[10·6/ − 0 06° kV.]

75 The diagram represents a 3-phase system supplied with 66 kV at *A* and loaded at *B*. If 5 000 kW at 0 8 power factor



lagging is supplied at *A*, calculate the voltage at *B* and the power loss in the system. The impedance per phase of each line is shown in ohms. [64 900 V ; 60 kW.]

CHAPTER XXI

FEEDERS AND TRANSMISSION LINES

D.C. Lines and Feeders

1 A direct-current motor is supplied from a point 100 yd away by means of a cable having a cross-section of 0·075 in^2 per conductor. If the voltage at the motor terminals be 500 V, find the voltage at the feeding point and the power dissipated in the cable when the motor is developing 50 h.p. at an efficiency of 90%. What is the transmission efficiency?
[505·4 V; 450 W; 98·9%.]

2 The voltage at the receiving end of a direct-current feeder 1 km in length is 700 V. What cross-section must each copper conductor have so that the voltage drop will not exceed 10% of the sending-end voltage when 100 A is flowing? Determine the relative weight and the relative cross-section if aluminium be used in place of copper. [44·8 mm^2; 0·50; 1·66.]

3 Calculate the cross-section of each core of a cable to deliver 1 000 kW at a direct voltage of 1 500 V to a point 1 mile from the generating station if the power loss is to be 10% of the power received. What will be the voltage at the sending end of the cable? Resistivity, 0·7 $\mu\Omega$-in. [0·395 in^2; 1 650 V.]

4 The cross-section of each core of a 2-core cable is 0·5 in^2 and the cable is 1 mile long. The power supplied to it is 100 kW at 500 V. Calculate the power delivered at the load end of the cable, the voltage at that end, and the efficiency of transmission. Resistivity, 0·7 $\mu\Omega$-in.
[92·92 kW; 464·6 V; 92·92%.]

5 If the resistance per mile of a 0·2-in^2, 2-core cable core be 0·2 Ω, to what distance can 50 kW at 400 V be transmitted through a 0 08-in^2 cable if the loss due to resistance is to be 20% of the power delivered? [0 64 mile.]

6 A direct-current line has a resistance of 0·8 Ω. A load of 10 kW is taken at the far end, while at the mid point a current of 40 A at 240 V is taken. Find the supply voltage. [274 V.]

7 A direct-current, overhead line, supplying a constant load of 600 kW at 600 V, is subject to extreme temperatures of − 10° C and 40° C If the total resistance of the line is 0·06 Ω at 20° C, express the energy dissipated at each temperature as a percentage of the load. The copper conductors have a resistance-temperature coefficient of 1/234·5 at 0° C. [8·82%; 10·8%.]

8 A direct-current, overhead line supplies a constant load of 600 kW at 600 V. The cross-sectional area of each conductor is 2 87 cm^2 and the line is 0·5 km long. Calculate the increase in cost of transmitting 7 500 kWh due to an average temperature rise from 10° C to 35° C. The specific resistance at 20° C and the resistance temperature coefficient at 0° C of the conductor material are 1/58 Ω per m and mm^2, and 1/234 5 respectively. Energy costs 1d per kWh. [6s 3d.]

9 Two substations together supply 1 000 A at 550 V to a load. The resistance of the feeder from the first substation to the load is 0·1 Ω, and that from the second is 0·2 Ω. Find the voltage required at the bus-bars of each substation so that the load shall be equally divided. [602 V ; 646 V.]

10 Three substations, A, B and C, together supply 300 A to a load. The substation voltages and the resistances of the feeders are $V_A = 250$ V, $R_A = 0\ 2\ \Omega$; $V_B = 255$ V, $R_B = 0{\cdot}15\ \Omega$; $V_C = 260$ V, $R_C = 0\ 1\ \Omega$. Find (*a*) the load voltage and (*b*) the current supplied by each substation.
[(*a*) 242·32 V ; (*b*) 38·4 A, 84·7 A, 176·8 A.]

Short Lines and Feeders : Single-phase

11 A 1-phase transmission line has a resistance of 0·22 Ω and an inductive reactance of 0·36 Ω. Find the voltage at the sending end to give 500 kVA at 2 000 V at the receiving end at load power factors of (*a*) unity ; (*b*) 0·707 lagging.
[(*a*) 2 056 V ; (*b*) 2 103 V.]

12 A voltage of 3 300 V is applied to a line of impedance 0 6 + j 0 8 Ω. Find the voltage at the receiving end when the load is 1 000 kVA at (*a*) unity power factor, (*b*) 0 8 power factor.
[(*a*) 3 096 V, (*b*) 2 976 V.]

13 A single-phase transmission line, connected with transformers at each end, delivers 250 kVA at 2 000 V and a power factor 0 8 lagging to the low-voltage bus-bars in the sub-station. The line has a total resistance of 10 Ω and an inductive reactance of 30 Ω. Each transformer has a ratio of 2 000/11 000 V, the resistance on the low- and high-voltage sides being 0 04 Ω and 1·3 Ω, and the reactance 0·125 Ω and 4·5 Ω. Calculate the bus-bar voltage and power factor at the generating station, and the efficiency of transmission. Neglect capacitance.
[2 169 V ; 0·77 ; 96·3%.]

14 A line having resistance of 16 Ω and a reactance of 12 Ω supplies a load of 5 000 kW at voltage V_r. The supply voltage is V_s. Find the power factor of the load when (*a*) $V_r = V_s = 33$ kV ; (*b*) $V_r = 30$ kV, $V_s = 31{\cdot}5$ kV. Draw the vector diagram in each case. [(*a*) 0·53 leading ; (*b*) 0·815 leading.]

15 A short 1-phase cable, having a capacitance of 0·7 μF, supplies a load at the receiving end of 150 kVA at 11 000 V, 50 c/s, and power factor 0 8 lagging. Calculate the current and power factor at the sending end, neglecting cable resistance and inductance. [12·3 A ; 0·89.]

16 A 1-phase feeder cable supplies 100 kW at 440 V and a power factor of 0·6 lagging. Each core has a resistance of 0·025 Ω. Calculate (*a*) the active and reactive components of the current ; (*b*) the respective heating losses due to each component ; (*c*) the total heating loss ; (*d*) the load that could be transmitted at unity power factor for the same heating.
[(*a*) 227 A, 303 A ; (*b*) 2 576 W, 4 590 W ; (*c*) 7 166 W ; (*d*) 167 kW.]

17 A feeder *OAB* has loads of 10 kW at power factor 0·8 at *A* and at *B*. The voltage at *B* is 400 V. The impedance of *OA* is $0{\cdot}3 + j\,0\,4\ \Omega$ and of *AB* is $0\,6 + j\,0{\cdot}8\ \Omega$. Find the value of the supply voltage at *O*. [462 V.]

18 The input to a short line of constants $R = 0{\cdot}4\ \Omega$ and $X = 0\,4\ \Omega$ is 2 000 kW at power factor of 0 8. If the load voltage is 3 000 V, find the load and the supply voltage.
[1 790 kW at power factor 0·81 ; 3 410 V.]

19 A 1-phase load of 2 000 kW at power factor 0 71 is to be supplied at 11 000 V, 50 c/s, over a distance of 5 km from a generating station by a pair of conductors having the following constants ; resistance of each conductor, 0·2 Ω per km : loop inductance, 0·002 H per km. Calculate the voltage to be maintained at the generating station bus-bars. Find the bus-bar voltage at the generating station, if a condenser be connected at the load end to raise the load power factor to 0 85.
[11 940 V ; 11 725 V.]

20 A 300-kW load has a power factor of 0·8 and is supplied by a short line with a total resistance of 0·5 Ω and a loop reactance of 1 Ω. If the supply is 2 200 V at 50 c/s and the load is shunted by a 51-μF capacitor, find the load voltage and supply power factor. [2 055 V ; 0 88.]

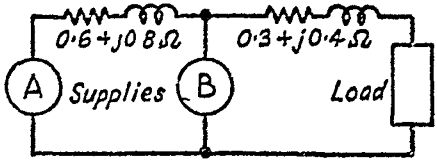
21 A feeder has a resistance of 8 Ω and a reactance of 11 Ω and supplies a 0 8 power factor load at a voltage of 6 370 V. Find the value of the load at which the supply voltage equals the load voltage when a 25-μF capacitor is connected across the load The supply frequency is 50 c/s. [207 kW.]

22 A 1-phase, 50-c/s generating station supplies an inductive load of 5 000 kW at a power factor of 0·71 by means of an overhead transmission line 5 miles long. The resistance per mile of each line is 0·0345 Ω and the loop inductance is 1·5 mH per mile.

The voltage at the receiving end is maintained constant at 10 kV, and a condenser is connected across the load to raise the power factor to 0 9 lagging. Calculate (*a*) the capacitance of the condenser; and the generating-station voltage when the condenser is (*b*) in use, (*c*) disconnected.

[(*a*) 82 μF ; (*b*) 10·8 kV ; (*c*) 11·39 kV.]

23 In the circuit shown, the two sources, *A* and *B*, supply the load, 10 kW at power factor 0 8 and 415 V. If *B* supplies 5 kW at power factor 0 707, find what *A* supplies.



[6·17 kVA at power factor 0 87 and 444 V.]

24 A load of 20 kW at power factor 0·707 is supplied by two short lines *A* and *B* in parallel. Calculate the input to each line when (*a*) $V_A = V_B = 440/0°$ V ; $Z_A = 0\ 8 + j\ 0{\cdot}6$ and $Z_B = 1\ 2 + j\ 0{\cdot}6$; (*b*) $V_A = 460/\overline{0°}$ V, $V_B = 440/\underline{0°}$, $Z_A = Z_B = 0\ 8 + j\ 0\ 6$. Draw vector diagram for (*a*).

[(*a*) $P_A = 12\ 8$ kW, $P_B = 9{\cdot}7$ kW ; (*b*) $P_A = 14{\cdot}5$ kW ; $P_B = 7{\cdot}63$ kW.]

Short Lines and Feeders : Three-phase

25 Estimate the distance over which a load of 15 000 kW at power factor 0·85 can be delivered by a 3-phase transmission line having conductors of steel-cored aluminium each of resistance 0 905 Ω per km. The voltage at the receiving end is to be 132 kV and the loss in transmission is to be 7·5% of the load.

[70 km.]

26 A load of 3 impedances, each $9 + j\ 21\ \Omega$, is supplied through a line to which a voltage of 415 V is applied. The impedance of each line is $2 + j\ 4\ \Omega$ Find the power input and output when the load is (*a*) star-connected, (*b*) mesh-connected.

[(*a*) 2 556, 2 091 W ; (*b*) 5 925 W, 3 555 W.]

27 The sending- and receiving-end voltages of a 3-phase transmission line are maintained at 33 and 31·2 kV respectively. The resistance and reactance per phase are 20 and 50 Ω respectively. Determine the maximum power obtainable at the receiving-end.

[12 3 MW.]

28 Find the sending-end voltage, the percentage voltage drop and the efficiency of a 20-km, 20-kV, 50-c/s, 3-phase transmission line delivering 5 000 kW at a power factor of 0 8. The conductors have a section of 95 mm² and an effective diameter of 12 mm ; they are arranged in equilateral triangular formation, the distance between conductor-centres being 1 m. Neglect capacitance. Temperature 20° C.

[22 2 kV ; 12% ; 93·3%.]

29 Reactor coils of reactance 1 Ω and of negligible resistance are inserted in each line of a 3-phase system supplying a load of 50 A at a power factor of (*a*) zero lagging; (*b*) unity; (*c*) zero leading. Calculate the line voltage at the load in each case when the supply voltage is maintained at 500 V.

[(*a*) 413 V; (*b*) 493 V; (*c*) 587 V.]

30 A 440-V, 3-phase, 4-wire service main supplies a star-connected load. The resistance of each line is 0·1 Ω and of the neutral, 0·2 Ω. The load impedances are $Z_R = 6 + j\,8\ \Omega$, $Z_Y = 10\ \Omega$, and $Z_B = 10 - j\,5\ \Omega$. Calculate the current in the neutral wire and the voltage across each load impedance. Phase sequence, *RYB*. [31·7 A; 255 V, 246 V, 256 V.]

31 A factory is supplied from a 3-phase, 4-wire, 440-V, symmetrical system. The resistance of each line is 0·2 Ω and of the neutral, 0·4 Ω. The power input to the main is as follows: 15 kVA at a lagging power factor of 0·8 to phase *R*; 15 kVA at a leading power factor of 0·8 to phase *Y*: and 15 kVA at unity power factor to phase *B*. Calculate the current in the neutral wire and the voltage between line *R* and the neutral at the factory. Phase sequence, *RYB*. [49·5 A; 236·5 V.]

32 A 12·5 h.p., 400-V, 3-phase motor with power factor 0·8 and efficiency 0·85 is supplied from a line having a resistance of 0·15 Ω per conductor. Lighting loads of 20, 15 and 5 A are taken from phases *R*, *Y* and *B* respectively. Calculate the voltage of phase *B* to neutral at the sending end. The neutral has a resistance of 0·4 Ω. [229 V.]

33 A 3-phase, 4-wire system has 420 V between lines at the sending end. The loads are: phase *R*, 40 A at power factor 0·8 lagging; phase *Y*, 30 A at unity power factor; phase *B*, 20 A at power factor 0·87 leading. The resistance of each conductor is 0·2 Ω, and of the neutral 0·4 Ω. Find the load voltages V_{RN} and V_{BR}. [236·4 V, 408 V.]

34 A 3-phase, 50-c/s, 3 300-V substation supplies a 500-kW load of 0·8 lagging power factor. Power is fed to the substation from a generating station 5 000 ft distant by a single-circuit, 3-phase line comprising 3 No. 2 S.W.G. copper conductors, effective diameter 0·138 in, spaced 30 in apart in a horizontal plane. The resistance is 0·408 Ω per conductor per 1 000 yd. Determine per conductor (line-to-neutral) (*a*) resistance, (*b*) inductance, (*c*) reactance, (*d*) impedance. Thence find (*e*) the voltage at the generating station, (*f*) the voltage regulation, no-load to full-load, (*g*) the power lost in the line, and (*h*) the efficiency of transmission.

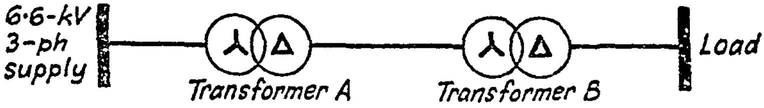
[(*a*) 0·68 Ω; (*b*) 0·002 H; (*c*) 0·628 Ω; (*d*) 0·926 Ω;
(*e*) 3 460 V; (*f*) 4·85%; (*g*) 24·2 kW; (*h*) 95·4%.]

35 A 3-phase voltage of 11 kV is applied to a line having $R = 8\ \Omega$ and $X = 11\ \Omega$ per conductor. At the end of the line is a balanced load of P kilowatts per phase at a leading power factor. At what value of P is the voltage regulation zero when the power factor of the load is (*a*) 0·707, (*b*) 0·8 ?

[(*a*) 1 965 kW ; (*b*) 210 kW.]

36 A 3-phase load of 1 000 kW at power factor 0 8 is supplied over a line of impedance $25 + j\ 12\ \Omega$ per phase. Calculate the supply voltage when the load voltage is (*a*) 30 kV, (*b*) 10 kV obtained by a 30/10 kV transformer. The equivalent resistance and reactance of the transformer on the 10-kV side are 0·8 and 2 5 Ω. [(*a*) 31·1 kV ; (*b*) 31·9 kV.]

37 In the accompanying diagram the phase turn-ratios of transformers *A* and *B* are 1 : 2·89 and 1·92 : 1 respectively. Find



the no-load line voltage on each side of *A* and of *B* ; also the line current when the load current is 1 000 A. Ignore magnetizing current and line drop.

[*A*: l.v. 6 600, h.v. 11 000 ; *B*: h.v. 11 000, l.v. 3 300 V.
A: l.v. 500, h.v. 300 ; *B*: h.v. 300, l.v. 1 000 A.]

38 A substation has a 10-MVA transformer in parallel with a 5-MVA transformer. Each has a turn-ratio of 5 and a reactive drop of 10%. The impedance of the feeder on the high-voltage side is $1 + j\ 5\ \Omega$ and on the low-voltage side is $0\ 05 + j\ 0\ 2\ \Omega$ per phase. Find the voltage regulation when 12 MW at a power factor of 0 8 and a line voltage of 33 kV is supplied to the high-voltage feeder. [15%.]

39 A substation receives 6 000 kVA at 6 000 V, power factor 0·8 lagging, on the low-voltage side of the substation transformer, from a generating station through a 3-phase cable system having a resistance of 7 Ω and reactance of 2 Ω per phase. Identical transformers of 6 600/33 000 V, delta/star connexion are installed at each end of the cable, having a resistance and reactance of 1 Ω and 9 Ω respectively, referred to the high-voltage side. Neglecting the capacitance of the cable, calculate the voltage on the low-voltage, generator bus-bars. [6 790 V.]

40 A 3-phase transmission line of impedance $16 + j\ 24\ \Omega$ is fed through a 1 : 3 transformer whose equivalent impedance on the secondary side is $2 + j\ 8\ \Omega$. The load current is 100 A at power factor 0 8, while the line voltage at the mid-point of the line is 33 kV. Find (*a*) supply voltage on the low-voltage side ; (*b*) the equivalent resistance and reactance of each phase of the load. [(*a*) 12·15 kV ; (*b*) 140 and 105 Ω/phase.]

41 The voltages of a switching station, transmission line and substation have nominal ratios corresponding to line values of 66, 132 and 22 kV. The substation load is 20 000 kVA at 20·5 kV (line) and p.f. 0 85 lagging. The transformers at each end of the transmission line have equivalent impedances of $9 + j\ 36\ \Omega$ per phase referred to the high-voltage side. The line-to-neutral impedance of the line is $21 + j\ 60\ \Omega$. Find the voltage at the switching station and the overall efficiency of transmission.
[70 2 kV ; 94·4%.]

42 A balanced 3-phase, 50-c/s load of 1 200 kW at power factor 0·8 and 15 kV is supplied at the end of a line 30 km long with the conductors spaced 1·5 m apart. Find the conductor diameter for a line loss of 5% of the load. If a similar load is added at the mid point *ab* of this line, find the voltage regulation at both loads and the total loss. Take the resistivity as 1/52·5 Ω per 1 m length and 1 mm² cross-section.
[11 mm ; 6·7% at *ab* ; 10 1% ; 146 kW.]

43 Two short 3-phase lines operating in parallel supply a 5 000-kW, 33-kV, balanced load having a power factor of 0·8 lagging. The resistance and reactance of each line are (*a*) 3 Ω and 5 Ω, and (*b*) 4 Ω and 4 Ω respectively. Determine the current, power, and power factor of each line.
[(*a*) 54·3 A, 2 230 kW, 0·72 ; (*b*) 55·8 A, 2 770 kW, 0·866.]

44 A 3-phase load at power factor 0·8 is supplied at 10 kV through a line $Z_A = 1 + j\ 2\ \Omega$ in series with $Z_B = 2 + j\ 4\ \Omega$ and $Z_C = 2 + j\ 6\ \Omega$ in parallel. Calculate the voltage supplied to *B* and *C* when the current in *B* is 100 A. [11 250 V.]

45 Two 11-kV, 3-phase substations are connected by a feeder of impedance $0{\cdot}2 + j\ 0\ 6\ \Omega$/phase, in parallel with a 33-kV feeder of impedance $1 + j\ 5\ \Omega$/phase. At each end of the 33-kV feeder is a 15-MVA, 33/11-kV transformer with 10% reactance. If 20 MW at 0·8 power factor is supplied to one substation, what will be the output at the other substation, and the current in each feeder? [19·3 MW at p.f. 0·84 ; $1\,020\underline{/-33^\circ}$, $99\underline{/-49^\circ}$.]

46 A 33-kV, 3-phase generating station is to supply 10 000-kW to a load at 31 kV and 0·9 power factor lagging. The efficiency of transmission is to be not less than 96%. What must be the resistance and reactance of the line? If the load carrying capacity is to be increased to 15 000 kW by erecting a parallel line, all the other conditions remaining the same, what must be the constants of the second line?
[3·25 Ω, 5·94 Ω ; 6·5 Ω, 11·9 Ω.]

47 A 10 000-kW load, power factor 0·8 lagging is received in a substation at 30 kV through two 3-phase overhead lines operating in parallel. Current supplied by line *A* is 100 A and the power delivered by line B is 5 500 kW. If each of the B lines has resistance and reactance of 8 Ω and 12 Ω respectively, what are the corresponding quantities for the A line?

[$R = 14{\cdot}6\ \Omega$, $X = 14{\cdot}4\ \Omega$.]

48 A 3-phase, star-connected, balanced load of 700 kW at power factor 0 866 lagging is supplied at 3 300 V from two 3-phase feeders A and B having impedances per line of $3 + j\,7$ and $2 + j\,7\ \Omega$. The difference in the sending-end voltages of corresponding phases of the two feeders is 300 V. Find the currents in each feeder and their power factors at the load end.

[82·7 A, 56·2 A ; 0 84, 0·96.]

49 The total power supply delivered to a 3-phase over-head line in parallel with a 3-phase cable is 250 A at 3·3 kV, power factor 0·8 lagging. Calculate the current distribution between the line and the cable, the power factor of the supply to line and to cable, and the power factor of the total supply delivered by the combination. The line and cable impedances per conductor are $4 + j\,6$ and $3 + j\,2\ \Omega$ respectively.

[84 6 A, 169 A ; 0 61, 0 87 ; 0·82.]

A.C. Long Lines

50 A long 3-phase transmission line is supplied from a transformer at the sending end, and a similar transformer is connected to the line at the receiving end. Each transformer has a reactance drop and resistance drop of 5% and 0 7% respectively of its normal voltage of 60 000 V on full-load current of 80 A. The load at the receiving and sending ends of the line is respectively 58 000 $\underline{/26^\circ}$ V, 80 $\underline{/0^\circ}$ A, and 62 000 $\underline{/19^\circ}$ V, 77 $\underline{/16^\circ}$ A. Calculate the overall percentage voltage drop of the combined line and transformers as a percentage of the sending-end voltage. Neglect magnetizing current and capacitance. [10%.]

51 A 1-phase transmission line delivers 1 000 kVA at power factor of 0 71 lagging, 22 kV, 50-c/s. The loop resistance is 15 Ω, the loop inductance 0 2 H and the capacitance 0 5 μF. Find (*a*) the voltage, (*b*) the current, and (*c*) the power factor at the sending end. Use the nominal-π method. (*d*) If the sending-end voltage be maintained unaltered, to what value will the received voltage rise on no-load?

[(*a*) 24·45 kV ; (*b*) 48 A ; (*c*) 0·71 ; (*d*) 24·5 kV.]

52 Find the regulation and efficiency of an 80-km, 3-phase, 50-c/s transmission line delivering 24 000 kVA at a power factor of 0·8 lagging and 66 kV to a balanced load. The conductors are of copper, each having resistance 0·12 Ω per km, 1·5 cm outside diameter, spaced equilaterally 2·5 m between centres. Neglect leakance and use the nominal-π method.

[14·5% ; 94%.]

53 The constants of a 3-phase, 80-mile transmission line are :— resistance, 0·125 Ω per mile per conductor : reactance, 0·375 Ω per mile, line-to-neutral ; admittance due to capacitance, $0{\cdot}5 \times 10^{-5}$ Ω per mile, line-to-neutral ; leakance, negligible. (*a*) Find the sending-end voltage, current and power factor, and the efficiency of transmission when the line delivers 80 000 kVA at power factor 0·8 lagging and 66 kV. (*b*) Estimate for this load the approximate rating of phase-modifier apparatus installed at the receiving end to enable the sending-end voltage to be maintained at 70 kV. Use the nominal-π method.

[(*a*) 78 kV, 252 A, 0·764, 92·3% ; (*b*) 18 300 kVA]

54 Find the characteristics of the load at the sending end and the efficiency of transmission of a 3-phase transmission line 100 miles long, consisting of three No. 000 S.W.G. hard-drawn copper conductors spaced in a 12-ft delta arrangement, when the receiving end delivers 20 000 kVA at 100 kV, 50 c/s, and power factor 0·9 lagging The resistance of the conductor is 0·4 Ω per mile and the effective diameter is 0·372 in. Neglect leakance and use the nominal-π method.

[112 kV ; 105 A ; power factor 0 948 ; 92 5%.]

55 A 3-phase, overhead line has resistance and reactance per phase of 25 Ω and 90 Ω. The supply voltage is 145 kV, while the load-end voltage is maintained at 132 kV for all loads by an automatically controlled synchronous phase modifier. If the kVAr of the modifier has the same value for zero load as for a load of 50 MW, find the rating of the modifier and the power factor of this load. [0·93, 20 000 kVA.]

56 A 3-phase, overhead line has resistance and reactance of 6 and 20 Ω respectively per phase. The sending-end voltage is 66 kV while the receiving-end voltage is maintained at 66 kV by a synchronous phase-modifier. Determine the kVAr of the modifier when the load at the receiving-end is 75 MW at power factor 0 8 lagging ; also the maximum load that can be transmitted. [93 MVAr, 149 MW.]

57 A 3-phase, overhead line has resistance and reactance per phase of 5 and 20 Ω respectively. The load at the receiving end is 25 000 kW, 33 kV, power factor 0·8 lagging. Find the voltage at the sending end. If a synchronous phase modifier is inserted at the receiving end and the voltage at both ends of the line is maintained at 33 kV, find the kVAr of the modifier when the load is 25 000 kW at power factor 0·8 lagging. Find also the maximum load that can be transmitted.

[48·1 kV ; 32·2 MVAr ; 41 500 kW.]

58 Using the nominal-π method, find (*a*) the sending-end voltage, efficiency and regulation of a 130-km, 3-phase, 50-c/s transmission line delivering 24 000 kVA at power factor 0·8 lagging to a balanced load at 66 kV. The conductors are 37/No. 17 S.W.G. copper, spaced equilaterally 2·5 m apart. Resistance 0·109 Ω per km, effective diameter 1·6 cm. The leakance is negligible. Draw a vector diagram for the line, and find (*b*) the reactive kilovolt-ampere rating on full-load and on no-load of a synchronous condenser to maintain the sending- and receiving-end voltages constant at 70 kV and 66 kV respectively.
[(*a*) 80·7 kV ; 91·4% ; 22·6% ; (*b*) 15 000 kVAr leading, 6 500 kVAr lagging.]

59 Using the nominal-T method and neglecting leakance, determine the regulation and efficiency of a 3-phase, 50-c/s transmission line when delivering a balanced load of 24 000 kVA, 0·8 power factor lagging and receiving-end voltage of 66 kV. Resistance, inductance and capacitance per phase are 9·6 Ω, 0·097 H, and 0 765 μF respectively. [14 7%, 93·6%.]

60 A 3-phase, 50-c/s line, 100 miles long, delivers 8 000 kVA at 60 kV and 0 8 power factor lagging. The line resistance is 0·2 Ω per mile and the reactance is 0 6 Ω per mile per conductor. If the capacitance of the line can be represented by a star-connected system of 1 μF condensers connected to the middle of the transmission line, calculate the voltage at the sending end of the line. [66·6 kV.]

61 A 3-phase transmission line has the following constants (line to neutral) ; $R = 10\ \Omega$, inductive reactance $= 20\ \Omega$, capacitive susceptance $= 4 \times 10^{-4}$ ℧. Using the nominal-T method, calculate the sending-end voltage, line current, power factor and the efficiency of transmission when supplying a balanced load of 10 000 kW at 66 kV, power factor 0 8 lagging.
[69·5 kV, 100 A, 0·85 lagging, 96·8%.]

62 Find (*a*) the sending-end voltage, current, power and power factor for a 100-mile section of single-circuit British "Grid" line delivering full load of 50 000 kVA at 132 kV and power factor 0 8 lagging. (*b*) Find also the efficiency of transmission. Resistance per line, 0·0479 Ω per 1 000 ft at 15° C ; conductor diameter 0 77 in ; spacings, 12 ft, 21 ft and 24 ft transposed. (*c*) Evaluate the constants *A*, *B*, *C* and *D* in the relations $V_s = V_rA + I_rB$, $I_s = I_rC + V_rD$. (*d*) Calculate the receiving-end regulation.
[(*a*) 153·5 kV, 197 A, 43 300 kW, 0·826 ; (*b*) 92·3% ;
(*c*) $A = C = 0{\cdot}986\underline{/0{\cdot}82^\circ}$, $B = 71{\cdot}3\underline{/69{\cdot}2^\circ}$,
$D = 4{\cdot}44.10^{-4}\underline{/90^\circ}$; (*d*) 17·9%.]

63 In a long line the sending-end voltage V_s and current I_s, are given in terms of the receiving-end conditions by $V_s = AV_R + BI_R$ and $I_s = CV_R + DI_R$, where $A = D = \cosh\sqrt{YZ}$, $B = \sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ}$, and $C = \sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ}$, where Y and Z are the distributed shunt admittance and series impedance of the line. Prove that $AD - BC = 1$. Such a line with $A = D = 0{\cdot}94\underline{/1{\cdot}5^\circ}$ and $B = 150\underline{/67{\cdot}2^\circ}\ \Omega$, has at the load end a transformer equivalent to a shunt admittance of $Y_T = 0{\cdot}00025\underline{/-75^\circ}$ ℧ across the receiving end of the line and a series impedance of $Z_T = 100\underline{/70^\circ}\ \Omega$ in series with the load. The load voltage and current are V_L and I_L. Obtain expressions for V_s and I_s in the form $V_s = A'V_L + B'I_L$ and $I_s = C'V_L + D'I_L$, and evaluate these four constants.
[$A' = 0{\cdot}977\underline{/1{\cdot}1^\circ}$, $B' = 247\underline{/68{\cdot}7^\circ}\ \Omega$, $C' = 0{\cdot}000606\underline{/86{\cdot}6^\circ}$ ℧,
$D' = 0{\cdot}885\underline{/3{\cdot}1^\circ}$.]

Interconnectors

64 Two 6 600-V generating stations, A and B, are interconnected by means of a 22 000-V transmission line. Each conductor has a resistance of 2·0 Ω and a reactance of 1 5 Ω. The station bus-bars are connected to the line at each end by groups of three 2 000-kVA, 1-phase transformers arranged star/star. Each transformer has an equivalent resistance and reactance of 2% and 8% respectively. Find the necessary increase in line voltage on the bus-bars at A in order that the bus-bars at B shall receive 5 000 kVA at unity power factor. [425 V.]

65 A 3-phase interconnector linking two stations has a 2% resistance and 10% reactance. Find the phase angle between the station bus-bar voltages when the interconnector carries full-load (a) for equal, and (b) for 105% and 100%, station voltages. Find for each case the percentage kilowatt and kilovar components at the receiving end.
[(a) 5·8° ; 97·0% ; 24·5% ; (b) 4·8° ; 95·0% ; 31·0%.]

66 A 3-phase interconnector linking two stations A and B has an impedance per phase of $3 + j\,2\ \Omega$. The supply voltages at both stations are 33 kV and are in phase. Find the boosting voltage, in magnitude and phase, required at station A to pass a load of 10 MW at unity power factor from station A to station B. [$525 + j\,350$ V/phase.]

67 The impedance between two points A and B in a 66-kV, 3-phase line is $10 + j\,20\ \Omega$/phase. A second circuit of impedance $5 + j\,30\ \Omega$/phase is to be installed between the two points. A total load of 18 MW, 66-kV, unity power factor, is delivered

to the circuits at *A*. What will be the currents, and their power factors, relative to voltage at *B*, in each circuit? How much in-phase and quadrature boost per phase is needed in the second circuit at *B* to make the currents supplied to the two circuits equal and both at unity power factor at the end *B*?
[91·7A, 67·6A ; 0·98 lead, 0·99 lag ; — 401 V, 787 V.]

68 An interconnector cable and a reactor, having a combined reactance of 2·5 Ω per phase with negligible resistance, link generating stations *A* and *B*. Consumers connected to station *A* take 6 000 kW at power factor 0·9 lagging at a bus-bar voltage of 11 000 V. Station *B* supplies 9 000 kW in its own area at power factor 0·75 lagging at the same voltage. The station loads are equalized by the flow of power in the interconnector cable. Estimate the station power factors and the phase angle between the two bus-bar voltages. [*A*, 0·93 ; *B*, 0·69 ; 1·8°.]

69 Two power stations *A* and *B* operate in parallel and are interconnected by a short transmission line. The station capacities are 10 000 kW and 5 000 kW respectively, and the generating sets have uniform speed regulations (no-load to full-load) of 2% and 4% respectively. Calculate the output of each station and the load on the interconnector when the load on each station bus-bars is 6 000 kW.
[*A*, 9 600 kW ; *B*, 2 400 kW ; 3 600 kW *A* to *B*.]

70 Two 50-c/s power stations are connected by means of an interconnector cable, so that the stations work in parallel. Station *A* has a full-load capacity of 10 000 kW and a uniform speed regulation (no-load to full-load) of 2·5% ; Station *B* has a capacity of 2 000 kW and a uniform speed regulation of 4%. If the load on the bus-bars of station *A* is 7 000 kW and of station *B* is 2 000 kW, due to consumers in their respective areas, calculate (*a*) the generated output of each station ; (*b*) the power transmitted by the interconnector ; (*c*) the frequency if the no-load frequency is 50 c/s.
[(*a*) *A*, 8 000 kW ; *B*, 1 000 kW ; (*b*) 1 000 kW *A* to *B* ; (*c*) 49 c/s.]

71 Two power stations *A* and *B* operate in parallel, being linked by an interconnector cable. *A* has a plant capacity of 10 000 kW and a uniform speed regulation of 2% ; *B*'s capacity 5 000 kW and its speed regulation is 3%. Calculate the load on the generators in *A* and *B* respectively and on the interconnector cable, (*a*) when consumers on *B*'s bus-bars take 8 000 kW, station *A* being on no-load except for the interconnector load ; (*b*) when there are consumers' loads of 10 000 kW and 2 000 kW on the bus-bars at *A* and *B* respectively.
[(*a*) 6 000 kW, 2 000 kW, 6 000 kW *A* to *B* ;
(*b*) 9 000 kW, 3 000 kW, 1 000 kW *B* to *A*.]

72 Two power stations A and B are linked by an induction motor-synchronous generator set. A has generators on its bus-bars having a total capacity of 10 000 kW, and B has generators totalling 5 000 kW. The speed regulation of each station from no-load to full-load is 2%. The induction motor is connected to the bus-bars of A and has a full-load rating of 1 000 kW at 2% slip. Calculate the load on the interconnector cable when there is a load of 5 000 kW on the B area, A's bus-bars being unloaded except for the interconnector load. Ignore losses. [770 kW A to B.]

73 An induction motor-synchronous generator set links two power stations A and B, the capacities of which are 20 000 kW and 10 000 kW, with speed regulations of 2% and 8% respectively. The induction motor connected to A's bus-bars has a 2 000-kW rating and a full-load slip of 4%. Find the load on the interconnector when there is a load of 10 000 kW on B's bus-bars due to its own consumers, A having no external load. [1 250 kW A to B.]

74 Two power areas operating at 50 and 60 c/s respectively are linked by an induction motor-synchronous generator set. The generating plant in the two areas comprises 10 000 kW and 5 000 kW respectively, each having a speed regulation (no-load to full-load) of 4%. The induction motor is a 50-c/s machine of 1 000 kW capacity with a full-load slip of 8%. Determine (*a*) possible speeds for which the motor-generator can be designed; (*b*) the load on the motor-generator when the only load is 4 000 kW on the 60-c/s station; (*c*) the load on the motor-generator when there are loads of 7 000 kW in each area. Ignore losses.

[(*a*) 600, 300 . . . rev per min; (*b*) 760 kW; (*c*) 680 kW.]

CHAPTER XXII

FAULTS AND PROTECTIVE DEVICES

Faults on D.C. Systems

1 A feeder cable 250 yd long has a fault to earth. The fault is localized by the following resistance measurements between earth and one end of the cable :—(*a*) distant end insulated, 6·95 Ω ; (*b*) distant end earthed, 1·71 Ω. The cable has a total resistance of 1·80 Ω. Find the resistance of the fault and its distance from the test end. [5·95 Ω ; 139 yd.]

2 On a 250-V supply a fault having a resistance of 20 Ω develops between the unearthed end of the winding of an electric cooker and the frame. If the resistance of the substation earth electrode is 4 Ω, that of the human body 2 000 Ω, and the safe maximum current through the body is 25 mA, what is the safe maximum resistance of the consumer's earth electrode ? [6 Ω.]

3 A 2-core cable AB, 30 miles long, has a partial breakdown between each core and earth, at a point P, x miles from A. The resistance between the cores at A with B open-circuited is 900 Ω. The resistance between core 1 and earth at A, with B earthed, is 200 Ω ; between core 2 and earth at A, with B earthed, is 240 Ω. The resistance of the cable is 10 Ω per mile per core. Find the distance x. [9·5 miles.]

4 The positive main of a 480-V d.c. system has an insulation resistance to earth of 0·5 MΩ : the negative main has an insulation resistance of 0·1 MΩ. A shunt motor connected to the system has a partial earth fault at the junction of two field coils one-quarter way between positive and negative mains. An 80 000-Ω voltmeter shows 162 V between the positive main and earth when the motor is running. What is the resistance of the fault ? [50 600 Ω.]

5 The middle wire of a 3-wire, direct-current system is earthed through a resistance of 10 Ω. There is a fault of 20 Ω resistance on the positive side and 18 Ω on the negative side. Calculate the potentials to earth of the three wires if the balancers maintain 200 V between each outer and the neutral.
[205·4 V + ; 5·3 V + ; 194·6 V —.]

6 Two accumulators having e.m.f.s of 100 V and 90 V respectively, and internal resistances of 0·1 Ω each, supply a 3-wire system. The resistance of each outer is 0 5 Ω and of the neutral 1·0 Ω. A 10-Ω load is across the outers at the supply end. Find

the voltage across this resistor when all three conductors at the distant end of the 3-wire system are short-circuited. [156 V.]

Faults on A.C. Systems :

Symmetrical Components

7 The line currents in a 3-phase supply to an unbalanced load are respectively $I_R = 12 + j\ 24$ A, $I_Y = 16 - j\ 2$ A, and $I_B = -4 - j\ 6$ A. The phase sequence is *RYB*. Find graphically the positive, negative, and zero phase-sequence currents, and check the results by calculation.

[$I_1 = 15{\cdot}1$ A ; $I_2 = 4{\cdot}75$ A ; $I_0 = 9{\cdot}61$ A.]

8 The currents in a 3-phase system are $I_R = 12 + j\ 6$ A, $I_Y = 12 - j\ 12$ A and $I_B = -15 + j\ 10$ A. Find symbolic expressions for the three symmetrical components.

[Positive : $I_{R_1} = 10{\cdot}85 + j\ 10{\cdot}13$, $I_{Y_1} = 3{\cdot}36 - j\ 14{\cdot}44$, $I_{B_1} = -14{\cdot}2 + j\ 4{\cdot}31$; Negative : $I_{R_2} = -1{\cdot}85 - j\ 5{\cdot}47$, $I_{Y_2} = 5{\cdot}7 + j\ 1{\cdot}13$, $I_{B_2} = -3{\cdot}83 + j\ 4{\cdot}36$; Zero : $3 + j\ 1{\cdot}33$ A.]

9 A dead earth fault occurs on one conductor of a 3-conductor cable supplied by a 10 000-kVA, 3-phase alternator with earthed neutral. The alternator has positive, negative, and zero phase-sequence impedances of $0{\cdot}5 + j\ 4{\cdot}7$, $0{\cdot}2 + j\ 0{\cdot}6$, and $j\ 0{\cdot}43\ \Omega$ per phase. The corresponding line-to-neutral values for the cable up to the fault position are $0{\cdot}36 + j\ 0{\cdot}25$, $0{\cdot}36 + j\ 0{\cdot}25$, and $2{\cdot}9 + j\ 0{\cdot}95\ \Omega$. Find (*a*) the fault current, (*b*) the sequence components in each line, and (*c*) the voltages of the sound lines to earth at the fault. The generator is excited to give 6 600 V between lines on open-circuit.

[(*a*) 1 370 A ; (*b*) $I_1 = I_2 = I_0 = 457$ A ; (*c*) 2 630, 3 250 V.]

10 A 10 000-kVA 50-c/s generator has reactances of 30%, 10%, and 5% to positive, negative, and zero phase-sequence currents respectively. It is connected to a line comprising three conductors of 1 cm diameter arranged in equilateral triangular spacing of 5 m side. The generator is excited to give 30 kV on open-circuit. Find the currents in the line when two lines are short-circuited at a distance of 20 km along the line. Neglect resistance and capacitance. [565, 565, 0 A.]

11 When the line conductors of a 3-phase system are short-circuited to earth, the three line currents are given by $I_A = 1\ 000\underline{/35^\circ}$ A, $I_B = 3\ 000\underline{/100^\circ}$ A, $I_C = 2\ 000\underline{/270^\circ}$ A. Resolve these into their symmetrical components. Draw these components to scale and show that they sum to the original unbalanced system.

[I_A : positive, $-1\ 070 - j\ 118$ A ; negative, $1\ 790 + j\ 183$ A ; zero, $99 + j\ 510$ A.]

12 Under fault conditions in a 3-phase system the following currents were recorded in the R, Y and B lines : $I_R = 1\,500\underline{/45°}$ A, $I_Y = 2\,500\underline{/150°}$ A, $I_B = 1\,000\underline{/300°}$ A. Calculate the values of the positive, negative and zero phase-sequence components. Show by vector diagrams the phase relations of these components.

[Positive : $I_R = 480\underline{/-87{\cdot}6°}$, $I_Y = 480\underline{/-207{\cdot}6°}$, $I_B = 480\underline{/-327{\cdot}6°}$;
Negative : $I_R = 1\,638\underline{/40{\cdot}4°}$, $I_Y = 1\,638\underline{/160{\cdot}4°}$, $I_B = 1\,638\underline{/-79{\cdot}6°}$;
Zero : $I_0 = 521\underline{/112{\cdot}9°}$ A.]

13 Two 11 000-V, 50 000-kVA, 3-phase alternators are connected in parallel and supply a substation by a feeder having an impedance of $0{\cdot}4 + j\,0{\cdot}7\ \Omega$ to positive and negative sequence currents and $0{\cdot}7 + j\,3{\cdot}0\ \Omega$ to zero sequence currents. The reactances of the alternators to positive, negative and zero sequence currents are $0{\cdot}6\ \Omega$, $0{\cdot}4\ \Omega$ and $0{\cdot}2\ \Omega$ respectively. Both machines have their neutrals earthed through resistances of $0{\cdot}2\ \Omega$. Calculate the fault currents in each line and the potential above earth attained by the alternator neutrals if an earth fault occurs simultaneously on the blue and yellow phases at the substation.

[$I_Y = -5\,230 - j\,960$ A ; $I_B = 4\,500 + j\,3\,230$ A ; 240 V.]

14 Three 6 600-V, 10 000-kVA, 3-phase alternators are connected to a common set of bus-bars. Each has a reactance to positive sequence currents of 15%. The reactances to negative and zero sequence currents are 75% and 30% of the positive sequence value. If an earth fault occurs on one bus-bar, determine the fault current (*a*) if all the alternator neutrals are solidly earthed, (*b*) if one only of the alternator neutrals is solidly earthed and the others are isolated, (*c*) if one of the alternator neutrals is earthed through a resistance of $0{\cdot}3\ \Omega$ and the others are isolated.

[(*a*) 25 600 A ; (*b*) 19 800 A ; (*c*) 10 700 A.]

Reactors

15 Define the percentage rating and the kVA rating of a current-limiting reactor. Two groups of generators A and B, each aggregating 20 000 kVA, have unity-power-factor loads on their respective section bus-bars of 10 000 kVA and 30 000 kVA. The section bus-bars are connected by a 20 000-kVA reactor with 40% reactance. Determine the phase angle between the voltages of the two sections A and B if each has the same terminal voltage of 10 000 V and the total load is equally divided between the two groups of machines. [11·5°.]

16 Find the value in ohms of the reactance per phase, external to a generator with 6% internal reactance, such that the steady

current on short-circuit shall not exceed 8 times full-load current. The generator is rated at 10 000 kVA, 10 000 V, 50 c/s. [0·65 Ω.]

17 A 150-MVA, 6·6-kV, 3-phase station has 15% reactance and a protective inductor of 8%. Find the ratio of the mechanical stresses on short circuit to the stresses on full load (*a*) without the inductor, (*b*) with the inductor. [(*a*) 44 : 1 ; (*b*) 19 : 1.]

18 A feeder reactor has a reactance of 3% at full-load. Find the percentage increase of bus-bar voltage required to compensate for the reactance drop when the feeder carries full-load current at a lagging power factor of 0·71. [2·1%.]

19 Show that a generating plant having N section bus-bars each rated at Q kVA with x% reactance, connected on the tie-bar system through bus-bar reactances of b%, has a total short-circuit kVA on one section of $[Q/x + Q(N-1)/(bN+x)]$ 100. If the section rating is 50 000 kVA, $x = 20\%$, and $b = 10\%$, find the short-circuit kVA with (*a*) 3 sections, (*b*) 9 sections. (*c*) Show that with a very large number of sections the short-circuit kVA would not exceed 750 000 kVA. [(*a*) 450 000 kVA ; (*b*) 614 000 kVA.]

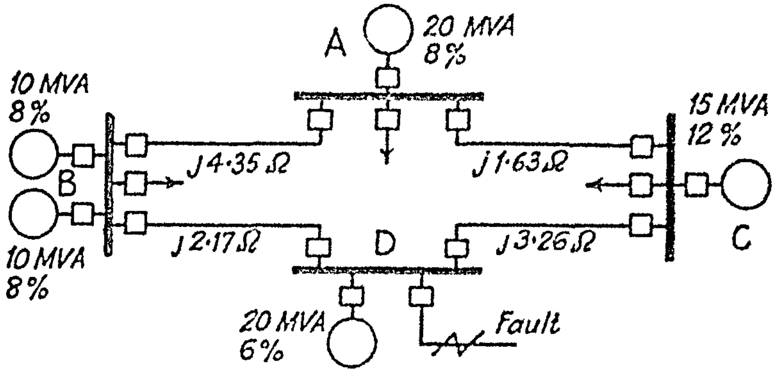
20 A generating station has five section bus-bars connected with a tie-bar through 7·5% reactors rated at 3 000 kVA. Generators aggregating 3 000 kVA with 10% reactance are connected to each section bus-bar. Find the total steady input to a dead short-circuit between the lines on one of the section bus-bars, (*a*) with and (*b*) without the reactors. [(*a*) 55 300 kVA : (*b*) 150 000 kVA.]

21 A generating station has 4 section bus-bars with the following generating plant :—

Bus-bar section	Generator kVA		Per cent reactance: Inherent	Per cent reactance: External
1 . . .	*A*	5 000	5	5
	B	20 000	15	–
2 . . .	*C*	7 000	5	6
	D	15 000	14	–
3 . . .	*E*	10 000	10	–
	F	10 000	10	–
4 . . .	*G*	20 000	18	–

Sections 1 and 2, 2 and 3, and 3 and 4 are connected through 20 000 kVA, 25% bus-bar reactors. Find the capacity of an oil switch for a feeder connected to section 3 in series with a 10 000-kVA, 8% feeder reactor. The feeder switch has no artificial time lag. [160 000 kVA.]

22 A 3-phase dead short-circuit occurs on the feeder on the bus-bar of station *D* in the 33-kV system shown. Estimate the



rating required of the circuit-breaker in the faulty feeder. The rated output of each machine with its percentage reactance is marked. [670 MVA.]

23 The plant capacity of a 3-phase generating station consists of two 8 000-kVA generators of 14.5% reactance each and one 4 000-kVA generator of 9.5% reactance. These are connected to a common bus-bar from which load is taken through a number of 8 000-kVA, step-up transformers each having a reactance of 4%. Determine the maximum MVA with which oil-switches on (*a*) the l.v. side, (*b*) the h.v. side, may have to deal.
[(*a*) 152 MVA ; (*b*) 50·2 MVA.]

24 The 33-kV bus-bars of a station are in two sections, *A* and *B*, separated by a reactor. *A* is fed from four 10-MVA generators having each 20% reactance, and *B* is fed from the grid through a 50-MVA transformer of 10% reactance. The circuit-breakers have each a rupturing capacity of 500 MVA. Find the reactance of the reactor to prevent the breakers being overloaded if a symmetrical short-circuit occurs on an outgoing feeder connected to *A*. [1·45 Ω.]

25 Three 20 000-kVA generators, each with 15% reactance, are connected through three 12 000-kVA reactors to a common bus-bar. Three feeders, each connected to the generator side of a reactor, have 200 MVA circuit-breakers. Find the appropriate percentage reactance of the reactors. [9%.]

26 A 3-phase station has two 30-MVA generators each with 15% reactance and one 10-MVA generator with 12.5% reactance at full load. A 3 300-V switchboard is supplied through a 5-MVA transformer. If the rupturing capacity of the switchgear is 100 MVA, find the safe minimum reactance of the transformer.
[4% = 0·087 Ω.]

27 Two section bus-bars *A* and *B* are linked by a bus-bar reactor rated at 5 000 kVA with 10% reactive drop. On bus-bar *A* there are two generators each of 10 000 kVA with 10% reactance, and on *B* two generators each of 8 000 kVA with 12% reactance. Find the steady kilovolt-ampere load fed into a dead short-circuit between all phases on *B* with the bus-bar reactor (*a*) in circuit, (*b*) short-circuited.

[(*a*) 173 000 kVA ; (*b*) 333 000 kVA.]

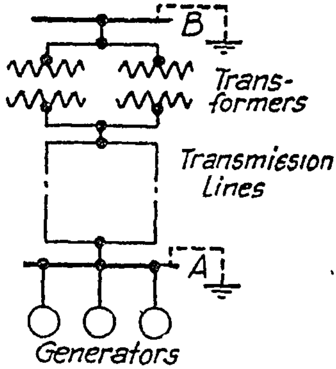
28 The estimated short-circuit MVA at the bus-bars of a generating station is 1 000 MVA, and of another station 670 MVA. The generated voltage at each station is 11 000 V. Calculate the possible short-circuit MVA at each station when they are linked by an interconnector cable having a reactance of 0·4 Ω per phase. [1 210 MVA ; 903 MVA.]

29 Two 10 000-kVA, 50-c/s, 11 000-V alternators, each with a 20% reactance, run in parallel on a section bus-bar *A* which is connected through a 30% bus-bar reactor rated at 20 000 kVA to a bus-bar *B* having two generators similar to those of *A*. A 5 000-kVA feeder having 5% reactance is connected to bus-bar *A*. Calculate the steady value of the fault current if a short-circuit occur between the three conductors at the far end of the feeder. [3 060 A.]

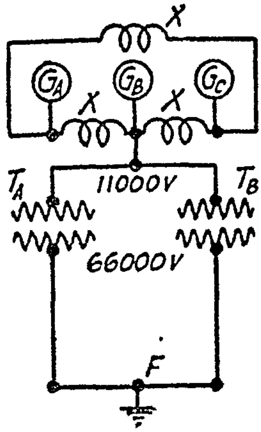
30 A 3-phase, 3 000-kVA, 6 600-V alternator having a 10% reactance is connected through a 2 000-kVA, 6 600/33 000-V transformer of 6% reactance to a transmission line having a resistance and reactance per mile of 0 15 Ω and 0·6 Ω respectively. Twelve miles along the line there occurs a symmetrical delta-connected fault, the fault impedance between each line being $24 + j\,60\ \Omega$. If the no-load alternator voltage is 7 200 V, find the alternator current. [1 075 A.]

31 A 3-phase, 11 000-V, 50-c/s system as shown is fed by three 2 500-kVA turbo-alternators each having a reactance of 12%. Energy is transmitted over a pair of circuits, each conductor of which has a reactance of 1·22 Ω to a sub-station, where the voltage is stepped down through two banks of transformers, one of 3 000 kVA and 6% reactance and the other of 2 000 kVA and 5% reactance. Find the value of the steady short-circuit current per generator if a dead-earth occur on all three phases (*a*) at *A*, (*b*) at *B*.

[(*a*) 1 093 A ; (*b*) 543 A.]



32 In the 3-phase power network schematically shown, the star-connected generators G_A, G_B and G_C are each rated at 20 000 kVA with a resistance of 2% and a reactance of 20%. The bus-bar reactors X have 1% resistance and 10% reactance on a 20 000-kVA rating. Transformer T_A has a resistance of 1% and a reactance of 10% on a 20 000-kVA rating, while T_B is a 10 000-kVA transformer with resistance and reactance of 1% and 10% respectively. The duplicate transmission lines at 66 000 V have each a resistance of 7·8 Ω and reactance of 21·75 Ω (line-to-neutral). Find the current output of each generator and the current fed into a dead-earth fault on all phases occurring at F. Make a diagram of the currents in the several branches of the system.



[G_A, 1 440 A ; G_B, 2 160 A ; G_C, 1 440 A ; F, 839 A.]

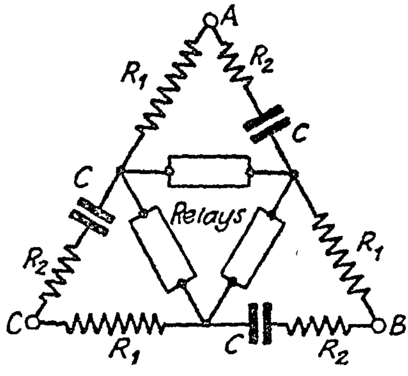
Protective Devices

33 A 6 600-V, 4 000-kVA, star-connected alternator has a reactance of 2 Ω per phase and negligible resistance. It is protected by Merz-Price balanced current gear which operates when the out-of-balance current exceeds 30% of the full-load current. If the star point is earthed through a resistance of 7·5 Ω, determine what proportion of the winding is unprotected against earth faults. Show that the effects of the alternator reactance can be ignored. [20 7%.]

34 Describe with the aid of a diagram of connexions, the Merz-Price circulating-current system for the protection of transformers. A 3-phase, 33 000/6 600-V transformer is connected star/delta and the protecting current transformers on the low-voltage side have a ratio of 300/5. What will be the ratio of the current transformer on the high-voltage side? [20·8 : 1.]

35 The neutral point of a 10 000-V alternator protected by the balanced circulating current system is earthed through a resistance of 10 Ω. The protective relay is set to operate when there is an out-of-balance current of 1 A in the pilot wires, which are connected to the secondary windings of 1 000/5 ratio current transformers. What percentage of the winding of each stator phase is protected against a fault to earth and what must be the minimum value of the earthing resistance to give protection to 90% of the winding of each phase? [65·5% ; 2·9 Ω.]

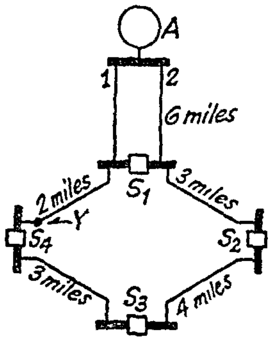
36 A, B and C are connected to a 100-V, 50-c/s symmetrical 3-phase supply having a positive phase sequence. Calculate the values of R_1, R_2 and C to ensure that 1 A flows through each arm and that there is no potential across the relays. If the phase sequence is reversed, what will be the potential across the relays? Assume the relays to have a high impedance compared with that of the network components.



[$R_1 = 57{\cdot}7\ \Omega$; $R_2 = 28{\cdot}9\ \Omega$; $C = 63{\cdot}7\ \mu F$; 100 V.]

37 The total flux in the pole-tip of a shaded-pole, induction-type, inverse-time, overcurrent relay varies with the current as shown in Fig. 4, where 100% represents 3 800 lines and 8 A. The operation time for 2 A is 5 sec. At what current will the relay operate in 0·5 sec? Assume that saturation does not alter the relative magnitude of the fluxes in the two halves of the pole-tip, and that the time taken to operate is inversely proportional to the torque. [11·2 A.]

38 The diagram represents a 3-phase, 33-kV ring main supplied by a 17 500-kVA alternator which, with its transformer, has a reactance drop of 15%. Overload protective gear is employed on each of the section switches S_1, S_2, S_3 and S_4. The overload relays are supplied by 400/5 current transformers and have an inverse-time characteristic given by the following data:



Current, A	7	9	11	15	20
Operating time in sec:					
S_3	3·1	1·95	1·37	0·975	0·78
S_1, S_2 and S_4 . .	4 0	2·55	1·8	1·27	1·02

The resistance and reactance per mile of line are 0·44 Ω and 0·71 Ω respectively. If line 1 is out of circuit and a 3-phase fault occurs at Y, find which switches will trip and the time taken. Assume the switches to trip immediately after the operation of the relays, and neglect doubling effect and current decrement. *[S_1 after 1·46 sec; S_3 1·4 sec later.]

39 A thyrite lightning arrester has a characteristic $RI^{0.72} = 72\,000$. Draw the volt-ampere characteristic of the arrester. What is the ratio of the voltages appearing at the end of a line having a surge impedance of 500 Ω due to a 500-kV surge (*a*) when the line is open-circuited, (*b*) when the line is terminated by the arrester? [2 0 : 1.]

40 Derive an expression for the fraction of the total voltage across the gap nearest the line end of a long multi-gap lightning arrester in terms of the capacitance of each unit with respect to earth, and of the capacitance of each unit to the next unit. If the ratio,

$$\frac{\text{capacitance to earth of each unit}}{\text{capacitance to next unit}} = 0{\cdot}12$$

and a surge voltage of 120 kV is applied to the arrester terminals, what is the voltage across the gap nearest to the line? [85 kV.]

41 A 132-kV, 3-phase, 50-c/s, transmission line 120 miles long consists of three conductors of effective diameter 19·9 mm, arranged in a vertical plane with 4 m spacing and regularly transposed. Find the inductance and kVA rating of the arc suppressor coil in the system. [1·97 H; 9 400 kVAr.]

CHAPTER XXIII

POWER FACTOR AND POWER-FACTOR CORRECTION

Power Factor of Mixed Loads

1 Find the power factor of a station supplying the following loads :—

250 kW at power factor unity,
1 500 kW „ „ „ 0·9 lagging,
1 000 kW „ „ „ 0·8 „
700 kW „ „ „ 0·9 leading.

If all these loads be carried by the same feeder cable, find what load at unity power factor the cable could carry with the same cable heating. [0·95 ; 3 630 kW.]

2 A star-connected, 550-V synchronous motor taking 100 kW operates in parallel with an induction motor load of 200 kW at a power factor of 0·71. If the resistance of the synchronous motor be 0·1 Ω per phase and of the cable feeding the load 0 03 Ω per conductor, calculate the power-factor of the total load, and the copper loss in the synchronous motor and in the cable, when the synchronous motor is operated at power-factors of (*a*) unity ; (*b*) 0·8 leading ; (*c*) 0·6 leading. (*d*) At what power factor should the synchronous motor work in order that the total copper loss shall be a minimum ? Neglect changes in excitation and iron losses.
[(*a*) 0·882, 3·3 kW, 12·9 kW ; (*b*) 0·923, 5·15 kW, 10·5 kW ; (*c*) 0·976, 9·2 kW, 9·4 kW ; (*d*) 0·91.]

3 A slow-speed alternator works in parallel with a turbo-alternator, the combined output being 2 500 kW at 0·8 power factor lagging. If the turbo-alternator provides 1 000 kW at unity power factor, at what power factor will the slow-speed machine work ? [0·625.]

4 Two alternators working in parallel supply a lighting load of 3 000 kW and a motor load aggregating 5 000 kW at a power factor of 0·71. One machine is loaded to 5 000 kW at power factor 0·8 lagging. What is the load and power factor of the other machine ? [3 00 kW ; 0·923 lagging.]

5 Two 6 600-V, star-connected alternators in parallel supply the following loads :—

400 kW at power factor unity.
1 000 kW „ „ „ 0·71 lagging.
400 kW „ „ „ 0·8 „
300 kW „ „ „ 0·9 „

The armature current of one machine is 110 A at power factor 0·9 lagging. Find the output, armature current and power factor of the other machine. [970 kW ; 116 A ; 0·784 lagging.]

6 A synchronous motor over-excited to take 600 kVA gives 860 h.p. at an efficiency of 0·9. It runs in parallel with a load of 1 000 kVA at power factor 0·8 lagging. Find resultant power factor. [0·997 lagging.]

7 A synchronous generator supplying 2 000 kW operates in parallel with an induction generator supplying 1 000 kW. The load has a lagging power factor of 0·8. The induction generator has a power factor of 0·95. Find the power factor at which the synchronous machine is working. [0·612.]

8 A substation equipped with induction generators giving a constant output of 2 000 kW at power factor 0·9 is connected in parallel with a station containing synchronous generators. At what power factor will the alternators work when the total load is (*a*) 10 000 kW, (*b*) 5 000 kW, each at a lagging power factor of 0·8 ? [(*a*) 0·685 ; (*b*) 0·54.]

9 The following loads are connected in a factory supplied from a 3-phase, 4-wire, 440-V distribution system : (*a*) a balanced 10-kW, star-connected load at a power factor 0 8 lagging, (*b*) a 5-kW load at a power factor 0·707 lagging between phase 1 and neutral, (*c*) a 2-kW load at a power factor 0 9 leading between phase 2 and neutral, (*d*) a 4-kW load at a power factor 0·867 lagging between phase 3 and neutral. Determine the current in each line and the power factor of the installation.
[$I_1 = 44$ A, $I_2 = 21·8$ A, $I_3 = 34·4$ A, $I_N = 21·4$ A, p.f. $= 0·886$.]

Power-factor Correction

10 A 1-phase motor takes a current of 40 A at power factor 0·7 lagging from a 440-V, 50-c/s supply. What value must a shunting condenser have to raise the power factor to 0·9 lagging, the load remaining the same ? [107 μF.]

11 A 500-W discharge lamp takes a current of 4 A at unity power factor. Calculate the inductance of a choke required to enable the lamp to work from 250-V, 50-c/s mains. Find also the capacitance of the condenser to be connected across the mains to bring the resultant power factor to unity. [0·172 H, 44 μF.]

12 A star-connected, 440-V, 50-c/s induction motor takes a line current of 40 A at a power factor 0·8 lagging. Three mesh-connected condensers are used to raise the power factor to 0·95. Find the kilovolt-ampere rating of the condenser bank, and the capacitance of each condenser. [10·8 kVA ; 56·5 μF.]

13 A bank of capacitors, each of 50 μF, is used to improve the power factor of a 3-phase, 50-c/s, 440-V load of 40 A at power factor 0·8. What will be the resultant power factor if the capacitors are (*a*) connected in Δ, (*b*) connected in star? Find the ratio (reduced line loss/original line loss) in each case.
[(*a*) 0·986 ; (*b*) 0·848. (*a*) 0·78 ; (*b*) 0·89.]

14 A 20-h.p., 50-c/s, 415-V, 3-phase induction motor of efficiency 0·87 and power factor 0·85 is supplied from a line of resistance 0·8 Ω per phase. Ignoring loss in capacitors, what capacitance must be installed to reduce the transmission loss to a minimum? What will be the percentage loss reduction?
[C = 66 μF per phase ; 28·3%.]

15 A 50-h.p. induction motor has power factor 0·9 and efficiency 0·9 at full load, power factor 0·6 and efficiency 0·7 at half load. At no-load the current is 25% of the full-load current and power factor 0·1. Capacitors are supplied to make the line power factor 0·8 at half load. With these capacitors in circuit, find the line power factor at (*a*) full load, (*b*) no load.
[(*a*) 0 994 lagging ; (*b*) 0·27 leading.]

16 A 3-phase, 300-kW, 650-V, 50-c/s load has a power factor 0·6. The power factor of the line is to be raised to 0·8 by means of a bank of Δ-connected, 2 200-V capacitors. Find (*a*) capacitance of each capacitor, (*b*) rating of 650/2 200-V transformer, (*c*) reduction in capacitance by raising voltage to 2 200 V, (*d*) percentage reduction in line loss at higher power factor.
[(*a*) 88·3 μF ; (*b*) 175 kVA ; (*c*) C is reduced 11·5 times ; (*d*) 43 7%.]

17 A long line from a hydroelectric station operating at 132 kV feeds a 66-kV system through a transformer. The load taken from the 66-kV windings of the transformer is 50 MVA at power factor 0·8 lagging. A tertiary winding on the transformer feeds a synchronous condenser at 11 kV. If the power factor at the receiving end of the line is to be unity, calculate the rating of each of the three windings. Neglect losses.
[40 MVA, 50 MVA, 30 MVA.]

18 A pump driven by a 440-V, 3-phase, induction motor delivers 10 000 gall of water per min against a head of 100 ft. At this load the power factor of the motor is 0 8 and the overall efficiency of the set 70 %. Calculate (*a*) the kilovar rating of the condensers required to raise the power factor to 0·9, and (*b*) the power factor of the installation with these condensers when the load has risen to 15 000 gall per min and the motor power factor to 0·89, the overall efficiency remaining unaltered. (86 kVAr ; 0·95.]

19 A 500-V, 50-c/s, 3-phase circuit takes 20 A at a lagging power factor of 0·8. A synchronous motor is used to raise the power factor to unity. Calculate the kilovolt-ampere input to the motor and its power factor when driving a mechanical load of 10 h.p. The motor has an efficiency of 80%. [13·9 kVA ; 0·67 lead.]

20 A generating station supplies power for the following loads :—

Lighting	100 kW.
Induction motor with 0·8 power factor and efficiency of 93%	400 h.p.
Synchronous converter with efficiency of 94% and output of	500 kW.

At what power factor must the synchronous convertor work in order that the station power factor shall be unity ? [0·911.]

21 An alternator supplying 800 kW at 0·8 power factor lagging has its power factor raised to unity by means of an over-excited synchronous motor. With the armature current unchanged, how much power can the alternator thus supply to the synchronous motor and what horse-power can the latter develop when operating at an efficiency of 90% ? Find at what power factor the motor runs. [200 kW ; 241 h.p. ; 0·316 lead.]

22 A substation operating at its full load of 1 000 kVA supplies a load of power factor 0·71. Calculate the permissible additional load at this power factor and the rating of the phase advancer to raise the substation power factor to 0·87.

[226 kVA ; 370 kVAr.]

23 A lagging load is supplied by a transmission line of resistance r ohms per phase. The transmission loss is to be reduced by installing a synchronous condenser, the armature resistance of which is R ohms per phase. Show that for the greatest overall saving in line and condenser armature, the leading reactive current of the condenser is $I_c = rI_r/(R + r)$, where I_r is the reactive component of the load current. If the 3-phase load is 1 000 kW at power factor 0·707, 3 000 V, and $R = 2r$, find the line power factor, the kVAr rating of the synchronous condenser and ratio (reduced line-loss/original line-loss).

[0·83 ; 333 kVAr ; 0·72.]

Phase Advancers

24 Given that the armature of a separately excited direct-current machine, when supplied with alternating current at a very low frequency, acts as a condenser, find the equivalent capacitance of a 4-pole, lap-wound machine so connected, if the total number of conductors is 44, the flux per pole 2 megalines (20 mWb), and the moment of inertia of the armature 80 kg-m-sec^2.

[15 000 F.]

25 A 1 000-h.p., 3-phase induction motor at full load has a power factor of 0·8, a slip of 2% and an efficiency of 95%. Estimate

the slip with, and the kVA rating of, a phase advancer connected in the rotor circuit to improve the power factor to 0 95 lagging. *[0 023 ; 8 kVA.]

26 A 3-phase induction motor at full load has a slip of 1·5% and a rotor current of 250 A. The star-connected rotor winding has a voltage between slip-rings at standstill of 500 V. What must be the rating of a shunt-wound phase advancer to cause the rotor current to lead the slip e.m.f. by 30° at full-load torque? Assume the voltage of the phase advancer to lead the slip e.m f. by 90° and ignore reactance. [2·17 kVA.]

CHAPTER XXIV

COSTS AND ECONOMICS

Energy

1 Find the cost of raising 5 tons a distance of 500 ft by means of an electric hoist having an overall efficiency of 65%. The cost of energy is 0·75d per kWh. [2·44d.]

2 A motor-driven water pump delivers 85 gall per min against a head of 35 ft. The motor takes 10 A at 200 V. Determine (*a*) the overall efficiency ; (*b*) the cost of raising 7 000 gall if energy costs 1d per kWh. [(*a*) 34% ; (*b*) $2\frac{3}{4}$d.]

3 Assuming 40 k-cal are required per hour per cubic metre, find the cost of heating a room continuously, with energy costing 0·5d per kWh. The room is 5 m × 4·5 m in plan and 3 m in height. [1·57d per hr.]

4 Calculate the resistance of a 250-V immersion heater required to raise the temperature of 25 gall of water from 15° to 85° C in 1 hour. Determine the cost of the operation if energy costs 1d per unit. Neglect losses. [6·78 Ω , $9\frac{1}{4}$d.]

5 Find the cost of energy consumed in the following cases :— (*a*) A 10-Ω resistor connected to 200-V mains for 30 min. (*b*) A 1 000-W resistor in circuit for 24 hours. (*c*) An appliance taking 30 A at 440 V for 4 hours. Tariff, 2d per kWh.
[(*a*) 4d ; (*b*) 4s 0d ; (*c*) 8s 10d.]

6 Compare the cost per 50 000 lumen-hours of incandescent gas mantles costing 1s each, burning 0·6 ft^3/hour, giving 300 lumens, and lasting 400 hours, with electric lamps giving 350 lumens, taking 40 W, costing 1s 6d each and lasting 1 000 hours. Electricity costs 2d/kWh and gas 18d/therm. 1 therm = 100 000 B.Th.U. The gas has calorific value of 500 B.Th.U./ft^3.
[Each costs 14d.]

7 Assuming gas and electric heating appliances to have an efficiency of 50% and 100% respectively, find the cost of gas per therm equivalent to electricity at 1d per kWh. [14·6d.]

8 A 50-c/s synchronous clock requires a torque of 1 g-cm at the spindle of the second reduction gear, which has a speed of 60 rev per min. What is the number of poles required if the first reduction gear effects a speed reduction of 2 5 : 1 ? What is the cost of running the clock for 1 000 hours if the efficiency of the motor and first reduction gear is 40% and electricity costs 0·3d per unit ? [40, 1·7d.]

9 A 125-kVA transformer with a copper loss of 1·5 kW at full load, and iron loss 1 kW, has the following equivalent annual load conditions :—

Full-load at power factor 0·8 for 2 000 hours
One-third load „ „ „ 8 000 „
No-load for the remainder of the year.

Find the annual cost of running with energy at 1d per kWh.
[£1 302.]

Generation

10 Plant having a first cost of £2 000 has an estimated salvage value of £200 at the end of a useful life of 20 years. What would be the valuation half-way through its life (*a*) on a basis of straight-line interim depreciation, (*b*) on a reducing balance basis, and (*c*) on a sinking-fund basis at 6% compounded annually ? [(*a*) £1 100 ; (*b*) £632 ; (*c*) £1 355.]

11 Estimate the generating cost per kWh delivered from a generating station from the following data :—plant capacity, 50 000 kW ; annual load-factor, 40% ; capital cost, £600 000 ; annual cost of fuel, oil, taxation, wages, and salaries, £80 000 ; interest 5% per annum ; depreciation 5% per annum of initial value. [0·192d.]

12 An electrochemical plant obtains 4 000 kW continuously from a direct-current generator driven by a synchronous motor fed through a transformer. Efficiencies : generator, 94% ; motor, 96% ; transformer, 99%. The installed plant cost £18 400, on which the fixed charges are 15% per annum. Repairs cost £100 per annum. High-voltage energy costs ½d per kWh. Find the cost per kWh at the direct-current bus-bars. [0·58d.]

13 A bulk supply is taken at £3 per kW and 0 2d per unit. Calculate the kWh rate for load factors of 100%, 50%, and 25%.
[0·282d, 0·365d, 0·529d.]

14 Taking installed plant costs at £45 per kW and 15% to cover all charges, find the generation cost per kWh at load factor (*a*) 90%, (*b*) 30%. Coal cost, 60s per ton. Fuel consumption, 1 lb per kWh. (*c*) If the thermal efficiency of the station is 29%, find the heat value of the coal used.
[(*a*) 0·53d ; (*b*) 0 94d ; (*c*) 11,770 B.Th.U./lb.]

15 Determine the daily load factor of a domestic lighting installation of 12 100-W lamps, switched on as follows :—

3 lamps from 6.30 p.m. till 8.00 p.m.
12 lamps from 8.00 p.m. till 8.30 m.
4 lamps from 8.30 p.m. till 1

Find also the improved load f W immersion heater is used from 1 p.m. till 6 p.m. re from 8.00 p.m. till 12.00 p.m. [8·5% ; 26·6%.]

Choice of Plant

16 A system with a maximum demand of 100 000 kW and a load factor of 30% is to be supplied by either (*a*) a steam station alone, or (*b*) a steam station in conjunction with a water storage scheme, the latter supplying 100×10^6 kWh per annum with a maximum output of 40 000 kW. The capital costs of the steam and storage stations are £20 per kW and £40 per kW respectively. The corresponding operating costs are 0·1d per kWh and 0 02d per kWh respectively. The interest on capital cost is 15% per annum. Calculate the average overall generating cost per kWh if no spare plant is required. [(*a*) 0 374d ; (*b*) 0·45d.]

17 A water supply system needs 250 000 gall daily at an average pressure of 50 lb per in², the demand varying from 500 to 50 gall per min. Estimate the motor rating (*a*) with no storage, (*b*) with elevated storage tank. Pump efficiency, 0·6. Which alternative will have the higher energy costs ?
[(*a*) 29 h.p., (*b*) 10 h.p. : energy consumption higher for (*a*) because of long periods at light load.]

18 Compare the annual costs of supplying a factory load having a maximum demand of 800 kW and a load factor of 28% by energy obtained from (*a*) a private oil-engine generating plant, and (*b*) a public supply. Private plant : cost, £25 000 ; cost of fuel oil, £4 per ton ; fuel consumption, 0 65 lb per kWh generated ; cost of lubricating oil, water and stores, 0·035d per kWh generated ; cost of repairs and maintenance, 0·03d per kWh generated ; wages, £900 per annum ; interest and depreciation, 12%. Public supply : £4 per kW of maximum demand plus 0·35d per kWh. [(*a*) £6 720 ; (*b*) £6 060.]

19 A 5-h.p. motor has to work daily on quarter load for 12 hours, and on full load for 12 hours. Two 230-V shunt motors, each costing the same, have the following losses at full load ;—

	Motor *A*	Motor *B*
Rotational loss, W	490	400
Shunt-field loss, W	200	150
Armature copper loss, W . .	240	380
Total losses, W	930	930

Which motor should be recommended and what is its all-day efficiency ? [*B* : 74·6%.]

20 A factory load of 800 kW at power factor 0·707 is to be increased by the installation of a 50-h.p. motor, and capacitors are to be installed at a cost of £2/kVAr to raise the overall power factor to 0·8. Compare the capital cost of the additions using the alternatives (*a*) an induction motor costing £150 with an efficiency of 0·9 and a power factor of 0·9, (*b*) a synchronous motor with an efficiency of 0·9 and a power factor of 0·8 leading, and costing £240.

[£278 with induction motor, £266 with synchronous motor.]

21 A grab-bucket hoist for unloading coal from barges has the following average duty-cycle :—

Duty	Close bucket	Hoist	Open bucket	Lower bucket	Rest
Duration, sec	6	10	3	10	16
Output, h.p.	40	80	30	45	0

(*a*) Specify the suitable continuous h.p.-rating of a dustproof enclosed motor, assuming the cooling conditions at rest to be the same as when running. (*b*) With energy costing 1d/kWh, interest charges of 5%, straight-line depreciation and write-off in 5 years, select the more suitable motor from the data given ;—

Fraction of full load	1/4	2/4	3/4	4/4	5/4	6/4	Cost, £
Efficiencies :							
Motor *A*	83 4	90·5	90 3	88·0	86·8	85 0	80
Motor *B*	81·3	88·6	90 3	90·6	90·3	89·6	90

The hoist is to be in use for an average of 2 000 hours per annum.

[50 h.p : motor *B*.]

22 Compare the total annual cost of a group drive with a motor costing £200, with that of five individual motors each costing £100. With group drive, annual consumption is 40 000 kWh. With separate drives, annual consumption is 30 000 kWh. Electric energy costs 1d per kWh. Depreciation, maintenance and other fixed charges amount to 15% in the case of the group drive, and 17·5% in the other case.

[Group drive, £197 ; separate drives, £212·5.]

23 Two 25-h.p., 3-phase motors have the following data :—

	Motor *A*		Motor *B*	
Load	Full	Half	Full	Half
Efficiency	0·89	0 83	0 88	0·8
Power factor	0·9	0·85	0·88	0·78

The daily duty cycle for each is 5 hours full and 3 hours half load for 300 days per annum. The tariff is £5 per annum per kVA of maximum demand plus 0 6d per unit. Motor *A* initially costs £10 more than *B*. Taking interest and depreciation charges at 12½%, find by how much the annual cost of running *B* exceeds that of *A*. [£4·3.]

24 Two 25-h.p. induction motors are each supplied on a 2-part tariff at £4 10s 0d per kVA of maximum demand per annum plus ½d per kWh. The service needed is 2 280 hours per annum at full load. Motor *A* has an efficiency 0·89 and power factor 0·83, plus condenser costing £4 per kVA to raise the power factor to 0·91. Motor *B* with efficiency 0·9 and power factor 0·91 costs £10 more than *A*. Assume interest and depreciation to be 12½% per annum for each motor and 8% per annum for the condenser. Compare the annual charges of each arrangement.

[Saving with *B* is £2 18s 0d.]

25 Tenders *A* and *B* for a 1 000-kVA, 0·8 power factor transformer are: *A*, full-load efficiency = 98 3% and core loss = 7 kW at rated voltage; *B*, 98·8% and 4 kW, but costs £125 more than *A*. The service needed is 1 800 hours per annum at 1 000 kVA, 600 hours at 600 kVA, and 400 hours at 25 kVA. Take the annual charges for interest and depreciation at 12½% of capital costs, and energy costs at ½d per kWh. Which is the better tender and what would be the annual saving?

[*B* : £6 12s 0d.]

26 Transformer *A* has a constant loss of 160 kWh and a load loss of 140 kWh daily, while the corresponding losses of transformer *B* are 80 and 240 kWh. If annual charges are 12½% and energy costs 0·75d per kWh, what should be the difference in cost so that the two transformers are equally economical?

[*B* should cost £182 less than *A*.]

Transmission, Kelvin's Law

27 A 550-V, 3-phase feeder, 1 500 yd long, is required to deliver 75 kW at a power factor of 0 85. If the voltage drop is to be 8% and equal to 1·5 times the resistance drop at this power factor, calculate the cost of the conductors required. The available sizes are :—

Cross-section, in^2 . . .	0·175	0·2	0 225	0·25	0·3
Weight, lb per 1 000 yd .	2 095	2 409	2 691	2 989	3 646

Copper costs £180 per ton. [£870.]

28 Find the most economical cross-section for a single-conductor

cable carrying 200 A for 3 500 hr per annum and open-circuited for the remainder of the year. Energy cost : 0·4d per kWh ; interest and depreciation, 10% ; resistivity, 0·7 $\mu\Omega$-in. The cable sizes available are :—

Cross-section, in^2 . . .	0·1	0·2	0·3	0·4	0·5
Cost, £ per 1 000 yd . .	200	310	415	520	630

[0·2 in^2.]

29 A 2-conductor cable, 0·5 mile long, is required to supply a constant load of 100 kW at 500 V. The cost of the cable including installation is £$(6a + 1{\cdot}3)$ per yd where a is the area of each conductor in in^2. Interest and depreciation total 10% and the cost of the energy wasted is 0·5d per kWh. Determine the most economical cable size and the corresponding voltage drop. [0·248 in^2 ; 35·8 V.]

30 The cost per mile of the copper conductors of section a in^2 for an overhead transmission line is £$(700a + 100)$. The load factor is 25% for the load current and 11% for the losses. The combined rate of interest and depreciation is 9·0% per annum, and the cost of energy wasted is 1d per kWh. Find the most economical current density for the line. [600 A per in^2.]

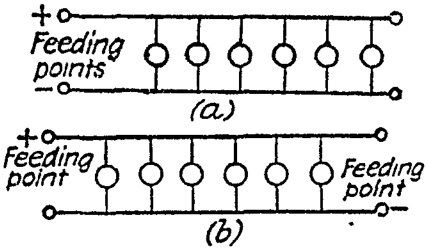
31 Determine the most economical cross-section for a 3-phase line, 4 miles long, to supply at a constant voltage of 30 kV, the following daily load cycle : 10 hr at 3 MW, 0·8 power factor ; 6 hr at 1·5 MW, 0·9 power factor ; 8 hr at 0·5 MW, 0·9 power factor. The line is in use 365 days yearly. The cost per mile of line, completely erected = £$(500 + 2\,600a)$, where a = cross-section in^2 in of each conductor. Interest = 8% of capital cost. Energy costs = 0·3d per kWh. The resistance per mile of conductor of cross-section a is $0{\cdot}043/a$ Ω. [0·18 in^2.]

32 A 6-mile, 3-phase line is required to transmit 150 kW at 0·9 power factor with an efficiency of 92%. Estimate the line voltage which will give a minimum capital cost and determine this cost if copper strung up costs £90 per ton and the supporting structures, placed 50 yd apart, cost (90 + 0·0915 V) shillings each. [2 560 V ; £4 678.]

Comparison of Systems

33 What is the percentage saving in feeder copper if the line voltage in a 2-wire, direct-current system be raised from 220 V to 500 V, for the same power transmitted ? State any assumptions made. [56%.]

34 The diagram shows two methods of feeding a 2-wire distributor having a uniformly-distributed lamp load of i amperes per yd. Compare the amounts of copper in the distributor conductors required in the two cases in order that there shall be the same maximum difference in voltage between the highest- and lowest-run lamps.



[(*a*) : (*b*) = 4 : 1.]

35 Compare the weights of copper required in a cable transmission scheme using (*a*) constant direct-current system, (*b*) 1-phase system, (*c*) 3-phase, 3-wire system. Assume the same transmitted power and maximum voltage between cable conductors and the same percentage loss in each case, also unity power factor and balanced load.

[(*a*) : (*b*) : (*c*) = 1 : 2 : 1·5.]

36 A 3-phase, 4-wire system is used for lighting. Compare the amount of copper required with that needed for a 2-wire, direct-current system with the same lamp voltage. Assume the same losses and balanced load. The neutral wire is one-half the cross-section of one of the respective outers. [0·292 : 1.]

37 Calculate the relative weights of copper required for the distributor cables in a low-voltage network with (*a*) a direct-current, 3-wire system, and (*b*) a 3-phase, 4-wire system. Assume the same voltage at consumers' terminals, the same percentage loss, unity power factor, and balanced load. Neglect loss in middle wires, which are of half the cross-section of corresponding outers. [(*a*) : (*b*) = 1 : 0·934.]

38 A 3-wire, direct-current network is converted to a 4-wire, 3-phase system by the addition of another conductor equal in section to one of the outers. For the same voltage at consumers' terminals and the same percentage power loss, find the percentage additional load that can be supplied. Assume balanced loads at unity power factor. [50%.]

39 A 1-phase load of 10 000 kW is transmitted by a pair of overhead conductors. If a third conductor of the same cross-section be added and a 3-phase supply be thus substituted for the original 1-phase, calculate the 3-phase load which can now be transmitted if the voltage between wires and the percentage loss in the line remain unchanged. [20 000 kW.]

40 A balanced and non-inductive load having a maximum value of 50 kW is to be delivered at a point 1 000 yards from the feeding point, at a voltage (received) of 250 V between any outer and the neutral. The maximum voltage drop in the cable is to be 5%. Calculate the cross-section of the outer conductors and the total weight of copper for (*a*) 3-wire, direct-current cable, (*b*) 4-wire, 3-phase cable. Take the neutral wire as having one-half the cross-section of one of the corresponding outers. Resistivity 0·7 $\mu\Omega$-in.

[(*a*) 0·202 in², 5 850 lb ; (*b*) 0·134 in², 5 440 lb.]

41 Find the direct voltage to be used for the overhead line of a railway in order that the same distance between substations shall be used as for a similar line equipped for single-phase working at 15 000 V, and requiring the same power as the direct-current system. Assume :—total resistance per mile, 0·2 Ω (a.c.) : 0·06 Ω (d.c.) ; allowable voltage drop, 20% (a.c.) : 30% (d.c.) ; power factor, 0·7. The load is assumed to be concentrated. [5 600 V.]

42 A given current is to be transmitted over a certain distance by means of bare conductors. On a basis of equal loss of power in the conductors, compare the weights, diameters and costs of solid aluminium and copper conductors. Take the price of aluminium and copper respectively as £118 and £60 per ton.

[Al/Cu :—weight, 0·5 ; diam., 1 29 ; cost, 0·98.]

Saving by Power-factor Correction

43 An industrial load has an average value of 500 kW at a power factor of 0·71 lagging for 2 000 hr per annum. The recorded maximum demand is 30% above the average load. The tariff is £5 per annum per kVA maximum demand plus 0·5d per unit. Find the annual saving in cost due to the installation of phase-advancing plant which costs £4 per kVA, and which raises the average power factor to 0·9 lagging. Allow 10% for interest and depreciation on the capital cost of the phase-advancer and neglect additional losses. [£682.]

44 A 3-wire feeder of resistance 0·009 Ω per line and negligible reactance supplies 400 kW at power factor 0·5 and 420 V to underloaded induction motors for 8 760 hr per annum. A 200-kVA bank of capacitors costing £1 100 is installed at the load. Allowing 12·5% for annual charges and 1d per kWh, in how many years will the phase-advancing plant pay for itself? [3.]

45 A consumer takes a steady load of 1 500 kW at a power factor of 0·71 lagging and pays £5 per annum per kVA of maximum demand. Phase-advancing plant costs £8 per kVAr. Determine the capacity of the phase-advancing plant required for minimum overall annual expenditure. Interest and depreciation total 10%. What will be the value of the new power factor of the supply? [1 257 kVAr ; 0·987 lagging.]

46 A fan is driven for 3 000 hr per annum by an induction motor developing 100 h.p. at an efficiency of 90% and a power factor of 0 8. To what value should the power factor be improved by phase-advancing plant, operating for the same time as the fan motor, costing £4 per kVA and having a loss of 120 W per kVA, in order that the total operating cost, including interest and depreciation of 10% per annum on the phase advancer shall be a minimum? The supply tariff is £4 per kVA plus 0·5d per kWh. [0·916.]

47 For a 100-h.p. drive a synchronous motor with an efficiency of 0·9 and power factor 0 95 lag costs £40 more than an induction motor with an efficiency of 0 91 and power factor 0·8. The motor has to work at full load for 3 000 hr per annum. Allowing 15% for investment charges and 1d per kWh, find the cost per kVAr per annum of power factor correction and the percentage reduction of transmission loss. [10s 2d ; 27·4%.]

48 For increasing the kW capacity of a power plant working at 0 7 lagging power factor, the necessary increase of power can be obtained by raising the power factor to 0 8 or by installing additional plant. What is the maximum cost per kVA of power factor correction apparatus to make its use more economical than additional plant at £10 per kVA? [£6 12s 2d.]

49 A load consisting of arc furnaces has a maximum demand of 800 kW, the corresponding power factor being of 0·65 lagging. The tariff in force is 0 75d per kWh in addition to a charge per kW of maximum demand of £5 plus 5s for every 0·1 by which the power factor is less than 0·95 lagging. Phase advancing plant costs £7 10s per kVAr and interest and depreciation charges total 12% per annum. What is the rating of the phase advancer to give a minimum total yearly cost? [584 kVAr.]

50 On a 240-V supply a consumer's load takes 1 987 kWh in a quarter (91 days). The maximum-demand indicator shows 10 A, which is charged at 5d per kWh for 2 hr daily. The remainder is 1d per kWh. Calculate the charge for the quarter and the average cost per kWh. [£15 7s 1d, 1·9d.]

Tariffs

51 A consumer takes a steady load of 200 kW at a power factor of 0 8 for 6 hr per day and 300 days per annum. Estimate his annual payment under each of the following tariffs :—(*a*) 1d per kWh plus £5 per annum per kVA ; (*b*) 1d per kWh plus £5 per annum per kW plus 0·25d per reactive kVAh.

[(*a*) £2 750 ; (*b*) £2 780.]

52 Calculate the annual cost of a yearly consumption of 4 682 000 kWh with a maximum demand of 1 240 kW on each of the following tariffs :—(*a*) a maximum demand charge of £5 per kW per annum plus 0·30d per kWh ; (*b*) a sliding scale graduated thus:—for the first 20 000 kWh, 1·5d per kWh ; for the next additional 70 000 kWh, 1·25d per kWh ; for the next additional 200 000 kWh, 1d per kWh ; for any subsequent number of kWh, 0·6d per kWh. [(*a*) £12 052 ; (*b*) £12 303.]

53 Obtain a two-part tariff for the consumers of a supply undertaking which generates 390×10^6 kWh per annum and has a maximum demand of 130 000 kW connected to it. The cost is distributed as follows ; Fuel £250 000, Generation £120 000, Transmission £250 000 and Distribution £170 000. Of these items 90%, 10%, 5% and 7% respectively are allocated to running costs, the remainder being a fixed charge. The total loss between the station and the consumers is 10% of the generated energy. If the load factor of the station is raised to 40% for the same maximum demand, find the percentage saving in the overall cost per kWh. [£4 1s 4d + 0·179d ; 9·7%.]

54 Two tariffs are offered : (*a*) £10 plus ½d per unit ; (*b*) a flat rate of 3d per unit. At what consumption is tariff (*a*) preferable ? [Above 960 units.]

55 Compare the costs of supply to a consumer taking 11 000 kWh per annum with a maximum demand of 7 kW, under the following tariffs :—(*a*) 12½% of the rateable value plus 0·5d per kWh ; (*b*) 10s per room plus 0·75d per kWh ; (*c*) £3 10s per kW maximum demand plus 0·25d per kWh. The house has eight rooms and a rateable value of £70 per annum. At what consumption will (*a*) and (*b*) be equal ?
[(*a*) £31 13s 4d ; (*b*) £38 7s 6d ; (*c*) £35 19s 2d ; 4 560 kWh per annum.]

56 A consumer has an annual consumption of 176 400 kWh. The charge is £7·5 per kW of maximum demand plus 0·75d per kWh. (*a*) Find the annual bill and the overall cost per kWh if the load factor is 36%. What is the overall cost per kWh (*b*) if the consumption were reduced 25% with the same load factor, (*c*) if the load factor were 27% with the same consumption as in (*a*) ?
[(*a*) £971 5s, 1·32d ; (*b*) 1·32d ; (*d*) 1·51d.]

57 A 10-room house with an assessed rental of £100 per annum has an annual consumption of 20 000 units. Find the total cost per annum and the overall cost per unit under each of the following tariffs : (*a*) fixed charge of 10% of assessed rental, day rate of 0·5d per unit on 12 500 units and night rate of 0·3d

per unit on 7 500 units. (*b*) Fixed charge of 15s per room and all units at 0·375d per unit. (*c*) Primary rate : 40 units for first room and 20 units for each additional room at 8d per unit. Secondary rate : (i) 9 times the number of units at primary rate at 0·6d per unit, (ii) all units thereafter at 0·25d per unit.
[(*a*) £45 8s 4d, 0·545d ; (*b*) £38 15s, 0·465d ; (*c*) £26 4s 10d, 0·315d.]

CHAPTER XXV

ARMATURE WINDINGS

D.C. Commutator Windings

1 In a two-layer winding, show the position in the slot and the number of the slot in which the following coil-sides lie :—

(a) Coil-side	53	when there are	4	coil-sides	per slot.
(b) ,,	74	,, ,,	6	,,	,,
(c) ,,	187	,, ,,	8	,,	,,
(d) ,,	196	,, ,,	10	,,	,,
(e) ,,	335	,, ,,	12	,,	,,

If there are u coil-sides per slot, show that the number of the last coil-side in slot n is un, and of the first coil-side in the nth slot is $un - u + 1$.

2 Draw a sketch showing the position in a two-layer winding of the first five coils in the slots when (a) $u = 6$ and $y_b = 31$; (b) $u = 8$ and $y_b = 45$. Find the condition that must be satisfied in order that the coils which have their top sides together in one slot have also their bottom sides together in another slot. [$(y_b - 1)/u$ = integer.]

3 A 6-pole, 2-circuit, wave-connected winding has 17 coils. Calculate the winding pitches and draw developed and sequence diagrams of the winding, showing the poles, the position and polarity of the brushes, and the direction of rotation and of the induced electromotive force. Make a table of the armature circuits with (a) 2 brushes ; (b) 6 brushes. [7, 5.]

4 Draw a developed diagram showing the lap-connected armature winding of a 4-pole generator with about 30 conductors. Show positions of poles and brushes, direction of rotation, induced electromotive force and current, and brush polarities. Make a table of the armature circuits.

5 Draw developed and sequence diagrams of wave-connected windings from the following data :—(a) 4 poles, 22 coil-sides, back-pitch = front-pitch = 5 ; (b) 4 poles, 24 coil-sides, back-pitch = front-pitch = 5. Insert positions of poles and brushes, and the direction of the induced electromotive force. Note, by the aid of tables of armature circuits, the difference between the two arrangements both when two and when four brushes are used. Indicate dummy coils, if any.

6 A 40-kW, 250-V generator has a 4-pole, lap-connected armature of 272 conductors. How would you change the connexions to the commutator to form two armature circuits and

what effect would this change have on the voltage, current and output of the machine?

[Lap : 250 V, 160 A ; Wave : 496 3 V, 80·6 A.]

7 A 4-pole winding with 1 470 conductors is to be wound on a standard armature with 37 slots and 147 commutator sectors. How will the winding be arranged?

[Wave connexions with 147 active coils and one dummy coil.]

8 A 4-pole, wave-connected armature has 2 940 conductors, 147 commutator sectors, and 37 slots. (*a*) How will the winding be arranged? (*b*) What will be the speed for an induced voltage of 500 V and a flux of 0·5 megaline (5 mWb)?

[(*a*) With 147 active coils and 1 dummy coil ; (*b*) 1 020 rev per min.]

9 A 4-pole armature has 103 commutator sectors and 35 slots. Arrange a 2-circuit, wave-connected winding for 500 V at 1 000 rev per min with a flux of 0·7 megaline (7 mWb) per pole.

[103 active coils with 10 turns per coil plus 2 dummy coils.]

10 Find possible windings for a 4-pole, 2-circuit armature. The number of conductors in series must not be greater than 400 nor less than 360 ; the number of slots is to be between 40 and 45 and the number of commutator sectors within 126 ± 4.

$$\begin{bmatrix} (a)\ C = 123,\ y_c = 61 \text{ or } 62,\ u = 6,\ S = 41. \\ (b)\ C = 129,\ y_c = 64 \text{ or } 65,\ u = 6,\ S = 43. \end{bmatrix}$$

11 State the conditions of symmetry in a commutator winding. An 8-pole, direct-current machine with a 4-circuit, wave-connected winding requires about 720 conductors. Using single-turn coils, find the nearest number of commutator sectors and a suitable number of slots that can be used to give a symmetrical winding with (*a*) 2 coil-sides per slot ; (*b*) 6 coil-sides per slot.

[(*a*) $C = S = 358$ or 362 ; (*b*) $C = 354, S = 118$ or $C = 366, S = 122$.]

12 Design a symmetrical 4-circuit, wave-connected winding for an 8-pole, 500-V generator with 6 conductors per slot. The mean voltage between commutator sectors must not exceed 12 V. [$C = 330$; $y_c = 82$ or 83 ; $y_b = 85$; $y_f = 81$ or 79.]

13 Select a 2-circuit, armature winding for a 4-pole, 1 000-rev per min, 500-V direct-current machine. The flux per pole is to be about 5 megalines (50mWb) and armature stampings with 41, 45, and 51 slots are available.

[6 coil-sides per slot ; 51 slots ; 51 commutator sectors ; 3 turns per coil.]

14 A commutator winding has 8 coils each of resistance 0·2 Ω. (*a*) What will be the resistance between two adjacent sectors? What type of fault is indicated if the resistance between adjacent

sectors is (*b*) less than in (*a*), (*c*) greater than in (*a*)? How would such faults be located?
[(*a*) 0·175 Ω; (*b*) short-circuit: lowest resistance across faulty coil; (*c*) open-circuit: resistance across faulty coil = 1·4 Ω.]

A.C. Double-layer Windings: Tapped

15 Draw out developed and sequence diagrams for the following windings:—(*a*) 4 poles, 22 coils, lap-connected; (*b*) 4 poles, 21 coils, wave-connected to form 2 circuits. Indicate poles, and brushes covering 1¼ commutator sectors. mark the coils short-circuited by the brushes. Add two equalizing rings in case (*a*). Show in tables of armature circuits the effect in each case of omitting a pair of brushes.

16 A 6-pole, lap-connected armature has 288 commutator sectors. Make a table showing to what sectors slip-rings must be connected for (*a*) 3-phase voltage; (*b*) 4-phase voltage; (*c*) 6-phase voltage.

17 Find what commutator sectors must be tapped to obtain a 4-phase voltage from a 2-circuit, 6-pole, wave-connected winding with (*a*) 100 coils, (*b*) 98 coils. The commutator pitch is 33 in each case. [(*a*) 1, 26, 51, 76—phases are symmetrical; (*b*) 1, 9, 50, 58—phases are unsymmetrical.]

18 An 8-pole, lap-connected winding with 352 commutator sectors is to have 8 equalizing rings. Draw up a table of connexions. [$y_{eq} = 88$; $y_{ph} = 11$.]

19 A lap-connected winding with single-turn coils for an 8-pole, direct-current generator, to give an induced voltage of 500 V at 350 rev per min, with a flux of about 5 6 megalines (56mWb) per pole, is to be tapped symmetrically for a 3-phase static balancer. For numbers of coil-sides per slot from 6 to 12, give a table of suitable numbers of coils and of slots, and the corresponding phase-pitches.

20 A 4-circuit, 8-pole generator running at 200 rev per min has 118 slots, 354 commutator sectors, 6 conductors per slot and a useful flux of 10 6 megalines (106mWb) per pole. Find (*a*) the induced voltage; (*b*) the tappings for a 3-phase voltage; (*c*) the commutator and winding pitches.
[(*a*) 500 V; (*b*) $y_{eq} = 177$, $y_{ph} = 59$; (*c*) $y_c = 88$ or 89; $y_b = 91$; $y_f = 85$ or 87.]

21 A 4-pole, 2-circuit, wave-connected winding has 177 commutator sectors, 59 slots and 1 062 conductors. Find (*a*) the number of coil-sides per slot; (*b*) the commutator pitch; (*c*) the coil-span and the winding pitches; (*d*) the number of turns per coil; (*e*) the sectors to be tapped for a 3-phase voltage.
[(*a*) 6; (*b*) 88 or 89; (*c*) coil-span = slot 1 to slot 16; $y_b = 91$, $y_f = 85$ or 87; (*d*) 3; (*e*) $y_{ph} = 59$.]

22 A 12-pole generator has 6 armature circuits, 177 slots and 531 commutator sectors. State (*a*) the commutator pitch ; (*b*) the winding pitches, using non-split groups of coils of approximately full pitch ; (*c*) whether the winding is symmetrical ; (*d*) the tappings for a 3-phase voltage.
[(*a*) 89 or 88 ; (*b*) 91, 87 or 85 ; (*c*) yes ; (*d*) $y_{eq} = 177$, $y_{ph} = 59$.]

23 A 6-pole armature winding with lap-connected coils has 72 slots and 6 coil-sides per slot. (*a*) How many commutator sectors are there ? (*b*) What are the winding pitches for full-pitch coils ? (*c*) Is the winding symmetrical ? (*d*) Tabulate the coils to be tapped for a 4-phase voltage. (*e*) How would 8 equalizer rings be connected ?
[(*a*) 216 ; (*b*) 73, 71 ; (*c*) yes ; (*d*) $y_{eq} = 72$, $y_{ph} = 18$; (*e*) $y_{hp} = 9$.]

24 A generator is to give 850 kW at 500 V when running at 200 rev per min. There are to be 8 poles, each with a flux of about 10 megalines (0·1Wb). Find a suitable armature winding given that the current per path should be between 150 and 300 A. Make the winding suitable for connexion to a 3-phase static balancer.
[$a = 2$, $Z = 756$; $C = 378$; $S = 126$; $y_{ph} = 63$.]

25 A 10-pole, lap-connected winding with approximately 550 conductors is required for an armature suitable for 84 to 96 slots. (*a*) Select a suitable number of slots and of conductors to give a winding capable of being symmetrically connected to 6 slip-rings. (*b*) Draw up a table of connexions to the slip-rings.
[(*a*) $S = 90$; (*b*) $Z = 540$.]

26 Find the useful flux per pole and make a skeleton table of the tappings for a 6-ring, 1 500-kW, 500-V, 50-c/s synchronous convertor running at 300 rev per min, and having a lap-connected winding with an average of 14 V between commutator sectors. [6·95 megalines (0·0695 Wb).]

27 A common wave winding has 10 poles, 144 coils and 4 coil-sides per slot. Find (*a*) tappings for 3 rings ; (*b*) tappings for 6 rings , (*c*) openings for 3-phase winding with 60° phase-spread. For (*c*) show diagrammatically the two parts of each phase in (i) series connexion, (ii) parallel connexion.
[(*a*) coils 1, 49, 97 ; (*b*) and (*c*) 1, 25, 49, 73, 97, 121.]

28 A 4-circuit wave winding has 8 poles, 342 coils and 6 coil-sides per slot. Find (*a*) tappings for 3 rings ; (*b*) openings for symmetrical 3-phase winding with 60° phase-spread. For (*b*) show diagrammatically the two circuits of each phase in (i) series connexion, (ii) parallel connexion.
[(*a*) tap coils : 1 and 172, 58 and 229, 115 and 286 ; (*b*) open coils : 1, 72, 58, 129, 115, 186 and 172, 243, 229, 300, 286, 15.]

29 How many identical circuits per phase is it possible to have in a 12-pole machine when (*a*) $N_{ph} = 3$; (*b*) $N_{ph} = 4$; (*c*) $N_{ph} = 6$? [(*a*) 3, 6 ; (*b*) 2,6 ; (*c*) 6.]

A.C. Double-layer Windings : Open

30 If *b* denote the number of a commutator sector, show that the numbers of the corresponding top and bottom coil-sides are $(2b - 1)$ and $(2b - 1 - y_f)$ respectively. Show how this relation can be used to find the phase connexions in a closed or opened commutator winding.

31 A 3-phase induction motor has 96 stator slots with 4 conductors per slot and 120 rotor slots with 2 conductors per slot. Find the number of turns per phase in stator and rotor when both are star-connected. Find the voltage across the rotor slip-rings when the rotor is open-circuited and at rest (ignoring leakage), the stator being connected to a 400-V supply. [64, 40 ; 250 V.]

32 A 50-c/s induction motor with 8 poles is operated on a 500-V supply and has a 3-phase, star-connected stator winding. Find the number of turns in series per phase (with a 60° phase-spread) for a flux of approximately 2·15 megalines (21·5 mWb). If the rotor has a total of 105 turns similarly arranged and connected in star, find the rotor slip-ring voltage on open-circuit at standstill. Ignore leakage. [64 turns ; 273 V.]

33 The stator of an induction motor has 48 coil-sides. Show how to connect these to obtain (*a*) a 4-pole, 2-phase winding ; (*b*) a 12-pole, 2-phase winding ; (*c*) a 4-pole, 3 phase, mesh-connected winding ; (*d*) an 8-pole, 3-phase, star-connected winding.

34 A 120-h.p., 500-V, 50-c/s, 3-phase induction motor, with a synchronous speed of 750 rev per min, has a star-connected stator winding accommodated in 63 slots with 6 conductors per slot. If the slip-ring voltage on open-circuit is to be about 400 V, find a suitable rotor winding, stating : (*a*) the number of slots ; (*b*) the number of conductors per slot ; (*c*) the coil-span ; (*d*) the slip-ring voltage on open-circuit ; and (*e*) the approximate full-load rotor current per phase.
*[(*a*) 72 ; (*b*) 4 ; (*c*) 9 slot-pitches ; (*d*) 381 V ; (*e*) 142 A.]

35 A wave winding for the rotor of an 8-pole induction motor has 45 coils and 2 coil-sides per slot. Work out the openings for connexion to 3 slip-rings. An internal star connexion is required, and the phase-spread should be 60°

[Joints to be opened : .1, 33, 31, 18, 16, 3.
Phases : coil-sides : 1, 61, 31.
Star-point : coil-sides : 65, 35, 5.
Connectors : coil-sides : 80–24, 54–20, 50–84.]

36 A simple wave-connected armature is to be opened at 6 places for a 3-phase winding. Given 8 poles, 57 slots and 6 coil-sides per slot, find (*a*) the coil-sides to which connexions are made; (*b*) the positions of these coil-sides in the slots.

[I_s coil-side 1 = *a* in slot 1;
II_s „ „ 115 = *a* „ „ 20;
III_s „ „ 229 = *a* „ „ 39;
I_f „ „ 215 = *e* „ „ 36;
II_f „ „ 329 = *e* „ „ 55;
III_f „ „ 101 = *e* „ „ 17.
Connect *f* in slot 50 to *d* in slot 29.
„ *f* „ „ 12 „ *d* „ „ 48.
„ *f* „ „ 31 „ *d* „ „ 10.

Coil-sides in slot:
a = top, left
e = top, right
d = bottom, middle
f = bottom, right]

A.C. Single-layer Windings

37 Draw out 4 pole-pitches of a 3-phase, concentric-coil, single-layer winding with 2-plane overhang, accommodated in 2 slots per pole per phase. Connect the phases in (*a*) star, and (*b*) mesh. Show that the mean coil-span is equal to a pole-pitch; also that when *p*, the number of pole-pairs, is odd, a cranked phase overhang is necessary.

38 Make a diagram of 2 pole-pitches of a 3-phase, single-layer winding with 3-plane overhang. Show that no cranked coils are necessary, and that the equivalent coil-span is equal to a pole-pitch.

39 Make a sketch of a 6-pole, single-layer winding with 6 slots per pole-pitch, showing coil connexions for a 3-phase winding with (*a*) one circuit per phase; (*b*) two circuits per phase. Connect the phases in star.

Winding Factors: Coil-span

40 Deduce an expression for the "coil-span factor" of an alternator armature winding short-chorded by an angle ϵ. A 3-phase alternator is to be free from third harmonics in its phase voltage. (*a*) By how much must the coils be short-chorded, and what other harmonics will this chording eliminate? (*b*) What phase-spread would produce the desired effect? (*c*) To what extent will the output of the winding be affected in (*a*) and (*b*), assuming the same specific electric loading? Assume a uniformly distributed double-layer winding.

[$\cos\frac{\epsilon}{2}$; (*a*) 60°; 9th, 15th, 21st . . .; (*b*) 120°;
(*c*) 13·3% reduction in each case.]

41 Draw out a 6-phase, double-layer winding with the coil-span equal to two-thirds of the pole-pitch. When the phases are

connected to form a 3-phase winding, show that no third harmonic of electromotive force is possible in the phase voltage. What relation will its output have to that of an ordinary winding for 3-phase working? [0·866.]

Winding Factors: Distribution

42 Prove that the "distribution factor" for a uniformly-distributed winding with full-pitch coils is $k_{m1} = \dfrac{\sin \frac{1}{2}\sigma}{\frac{1}{2}\sigma}$ for the fundamental electromotive force, and $k_{mn} = \dfrac{\sin n\frac{1}{2}\sigma}{n\frac{1}{2}\sigma}$ for the electromotive force harmonic of the n^{th} order. Calculate k_{m1} and k_{m3} for phase-spreads $\sigma = 60°$, 90°, 120° and 180°.

[σ =	60°	90°	120°	180°
k_{m1} =	0·955	0·9	0·827	0·636
k_{m3} =	0·636	0·3	0	−0·212]

43 A flux-distribution curve contains harmonics of the order $2g \pm 1$, where g denotes the number of slot-pitches per pole-pitch. Show that these harmonics will equal the same percentage of the fundamental in the voltage wave as in the flux wave.

44 A winding with g slots per pole-pitch has m slots per pole per phase. Prove that the "distribution factor" is

$$k_{m1} = \frac{\sin \frac{m}{g}\frac{\pi}{2}}{m \sin \frac{1}{g}\frac{\pi}{2}} \text{ and } k_{mn} = \frac{\sin n \frac{m}{g}\frac{\pi}{2}}{m \sin \frac{n}{g}\frac{\pi}{2}}$$

for the fundamental electromotive force and for the electromotive force harmonic of the n^{th} order respectively. Calculate k_{mn} up to $n = 19$ for $g = 9$ and $m = 3$.

[n =	1	3	5	7	9
k_{mn} =	0 96	0·667	0·217	−0·177	−0·333

n =	11	13	15	17	19
k_{mn} =	−0·177	0·217	0·667	0·96	0·96]

45 With g slots per pole-pitch, prove that the distribution factor for harmonics of the orders $2g \pm 1$ is the same as for the fundamental.

46 Define "distribution factor" and find its value for a 3-phase winding with (*a*) 120° phase-spread, and (*b*) 60° phase-spread, when the winding is (i) uniformly distributed and (ii) occupies 6 slots per pole. Tabulate the values for both fundamental and third harmonic.

[(*a*) (i) 0 827, 0; (ii) 0·836, 0; (*b*) (i) 0 955, 0·636; (ii) 0·966, 0·707.]

47 Find analytically the distribution factor of a double-layer winding for the fundamental and third-harmonic electromotive forces for 1-phase, 3-phase and 6-phase connexions, (*a*) when the winding is uniformly distributed, and (*b*) when there are 6 slots per pole, all wound.

		1-ph.	3-ph.	6-ph.
(*a*)	$k_{m1} =$	0·636	0·827	0·955
	$k_{m3} =$	− 0·212	0	0·636
(*b*)	$k_{m1} =$	0·644	0·836	0·966
	$k_{m3} =$	− 0·236	0	0·707

48 An alternator has a uniformly-distributed winding with full-pitch coils.
Find (*a*) the distribution factor for the fundamental electromotive force when two-thirds of the slots are wound;
(*b*) the distribution factor for the third harmonic e.m.f. when two-thirds of the slots are wound;
(*c*) the same as (*a*) and (*b*) when all slots are wound;
(*d*) *ratio of outputs with two-thirds and all slots wound*;
(*e*) ratio of 1-phase output with two-thirds of slots wound to 3-phase output;
(*f*) ratio of 1-phase output with all slots wound to 3-phase output.

[(*a*) 0·827; (*b*) 0; (*c*) 0·636, − 0·212; (*d*) 1 : 1·15; (*e*) 1 : 1·73; (*f*) 1 : 1·5.]

Magnetomotive Force

49 Plot to scale the armature magnetomotive force in the air-gap of a machine having a uniformly-distributed 1-phase winding occupying (*a*) the whole periphery; (*b*) two-thirds of each pole-pitch. Assume sinusoidal current distribution and full-pitch coils.

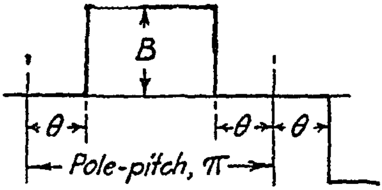
50 Plot to scale the armature magnetomotive force in the air-gap of a machine having a uniformly-distributed 2-phase winding (each phase occupying one-half a pole-pitch), (*a*) when the current in phase 1 is a maximum; (*b*) $\pi/4$ later than (*a*); (*c*) $\pi/2$ later than (*a*). Assume sinusoidal current variation and full-pitch coils.

51 Plot to scale the ampere-conductor distribution over the armature of a machine having a uniformly-distributed 3-phase winding with a phase-spread of 60°, (*a*) when the current in phase I is a maximum; (*b*) $\pi/6$ later than (*a*). Assume sinusoidal current variation and full-pitch coils.

52 Deduce the shapes of the magnetomotive force distribution of a uniformly-distributed 3-phase winding with 60° phase-spread for the instants (*a*) $i_1 = \max$; (*b*) $i_2 = 0$. Prove that the amplitudes of the fundamentals of these two curves are equal.

53 A 3-phase stator has 4 slots per pole per phase and there are 4 conductors in series per slot. Plot to scale the distribution of magnetomotive force in the air-gap (*a*) when the current in phase I is a maximum ; (*b*) $\pi/12$ later than (*a*) ; (*c*) $\pi/6$ later than (*a*). The maximum current per phase is 10 A, and the variation with time is sinusoidal.

54 The diagram shows the field form of a salient-pole alternator. (*a*) Calculate the amplitude of the fundamental of the flux-wave, and (*b*) evaluate this when θ has the value $\pi/6$.



[(*a*) $(4B\cos\theta)/\pi$; (*b*) $2\sqrt{3}B/\pi$.]

55 A 1 500-rev per min 50-c/s, star-connected, salient-pole, 3-phase alternator has a single-layer stator winding with 300 turns per phase and a spread of 60°. If there are 800 exciting turns on each pole, calculate the change in field current necessary to maintain the same terminal voltage when the output of the machine is varied from zero to 50 A at zero power factor, assuming the stator winding (*a*) uniformly distributed, (*b*) accommodated in 3 slots per pole per phase, (*c*) accommodated in 2 slots per pole per phase. Take the field m.m.f. to be a rectangle extending over polar arc, and the ratio pole-arc : pole-pitch = 2 : 3.

[(*a*) 10·0 ; (*b*) 10·05 ; (*c*) 10·1 A.]

CHAPTER XXVI

EXCITING COILS

1 A field coil has to produce 6 000 ampere-turns at a mean coil temperature of 60° C. The voltage across the coil is 50 V and the mean length of turn is 50 cm. Determine what diameter of wire must be used. [1·24 mm.]

2 Show that the cross-section a of the conductor necessary to produce AT ampere-turns per pole when the voltage V is applied to each coil of a field winding is

$$a = \frac{\rho L_{mt} AT}{V} \text{ mm}^2$$

where L_{mt} is the mean length of turn in centimetres and ρ is the resistivity of the material. Find the diameter of round wire for an 8-pole machine to give 7 500 ampere-turns per pole with a bus-bar voltage of 500 V, assuming 30% of the latter to be absorbed in the field regulator. The mean length of turn is 110 cm, and the winding has a mean temperature of 75° C. [2·24 mm.]

3 A coil of 4 000 turns has an outside radius of 2 in and an inside radius of 0 75 in. How many turns must be removed from the outside to reduce the resistance by 20% ? [576.]

4 The copper coil of a 0–150 V moving-iron voltmeter has a resistance equal to one-tenth of the total instrument resistance. Mean length of turn = 14·5 cm and 240 ampere-turns are needed to produce full-scale deflexion. Calculate the cross-sectional area of the wire of the coil. [0 04 mm².]

5 A coil of 300 m of copper wire has a resistance of 10 Ω at 15° C. Find the average temperature rise of the coil when its resistance is 11·5 Ω and the ambient temperature is 20° C. Calculate the diameter of the wire. [32° C ; 0·8 mm.]

6 Prove that the maximum AT that can be produced by an exciting winding with given overall dimensions is independent of the exciting voltage. Assume fixed values of temperature rise, cooling coefficient and space factor.

7 Show that the I^2R loss per kg of copper in a winding is proportional to the square of the current density. Find the I^2R loss per kg of copper in a coil worked at a current density of 1·5 A per mm² and at a mean temperature of 60° C. [5·05 W per kg.]

8 Design a cylindrical field coil from the following particulars :— Conductor, No. 13 S.W.G., diameter 2·34 mm ; potential difference across coil, 47 V ; outside diameter 42 cm ; length of bobbin, 26 cm ; length of mean turn, 120 cm ; permissible loss per unit surface, 0·1 W per cm^2. Find the number of layers and the number of turns per layer. Allow 6% for slack in each layer. Thickness of wire insulation is 0·18 mm ; $\rho = 0{\cdot}02\ \Omega$ per m and mm^2.

*[13 layers × 90 turns per layer = 1 170 turns.]

9 Design the field coils for a 250-V, 2-pole shunt motor to give about 2 500 ampere-turns per pole with coils in series. A maximum of 150 W per coil can be dissipated without exceeding the specified mean temperature of 65° C. State (*a*) the most suitable S.W.G. No. (see Prob. 14) ; (*b*) the coil resistance (hot) ; (*c*) the actual ampere-turns per pole ; (*d*) the weight of wire used (mean length of turn = 100 cm).

[(*a*) No. 22 ; (*b*) 104 Ω ; (*c*) 2 420 ; (*d*) 14·3 kg.]

10 Each pole of a direct-current generator is required to produce 19 000 ampere-turns. The gap flux is 20 megalines (0·2 Wb) and the density in the pole core (of circular cross-section) is 15 000 lines per cm^2 (1·5 Wb/m^2). The leakage coefficient is 1·2. The field coil has a radial depth of 15 cm and can dissipate 0·05 W per cm^2 of outside cylindrical surface without overheating. Find (*a*) the necessary diameter of round copper wire at 65° C ; (*b*) the number of turns ; (*c*) the length of the coil ; (*d*) the field current. The voltage across a coil is 60 V and the space factor is about 0·7.

*[(*a*) 3·93 mm ; (*b*) 2 900 ; (*c*) 33 cm ; (*d*) 6·55 A.]

11 An exciting coil of external surface A cm^2 can dissipate w watts per cm^2. The potential difference across the coil is V volts, the section of the conductor is a mm^2 and the length of the mean turn is L_{mt} cm. The resistivity is ρ in ohms per m and mm^2. Find the number of turns T in the coil in terms of A, w, V, a, ρ and L_{mt}. Find T when $A = 87$ cm circumference by 16 cm deep ; $w = 0{\cdot}25$; $V = 3$; $a = 44$ and $L_{mt} = 78$. Assuming a strip-on-edge winding with 0·2 mm insulation between turns and 4% for slack, find the dimensions of the conductor. Take the resistivity of copper at working temperature as 0·02 Ω per m and mm^2. [$T = 73$; $a = 1{\cdot}9$ mm × 23 mm.]

12 A coil is to be wound on a cylindrical bobbin, with a diameter of 8 cm and a length of 8 cm. The radial depth of winding space is 1·5 cm. The power dissipated must not exceed 100 W at a terminal voltage of 100 V. Assuming the insulated copper wire to have a space factor of 0·7, find the maximum number of ampere-turns that can be accommodated. Assume the copper temperature to be 60° C. *[5 455.]

13 A relay requires 5 000 AT for its operation. Each of two bobbins is 6 cm long and has a radial depth of 2 cm with a mean length of turn of 12 cm. The space factor is 0·7 and the operating voltage is 100 V. Find the power required to operate the relay. Mean copper temperature, 60° C. Specify a suitable size of wire and the number of turns required on each bobbin.
[35·7 W ; 0·12 mm^2, 7 000.]

14 A magnet requires an excitation of about 10 000 ampere-turns. The mean length of turn is 15 in, the terminal voltage is 110 V, and the mean temperature 60° C. If a current density of about 3 A per mm^2 can be allowed, find the nearest size of wire (S.W.G.) and the number of turns. Suitable gauges are :—

S.W.G. No.	18	19	20	21	22	23
Diam., mm	1·22	1·02	0·914	0·813	0·711	0·610

[No. 20 ; (0·657 mm^2 ; 0·914 mm) ; 4 880.]

15 A direct-current lifting magnet is designed to dissipate 6 kW at 110 V. The net winding space (after allowing for coil wrapping, etc.) is 22·5 cm × 18·5 cm and the mean length of turn is 195 cm. Find the number of turns, the cross-section of conductor and the total excitation. Sketch the arrangement of the coil. Assume a working temperature of 100° C and a space factor of 0·6.
[1 065 turns ; 23 4 mm^2 ; 58 000 ampere-turns.]

16 A magnet coil has a resistance of 0·02 Ω per turn. Show that the total ampere-turns will be 2 500 for an applied voltage of 50 V.

17 A lifting magnet is wound with 10 000 turns of cross-section a and total length l. By stripping and re-winding with wire of section 2 a, the turns become 5 500 and the length 0·48 l. Find how the ampere-turns and lifting power compare with the original winding, when the magnet is connected to the same supply. Ignore saturation. [2·29 ; 5·25.

18 A rectangular field coil is to produce 7 500 ampere-turns when dissipating 220 W at a temperature of 60° C. The inner dimensions of the coil are 10 cm × 24 cm × 15 cm high. The heat dissipation is 0 003 W per cm^2 per 1° C from the outer surface, neglecting the top and bottom of the coil. Temperature of ambient air is 20° C. Calculate the thickness of the coil, the space factor and the current density required.
[6·8 cm ; 0·475 ; 1·55 A/mm^2.]

19 A small test coil of 0 15-mm diameter copper wire is embedded in an exciting coil. If the test coil has a resistance of 20 Ω at 20° C, what is the hot-spot temperature when its resistance is 24 Ω ? Find the length of wire in the test coil. [71° C ; 20 m.]

20 A field winding has a resistance of 10 Ω at 20° C. Assuming this ambient temperature and a constant heat dissipation per 1° C temperature rise, find (*a*) the steady temperature rise when a voltage of 120 V produces a current of 10 A; (*b*) the steady temperature rise at 140 V; (*c*) the voltage to produce a steady rise of 40° C. Take the resistance-temperature coefficient as 0 004 $\Omega/\Omega/°$ C at 20° C. [(*a*) 50° C; (*b*) 65° C; (*c*) 105·6 V.]

CHAPTER XXVII

D.C. MACHINES

Output Coefficient

1 Prove that the kW rating of a direct-current generator is given by $\pi^2 \bar{B}\, ac\, D^2\, L\, n\, 10^{-11}$, where $\bar{B}$, ac, D, L and n are respectively the mean flux-density in gauss over the pole pitch, the ampere-conductors per cm of periphery, the diameter and length in cm, and the speed in rev per sec of the armature. Calculate the diameter and length of an armature suitable for a 4-pole, 20-kW, 1 500-rev per min, direct-current generator. Assume $ac = 180$, $\bar{B} = 3\,800$, and make the length of the armature equal to the pole-pitch. *[$D = 25$ cm ; $L = 19$ cm.]

2 Prove that the output of a direct-current machine with single-turn coils is given by $\dfrac{3\, a\, \bar{E}\, v\, ac}{pN}$ kW, where a, $\bar{E}$, v, ac, p and N denote pairs of armature circuits, average voltage between commutator sectors, peripheral speed of armature in m per sec, ampere-conductors per cm of periphery, pole-pairs and rev per min respectively.

Taking as limiting values $\bar{E} = 20$, $v = 50$, and $ac = 500$, find the largest output for a direct-current machine running at (*a*) 300 rev per min, (*b*) 1 000 rev per min.

[(*a*) 5 000 kW ; (*b*) 1 500 kW.]

3 If constant values are assumed for the magnetic and electric loadings of a machine, show that the weight of active material is approximately proportional to the full-load torque.

4 Find the minimum number of poles for a 1 200 kW generator if the average voltage between commutator sectors is not to exceed 15 V and the armature ampere-turns per pole are not to exceed 10 000. [8.]

Magnetic Circuit

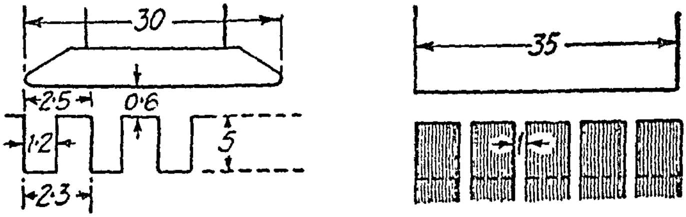
5 A laminated tooth of armature steel in an electrical machine is 3 cm long and has a taper such that the maximum width is 1·4 times the minimum. Estimate the ampere-turns required for a mean flux density of 19 000 lines per cm² (1·9 Wb/m²) in this tooth. Use Simpson's rule. *[835.]

6 Develop an expression connecting the apparent and true flux densities in highly saturated armature teeth.

The apparent density in the net iron section of an armature tooth is 23 000 lines per cm² (2·3 Wb/m²) at a section of slot-pitch 2·5 cm and slot-width 1·4 cm. The armature is 33 cm long and

includes 3 ducts of 1 cm width each. Estimate the true flux density and the ampere-turns per cm for the tooth at the particular section. Use the magnetization curve for armature plates.
*[604 AT/cm ; 21 650 gauss (2·165 Wb/m^2).]

7 Find the ampere-turns per pole required to drive the flux through (*a*) the teeth and (*b*) the gap of the machine shown



(i) when $\Phi = 7$ megalines (70 mWb), (ii) when $\Phi = 8$ megalines (80 mWb). Use Simpson's rule, the curves for armature steel stampings, and Carter's coefficient, Fig. 5. Dimensions are in cm.
*[(*a*) (i) 405, (ii) 1 400 ; (*b*) (i) 3 800, (ii) 4 340.]

8 Find the ampere-turns per pole of field excitation required for the single gap of a 2-pole generator with the following particulars :—200 V on no-load at 2 000 rev per min ; armature length, 32 cm ; effective gap length, 0·8 cm ; pole-arc, 30 cm ; 200 armature conductors. [2 000.]

9 Calculate the ampere-turns required for one air-gap of a machine with an axial length of 20 cm (no ducts) and a pole-arc of 18 cm. The slot-pitch is 27 mm, slot-opening 12 mm, air-gap 6 mm, and the useful flux per pole 2·5 megalines (25 mWb). Take Carter's coefficient from Fig. 5. *[3 800.]

10 The voltage induced in a 4-pole, separately excited generator, having a lap-connected armature winding with 750 conductors, and running at 1 000 rev per min varies as shown in Fig. 4, where 100% represents 250 V and an exciting current of 1 A. Plot a curve showing the flux per pole crossing the air-gap to a base of field ampere-turns per pole. Each field coil has 1 200 turns.

11 A shunt generator with lap-connected armature and a normal speed of 450 rev per min has the following data relating to its magnetic circuit :—

Parts	Area, cm^2	Length, cm
Yoke (cast steel)	284	66
Pole (cast steel)	506	35·5
Gap (effective)	658	0·8
Teeth (mean)	418	4·1
Armature core	684	38

Leakage coefficient 1·17. Plot the ampere-turns per pole for gap fluxes of 3, 5 and 7 megalines (30, 50 and 70 mWb). Show scales for induced e.m.f. and field current, taking 1 800 turns per pole and a total of 696 armature conductors. Determine the field current for an e.m.f. of 350 V. *[4·6 A.]

12 A 1-kW, 110-V, 2-pole shunt generator has a magnetic circuit having the following data :—gap length, 0·3 cm ; gap density, 4 400 lines per cm^2 (0·44 Wb/m^2) ; armature path length, 13 cm ; core density, 10 000 lines per cm^2 (1 Wb/m^2) ; tooth density, 20 000 lines per cm^2 (2 Wb/m^2) ; tooth length, 1·5 cm ; pole and yoke length, 80 cm ; pole and yoke net area, 200 cm^2 ; total flux in armature, 1 megaline (10 mWb) ; leakage coefficient, 1·4 ; power for field excitation, 12% of output ; mean length of turn in coil, 24 in. The field system is of armature steel plates. Find the number of ampere-turns per pole required, the S.W.G. No. of the wire, and the necessary number of turns per coil at a mean copper temperature of 60° C. Suitable wire gauges are given in Prob. 14, Ch. XXVI.

*[1 580 ampere-turns ; No. 22 ; 1 400 turns.]

Induced E.M.F.

13 The armature of a 2-pole, 200-V generator has 400 conductors and runs at 300 rev per min. Calculate the useful flux per pole. If the number of turns in each field coil is 1 200, what is the average value of the electromotive force induced in each coil on breaking the field if the flux dies away completely in 0·1 sec ?

[10 megalines (0·1 Wb) ; 1 200 V.]

14 (*a*) A 6-pole, 2-circuit, wave-connected armature has 250 conductors and runs at 1 200 rev per min. The electromotive force generated on open-circuit is 600 V. Find the useful flux per pole.

(*b*) An 8-pole, lap-connected armature has 960 conductors, a flux of 4 megalines (40 mWb) per pole, and a speed of 400 r.p.m. Calculate the e.m.f. generated on open-circuit.

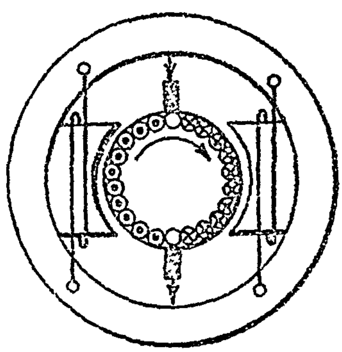
(*c*) If the armature in (*b*) were wave-connected for 2 circuits, at what speed must it be driven to generate 400 V ?

(*d*) A 4-pole generator has a flux of 4 megalines (40 mWb) per pole and a lap-connected armature with 740 conductors. Find the e.m.f. generated on open-circuit at 1 000 r.p.m.

[(*a*) 4 megalines (0·04 Wb) ; (*b*) 256 V ; (*c*) 156 r.p.m. ; (*d*) 494 V.]

15 A shunt machine, connected to 250-V mains, has an armature resistance (including brushes) of 0·12 Ω, and the resistance of the field circuit is 100 Ω. Find the ratio of the speed as a generator to the speed as a motor, the line current in each case being 80 A. [1·08.]

16 The sketch shows a bipolar direct-current machine; the direction of rotation, and the direction of the armature currents when the main current has a given direction, are indicated. Show by means of sketches how to connect this machine as (*a*) a shunt generator; (*b*) a series generator; (*c*) a shunt motor; (*d*) a series motor. Assuming clockwise rotation in each case, indicate the polarity of the terminals and main poles.



Insert commutating poles in series with the armature (*e*) for a shunt generator, (*f*) for a shunt motor.

Add series turns to the main poles for operation (*g*) as a compound generator, and as a motor compounded (*h*) cumulatively; (*j*) differentially.

17 A 1 500-kW, 550-V, 16-pole generator runs at 150 rev per min. What must be the useful flux per pole if there are 2 500 conductors lap-connected and the full-load copper losses are 25 kW? Calculate the area of the pole shoe if the gap density has a uniform value of 9 000 lines per cm^2 (0·9 Wb/m^2) and find the no-load terminal voltage, neglecting armature reaction and change in speed.

[(*a*) 8·95 megalines (0 0895 Wb); (*b*) 994 cm^2; (*c*) 559·2 V.]

18 A 480-V, 4-pole, shunt motor runs at 600 rev per min and has 46 slots with 6 conductors per slot, wave-connected to the commutator. How must the armature and field windings be re-connected to enable the motor to run at its original speed on a 240-V supply? If the field coils be replaced by others of the same size and space-factor, but wound with larger wire and connected in series, show that the diameter of the new wire must be $\sqrt{2}$-times that of the wire in the original coils.

[Lap winding; two parallel groups of two field coils in series.]

19 A 500-V, 750-rev per min generator has 83 slots with 12 conductors per slot. The field winding has 6 500 turns of 22 S.W.G. per pole. Show how to adapt the machine for 220 V at the same speed.

[Halve number of armature conductors and join two halves of field winding in parallel.]

Armature Reaction

20 A 4-pole generator supplies a current of 143 A. It has 492 armature conductors (*a*) wave-connected, (*b*) lap-connected. When delivering full load, the brushes are given an actual lead

of 10°. Calculate the demagnetizing armature ampere turns per pole. The field winding is shunt connected and takes 10 A; find the number of extra shunt field turns necessary to neutralize this demagnetization.

[(*a*) 1 040; 104 turns; (*b*) 520; 52 turns.]

21 Determine per pole the number (*a*) of cross ampere-turns, (*b*) of back ampere-turns, and (*c*) of series turns to balance the back ampere-turns in the case of a direct-current generator having the following data:—500 conductors; total current, 200 A; 6-pole, 2-circuit wave winding: angle of lead, 10°; leakage coefficient, 1·8. [(*a*) 2 778; (*b*) 1 390; (*c*) 9.]

22 Determine the effect of armature reaction by drawing a curve showing the flux-density over the polar arc of a generator, details of which are as follows: the ampere-turns of the field coil are 4 500; the armature reaction is equivalent to 8 000 ampere-turns per pole; gap length, 0·6 cm (uniform); ratio (pole arc/pole pitch) is 0·7. Neglect slotting, fringing, and the iron parts of the magnetic circuit.

Interpoles, Commutation

23 Estimate the number of turns needed on each commutating pole of a 6-pole generator delivering 200 kW at 200 V, given:—number of lap-connected armature conductors = 540, interpole air-gap = 1·0 cm, flux density in interpole air-gap = 8 000 lines per cm^2 (0·8 Wb/m^2). Ignore the effect of iron parts of the circuit and of leakage. [10.]

24 Calculate the ampere-turns for each commutating pole of an 8-pole generator with 107 slots, each containing 1 000 ampere-conductors. The interpole air-gap is 1·2 cm. The flux-density in the gap is to be 8 200 lines per cm^2 (0·82 Wb/m^2). Neglect the iron parts of the circuit, and leakage. [9 750.]

25 A compensated generator has 12 000 armature ampere-turns per pole, the ratio (pole-arc/pole-pitch) = 0·7, the length of interpole air-gap = 1·25 cm and the flux-density in the interpole air-gap = 8 000 gauss (0·8 Wb/m^2). Find the ampere-turns per pole for the compensating winding and for the interpole winding.

[8 400; 6 600.]

26 A 350-kW, 500-V generator has 8 poles, an armature diameter of 180 cm and a core length of 35 cm. A 4-circuit, wave-connected winding is accommodated in 114 slots with 6 coil-sides per slot. Taking commutating poles with an axial length of 20 cm and a gap of 1 cm, and assuming a specific permeance = 6, find the necessary excitation of, and number of turns for, each interpole. [9 980: 14.]

27 A commutator having a diameter of 30 in rotates at 600 rev per min. Find the approximate time of cummutation if the brush width be 1·5 cm. [$0{\cdot}625 \times 10^{-3}$ sec.]

28 A single-turn armature coil has, in the commutating zone, an inductance of 0·02 mH. Find what commutating field is required for the straight-line commutation of 25 A.
[50 000 lines (0·05 mWb).]

Field Rheostats and Diverters

29 A 220-V generator is to have a maximum exciting current of 5 A and a minimum exciting current of 2·5 A. The regulator is to have 10 steps with equal current changes per step. The following high-resistance wires are available: 5·4-A wire of 0·6 Ω per ft, 3·6-A wire of 1·0 Ω per ft, and 2·8-A wire of 1·5 Ω per ft. Find the minimum quantity of material to be used.
[5·4-A wire, 24·5 ft ; 3·6-A wire, 14·6 ft ; 2·8-A wire, 9·8 ft.]

30 A 54-Ω field rheostat can carry 5 A when all resistance is out and 2·5 A when all resistance is in. Find the minimum resistance of field winding that can be used with the rheostat without exceeding these limits. If the rheostat has 20 steps designed for equal current changes when used with the field-winding calculated above, determine the resistance between the first pair of contacts and between the last pair of contacts. Supply voltage, 250 V. [50 Ω ; 5 Ω ; 1·3 Ω.]

31 A 100-V shunt motor with a normal speed of 500 rev per min has an exciting current of 10 A when running light. Using the O.C.C. in Fig. 4, find the resistance steps in the field rheostat to raise the speed in 5 equal stages to 1 000 rev per min.
[4·3, 3·9, 4·0, 2·8, 2·8 Ω.]*

32 Calculate the value of the resistance R_D of a diverter in terms of the field resistance R_F to reduce the exciting current to n% of its full value. Compare the total I^2R loss in the two cases.
[$R_D = R_F n/(100 - n)$; $100/n$.]

33 The field current of a 4-pole series motor is to be halved, (*a*) by series-parallel connexion of the field coils, (*b*) by a diverter. Compare the total I^2R losses in the two arrangements with the original loss. [1 : 2 : 4.]

34 A 6-pole generator has 1 000 shunt turns and 12 series turns per pole. For a terminal voltage of 220 V on no load and 240 V when supplying 300 A as a shunt machine, the exciting current has to be increased from 2 to 5 A. Find the resistance of the diverter when the generator supplies this load as a compound machine. The series winding has a resistance of 0·005 Ω/pole.
[0·108 Ω.]

Armature Resistance

35 Develop a general expression for the resistance of a direct-current armature winding with Z conductors each having a length L (including overhang) and cross-section s arranged in a pairs of circuits and p pole-pairs, the resistivity of the winding material being ρ. Hence write down the expressions for simple lap and simple wave windings.
[$\rho ZL/4sa^2$; $\rho ZL/4sp^2$; $\rho ZL/4s$.]

36 Calculate the resistance of a 6-pole, lap-connected armature winding from the following data: Number of slots, 150; conductors per slot, 8; mean length of one turn, 250 cm; cross-section of each conductor, 10 mm × 2 5 mm; working temperature, 80° C. [0·0856 Ω.]

Performance

37 A 12-pole generator has a wave-connected armature with 6 parallel paths, and delivers 500 kW at 500 V. There are 1 062 conductors, each 90 cm in length (including overhang) and 15 mm × 2·5 mm. in cross-section. Calculate (*a*) the armature resistance at 20° C; (*b*) the ohmic drop in the armature winding at 20° C; (*c*) the loss in the armature winding at full load and a temperature of 75° C.
[(*a*) 0·0122 Ω; (*b*) 12·2 V; (*c*) 14·9 kW.]

38 Calculate the armature I^2R and the brush friction losses in a 6-pole, 125-V, 100-kW, 750-rev per min, lap-wound, direct-current shunt generator having the following data. Field current, 10 A. Armature winding: single-turn coils; mean length of turn, 80 cm; conductor cross-section, 0·25 cm²; temperature, 75° C. Commutator: diameter, 45 cm; number of sectors, 162; total volt drop at brushes, 1 7 V; brush current-density, 10 A per cm²; coefficient of friction, 0·2; brush pressure, 0·15 kg per cm². Take the resistivity of copper as 1/58 Ω per m and mm² at 20° C. [4·2 kW.]

39 Show that the rate of heat development per unit area of cylindrical surface of an armature due to copper loss is given by $ac\,\alpha\,\rho$ W, where ac = specific electric loading, α = current density in, and ρ = resistivity of, the armature conductors. Find the minimum permissible conductor area for a 1 000-kW, 500-V, 8-pole d.c. generator if the loss is not to exceed 0 3 watt per cm² of armature surface. The armature diameter is 210 cm and there are 760 lap-connected conductors. Assume ρ as 0·02 Ω per m per mm². [48 mm².]

40 Calculate the hysteretic torque and the horse-power thereby lost in a 6-pole dynamo running at 1 000 rev per min if the loss per cycle be 5 500 ergs per cm³. The volume of the armature

can be obtained from the following data :—Length, 30 cm ; depth of core, 9 cm ; mean diameter of core, 35 cm.

[5·74 lb-ft ; 1·09 h.p.]

41 The hysteresis and eddy-current losses in a motor running at 1 000 rev per min are 250 W and 100 W respectively. If the flux remains constant, at what speed will the total iron losses be halved ? [570 rev per min.]

42 Two 10-h.p. motors have each a full-load efficiency of 0·84. The running-light losses of motor *A* are 480 W, and of motor *B* 520 W. Find the half-load efficiency of each motor.

[*A*, 0·846 ; *B*, 0·833.]

43 A constant-speed motor works on following duty cycle: 50 h.p. for 10 sec, 100 h.p. for 10 sec, 150 h.p. for 6 sec, 120 h.p. for 20 sec, idling for 14 sec. What rating of motor should be installed ? [100 h.p.]*

44 One of two similar 250-V shunt machines *A* and *B*, running light, takes 6 A. When *A* is mechanically coupled to *B*, the input to *A* is 7 A when *B* is unexcited, and 9 A when it is separately excited to generate 250 V. Calculate the friction and windage loss and core loss of each machine. [250 W ; 500 W.]

45 A shunt motor takes a current *I* at speed *n*. Find the current at speed 3*n* (*a*) with field control, (*b*) with voltage control, when (i) the output is constant, (ii) the torque is constant. Ignore all losses. [(*a*) (i) *I* ; (ii) 3*I*. (*b*) (i) *I*/3 ; (ii) *I*.]

46 A 500-V compound generator has a constant (excitation + rotational) loss of 18·0 kW. Its resistances are : armature winding, 0·01 Ω ; series winding, 0 002 Ω ; shunt winding, 33 Ω. Find the maximum efficiency and the load at which it occurs.

[605 kW ; 0·945.]

Generators : Separately-excited

47 The terminal voltage/armature current curve of a separately-excited generator is given by :—

Current . .	0	40	80	120 A
Terminal voltage .	220	218	214	208 V

and the armature resistance is 0·05 Ω. Plot this curve and derive therefrom the internal characteristic. Neglect brush contact drop.

48 The open-circuit characteristic of a separately-excited generator driven at 600 rev per min is as given in Fig. 4, where 100% represents 517 V and a field current of 8 A. Find the voltage

to which the machine will excite as a shunt generator on op-circuit with a field resistance of 60 Ω (*a*) at 600 rev per mi (*b*) at 500 rev per min. (*c*) At what speed will the machine ju fail to excite?

[(*a*) 545 V; (*b*) 390 V; (*c*) about 350 rev per mi

49 Estimate the reduction in speed of a dynamo working wi constant excitation on 500-V bus-bars to decrease its load fro 500 to 250 kW. The resistance between terminals is 0·015 Ω neglect armature reaction. [1·45%

50 A separately-excited generator, when running at 1 200 r per min, supplies 200 A at 125 V to a circuit of constant resis ance. What will be the current when the speed is dropped 1 000 rev per min if the field current is unaltered? Armatu resistance: 0 04 Ω; total drop at brushes: 2 V; ignore chan in armature reaction. [166 A

Generators: Shunt

51 A shunt generator has an induced voltage on open-circuit 127 V. When the machine is on load the terminal voltage 120 V. Find the load current if the field-circuit resistance 15 Ω and the armature resistance 0 02 Ω. Ignore armatu reaction. [342 A

52 A 4-pole shunt generator with lap-connected armature havi field and armature resistances of 50 Ω and 0·1 Ω respective supplies sixty 100-V, 40-W lamps. Calculate the total armatu current, the current per armature path, and the generat electromotive force. Allow a contact drop of 1 V per brush.

[26 A; 6 5 A; 104·6 V

53 The open-circuit characteristic of a shunt generator runni at 850 rev per min is as given in Fig. 4, where 100% represen an induced electromotive force of 100 V and field current of 4 Find the open-circuit voltage of the machine with a field resis ance of (*a*) 22 Ω; (*b*) 30 Ω. (*c*) Find the field resistance R_f f normal voltage at normal speed. (*d*) Find the critical spe with R_f in circuit. (*e*) Find the critical resistance at norm speed.

[(*a*) 110 V; (*b*) 81 V; (*c*) 25 Ω; (*d*) 575 rev per min; (*e*) 37 Ω

54 A 200-V shunt generator is to maintain constant termin voltage at all loads. At full load the speed falls 10% and t voltage drop in the armature winding is 10 V. The no-lo exciting current is 4 A and the O.C.C. is shown in Fig. 4. Fi the change in field resistance from no load to full load.

[50 to 34·5 Ω

55 The open-circuit curve of a direct-current generator run normal speed is as shown in Fig. 4, where 100% represents

induced electromotive force of 400 V and a field current of 5 A. If the field circuit is adjusted to (*a*) 80 Ω, (*b*) 70 Ω resistance, and the armature resistance is 0·2 Ω, plot the two complete external characteristics. Take armature reaction into account by an addition to the armature resistance R_a, making the effective resistance $(R_a + 2 \times 10^{-4} I)$, where I is the load current. Find the load current when the terminal voltage is 350 V. *[(*a*) 112 A ; (*b*) 205 A.]

56 A 100-kW, belt-driven shunt generator running at 300 rev per min on 220-V bus-bars continues to run as a motor when the belt breaks, then taking 10 kW. What will be its speed? Armature resistance = 0·025 Ω; field resistance = 60 Ω; contact drop under each brush = 1 V. Ignore armature reaction. [279 rev per min.]

Generators : Series

57 A series generator, having an external characteristic which is a straight line through zero to 50 V at 200 A, is connected as a booster between a station bus-bar and a feeder of 0·3 Ω resistance. Calculate the voltage between the far end of the feeder and the bus-bar at a current of (*a*) 160 A, (*b*) 50 A. [(*a*) 8 V ; (*b*) 2·5 V.]

Generators : Compound

58 A long-shunt, compound generator delivers a load current of 50 A at 500 V, and has armature, series-field and shunt-field resistances of 0·05 Ω, 0·03 Ω and 250 Ω respectively. Calculate the generated electromotive force and the armature current. Allow 1·0 V per brush for contact drop. [506·2 V ; 52 A.]

59 In a 110-V, compound generator the resistances of the armature, shunt and series windings are 0·06 Ω, 25 Ω and 0·04 Ω respectively. The load consists of 200 lamps each rated at 55 W, 110 V. Find the total electromotive force and armature current when the machine is connected (*a*) long-shunt ; (*b*) short-shunt. (*c*) How will the ampere-turns of the series winding be changed if in (*a*) a diverter of resistance 0·1 Ω be connected in parallel with the series field winding? Ignore armature reaction and brush contact drop.
[(*a*) 120·4 V, 104·4 A ; (*b*) 120·3 V, 104·6 A ; (*c*) reduced to 71·5%.]

60 Estimate the number of series turns per pole for a 500-kW, compound generator required to develop 500 V at no-load and 550 V at full-load, the requisite ampere-turns per pole being 7 900 and 11 200 respectively. The shunt winding is designed to give 500 V at no-load when its temperature is 20° C. Its final temperature on load is 60° C. [4.]

61 A 230-V compound generator runs at 1 200 rev per min on no load. The combined resistance of armature and series windings is 0·09 Ω and the field resistance is 104 Ω. The shunt winding has 2 000 turns per pole. When the load is 65 A the speed falls to 1 150 rev per min. (*a*) Find the terminal voltage if the series winding has 6 turns per pole. (*b*) Find how many series turns per pole are needed to maintain 230 V at the stated load if the combined resistance of armature winding, series winding and line is 0·2 Ω. Ignore armature reaction. Use the O.C.C. in Fig. 4. [(*a*) 229 V ; (*b*) 9.]

62 A 200-kW, 500-rev per min, 6-pole, lap-wound, long shunt, compound generator has a magnetization curve given by Fig. 4, where 100% represents 5·5 megalines (55 mWb) per pole and 6 500 ampere turns respectively. The armature, series and shunt field resistances are 0·035 Ω, 0·015 Ω, and 100 Ω respectively. There are 1 260 armature conductors and 1 000 turns on each shunt field coil. If the field ampere-turns to compensate armature reaction amount to 10% of the armature ampere-turns, determine the number of series turns per pole required, so that full load is delivered at 500 V. There is a volt drop of 2 V at the brushes. Estimate also the no-load voltage. *[4 ; 500 V.]

63 A compound generator is level-compounded at 300 rev per min. It is rated at 500 kW, 500 V, and has 2 series turns on each pole. The excitation in ampere-turns per pole is 8 000 for no-load and 10 000 for full load, the magnetization curve being that shown in Fig. 4, where 100% = 10 000 ampere-turns. Determine approximately the external characteristic at 250 rev per min for the same current range. Armature resistance 0·02 Ω.

64 A long-shunt compound generator gave the following external characteristic :—

Main current, A .	0	4	8	12	16	20
Terminal voltage, V	206	206·5	207	207	206	203

The field resistances are 0·25 Ω and 90 Ω, and the armature resistance 0·45 Ω. Deduce the internal characteristic, neglecting brush-contact drop.

65 A 110-kW, short-shunt compound generator has a no-load voltage of 500 V and is overcompounded 10%, producing a linear external characteristic. If the armature resistance be 0·09 Ω, series field resistance 0·04 Ω, and shunt resistance 100 Ω, plot the internal characteristic.

Motors : Shunt

66 A 4-pole, 500-V, shunt motor has 720 wave-connected conductors on its armature. The full-load armature current is 60 A, and the flux per pole 3 megalines (0·03 Wb). The armature resistance is 0·2 Ω, and the contact drop is 1 V per brush. Calculate the full-load speed of the motor. [675 rev per min.]

67 A 250-V shunt motor on no-load runs at 1 000 rev per min and takes 5 A. The total armature and shunt field resistances are respectively 0·2 Ω and 250 Ω. Calculate the speed when loaded and taking a current of 50 A, if armature reaction weakens the field by 3%. [994 rev per min.]

68 A direct-current shunt machine generates 250 V on open-circuit at 1 000 rev per min. Armature resistance, including brushes, 0·5 Ω; field resistance, 250 Ω; input to machine running as a motor on no-load, 4 A at 250 V. Calculate the speed and efficiency of the machine as a motor taking 40 A at 250 V. Armature reaction weakens the field by 4%.
[960 rev per min; 82·5%.]

69 Find the useful flux per pole on no-load of a 250-V, 6-pole, shunt motor having a 2-circuit, wave connected, armature winding with 110 turns. At normal working temperature the overall armature resistance including brushes is 0·2 Ω. The armature current is 13·3 A at the no-load speed of 908 rev per min. [2·48 megalines (24·8 mWb).]

70 A 4-pole, shunt motor has 472 wave-connected armature conductors. The flux per pole for a field current of 2 A is 3·6 megalines (36 mWb), and for 4 A is 4·2 megalines (42 mWb). The field resistance is 125 Ω. Calculate the no-load speed of the motor with a terminal voltage of (*a*) 500 V; (*b*) 250 V. The no-load armature resistance drop is negligible.
[(*a*) 756 rev per min; (*b*) 441 rev per min.]

71 The following data refer to a direct-current shunt machine :— total armature resistance, 0·3 Ω; field resistance, 40 Ω; the open-circuit characteristic at 1 000 rev per min is such that the field current is 5 A for an electromotive force of 220 V and 4 A for 200 V. Neglecting armature reaction, find the resistance in circuit in the shunt regulator to obtain a speed of 1 000 rev per min when running as a motor on a 220-V supply at the full-load armature current of 35 A. [9·2 Ω.]

72 A 240-V, unsaturated shunt motor has an armature resistance (including brushes and interpoles) of 0·04 Ω and a field resistance of 100 Ω. (*a*) Find what resistance must be added to the field circuit to increase the speed from 1 200 to 1 500 rev per min when the supply current is 200 A. (*b*) With the field resistance as in (*a*), find the speed when the supply current is 100 A. If the

machine is run as a generator to give 200 A at 240 V, find (*c*) the field current at 1 200 r.p.m., (*d*) the speed when the field current is 2 A. [(*a*) 25 Ω ; (*b*) 1 525 rev per min ; (*c*) 2 56 A ; (*d*) 1 540 rev per min.]

73 Find the no-load and full-load speeds, and the speed regulation, expressed as a percentage of the no-load speed, for the 4-pole, 220-V, 24-h p. shunt motor having the following data :— Field current, 5 A ; armature resistance, 0·04 Ω ; flux, 4 megalines (40 mWb) ; 160 armature conductors ; 2-circuit wave connexion ; full-load current, 95 A ; no-load current, 9 A. Neglect armature reaction. [1 030, 1 014 rev per min ; 1·55%.]

74 A shunt generator delivers 50 kW at 250 V and 400 rev per min. The armature and field resistances are 0·02 Ω and 50 Ω respectively. Calculate the speed of the machine running as a shunt motor and taking 50 kW input at 250 V. Allow 1 V per brush for contact drop. [382 rev per min.]

75 A 250-V, 4-pole, shunt motor has an armature with 33 slots and 30 conductors per slot, with 2-circuit wave connexions. In the magnetization curve Fig. 4, 100% = 1 megaline (10 mWb) gap flux and 1·6 A field current. The field resistance is 150 Ω, and a 100-Ω variable rheostat is connected in series with it. Plot no-load speed to a base of field-rheostat resistance between 0 and 100 Ω. Find the speed when 50 Ω of the rheostat are in circuit, Neglect ohmic drop in the armature. [860 rev per min.]

76 A 250-V shunt motor has an armature resistance of 0 5 Ω and a field resistance of 250 Ω. When driving at 600 rev per min a load, the torque of which is constant, the armature takes 20 A. If it be desired to raise the speed from 600 to 800 rev per min, what resistance must be inserted in the shunt field circuit, assuming the magnetization curve to be a straight line ? [88 Ω.]

77 A 220-V shunt motor with an armature resistance of 0·5 Ω is excited to give constant main field. At full-load the motor runs at 500 rev per min and takes an armature current of 30 A. If a resistance of 1 Ω is placed in the armature circuit, find the speed at (*a*) full-load torque, (*b*) double full-load torque ; (*c*) find also the stalling torque. [(*a*) 427 rev per min ; (*b*) 317 rev per min ; (*c*) 5 × full-load torque.]

78 A 500-V, 10-h.p., shunt motor has a full-load efficiency of 85%. With the same field and armature currents it is desired to reduce the speed by 30% by insertion of resistance in the armature circuit. Assuming that all losses except copper losses vary directly with the speed, calculate the value of the inserted resistance and the efficiency of the motor when running at the reduced speed. The resistances of the field and armature are 400 Ω and 0·25 Ω respectively. [9·12 Ω ; 59·6%.]

79 The armature of a 4-pole shunt motor has a lap winding accommodated in 60 slots, each containing 20 conductors. If the useful flux per pole be 23 mWb, calculate the total torque developed (*a*) in pounds-feet, (*b*) in newton-metres, when the armature current is 50 A. [(*a*) 162 lb.-ft. ; (*b*) 218 Nw-m.]

80 A direct-current shunt motor develops 10 b.h.p. at 600 rev per min when drawing a line current of 18 A at 500 V. Find the efficiency at this load, and the useful torque in pounds-feet. [83% ; 87·5 lb.-ft.]

81 The armature of a 6-pole, 6-circuit shunt motor takes 400 A at a speed of 350 rev per min. The flux per pole is 8 megalines (80 mWb), the number of armature turns is 600, and 3% of the torque is lost in windage, friction and iron loss. Calculate the brake horse-power. [291 h.p.]

82 Calculate the mean force in newtons on each conductor of a 440-V, 14-pole shunt motor with lap-connected armature winding, when running at 100 rev per min and taking a current of 1 550 A. Each conductor has an active length of 33 cm and the armature diameter is 130 cm. The total gap flux per pole is 70mWb, and the mean flux density in the slots is 0·05 Wb/m^2. What would be the mean force on each conductor if the armature were of the smooth-core, unslotted type ? [1·82, 26·5 Nw.]

83 A 2-pole shunt motor has an armature diameter of 8 in and a core length of 10 in : the pole-arc is 120°. Find the total torque developed in kilogram-metres, and the total mechanical horse-power when the armature carries 40 A. The speed is 1 200 rev per min, the gap density is 3 500 lines per cm^2 (0 35 Wb/m^2), and the armature has 220 conductors. Neglect fringing. [2·69 kg-m ; 4·46 h.p.]

84 A 100-h.p., 500-V shunt motor has 4 poles and a 2-circuit wave-connected armature winding with 492 conductors. The flux is 5 megalines (50 mWb) per pole and the full-load efficiency 92%. The armature and commutating-pole windings have a total resistance of 0·1 Ω. The shunt field resistance is 250 Ω. Calculate for full load (*a*) the speed ; (*b*) the useful torque in pounds-feet. [(*a*) 590 rev per min ; (*b*) 892 lb.-ft.]

85 A 240-V, 4-pole shunt motor running at 1 000 rev per min gives 15 h.p. with an armature current of 50 A and a field current of 1·0 A. The armature winding is wave-connected and has 540 conductors. Its resistance is 0 1 Ω and the drop at each brush is 1 V. Find (*a*) useful torque ; (*b*) total torque ; (*c*) useful flux per pole ; (*d*) rotational losses ; (*e*) efficiency.
[(*a*) 78·8 lb-ft ; (*b*) 82 lb-ft ; (*c*) 1·3 megalines (13 m Wb) ; (*d*) 450 W ; (*e*) 0·915.]

86 A 500-V, 6-pole, shunt motor has a wave-connected armature winding with 1 200 conductors. The useful flux per pole is 2 megalines, and the armature and field resistances are 0·5 Ω and 250 Ω respectively. Ignoring the effect of armature reaction, find the speed and the total developed torque in pounds-feet when a current of 20 A is taken from the mains. If the iron, friction and windage losses aggregate 900 W, find the useful torque, brake horse-power, and efficiency at this speed.
[410 rev per min ; 152 lb-ft ; 136 lb-ft ; 10·6 h.p. ; 79·4%.]

87 A 200-V shunt motor develops 23 h.p. when taking 20·2 kW. The field resistance is 50 Ω and armature resistance 0 06 Ω. What is the efficiency and the power input when the output is 10 h.p. ? [74·6% ; 10·0 kW.]

88 A 250-V, shunt motor runs at 910 rev per min when cold. After 80 min the speed is 1 020 rev per min. Determine the average temperature rise of the field coils. Assume that the flux varies directly as the exciting current. Ambient temperature, 20° C. [81° C.]

89 A 440-V motor, when cold, with a field resistance of 100 Ω, and an armature resistance of 0·04 Ω, runs at 1 200 rev per min unloaded. Calculate its speed after reaching a steady temperature rise of 85° C when taking 90 A. State any assumptions made. [1 360 rev per min.]

90 A 250-V shunt motor has an armature current of 20 A when running at 1 000 rev per min against full-load torque. The armature resistance is 0·5 Ω. What resistance must be inserted in series with the armature to reduce the speed to 500 rev per min at the same torque, and what will be the speed if the load torque is halved with this resistance in circuit? Assume the flux to remain constant throughout. [6 Ω ; 771 rev per min.]

91 A 250-V shunt motor giving 20 h.p. at 1 000 rev per min, takes an armature current of 75 A. The armature resistance is 0·25 Ω and the load torque remains constant. If the flux is reduced by 20% of its normal value before the speed changes, find the instantaneous value of the armature current and the torque. Determine the final value of armature current and speed. [260 A, 292 lb-ft ; 93·7 A, 1 225 rev per min.]

92 A 220-V shunt motor has an armature resistance of 0·5 Ω and takes a current of 40 A on full load. By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant ? [37·5%.]

93 A 10-h.p., 460-V shunt motor has an input of 8 500 W when developing a shaft torque of 58 lb-ft at 900 rev per min. By what percentage must the field be reduced to raise the speed to

1 050 rev per min with a shaft torque of 45 lb-ft? The armature resistance is 1 Ω, field-circuit resistance at 900 rev per min is 770 Ω, and the core and mechanical losses are constant. Ignore armature reaction. [14%.]

94 A 250-V shunt motor with a constant main field drives a load, the torque of which varies as the cube of the speed. When running at 500 rev per min it takes 40 A. Find the speed at which it will run if a 25-Ω resistor is connected in series with the armature. Neglect motor losses. [250 rev per min.]

95 A 500-V, shunt motor, taking an armature current of 240 A while running at 800 rev per min, is braked by disconnecting the armature from the supply and closing it on a resistance of 2·02 Ω, the field excitation remaining constant. Calculate (*a*) the initial braking current, (*b*) the torque at 600 rev per min as a percentage of the torque at 800 rev per min. The armature resistance is 0·05 Ω. [(*a*) 236 A; (*b*) 74%.]

96 A fan has the following speed/load characteristic :—

Speed, rev per min.	700	800	900	1 000	1 100
Input, h.p.	2·9	3·9	5·2	6·7	8·6

It is coupled to a 4-pole, 230-V shunt motor having a 2-circuit armature winding with 666 conductors and a flux per pole of 1·0 megamaxwell (0·01 Wb). The armature resistance is 0 27 Ω and the no-load loss is 600 W. Find the operating speed of the set and the efficiency of the motor.

[1 005 rev per min; 0·87.]

97 The speed of a 500-V shunt motor, when running light, is 1 000 rev per min. Its armature resistance, including brushes, is 2 Ω. To reduce the speed the armature voltage V is tapped off a 50-Ω resistor across the supply mains. With V tapped off 25 Ω, at what speed will the motor run (*a*) when the supply current is 15 A, (*b*) when the armature current is 15 A? What value of resistance must be tapped to give a speed of 500 rev per min, (*c*) when the armature current is 15 A, (*d*) when the supply current is 15 A? Ignore field current and assume constant flux.

[(*a*) 212 rev per min; (*b*) 65·5 rev per min; (*c*) 39·9 Ω; (*d*) 34·2 Ω.]

Motors: Series

98 A series motor of resistance 1 Ω between terminals runs at 800 rev per min at 200 V with a current of 15 A. Find the speed at which it will run when connected in series with a 5-Ω resistance and taking the same current at the same supply voltage. [476 rev per min.]

99 A 240-V series motor takes 40 A when giving its rated output at 1 500 rev per min. Its resistance is 0·3 Ω. Find what resistance must be added to obtain rated torque (*a*) at starting, (*b*) at 1 000 rev per min. [(*a*) 5·7 Ω ; (*b*) 1·9 Ω.]

100 A 440-V motor has a regulating resistance R in series. The motor resistance is 0·3 Ω. When $R = 0$ and the current I is 20 A, the motor runs at 1 200 rev per min. Find the speed when $R = 3$ Ω and $I = 15$ A ; also the ratio of the total mechanical output in the two cases. Given that the flux with $I = 15$ A is 80% of that with $I = 20$ A. [1 350 rev per min, 1·48.]

101 A 240-V series lift motor has a resistance of 0·2 Ω. At a speed of 1 800 rev per min it takes 40 A. Find the resistance to be added (*a*) to limit the speed to 3 600 rev per min when the current is 10 A, assuming flux proportional to current between 10 and 40 A ; (*b*) to make the speed 900 rev per min when the current is 60 A, given that the flux at 60 A is 18% greater than that at 40A. At what speed will the motor run when connected directly to the mains and taking 60 A ?
[(*a*) 12·2 Ω ; (*b*) 1·52 Ω ; 1 500 rev per min.]

102 A 4-pole, series-wound fan motor runs normally at 600 rev per min on a 250-V supply, taking 20 A. The field coils are connected all in series. Estimate the speed, and the current taken by the motor, if the coils are re-connected in two parallel groups of two in series. The load torque increases as the square of the speed. Assume that the flux is directly proportional to the current and ignore losses. [714 rev per min ; 33·5 A.]

103 A 250-V series motor runs at 1 000 rev per min, gives 5 h.p., and takes a current of 20 A, when the two sections of its field winding are connected in series. What current will the motor take for the same output when the two sections are connected in parallel, and what are the corresponding torque and speed ? The resistance of the armature is 1·1 Ω, and that of each section of the field winding is 0·4 Ω. The magnetization curve is given in Fig. 4. *[18·9 A ; 15·6 lb-ft, 1 700 rev per min.]

104 The magnetization curve of a 500-V, 35-h.p., 4-pole railway motor with 44 turns per pole, 744 armature conductors with 2-circuit wave connexions, armature resistance 0·32 Ω and field resistance 0 18 Ω, is as given in Fig. 4, where 100% represents 3 megalines (30 mWb) and 3 000 ampere turns per pole. Find the speeds for currents of 30 A and 60 A from a 500-V supply (*a*) connected in series with a 5-Ω resistor ; (*b*) connected direct; and (*c*) connected direct, with a diverter of resistance 0·36 Ω shunted across the field winding.
*[(*a*) 750, 240 ; (*b*) 1 080, 670 ; (*c*) 1 560, 860 rev per min.]

105 A 500-V series motor runs at 400 rev per min. The efficiency

is 90% and the shaft torque is 144 lb-ft. What is the current taken? [18·2 A.]

106 In a series motor, when giving 50 h.p. at 600 rev per min, the I^2R losses are 8% of input; the other losses, 3% of input. Find the output and speed when the current is halved. Use magnetization curve in Fig. 4. [26 h.p., 980 rev per min.]

107 A 110-V series motor when running at 1 500 rev per min and taking 35 A develops a gross torque of 15 lb-ft. The resistance of the motor is 0·3 Ω. At what speed will the motor run (*a*) when the developed torque is 10 lb-ft, assuming the flux to be proportional to the current; (*b*) when the current is doubled, assuming the flux to be increased by 30%?

[(*a*) 1 870; (*b*) 1 030 rev per min.]

108 Determine (*a*) the total torque developed; (*b*) the useful torque of a 250-V, 4-pole series motor with 782 conductors wave-connected, developing 11 b.h.p. and taking 40 A, with a flux per pole of 2·5 megalines (25 mWb). The resistance of the motor is 0 75 Ω.

[(*a*) 184 lb-ft; (*b*) 171 lb-ft.]

109 A 250-V, 4-pole series motor has a lap-connected armature winding accommodated in 43 slots with 28 conductors per slot. The gap-flux per pole is 2 megalines (20 mWb) when the current is 50 A. The armature and field resistances are 0·5 Ω and 0 2 Ω respectively. The iron and friction losses aggregate 750 W. Calculate the pull in pounds at the rim of the 14-in diameter motor pulley. [225 lb.]

110 A 4-pole, series motor has 944 wave-connected armature conductors. At a certain load the flux per pole is 34 6 mWb and the total mechanical torque developed is 209 Nw-m. Calculate the line current taken by the motor and the speed at which it will run with an applied voltage of 500 V. Total motor resistance is 3 Ω. [20 A; 403 rev per min.]

111 A 250-V, 4-pole series motor has a 2-circuit, wave-connected winding with 105 slots, each containing 12 conductors. The gap-flux per pole is 0 02 Wb when the motor is on full load and taking a current of 45 A. The armature and field resistances are 0 2 Ω and 0 1 Ω respectively, and the iron, friction and windage losses total 700 W. Calculate the available shaft torque in British and in M K.S. units, the speed, the brake horse-power and the efficiency on full load.

[248 lb-ft, 336 Nw-m; 282 rev per min; 13·3 h p.; 88%.]

112 A series motor, with an unsaturated magnetic circuit and with negligible resistance, when running at a certain speed on a given load, takes 50 A at 500 V. If the load torque varies as

the cube of the speed, find the resistance necessary to reduce the speed by (*a*) 50% ; (*b*) 20%. [(*a*) 23·3 Ω ; (*b*) 6 Ω.]

113 A series motor having a resistance of 1 Ω between terminals drives a fan for which the torque varies as the square of the speed. At 220 V the set runs at 300 rev per min and takes 25 A. The speed is to be raised to 400 rev per min by increasing the voltage. Find the voltage and current for the limiting cases when the field is (*a*) saturated (i.e. flux constant) ; (*b*) unsaturated (i.e. flux directly proportional to current).
[(*a*) 44·3 A. 304·5 V ; (*b*) 33·3 A, 378·3 V.]

114 A 500-V series motor with a resistance of 0·5 Ω ran at 160 rev per min with a current $I = 40$ A, and at 140 rev per min with $I = 50$ A. Calculate (*a*) total resistance R at starting for $I = 50$ A, (*b*) speed with R in circuit when $I = 40$ A, (*c*) total resistance with speed as in (*b*) and $I = 50$ A.
[(*a*) 10 Ω ; (*b*) 33·3 rev per min ; (*c*) 7·74 Ω.]

115 A 4-pole, 250-V series motor has the magnetization curve of Fig. 4, where 100% represents 2 megalines (0·02 Wb) per pole and 50 A. The armature has 1 100 conductors, lap-connected, and the resistance between terminals is 0·26 Ω. Plot curves of total developed mechanical torque (in lb-ft), speed, and total horse-power, to a base of current.

116 A series motor gave the following data from a separately-excited open-circuit test at 700 rev per min :—

Field current, A .	5	10	15	20	25	30	40
Voltage across armature brushes, V .	73	134	171	193	209	220	238

The armature resistance is 0·3 Ω and the field resistance 0·2 Ω. Plot the speed/current and the speed/total torque (rev per min/lb-ft) curve for a terminal voltage of 250 V.

117 The magnetization curve of a series motor is given in Fig. 4. Deduce from it the speed/current curve and the speed/torque curve (*a*) ignoring armature resistance, (*b*) assuming that the voltage drop is 10% at full load and proportional to the current.

118 A series motor on test is found to give full-load speed and torque at current values higher than the rated full-load current. Assuming this to be due to armature reaction, determine the percentage resistance-drop required at rated current to make rated torque and speed occur at the same current value. Assume armature reaction to be proportional to the load current and to weaken the flux by 4% at full load. The magnetization curve is given in Fig. 4. *[2·9%.]

119 Two identical series motors A and B are connected in seri across the supply mains. If the output of A is $K_A n_A{}^P$ and B is $K_B n_B{}^Q$, where n_A and n_B are the speeds of machine A an B respectively, and K_A and K_B are constants, find an equatio connecting n_A, n_B, P, Q, K_A, and K_B.

[$n_A{}^{(P-1)}/n_B{}^{(Q-1)}) = K_B/K_A$

120 Two identical series motors are connected in series across th supply mains. Show that the speeds of the two machines a in the same ratio as their loads.

121 Two identical 20 h.p., 1 000 rev per min, 250-V, seri motors A and B are connected in series across a 250-V suppl They drive the same shaft through reduction gearing 5 : 1 an 4 : 1 respectively. If the total load torque on the shaft is 60 lb-ft, calculate the speed of the shaft, the voltage across eac motor, and the current from the supply main. Neglect losse and assume the magnetic circuits to be unsaturated.

[140 rev per min : 111 V, 139 V ; 47·5 A

122 A crane operated by a 250-V series motor works under th following full- and light-load conditions : full load, 3 tons lifte 100 ft per min at an overall efficiency of 80% : light load (spee raised by a diverter resistor connected in parallel with mot field), 1 ton lifted 250 ft per min at an overall efficiency of 75% Resistances of armature and motor field, 0·4 Ω and 0·2 respectively. Calculate the resistance of the diverter. Negle magnetic saturation. [0·182 Ω

123 The 100-h.p. series motor of a mine haulage system co trolled by a liquid rheostat exerts a constant torque of 1 times full-load torque during the starting period of 20 sec, i which the motor is accelerated uniformly. If 6 starts are mac per hour and the efficiency of the motor during the startin period is constant at 85%, find the mean rate of heat dissipatio by the controller. Assume the magnetic circuit to be saturate

[2 500 J/sec

124 The speed of a 4-pole, 500-V, wave-wound series motor havi 654 armature conductors is controlled by a diverter connect across the field. Assuming that the input current remai unaltered at 40 A, draw a curve of speed to a base of divert resistance. The resistances of the field and armature are 0·2 and 0·5 Ω respectively. The magnetization curve is given b the following data :—

Current, A .	5	10	15	20	25	30	40
Flux, mWb	4·78	8·79	11·2	12·65	13·7	14·4	15·6

125 A series motor develops full load at 400 rev per min. A diverter is connected across the field. Plot curves of speed and input current to a base of diverter resistance if the load torque remains unaltered. The field resistance is 0 2 Ω and the Φ/I characteristic of the machine is given in Fig. 4, where 100% represents the flux per pole and the full-load current of the machine at a speed of 400 rev per min. Ignore losses.

126 A series motor is required to give a constant torque between 80% and 120% full-load current. Show graphically the variation of the diverter current expressed as a percentage of the load current, and find the diverter current at 100% load current. Draw also the speed/current graph of the machine. The magnetization curve is given in Fig. 4. Neglect motor resistance.
*[43%.]

127 A 4-pole, 500-V, series motor with all field coils connected in series has the following characteristic :—

Current . .	21	17·5	14	10 5	7	A
Speed . .	450	495	560	660	800	r.p.m.
Gross torque .	128	104	76·6	50·4	28·4	lb-ft

When driving a fan it takes 14 A. To increase the fan output the field coils are connected in two parallel groups each of two coils in series. Calculate the speed and current under these conditions, assuming torque and losses to vary as the square of the speed. Armature resistance = 0 8 Ω, field resistance = 0 05 Ω per coil. [618 rev per min ; 19·5 A.]

128 The magnetization curve of a 550-V series motor running at 500 rev per min is as follows :—

Exciting current .	40	80	120	160	200 A
Induced e m.f. .	170	302	388	430	472 V

The armature resistance is 0·2 Ω and the field resistance is 0·3 Ω. Plot curves of speed against current and speed against torque. When the motor is running at 500 rev per min, the exciting current is reduced suddenly to two-thirds. Find (*a*) the change in line current at this instant, and (*b*) the ultimate speed and current, the torque remaining unaltered throughout.
[(*a*) Increase of approx. 76 A ; (*b*) 572 rev per min, 194 A.]

129 A coach is driven by two similar series motors *A* and *B*. *A* drives 42-in wheels and *B* drives 40-in wheels. Characteristics of motor *A* are as follows :—

Current . .	200	300	400	500 A
Speed . .	31·4	26·4	23·9	22·1 m.p.h.
Tractive effort .	1 600	2 950	4 830	5 700 lb

Calculate the input to, and tractive effort of, each motor when (*a*) motors are in parallel and speed is 25 m.p.h., (*b*) motors are in series taking 400 A For each motor $R = 0\ 08\ \Omega$.
[(*a*) *A* :— 210 kW, 3 650 lb ; *B* :— 183 kW, 3 150 lb.
(*b*) *A* :— 117 kW, 4 330 lb ; *B* :— 123 kW, 4 550 lb.]

130 A 20-h.p., 500 rev per min, 400-V series motor with a full-load efficiency of 0·86 starts against a constant torque of 160 lb-ft. The current is kept constant at 40 A. If the kinetic energy stored in armature and load at 500 rev per min is 40 000 ft-lb, find the time taken to reach full speed.

Given :				
% full-load current	87	100	112	125
% full-load flux	93	100	104	106

[61·1 sec.]

Motors : Compound

131 Deduce the speed/current and speed/torque curves of a compound motor when the shunt ampere-turns are equal to (*a*) $\frac{1}{3}$, (*b*) $\frac{1}{2}$, of the normal excitation. In both cases find the starting torque in terms of full-load torque if the starting current is (i) 75%, (ii) 135% normal current. Neglect resistance, and use the magnetization curve in Fig. 4.
*[(i) (*a*) 68%, (*b*) 70% ; (ii) (*a*) 148%, (*b*) 145%.]

132 The shunt winding of a compound motor is to limit the speed to twice normal speed. Calculate, in terms of the excitation at normal load and speed, the ampere-turns for which the shunt winding must be designed. At top speed, assume the motor to be unsaturated, with a voltage-drop of 2%. At normal load and speed, assume that 75% of the excitation is needed for the air-gap, and the voltage-drop is 10%. [26%].

133 A 220-V compound motor, at no load, runs at 1 200 rev per min. The shunt winding has a resistance of 180 Ω and 2000 turns per pole. Find the number of series turns per pole to reduce the speed to 950 rev per min, the corresponding current being 75 A. Assume the loss at this current to be 750 W in the armature winding and brushes and 875 W in the series winding. Use Fig. 4. [16.]

134 A 50-h.p., 500-V, shunt motor has a full-load efficiency of 0·87 and runs at 750 rev per min. A series winding is added to raise the speed to 800 rev per min. Find the armature current and the efficiency under these conditions. Armature resistance

= 0·4 Ω, series-winding resistance = 0·1 Ω, shunt-winding resistance = 250 Ω. Assume that the load and the "constant" losses remain constant. [85 A, 85·8%.]

135 The magnetization curve of a compound motor is given in Fig. 4. The shunt ampere-turns amount to 40% of the normal excitation. Plot the speed/current and the speed/torque curve for the motor when the length of its air-gap is doubled, all other conditions remaining unaltered. What is the speed of the motor when taking full-load current? *[147% normal speed.]

136 A cumulatively-compounded motor is required to start against 140% of full-load torque with a starting current of 130% full-load current. Determine the ampere-turns required on the shunt field in terms of the total ampere-turns at full load. Find also the no-load speed of the machine if there is a resistance-drop of 12% at full load and 2% at no load. *[39%, 203%.]

137 A differential-compound motor is required to be level compounded at full load. The volt-drop at no load is 1% of the applied voltage and at full load it is 12%, being proportional to the load current. Calculate the ampere-turns required for the series field winding. expressed as a percentage of the shunt field winding, and find the speed at 100% over-load current. The magnetization curve is given in Fig. 4. *[21%, 105%.]

Testing of Motors: Brake Method

138 In a brake-test on a small shunt motor the speed was 1 500 rev per min, the load on one side of the brake band was 6·5 lb and on the other 0 375 lb. The diameter of the brake pulley was 6 in. If the input current was 2 A at 250 V, calculate the torque in kilogram-metres, the efficiency and the brake horse-power. [0·212 kg-m; 65·2%; 0·436 h.p.]

139 A 240-V motor on brake test took 52 A when running at 1 500 rev per min. The spring balance at the end of the 30-in brake arm read 21 lb. Calculate the efficiency. [0·9.]

140 The following test figures were obtained for a 250-V shunt motor. Full-load brake-test: $V = 250$ V, $I = 21$ A, $N = 1\,490$ rev per min, spring balance readings, $w_1 = 41{\cdot}8$ lb., $w_2 = 2{\cdot}1$ lb, pulley diameter 12 in. No-load test: $I_a = 1{\cdot}24$ A, $I_f = 1{\cdot}2$ A; resistance of armature determined from $I_a = 21$ A, $V_a = 15{\cdot}1$ V. The accuracy obtainable from each spring balance and from the tachometer is 1·5% and from each voltmeter and ammeter 1%. Determine the possible upper and lower limits, due to instrument errors, of the figures obtainable in the calculation of efficiency (*a*) by the load method, and (*b*) by the summation of losses method. [(*a*) 84%, 76%; (*b*) 83·3%, 82·7%.]

Loss Method

141 A 500-V shunt motor takes 4 A on no-load. The armature resistance including that of the brushes is 0·2 Ω and the field current is 1 A. Estimate the output and efficiency when the input current is (*a*) 20 A; (*b*) 100 A.
[(*a*) 10·6 h.p., 79·3%; (*b*) 61·8 h.p., 92·1%.]

142 A 250-V shunt motor having an armature resistance of 0·8 Ω and field resistance of 250 Ω takes 4 A when running unloaded. Calculate its efficiency when taking currents of 20, 40 and 60 A; also the full-load speed regulation, expressed as a percentage of the no-load speed, full-load current being 60 A. State any assumptions made.
[77·8%; 85·4%; 86·4%; 6·75%.]

143 Calculate the efficiency of a 600-V shunt motor when taking 700 A from the following data taken when the motor was hot. Motor stationary: voltage drop in armature winding 15 V when armature current was 510 A; field current 9 A at normal voltage. Motor running at normal speed unloaded: armature current was 22·5 A when applied voltage was 550 V. Allow 2 V for brush-contact drop and 1% of rated output of 500 h.p. for stray loss. [0·91.]

144 A 480-V, 25 h.p. shunt motor took 2·5 A when running light. Taking armature resistance to be 0·6 Ω, field resistance 800 Ω and brush drop 2 V, find the full-load efficiency. [0·885.]

145 A 12-h.p., 500-V shunt motor furnished the following test data:—

Armature current, A	3	5	10	20	30
Armature resistance, Ω	1·885	1·77	1·70	1·61	1·59

Input on no load:—

Speed, rev per min.	1 030	1 440	1 750
Total current, A	2·32	1·85	1·93
Field current, A	0·95	0·38	0·28

Plot curves of efficiency to a base of input current for the three given settings of the field regulator.

Retardation Method

146 A retardation test is made on a separately-excited motor. The induced voltage falls from 400 to 380 V (*a*) in 65 sec on opening the armature circuit, (*b*) in 40 sec on suddenly changing the armature connexions from the supply to a resistance taking 10 A. Calculate the " constant " losses of the motor. [6·24 kW.]

147 A retardation test on a direct-current motor gave the following results ; With field unexcited, the speed fell from 1 525 to 1 475 rev per min in 43 sec ; with field normally excited the same speed drop occurred in 26 sec ; with an average load of 1·1 kW supplied by the armature, the same speed drop occurred in 20 sec. Determine the moment of inertia of the rotating parts (in kg-m² units) at 1 500 rev per min, and the core loss for normal excitation at this speed. [117 kg-m² ; 1·46 kW.]

Back-to-back Method

148 A Field test on two mechanically-coupled similar tramway motors [with their field windings connected in series, and with one machine running as a motor and the other as a generator] gave the following data :—Motor : armature current, 56 A ; armature voltage, 510 V ; drop across field winding, 40 V. Generator : armature current, 44 A ; armature voltage, 400 V ; drop across field winding, 41 V. Resistance of each armature, 0·3 Ω. Calculate the efficiency of each machine with its gearing at this load. [74·5% ; 70·0%.]

149 In a differential test of two series motors, the readings were : motor supply voltage, 500 V ; generator e.m.f., 427 V ; circulating current, 72 A ; make-up current, 9 A. Find the friction and core loss per machine ; also the motor efficiency, given its resistance to be 0·975 Ω. [1 676 W ; 80%.]

150 In a Hopkinson test on two 200-V shunt motors, each having armature and field resistance of 0·12 Ω and 125 Ω respectively, the following readings were recorded on the ammeter A_1 in the main circuit and A_2 in the common lead to the motor :—

A_1	A_2	A_1	A_2	A_1	A_2
6·8	3 4	7·3	24·6	8·7	40·8
6 9	13·0	7·8	31·3	9·8	48·0

Plot on a base of mean current the efficiency (*a*) of each machine ; (*b*) of the combination. Compare the results with values obtained by the loss method.

151 The Hopkinson test on two shunt machines gave the following results for full load :—Line voltage, 250 V ; line current, excluding field currents, 50 A ; motor armature current, 380 A ; field currents, 5 A and 4·2 A. Calculate the efficiency of each machine. Armature resistance of each machine 0·02 Ω.

[Motor, 92·04% ; Generator, 92 02%.]

Acceleration and Braking

152 Show that the time taken by a motor to accelerate to a speed of ω radians per sec is $t = (20\pi J\omega a/pIZ\Phi)$ sec, where a = number of pairs of parallel paths, p = number of pole-pairs, J = moment of inertia (g-cm²), Φ = constant flux per pole, Z = number of armature conductors, I = accelerating current, assumed constant.

153 A 500-V, 50-h.p., shunt motor takes 81 A when running on full load at 500 rev per min. The moment of inertia of the rotating system is 20 kg-m². Find the time to attain full speed when started from rest against full-load torque. The starting current fluctuates between 1·1 and 1·4 times normal rated current. Assume efficiency constant at full-load value during starting period. [5·9 sec.]

154 Find the time taken for a shunt motor to fall in speed from 750 rev per min to 200 rev per min, with the excitation maintained constant and the armature connected across a resistance of 10 Ω. The electromotive force induced at 750 rev per min is 200 V. Assume (*a*) negligible motor losses ; (*b*) a friction loss equivalent to a constant frictional torque of 10 lb-ft. The moment of inertia of the armature is 600 lb-ft².

[(*a*) 51·5 sec ; (*b*) 33·8 sec.]

155 A 200-V shunt motor drives a load, the torque of which is constant. The speed of the combination is raised uniformly from 500 to 1 000 rev per min in 20 sec by weakening the field of the motor. The kinetic energy of the rotating system at 1 000 rev per min is 90 ft-tons. If the motor takes a current of 100 A when running at 500 rev per min, determine the average current taken during acceleration. Neglect losses. [201·5 A.]

156 Calculate the minimum time required to accelerate a flywheel having a moment of inertia of 50 000 lb-ft² to a speed of 500 rev per min by a direct-coupled 100-h.p. shunt motor, if the current taken by the motor during the starting period is constant and equal to twice full-load current.

[38·7 sec.]

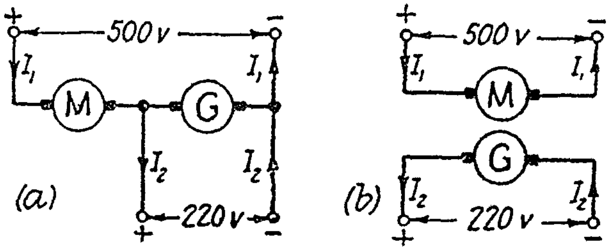
157 Direct-current motors may be braked by reversing the connexions to the armature and at the same time introducing a resistance in series with the armature. What resistance must be so connected in series with the armature of (*a*) a 500-V, 57-h.p. shunt motor, (*b*) a 500-V, 57-h.p. series motor, to limit the initial current to 150 A ? Calculate the braking torque so obtained ; also the torque when the speed of the motor has fallen by 30%. Shunt motor : armature resistance 0·1 Ω ; full-load current, 100 A. Series motor : armature and field resistances 0·1 Ω and 0·2 Ω respectively ; full-load current, 100 A. Full-load speed,

700 rev per min. Magnetization curve as shown in Fig. 4, where 100% represents the armature current and voltage at full-load normal voltage.
[(*a*) 6·5 Ω, 641 lb-ft, 546 lb-ft; (*b*) 6·7 Ω, 750 lb-ft, 460 lb-ft.]

158 The magnetization curve of a series crane motor running at 640 rev per min is given by Fig. 4, where 100% represents an e.m.f. of 490 V and a current of 60 A. Estimate the resistance which must be connected across the motor when the supply is cut off and rheostatic braking is employed to limit the speed to 600 rev per min if the descending load exerts a torque of 200 lb-ft. Total motor resistance, 1 Ω. *[8·0 Ω.]

Mechanical Coupling

159 Diagrams (*a*) and (*b*) show two ways of connecting a direct-current voltage-changer set. If 20 kW are to be supplied to the



low-voltage network, find the rating of the machines in each case, taking the efficiency of each machine as 87%.
[11·8 and 9·8 kW; 23 and 20 kW.]

160 A 500-V series motor of resistance 0·5 Ω runs at 600 rev per min when the current is 50 A. If an identical motor is mechanically coupled and joined in series, what will be the current when the set runs at 200 rev per min and exerts twice the original gross torque? [34 A.]

161 Two series motors with different air-gaps but otherwise identical run at 700 and 750 rev per min respectively when taking 50 A at 500 V, the terminal resistance of each motor being 0·32 Ω. If mechanically coupled and connected in series to a 500-V supply, and taking a current of 50 A, calculate (*a*) the speed at which the machines will run; (*b*) the voltage across each machine.
[(*a*) 350 rev per min; (*b*) 258 V, 242 V.]

162 A mechanical-electrical drive consists of a 1 000-V shunt generator and two 500-V shunt motors coupled to separate loads, with all armatures connected in series. The field strength of one motor is progressively decreased and that of the other maintained constant. If the speed, voltage and power input of the generator remain unaltered, plot the relative speed, power output and torque of each motor to a base of the relative field strength of the motor of which the field is varied.

CHAPTER XXVIII

TRANSFORMERS

Magnetic Circuit

1 Estimate the active cross-sectional area of the core of a 20-turn, 1-phase, inductor for a terminal voltage of 100 V at 50 c/s. The flux-density is to be about 10 000 lines per cm^2 (1 Wb/m^2). Give suitable dimensions for a square core.
[225 cm^2 ; 16 cm square.]

2 Calculate the core and window areas required for a 1 600-kVA, 6 600/440-V, 50-c/s, single-phase, core-type, power transformer. Assume a maximum flux density of 12 000 gauss (1·2 Wb/m^2) and a current density of 3 A per mm^2. Induced voltage per turn, 30 V ; window space factor, 0·32.
[1 125 cm^2 ; 1 110 cm^2.]

3 The cores of a core-type transformer having cylindrical coils are enclosed in circles of diameter d cm. Compare the maximum available gross core sections if (*a*) square, (*b*) single-stepped (cruciform) shapes are used. [(*a*) 0·5d^2 ; (*b*) 0·616d^2.]

4 Estimate the reduction in volume, expressed as a percentage of the original volume, of (*a*) core iron, (*b*) copper, in a transformer when ordinary steel plates worked at a flux density of 8 000 gauss are replaced by others of silicon steel worked at 12 000 gauss, assuming the total flux to remain unchanged. State any assumption made. [(*a*) 33% ; (*b*) 18·4%.]

5 Calculate the effective length and cross-sectional area of the air-gap in an inductor with 300 turns, required to have an induced electromotive force of 100 V with a current of 10 A at 50 c/s. Assume maximum gap density of 10 000 lines per cm^2 (1 Wb/m^2) and that the iron requires 10% of the total ampere-turns. Ignore leakage and fringing. [4·8 mm ; 15 cm^2.]

6 An air-gap inductor is to take a current of 10 A when supplied from 250-V, 50-c/s mains. Details of the coil are as follows : Net cross-section of core, 15 cm^2 ; mean magnetic length, 20 cm ; number of turns, 400 ; resistance of winding, 15 Ω. Calculate the length of the air-gap, using the magnetization curve for armature steel plates. *[0·485 cm.]

Windings

7 A 1-phase transformer has 400 primary and 1 000 secondary turns. The net cross-sectional area of the core is 60 cm^2. If

the primary winding be connected to a 50-c/s supply at 500 V, calculate (*a*) the peak value of the flux density in the core, and (*b*) the voltage induced in the secondary winding.

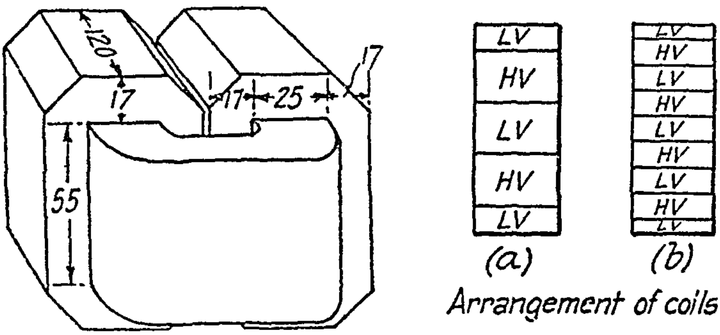
[(*a*) 9 400 lines per cm² (0 94 Wb/m²) ; (*b*) 1 250 V.]

8 The required no-load ratio in a 1-phase, 50-c/s, core-type transformer is 6 000/250 V. Find the number of turns in each winding if the flux is to be about 6 megamaxwells (0 06 Wb).

[480, 20.]

9 A 1-phase, 50-c/s, core-type transformer has square cores of 20-cm side. The permissible maximum density is 10 000 gauss (1 Wb/m²). Calculate the numbers of turns per limb on the high- and low-voltage sides for a 3 000/220-V ratio. [191, 14.]

10 A 1-phase, 25-c/s, shell-type transformer with sandwich coils has the dimensions in cm as shown. The ratio is 20 000/4 000 V and the flux density is not to exceed 12 000 gauss



Arrangement of coils

(1·2 Wb/m²). Find the number of turns in the several sections in the alternative arrangements

[(*a*) $T_1 = 420 = 2 \times 210$; $T_2 = 84 = 2 \times 21 + 42$;
(*b*) $T_1 = 440 = 4 \times 110$; $T_2 = 88 = 2 \times 11 + 3 \times 22$.]

11 The cruciform (single-stepped) cores in a 200-kVA, 6 600/440-V, 25-c/s, 1-phase, core-type transformer are enclosed in circumscribed circles of 37 cm diameter. Find the number of turns for a flux-density of about 12 000 gauss (1·2 Wb/m²) and suitable conductor sections for a current-density of about 2 A per mm². [660/44 turns ; 15/230 mm².]

12 The core of a 230/260 V continuously variable auto-transformer consists of circular laminations, internal diameter 6 cm, external diameter 14 cm, built to a height of 8 cm. Allowing 10% for insulation and a maximum flux-density of 12 000 gauss (1·2 Wb/m²), find the required number of turns and size of wire for a single-layer winding.

[339 turns tapped at 300 ; No. 18 S.W.G. enamelled.]

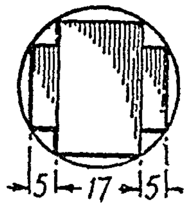
13 Determine the number of turns per phase in each winding of a 3-phase transformer with a ratio of 20 000/2 000 V, to work on a 50-c/s network. The high-voltage winding is delta-connected and the low-voltage is star-connected. Each core has a gross section of 560 cm². Assume a flux-density of about 12 000 lines/cm² (1·2 Wb/m²). [1 490/86.]

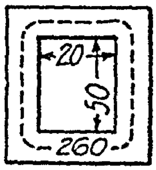
14 A 3-phase, 50-c/s transformer of the shell-type has an iron cross-section of 400 cm² (gross). If the flux-density be limited to 12 000 gauss (1·2 Wb/m²), find the numbers of turns per phase on high- and low-voltage windings. The voltage ratio is 11 000/ 550 V, the high-voltage side being connected in star and the low-voltage in mesh. [670/58.]

15 A 50-c/s 3-phase, core-type transformer is to be built for a 10 000/500-V ratio, connected star/mesh. The cores are to have a square section and the coils are to be circular. Taking an induced electromotive force of about 15 V per turn, and a maximum core density of about 11 000 gauss (1·1 Wb/m²), find the cross-sectional dimensions of the core, the diameter of the circumscribing circle, and the numbers of turns per phase. [26 cm × 26 cm ; 37 cm ; 392/34.]

Magnetizing Current

16 A 500-kVA, 6 600/520-V, 50-cycle, 1-phase transformer has a core built up as shown from transformer steel plates. (*a*) Plot the magnetization curve showing voltage/ampere-turn scales for both high- and low-voltage sides. The turn ratio is 456/36. (*b*) State the approximate magnetizing currents of the high- and low-voltage windings for normal voltage. (*c*) Using the hysteresis loop shown in Fig. 3, draw out a curve showing the current wave-form for a sinusoidal applied voltage on the primary side. Neglect air-gaps and assume the yokes to have the same cross-section as the cores. *[(*b*) 3·1 A, 39·2 A.]





17 Calculate the value of the magnetizing current for a 6 600/384-V, 1-phase, 50-c/s, core-type transformer. The net cross-section of both core and yoke is 775 cm², the length of the mean path of the flux is 250 cm, and there are 378 turns on the high voltage winding. The magnetization curve is given in Fig. 1. Ignore the effects of joints. *[2·34 A.]

18 Estimate the active and reactive components of the no-load current of a 400-V, 50-c/s, 1-phase transformer, the particulars of which are as follows : core of transformer steel, length of mean magnetic path 200 cm ; gross cross-section, 100 cm² ; joints equivalent to 0·1 mm air-gap ; maximum density, 7 000 gauss (0·7 Wb/m²) ; specific loss at 50 c/s and 7 000 gauss, 0·5 W per kg. Space factor, 0·9. *[0·169 A ; 1·23 A.]

Connexions: Star, Mesh

19 Three 1-phase transformers connected in mesh supply 100 A per line to a 3-phase, 3-wire system. (*a*) What is the current in each transformer? One unit develops a fault and is removed: (*b*) by how much is the capacity of the set reduced for the same temperature-rise? (*c*) Find the current in each of the remaining transformers if the line current corresponds to the rating in (*b*). Draw the vector diagram.

[(*a*) 57·8 A; (*b*) 42 2%; (*c*) 57·8 A.]

20 A 3-phase, step-down transformer is connected to 6 600-V mains, and takes 10 A. Calculate the secondary line voltage, line current and output for the following connexions:—(*a*) mesh/mesh; (*b*) star/star; (*c*) mesh/star; (*d*) star/mesh. The ratio of turns per phase is 12. Neglect losses.

[(*a*) and (*b*) 550 V, 120 A; (*c*) 953 V, 69·4 A; (*d*) 318 V, 208 A; Output, 114·3 kVA.]

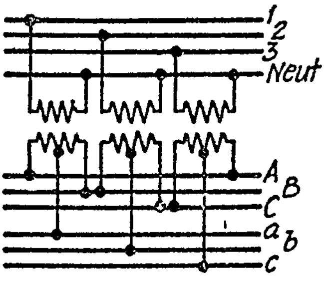
21 A lighting load of *I* amperes is taken from one phase of a 3-phase transformer bank. Find the current distribution in the phases and in the lines (*a*) when the transformers are connected mesh/mesh; (*b*) when the primary side is connected in star (3-wire) and the secondary in mesh; and (*c*) when both sides are connected in V or open-delta.

22 A 50-h.p., 440-V, 3-phase induction motor with an efficiency of 0·9 and a power factor of 0·85 on full load is supplied from a 6 600/440-V, mesh/star-connected transformer. Ignoring the magnetizing current, calculate the currents in the high- and low-voltage transformer phases when the motor is running at full load.

[2·46 A; 64 A.]

23 Mid-points of the secondaries of a bank of 4 000/440-V transformers are used to provide a lower voltage, as shown in the diagram. Find (*a*) the voltage of the bus-bars *a b c*, and (*b*) the current in the transformer secondaries when currents of 100 A at unity power factor are taken from each of the two sets of low-voltage bus-bars, *a b c* and *A B C*.

[(*a*) 200 V; (*b*) 76 A.]



Two-phase to Three-phase

24 A Scott-connected transformer is fed from a 6 600-V, 2-phase network and supplies 3-phase power at 500 V between lines (outer) on a 4-wire system. If there are 500 turns per phase on the 2-phase side, find the number of turns in the low-voltage

windings, and the position of the tapping of the neutral wire. Show that if the load is balanced on one side it will also be balanced on the other side.

[38, 33, 22 from outer in phase with 33 turns.]

25 Two 1-phase Scott-connected transformers supply a 3-phase, 4-wire, 50-c/s distribution system with 250 V between lines and neutral. The high-voltage windings are connected to a 2-phase system with a phase voltage of 11 000 V. Allowing a maximum flux density of 12 000 gauss (1·2 Wb/m²) in a gross core section of 550 cm², determine the number of turns in each section of the high-voltage and low-voltage windings, and the position of the neutral point.

[886 ; 35, 30, 20 from outer in phase with 30 turns.]

26 Two 1-phase furnaces *I* and *II* are supplied at 80 V by means of a Scott-connected transformer combination from a 3-phase, 6 600-V system. The voltage of furnace *I* is leading. Calculate the line currents on the 3-phase side when the furnaces take 500 kW and 800 kW respectively (*a*) at unity power factor ; (*b*) furnace *I* at unity p f., furnace *II* at 0·7 p.f. lagging. Draw the corresponding vector diagrams.

[(*a*) 129, 129 and 87·6 A ; (*b*) 207, 145 and 87·6 A.]

Tertiary Windings

27 The ratio of the numbers of turns per phase in the primary, secondary, and tertiary windings of a transformer is 10 : 2 : 1. With lagging currents of 45 A at power factor 0·8 in the secondary, and 50 A at power factor 0·71 in the tertiary winding, find the primary current and power factor. [14·0 A, 0·77.]

28 A star/star/mesh transformer, with primary, secondary and tertiary voltages of 11 000 V, 1 000 V and 400 V, has a magnetizing current of 3 A. There is a balanced load of 600 kVA at 0·8 power factor lagging on the secondary winding and a balanced load of 150 kW on the tertiary winding. Neglecting losses, find the primary and tertiary phase currents if the primary power factor is 0·82 lagging. [40·3 A ; 126 A.]

Auto-transformers

29 Derive an expression for the approximate relative weights of copper in an auto-transformer and a 2-winding transformer, the primary voltage being V_1 and the secondary voltage V_2. Compare the weights of copper when the transformation ratio is 3. Ignore the magnetizing current.

[Ratio $= 1 - (V_2/V_1)$; $\frac{2}{3}$.]

30 A single-phase auto-transformer is to transmit a power of W kW from the primary network at E_1 volts to the secondary at E_2 volts. Determine the rating of each section of the winding

and compare it with the rating of a two-winding transformer, assuming an efficiency of η in both cases. Evaluate for $E_2 = 90\% \; E_1$, $W = 100$ kW and $\eta = 0\ 97$. Ignore magnetizing current. [Primary 103·2/10 32, Secondary 100/7·22.]

31 An 11 500/2 300-V transformer is rated at 100 kVA as a 2-winding transformer. If the two windings are connected in series to form an auto-transformer, what will be the voltage ratio and output.

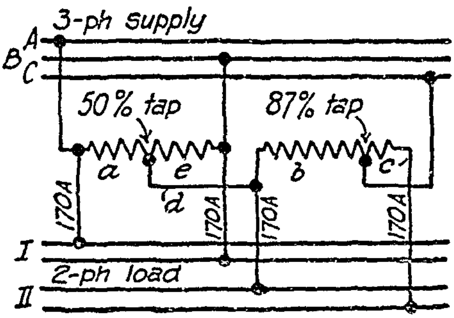
[13 8/11 5 kV, 600 kVA or 13·8/2 3 kV, 120 kVA.]

32 An auto-transformer is used to provide a balanced 250-V, 3-phase supply from a 250-V d.c. generator. Draw a connexion diagram, indicating how a neutral wire could be obtained for the generator, and find the percentage tapping required for the auto-transformer. [61·2% from the starpoint.]

33 Find the values of the currents flowing in the various branches of a 3-phase, star-connected auto-transformer loaded with 500 kW at power factor 0 8 lagging, and having a ratio of 440/500 V. Neglect voltage drops and all losses in the transformer; also the magnetizing current.

[Line currents, 725 and 822 A; Winding currents, 725 and 97 A.]

34 Two 1-phase auto-transformers are Scott-connected and loaded as shown in the diagram. Determine the currents in the parts *a*, *b*, *c*, *d*, *e*, and the 3-phase load current. Draw a schematic diagram showing the current directions in all circuits. [$I_a = 98$ A; $I_b = 26{\cdot}4$ A; $I_c = 170$ A; $I_d = 196$ A; $I_e = 98$ A; $I_A = I_B = I_C = 196$ A.]



Equivalent Circuits

35 A 1-phase transformer has 180 and 90 turns respectively in its secondary and primary windings. The respective resistances are 0 233 Ω and 0 067 Ω. Calculate the equivalent resistance of (*a*) the primary in terms of the secondary winding, (*b*) the secondary in terms of the primary winding, and (*c*) the total resistance of the transformer in terms of the primary.

[0 268 Ω; 0 058 Ω; 0·125 Ω.]

36 Calculate in terms of the primary the effective (equivalent) resistance and the leakage reactance of a transformer which gave the following data on test with the secondary terminals short-circuited :—Applied voltage, 60 V ; current, 100 A ; power input, 1 2 kW. [0·12 Ω ; 0·59 Ω.]

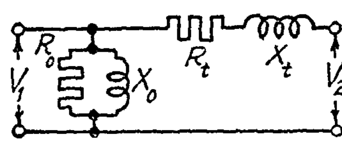
37 A 40-kVA transformer with a ratio of 2 000/250 V has a primary resistance of 1·15 Ω and a secondary resistance of 0·0155 Ω. Calculate (*a*) the total resistance in terms of the secondary winding, (*b*) the total resistance drop on full load, and (*c*) the total copper loss on full load.
[0·0334 Ω ; 5·35 V ; 856 W.]

38 A 50-c/s, 1-phase transformer has a turn-ratio of 6. The resistances are 0·90 Ω and 0·03 Ω, and the reactances 5 Ω and 0·13 Ω for high-voltage and low-voltage windings respectively. Find (*a*) the voltage to be applied to the high-voltage side to obtain full-load current of 200 A in the low-voltage winding on short-circuit. (*b*) the power factor on short-circuit.
[(*a*) 330 V ; (*b*) 0·2.]

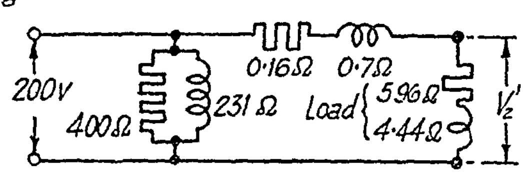
39 The high- and low-voltage windings of a 6 600/250-V, 50-c/s, 1-phase transformer have resistances of 0 21 Ω and 2 72 $\times 10^{-4}$ Ω and reactances of 1·0 Ω and 1·3 $\times 10^{-3}$ Ω respectively. Determine the current and power input when the high-voltage winding is connected to a 400-V, 50-c/s supply, the low-voltage winding being short-circuited. [206 A ; 17 kW.]

40 A 3-phase transformer is supplied at 6 000 V on the delta-connected side. The terminal voltage on the star-connected side when loaded at power factor 0·8 is 415 V. The equivalent resistance and reactance drops are 1 and 5%. Find the turn ratio. [24.]

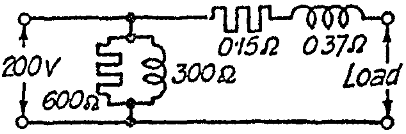
41 Calculate the values of R_0, X_0, R_t and X_t in the diagram for the equivalent circuit of a 1-phase, 4-kVA, 200/400-V, 50-c/s transformer of which the following are test results :—Open-circuit : 200 V, 0 7 A, 70 W on low-voltage (primary) side. Short-circuit : 15 V, 10 A, 80 W on high-voltage (secondary) side. [571, 330, 0·2 and 0·317 Ω.]



42 The diagram shows the equivalent circuit for a 1-phase transformer. Figures give resistances and reactances in ohms in terms of the primary side. The ratio of secondary to primary turns is 10 and the load is inductive. Find (*a*) the secondary terminal voltage ; (*b*) the primary current ; (*c*) the efficiency. [(*a*) 1 860 V ; (*b*) 25·9 A ; (*c*) 94·9%.]



43 The equivalent circuit shown refers to a 200/400-V, 1-phase, 50 c/s, 4-kVA transformer, the values given being reduced to the low-voltage side. For a high-voltage current of 10 A at a lagging power factor of 0 8, calculate (*a*) the low-voltage input current; (*b*) the efficiency; (*c*) the voltage at the terminals of the high-voltage side.



[(*a*) 20·65 A; (*b*) 96·08%; (*c*) 386 V.]

44 The efficiency, at unity power factor, of a 6 600/384-V, 200-kVA, 1-phase transformer is 98% both at full load and half load. The power factor on no load is 0·2 and the full load regulation at a lagging power factor of 0·8 is 4%. Draw the equivalent circuit referred to the l.v. side, and insert all values.

[$R_o = 108\ \Omega$; $X_o = 22{\cdot}1\ \Omega$; $R = 0{\cdot}01\ \Omega$; $X = 0\ 036\ \Omega$.]

Performance: Regulation

45 Calculate the regulation of a transformer in which the ohmic loss is 1% of the output and the reactance drop 5% of the voltage, when the power factor is (*a*) 0·8 lagging; (*b*) unity; (*c*) 0·8 leading. [(*a*) 3·8%; (*b*) 1%; (*c*) − 2·2%.]

46 A 100-kVA, 6 600/330-V, 50 c/s, single-phase transformer took 10 A and 436 W at 100 V in a short-circuit test, the figures referring to the high-voltage side. Calculate the voltage to be applied to the high-voltage side on full load at power factor 0·8 lagging when the secondary terminal voltage is 330 V.

[6 784 V.]

47 A 4-kVA, 200/400-V, 50-c/s, single-phase transformer gave the following test figures:—No load: low-voltage data, 200 V, 0·7 A, 60 W. Short-circuit: high-voltage data, 9 V, 6 A, 21·6 W. Calculate (*a*) the magnetizing current and the component corresponding to iron loss at normal voltage and frequency.; (*b*) the efficiency on full load at unity power factor; (*c*) the secondary terminal voltage on full load at power factors of unity, 0·8 lagging, and 0 8 leading.

[(*a*) 0 63 A, 0·3 A; (*b*) 97·1%; (*c*) 394 V, 387 V, 403 4 V.]

48 A 110-kVA, 1-phase transformer has a ratio of 11 000/440 V. The iron loss measured on open-circuit is 1 100 W. With the secondary winding short-circuited, a voltage of 500 V at normal frequency applied to the primary produces full-load current, the wattmeter reading 1 000 W. Calculate (*a*) the secondary terminal voltage, (*b*) the efficiency, when a current of 250 A at a lagging power factor of 0·8 is taken by a load connected to the low-voltage terminals, the primary voltage being 11 000 V.

[(*a*) 425 V; (*b*) 97·6%.]

49 Calculate (*a*) the full load efficiency at unity power factor; (*b*) the voltage at the secondary terminals when supplying full load secondary current at power factors (i) unity, (ii) 0·8 lagging, (iii) 0·8 leading, for the 4-kVA, 200/400-V, 50-c/s, single-phase transformer, of which the following are the test figures :—Open-circuit with 200 V applied to primary winding : current 0·8 A, power 70 W. Short-circuit with 17·5 V applied to secondary (high-voltage) side : current 9 A, power 50 W.

[(*a*) 96·8% ; (*b*) (i) 393·7 V ; (ii) 384 V ; (iii) 406 V.]

50 A transformer has resistance and reactance drops of 2·5% and 5% respectively. Plot a curve showing the percentage voltage regulation on full load as a function of power factors between zero lagging and zero leading. The simplified formula may be used. Find the maximum regulation and the power factor at which this occurs. [5·6% ; 0·45 lagging.]

51 The maximum efficiency of a 500-kVA, 3 300/500-V, 50-c/s, single-phase transformer is 97% and occurs at $\frac{3}{4}$ full load, unity power factor. If the impedance is 10%, calculate the regulation at full load, power factor 0·8 lagging. [7·51%.]

52 A 40-MVA tap-changer transformer has a percentage impedance of $0{\cdot}6 + j\,8{\cdot}0$. What tapping must be used to maintain the rated voltage at the following loads :—(*a*) 35 MVA at power factor 0·8 lagging ; (*b*) 45 MVA at power factor 0·7 lagging ?

[(*a*) 4·9% tap down ; (*b*) 7·1 % tap down.]

53 A tap-changer transformer, of nominal ratio 11/33 kV, supplies a load of 5 000 kVA with a lagging power factor of 0·8. The voltage at the load end of the line is 30 kV, while $R = 3\ \Omega$ and $X = 2\ \Omega$ for each conductor. The mesh-connected primary of the transformer has $R = 0{\cdot}17\ \Omega$ and $X = 2{\cdot}0\ \Omega$ per phase, while the star-connected secondary has $R = 0{\cdot}5\ \Omega$ and $X = 6\ \Omega$ per phase. Calculate the voltage at the secondary terminals and the setting of the tap-changer. [30·6 kV ; 3% tap down.]

54 A 3-phase load of 1 500 kW, power factor 0·8 lagging is supplied from a tap-changer transformer, 11/33 kV, delta on l.v. and star on h.v. side. R per phase $= 0{\cdot}5\ \Omega$ on l.v. and $1{\cdot}5\ \Omega$ on h.v. X per phase $= 6{\cdot}5$ and $20\ \Omega$ respectively. Each line conductor has $R = 10$ and $X = 6\ \Omega$. If the l.v. side is supplied at 11 kV, find setting of tap-changer to give 33 kV at full load.

[Tap up 6·85%.]

Efficiency, Losses

55 Find the efficiency of a 150-kVA transformer at 25%, 33% and 100% full load, (*a*) at unity power factor, (*b*) at power factor 0·8 lagging, if the copper loss is 1 600 W at full load and the iron loss is 1 400 W. Ignore the effects of temperature rise and magnetizing current.

[(*a*) 96·15, 96 94, 98 04% ; (*b*) 95·23, 96·21, 97·56%.]

56 In a 25-kVA, 2 000/200-V transformer the iron and copper losses are 350 and 400 W respectively. Calculate the efficiency on unity power factor at (*a*) full load and (*b*) half load. (*c*) Determine the load for maximum efficiency and the iron and the copper loss in this case.

[(*a*) 97·1% ; (*b*) 96·5% ; (*c*) 23·4 kVA, 350 W each.]

57 A transformer has its maximum efficiency of 0·98 at 15 kVA at unity power factor. During the day it is loaded as follows :—

12 hours— 2 kW at power factor 0·5
6 „ 12 kW at power factor 0·8
6 „ 18 kW at power factor 0·9

Find the "all-day" efficiency (output in kWh/input in kWh).

[0·97.]

58 Calculate the efficiencies at half, full and 1¼ load of a 100-kVA transformer for power factors of (*a*) unity ; (*b*) 0·8. The copper loss is 1 000 W at full load and the iron loss is 1 000 W.

[(*a*) 97·56, 98·04, 97·98% ; (*b*) 96·96, 97·56, 97·50%.]

59 A 100-kVA, 1 000/10 000-V, 50-c/s transformer has an iron loss of 1 200 W. The copper loss with 6 A in the high-voltage winding is 500 W. Calculate the efficiencies at (i) 25%, (ii) 50% and (iii) 100% of normal load, for power factors of (*a*) 1·0 and (*b*) 0·8, the output terminal voltage being maintained at 10 000 V. Find also the load for maximum efficiency at both power factors.

[(*a*) (i) 95·10%, (ii) 97·00%, (iii) 97·47% ; 93 kVA ;
(*b*) (i) 93·95%, (ii) 96·27%, (iii) 96·86% ; 93 kVA.]

60 The efficiency of a 400-kVA, 1-phase transformer is 98·77% when delivering full load at 0·8 power factor, and 99·13% at half-load and unity power factor. Calculate (*a*) the iron loss, (*b*) the full load copper loss. [(*a*) 1 kW ; (*b*) 3 kW.]

61 A 16 000-kVA, 3-phase transformer with a voltage ratio of 4 000/56 000 V, star/star, gave the following test results :— Resistance per phase, 0·004 and 0·6 Ω ; measured loss on short-circuit with full load current, 134 kW. Calculate the average eddy-loss ratio, i.e. the ratio of effective resistance/ohmic resistance. [1·185.]

62 The following data were obtained from the open-circuit test of a small transformer :—

Terminal voltage, V	200	150	100	50
Frequency, c/s	50	37·5	25	12·5
Power input, W	55	38	23	10 25

Calculate the values of the hysteresis and eddy-current iron losses for frequencies of (*a*) 60 ; (*b*) 50 ; and (*c*) 25 c/s.
*[(*a*) 44·4, 25·9 W ; (*b*) 37, 18 W ; (*c*) 18·5, 4·5 W.]

63 In a back-to-back test on two 20-kVA, 1-phase transformers, the two primaries are fed from a 230-V supply and the secondaries are connected in series opposition. A booster transformer, the primary of which is across the same 230-V supply, is used to circulate 20 A in the secondaries. If the iron losses are 300 W per transformer, and each has a reactance equal to twice its resistance, calculate the reading of a wattmeter connected to measure the power input to the primaries. The turn-ratios are 230 : 1 000 and 230 : 1 010. [690 or 510 W.]

64 Two similar 200-kVA, 1-phase transformers gave the following results when tested by the back-to-back method : W_1 in the supply line, 4 kW ; W_2 in the primary series circuit, when full load current circulated through the secondaries, 6 kW. Draw a diagram of connexions and calculate the efficiency of each transformer. [97·56%.]

65 A 400/100-V, 200-kVA, 3-phase, 50-c/s transformer, delta/delta connected, is tested by the double-delta method, using a 1-phase induction regulator to circulate the load current. The regulator, when connected in the secondary, injects 12 V to cause full load current to flow in the primary. Calculate the percentage reactance of the transformer, ignoring resistance of windings. [4·0%.]

66 A 10 000-V, 25-c/s transformer has copper, hysteresis and eddy-current losses of 1·5, 0·7 and 0·4% of the output. What will the percentage losses become if the transformer be used on a 20 000-V, 50-c/s system, assuming the full load current to be the same ? Compare the full load efficiencies at the two frequencies. [0·75, 0·7 and 0·8% ; 97·47 and 97·80%.]

67 A 40-c/s transformer is to be used on a 50-c/s system. Assuming Steinmetz's coefficient as 1·6. and the losses at the lower frequency 1·2%, 0·7% and 0·5% for copper, hysteresis and eddy currents respectively, find (*a*) the losses on 50 c/s for the same supply voltage and current ; (*b*) the output at 50 c/s for the same total losses as on 40 c/s.
[(*a*) 1 2, 0·61, 0·5% ; (*b*) 103·5% of original.]

68 Find the eddy-current loss in a 50-c/s transformer core with a maximum density of 10 000 gauss (1 0 Wb/m²). The core, of section 8 cm × 6 cm and total effective length 50 cm, is constructed of laminations of thickness (uninsulated) of 0 04 cm, having a resistivity of 25 $\mu\Omega$-cm. The space factor is 0 9.
[5·68 W.]

69 A voltage of (2 000 sin ωt — 500 sin 3 ωt) is applied to a 250-turn transformer winding having negligible resistance and leakage reactance. Deduce an expression for the flux and find its maximum value. By what percentage will the eddy-current loss in the iron be reduced if the applied voltage is altered to 2 000 sin ωt? $\omega = 100\pi$.

$$\left[\frac{2}{\pi}\left(4\cos\omega t - \frac{1}{3}\cos 3\,\omega t\right)\text{ megamaxwells ;}\right.$$
$$\left. 2{\cdot}76 \text{ megamaxwells } (27{\cdot}6 \text{ mWb}) : 5{\cdot}9\%. \right]$$

CHAPTER XXIX

SYNCHRONOUS MACHINES

Speed and Frequency Relations

1 Draw up a table showing the speed at which an alternator must run in order that the frequency of the induced electromotive force shall be (*a*) 60, (*b*) 50, (*c*) 40, (*d*) 25 c/s. Consider all numbers of pole-pairs from 1 to 20.

2 A 50-c/s synchronous clock is to have a rotor speed between 150 rev per min and 3 000 rev per min. Available gears give overall ratios of 3 000 : 1, 2 000 : 1 and 1 000 : 1, 900 : 1, 800 : 1, down to 200 : 1, between the rotor spindle and the second hand, which makes one revolution per minute. What numbers of pole-pairs are possible with these gears? [1, 3, 5, 6, 10, 15.]

3 Find the three highest speeds at which synchronous motor-generator sets could run to link up a 25-c/s with a 60-c/s system. [300, 150, 100.]

4 Find the highest speed and the corresponding number of poles for two direct-coupled alternators required to give frequencies of (*a*) 15 and 42 c/s; (*b*) 42 and 50 c/s.
[(*a*) 10 and 28; 180 rev per min; (*b*) 42 and 50; 120 rev per min.]

5 A 50-c/s synchronous clock is correct with G.M.T. at 7 a.m. From 7 a.m. to 1 p.m. the average frequency is 49 95 c/s; from 1 p.m. to 6 p.m. it is 49 90 c/s. What must be the average frequency for the remainder of the 24 hours in order that the clock be correct again at 7 a.m.? By how much is the clock incorrect at 6 p.m.? [50·062 c/s; 57 6 sec slow.]

6 What is the minimum permissible value for the average supply-frequency between 8 a.m. and 6 p.m. if a 50-c/s synchronous clock, correct at the former hour, is not to lose more than 5 sec during this period? By what percentage must the frequency be raised above 50 c/s if the clock is to be correct again by 8 a.m? [49 993 c/s; 0 00994%.]

Induced E.M.F.

7 The armature of a 1-phase alternator is completely wound with T single-turn coils distributed uniformly. The induced electromotive force in each turn is 2 V (root-mean-square). What is the electromotive force of the whole winding with all T coils in series? [$4T/\pi$.]

8 Calculate the phase and line voltages of a 3-phase, star-connected alternator with uniformly-distributed winding of T

full-pitch turns, in each of which e_m volts (peak value) is induced, when each phase has a spread of two-thirds of the pole-pitch. If the machine is re-connected for 6 phases each with a spread of one-third of a pole-pitch, find the phase and line voltages when opposite phases are connected in series to form a 3-phase, star-connected winding.

[0 195 Te_m ; 0·338 Te_m ; 0·225 Te_m ; 0·390 Te_m.]

9 A 4-pole, 50-c/s alternator has single-turn, full-pitch coils. Stator bore, 50 cm , core length, 35 cm. The e.m.f. induced in each coil is constant over the middle two-thirds of each half-cycle and varies uniformly over the remainder ; while its r.m.s. value is 18 V. Calculate the useful flux per pole.

[8 5 megamaxwells (0 085 Wb).]

10 A 3-phase, 16-pole alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 3 megamaxwells (0 03 Wb), sine-distributed, and the speed is 375 rev per min. Find the frequency, and the phase and line electromotive forces. [50 c/s ; 1 530 V ; 2 650 V.]

11 Find the number of armature conductors in series per phase required for the armature of a 3-phase, 50-c/s, 10-pole alternator with 90 slots. The winding is to be star-connected to give a line voltage of 11 000 V. The flux per pole is about 16 megalines (0 16 Wb). [360.]

12 A 3-phase, 10-pole, star-connected alternator runs at 600 rev per min. It has 120 stator slots with 8 conductors per slot, and the conductors of each phase are connected in series. Determine the phase and line electromotive forces if the flux per pole is 5·6 megalines (56 mWb). What harmonics due to the slots might occur in the phase and line voltages ?

[1 900 V ; 3 290 V ; 23rd and 25th.]

13 The flux-distribution curve of a smooth-core, 50-c/s generator is $\beta = 10 \sin \theta + 2 \sin 3 \theta + 2 \sin 5 \theta + 2 \sin 7 \theta$ kilogauss, when θ is measured from the neutral axis. The pole pitch is 35 cm, the core length 32 cm, and the stator coil span four-fifths of the pole pitch. Determine the equation for the e.m.f. induced in one turn, and its r.m.s. value.

[$e = 21{\cdot}3 \sin \theta + 2{\cdot}64 \sin 3 \theta - 2{\cdot}64 \sin 7 \theta$ V ; 15·3 V.]

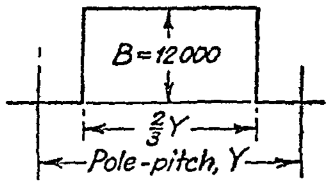
14 The field form of a star-connected, 3-phase machine is given by $100 \sin \omega t + 40 \sin 3\omega t + 20 \sin 5\omega t$. If the stator has 9 slots per pole, determine in terms of $\hat{v}$, the amplitude of the fundamental of the phase voltage : (*a*) the r.m.s. value of the line voltage, (*b*) the r.m.s. value of the phase voltage. Coil-span = pole-pitch. [(*a*) 1·228 $\hat{v}$; (*b*) 0 788 $\hat{v}$.]

15 The flux distribution in the air-gap of a 50-c/s salient-pole alternator may be taken as rectangular, the base being two-

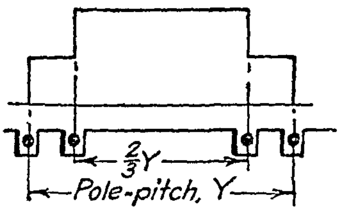
thirds of the pole-pitch. Calculate the r.m.s. value of the fundamental phase e.m.f. The following particulars are given: turns per phase. 120; phase-spread, 60°; pole-pitch, 50 cm; stator length, 75 cm; maximum value of the flux density in the air-gap, 7 000 gauss (0·7 Wb/m²). Find also the phase e.m.f. if the same total flux had a sinusoidal distribution.

[4 670 V, 4 450 V.]

16 The diagram shows the field of a salient-pole alternator. Draw one complete cycle of the e.m.f. induced in a single-turn coil which is short-chorded by 30° and moves with a uniform velocity of 20 m per sec. The active conductor length is 2 m. Calculate the r.m.s. value of the induced voltage. [78·3 V.]



17 The m.m.f. wave of a 3-phase alternator can be represented as shown. Each slot contains 1 000 ampere-conductors. The area and length of the air-gap are 500 cm² and 0·2 cm respectively. Calculate the amplitude of the fundamental of the phase e.m.f. if there are 200 turns per phase and the spread is 60°. Frequency, 50 c/s. Neglect the reluctance of the iron circuit. [2 085 V.]



18 Determine the root-mean-square value of the individual harmonic components and of the total induced electromotive force per phase of a 50-c/s, 3-phase, alternator from the following data.—Number of poles, 10; slots per pole per phase, 2; conductors per slot (2 layers), 4; coil-span, 150°; flux per pole, fundamental, 12 megalines (0·12 Wb). The analysis of the gap flux density shows a 20% third harmonic. All the coils of a phase are connected in series.

[995 V; 106 V; 1 000 V.]

Design

19 Determine approximate stator core dimensions for a 23 400-kVA, 50-c/s, 3-phase turbo-alternator, under the following limitations:—Mean gap density, 5 000 gauss (0·5 Wb/m²); ampere-conductors per cm of periphery, 560; peripheral speed, 144 m per sec; air gap, 3 cm. [98 cm dia. × 167 cm length.]

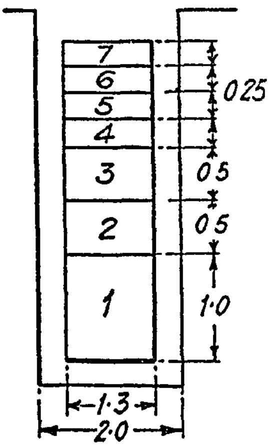
20 Prove that the kVA-rating of a synchronous machine is equal to $10{\cdot}4\ \bar{B}\ ac\ D^2\ L\ n\ 10^{-11}$, when $\bar{B}$, ac D, L and n are the mean flux density over the pole-pitch in gauss, the ampere-conductors per cm of periphery, the diameter and length of the stator in cm, and the rev per sec respectively. Assume that the stator winding is uniformly distributed with a 60° phase-spread and the average coil-span equals a pole-pitch. Determine approximately the diameter and length of the stator of a 2 500-kVA, 50-c/s, 2-pole, turbo-alternator, given: $\bar{B} = 4\,500$, $ac = 250$, air-gap 2 cm. The peripheral speed must not exceed 100 m per sec. *[67·7 cm ; 94 cm.]

21 A 2-pole 3 000-rev per min alternator has a core length of 1·5 m. Selecting a mean flux-density over the pole-pitch of 5 000 gauss (0·5 Wb/m^2), a specific loading of 260 ampere-conductors/cm, a peripheral velocity of 99 m/sec, and an air-gap of 2·5 cm, determine the output obtainable when the average span of the stator coils is (*a*) a pole-pitch, (*b*) two-thirds of a pole-pitch. [(*a*) 4 700 ; (*b*) 4 100 kVA.]

Apparent Resistance : Eddy Currents in Conductors

22 A slot, 20 mm wide, has four layers of solid copper conductors each 14 mm wide and 8 mm deep. Determine for a frequency of 50 c/s the eddy-current loss ratio in each layer and the average loss ratio in the four layers together. Find also the critical depth for minimum loss and the loss ratio for this depth. Take $\alpha = \sqrt{\text{copper width/slot width}}$, and use the simplified formulæ. [1·0, 1·133, 1·4, 1·8 ; 1·383 ; 0·78 cm ; 1·33.]

23 A slot, 50 mm wide, is to have 5 equal layers of single solid conductors each 35 mm wide. What depth of conductor would give the minimum loss ? [7 mm.]



24 Calculate the eddy-current loss ratio in each layer, and the average loss ratio, in the case shown. Deal with the slot portion only. Frequency 50 c/s. In which layer is the loss ratio the greatest ? [Layer *1*, 1·038 ; *2*, 1·055 ; *3*, 1·108 ; *4*, 1·039 ; *5*, 1·049 ; *6*, 1·060 ; *7*, 1·072 ; Average, 1 058 ; 3rd.]

Regulation, Ampere-turn Method

25 A 1 500-kVA, 6 600-V, 3-phase, synchronous generator has an open-circuit characteristic as given in Fig. 4, where 100% represents 6 600 V and an excitation of 5 000 ampere-turns per pole. Find the requisite pole excitation when the machine is running on full load at power factor 0 8 lagging, if the reactance drop be 8% and the resistance drop 2% of normal voltage.

Full-load current is obtained on short-circuit by a field excitation of 2 500 ampere-turns. Use the ampere-turn method.
*[7 400 ampere-turns.]

26 The magnetization curve of a 3-phase, 5 000-kVA, 6 600-V, star-connected alternator is given in Fig. 4, where 100% represents a phase voltage of 4 000 V and a field current of 100 A. Full load current is obtained on short-circuit with a field current of 20 A. Find the range of excitation required to give a constant terminal voltage from no load to full load at a power factor 0·6 lagging. Armature leakage reactance and resistance per phase are 1 Ω and 0 2 Ω respectively. If iron, friction and windage losses aggregate 100 kW and the field coils are supplied at 200 V, estimate the efficiency at full load and power factor 0 6.
*[90 A – 124 A ; 92·6%.]

27 Find approximately the exciting current and regulation on full load at power factor 0·8 lagging of a 3-phase, 1 500-kVA, 6 600-V alternator having 9 slots per pole and 6 conductors per slot. The inductive drop is 10% on full load, the resistance being negligible. The open-circuit characteristic is given in Fig. 4, where 100% represents 6 600 V and a field current of 25 A. The rotor has salient poles wound with 125 turns each.
*[45 A ; 23%.]

28 The open-circuit characteristic of a 500-kVA, 4 000-V, 8-pole, 3-phase, 50-c/s synchronous generator is given in Fig. 4, where 100% represents 4 000 V at a field excitation of 5 000 ampere-turns per pole. The equivalent armature reaction expressed in ampere-turns per pole is (1·1 × ampere-conductors per pole per phase). There are 240 conductors per phase in series. Determine (*a*) the short-circuit characteristic, and the field excitation and regulation for full load at 0 8 power factor, (*b*) leading and (*c*) lagging. The inductive voltage drop is 8% on full load and the resistance drop is negligible.
*[(*a*) 2 680 ampere-turns for 72 A ; (*b*) 3 750, – 13% ; (*c*) 7 400, 17·5%.]

Regulation, Zero-power-factor Method

29 The table give data for open-circuit, short-circuit, and full-load zero-power-factor measurements on a 6-pole, 440-V, 50-c/s, 3-phase, star-connected alternator. The effective ohmic resistance between any two terminals is 0 3 Ω.

Field current, A	2	4	6	7	8	10	12	14	16	18
Open-circuit terminal voltage, V	156	288	396	440	474	530	568	592	—	—
Short-circuit line current, A	11	22	34	40	46	57	69	80	—	—
Zero-p.f. terminal voltage, V	—	—	—	0	80	206	314	398	460	504

(*a*) Plot to a base of field current the terminal voltage on open-circuit and the line current on short-circuit. Derive therefrom a curve of synchronous reactance.

(*b*) Plot to a base of field current the terminal voltages on open-circuit and on zero-power-factor load. Derive therefrom a curve of synchronous reactance and compare with that found in (*a*).

(*c*) Find by the zero-power-factor method for (*b*), and by the synchronous-impedance method for (*a*), the percentage regulation for full-load output of 40 A at rated voltage and 0·8 power factor lagging. [(*a*) 31% ; (*b*) 34%.]

30 A 1 000-kVA, 11 000-V, 3-phase, star-connected alternator has an effective resistance of 2 Ω per phase. The characteristics on open-circuit and with full load current at zero power factor, and the open-circuit core losses, are :—

Field current, A	Open circuit terminal voltage, V	Core loss, kW	Saturation curve, zero p.f., V
40	—	—	0
50	7 000	7·5	—
110	12 500	16·6	8 500
140	13 750	22·4	10 500
180	15 000	33·5	12 400

Deduce by the zero power factor method (*a*) the percentage regulation for full load at a lagging power factor of 0·8. Find also (*b*) the efficiency at this load, given that the field circuit has a resistance of 0·5 Ω and that the mechanical and additional losses amount to 10 kW. *[(*a*) 22% ; (*b*) 93·5%.]

31 A 5 000-kVA, 6 600-V, 3-phase, star-connected alternator has a resistance of 0·075 Ω per phase. Estimate by the zero power factor method the regulation for a load of 500 A at power factor (*a*) unity, (*b*) 0·9 leading, (*c*) 0·71 lagging, from the following open-circuit and full load, zero-power-factor curves :—

Field current, A	Open-circuit terminal voltage, V	Saturation curve, zero p.f., V
32	3 100	0
50	4 900	1 850
75	6 600	4 250
100	7 500	5 800
140	8 800	7 000

*[(*a*) 6·3% ; (*b*) − 7·9% ; (*c*) 20·2%.]

Regulation, Synchronous-impedance Method

32 The effective resistance of a 2 200-V, 50-c/s, 440-kVA, 1-phase alternator is 0·5 Ω. On short-circuit a field current of 40 A gives the full load current of 200 A. The electromotive force on open-circuit with the same field excitation is 1 160 V. Calculate the synchronous impedance and reactance.
[5·80 Ω ; 5·77 Ω.]

33 If a field excitation of 10 A in a certain alternator gives a current of 150 A on short-circuit and a terminal voltage of 900 V on open-circuit, find the internal voltage drop with a load current of 60 A. [360 V.]

34 A 550-V, 55-kVA, 1-phase alternator has an effective resistance of 0·2 Ω. A field current of 10 A produces an armature current of 200 A on short-circuit and an electromotive force of 450 V on open-circuit. Calculate (*a*) the synchronous impedance and reactance, and (*b*) the full load regulation with power factor 0·8 lagging. [(*a*) 2·25 Ω, 2·24 Ω ; (*b*) 31%.]

35 Determine the voltage regulation of a 2 000-V, 1-phase alternator giving a current of 100 A at (*a*) unity power factor ; (*b*) power factor 0·8 leading ; and (*c*) power factor 0·71 lagging. From the test results :—full load current, 100 A, is produced on short-circuit by a field excitation of 2 5 A ; an electromotive force of 500 V is produced on open-circuit by the same excitation. The armature resistance is 0 8 Ω.
[(*a*) 7% ; (*b*) – 8·9% ; (*c*) 21·5%.]

36 A 1 500-kVA, 6 600-V, 3-phase, star-connected alternator with a resistance of 0·4 Ω and reactance of 6 Ω per phase, delivers full load current at power factor 0·8 lagging, and normal rated voltage. Estimate the terminal voltage for the same excitation and load current at 0·8 power factor leading.
[8 220 V.]

37 A 3-phase, star-connected alternator is rated at 1 600 kVA, 13 500 V. The armature effective resistance and synchronous reactance are 1·5 Ω and 30 Ω respectively per phase. Calculate the percentage regulation for a load of 1 280 kW at power factors of (*a*) 0·8 leading ; (*b*) unity ; (*c*) 0·8 lagging.
[(*a*) – 11 8% ; (*b*) 3·2% ; (*c*) 18·6%.]

Mechanical Coupling

38 Two identical delta-connected alternators, mechanically coupled to one prime mover, have each an armature resistance and synchronous reactance per phase of 0·3 Ω and 4 Ω respectively, and their induced electromotive forces are in phase. The machines are connected in parallel and the field excitations are so adjusted that the armatures develop 250 V and 350 V

per phase respectively. Find (*a*) the circulating armature current, (*b*) the terminal voltage, (*c*) the electrical output of the generating machine, and (*d*) the power supplied by the prime mover if the mechanical and core losses total 2 kW.

[(*a*) 12·4 A ; (*b*) 300 V ; (*c*) 0·84 kW ; (*d*) 2·28 kW.]

39 Two 50-c/s, 100-V alternators are mechanically coupled and connected in series to a circuit of resistance 5 Ω and inductance 0·0159 H. How much power will be given by each machine to the circuit when the phase difference of the two terminal voltages is (*a*) 30° ; (*b*) 45° ; (*c*) 90° ?

[(*a*) 1·65, 2·86 kW ; (*b*) 1·21, 2·92 kW ; (*c*) 0, 2·42 kW.]

Synchronous Motors

Performance : Efficiency, Losses, Power Factor, Braking

40 A 100-h.p., 500-V, 3-phase, star-connected synchronous motor has a resistance and synchronous reactance per phase of 0·03 Ω and 0·3 Ω respectively. Calculate for full load and 0·8 power factor (i) leading and (ii) lagging, the following :—(*a*) electromotive force per phase, (*b*) total mechanical power developed (watts). Assume an efficiency of 93%.

[(i) (*a*) 309 V ; (*b*) 79 200 W ; (ii) (*a*) 266 V ; (*b*) 79 200 W.]

41 The input to an 11 000-V, 3-phase, star-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are respectively 1 Ω and 30 Ω. Find (*a*) the power supplied to the motor, and the induced electromotive force for a power factor of 0·8, (*b*) leading, and (*c*) lagging. [(*a*) 915 kW ; (*b*) 13 000 V ; (*c*) 9 860 V.]

42 A 2 000-V, 3-phase, star-connected synchronous motor has an effective resistance and synchronous reactance of 0·2 Ω and 2·2 Ω per phase respectively. The input is 800 kW at normal voltage and the induced line electromotive force is 2 500 V. Calculate the line current and power factor.

[254 A ; 0·91 leading.]

43 A 3-phase synchronous machine was driven at synchronous speed by a calibrated motor. The output of the driving motor was 1 200 W with the machine under test unexcited, and 2 900 W with the armature short-circuited and the normal current of 6 A flowing. At a temperature of 25° C, the ohmic resistance per phase was 10 Ω. Calculate the effective resistance at 75° C.

[16·7 Ω.]

44 What is the output corresponding to maximum input of a 3-phase, mesh-connected, 250-V, 20-h.p. synchronous motor when the generated electromotive force is 320 V ? The effective resistance and synchronous reactance per phase are respectively 0·3 Ω and 4·5 Ω. The friction, windage, iron and excitation

losses total 800 W, and are assumed to remain constant. Give values for (*a*) horse power, (*b*) line current, (*c*) power factor.
[(*a*) 63 7 h.p. ; (*b*) 161 A ; (*c*) 0·804]

45 The excitation of a 415-V, 3-phase, mesh-connected synchronous motor is such that the induced e.m.f. is 520 V. The impedance per phase is $0{\cdot}5 + j\ 4\ \Omega$. If the friction and iron losses are constant at 1 000 W, calculate the horse-power output, line current, power factor and efficiency for (*a*) maximum power output, (*b*) maximum power input.
[(*a*) 180 h.p., 268 A, 0 89, 78·4% ; (*b*) 177 h.p., 303 A, 0·815, 74·6%.]

46 A 400-V, 6-pole, 3-phase, 50-c/s, star-connected synchronous motor has a resistance and synchronous reactance of 0·5 Ω and 4 Ω per phase respectively. It takes a current of 15 A at unity power factor when operating with a certain field current. If the load torque is increased until the line current is 60 A, the field current remaining unchanged, find the gross torque developed, and the new power factor. [262 lb-ft, 0·93.]

47 The armature winding of a 2 000-V, 3-phase, star-connected synchronous motor is rated to carry a current of 300 A. Estimate the output, field current and efficiency of the motor when working at power factors (*a*) unity, (*b*) 0·71 leading, (*c*) 0·71 lagging. Data :—Armature resistance and leakage reactance per phase, 0·2 Ω and 0 87 Ω respectively ; field resistance, 1·5 Ω ; friction, windage and exciter losses, 10 kW ; open- and short-circuit tests with corresponding core loss :—

Field current, A	Open-circuit terminal voltage, V	Core-loss, kW	Short-circuit armature current, A
10	800	—	—
20	1 500	6·5	200
25	1 760	9 0	250
30	2 000	12·0	300
40	2 350	20 0	—
50	2 600	—	—

Use the ampere-turn method to find the field current.

[(*a*) 1 295 h.p. ; 42 A ; 92·5%.
(*b*) 870 h.p. ; 59 A ; 88·2%.
(*c*) 885 h.p. ; 43 A ; 89·7%.]

48 Find the time taken to stop, and the number of revolutions made during the retardation period, when a 500-rev per min,

100-h.p. synchronous motor and its driven machine, having a stored energy of 4 200 ft-lb per rated horse power at normal speed and a constant frictional torque of 100 lb-ft, are braked by the following methods: (*a*) a constant braking torque of 1 000 lb-ft, (*b*) rheostatic braking, with constant field current and braking resistance adjusted to give an initial electric braking torque of 4 000 lb-ft.

[(*a*) 14 6 sec, 61 rev; (*b*) 15 sec, 30·4 rev.]

49 The open-circuit characteristic of a 3 300-V, star-connected, 3-phase synchronous machine is as follows:—

Field current .	31·5	86	101	122	150 A
e.m.f. . .	1 500	3 300	3 600	3 900	4 100 V

On short-circuit, an armature current of 219 A is obtained with a field current of 45 A. The resistance and leakage reactance per stator phase are 0 14 and 1·2 Ω respectively. Find the field current when the machine is running as a motor on 3 300-V bars, taking 1 000-kW at a power factor of 0·8 leading. [122 A.]

50 A 6 600-V, star-connected, 3-phase, synchronous motor works at constant voltage and constant excitation. Its synchronous impedance is $2\ 0 + j\ 20\ \Omega$ per phase. When the input is 1 000 kW, the power factor is 0·8 leading. Find the power factor when the input is increased to 1 500 kW.

[0 925 leading.]

V-curves and Locus Diagrams

51 A synchronous motor has a resistance drop of 2% and a synchronous reactance drop of 10%. Draw V-curves for 10%, 50% and 100% of full load input, using the magnetization curve given in Fig. 4: a field excitation of 100% gives an electromotive force equal to the terminal voltage (100%).

52 A 6 600-V, synchronous machine has an open-circuit characteristic given by:—

Field current . .	60	80	100	120	140	160	180 A
Armature e.m.f. .	5 300	6 200	6 800	7 200	7 500	7 700	7 900 V

On short-circuit, a field current of 80 A gives an armature current of 350 A. Determine the synchronous reactance and, neglecting resistance, construct a graph of armature current to a base of field current for the machine running as a motor and developing 400 h.p. Neglect losses. [17·75 Ω.]

53 A bipolar, 3-phase, star-connected turbo-alternator is rated at 3 000 kW, 6 600 V, 50 c/s. The open-circuit characteristic is

given in Fig. 4, where 100% represents 6 600 V and a field current of 100 A. On short-circuit a field current of 75 A gives a phase current of 300 A. The phase resistance and leakage reactance are respectively 0·1 Ω and 1·25 Ω. Draw the V-curve of the machine running as a synchronous motor with a constant full load input. Take values of the field current between the limits 60 A and 180 A.

54 Draw curves showing the relation between (*a*) the armature and field currents, and (*b*) the power factor and field current, for a synchronous motor with a constant power input of 1 000 kW at 10 000 V. The equivalent resistance and synchronous reactance are respectively 2 Ω and 25 Ω. Use the magnetization curve in Fig. 4, where 100% represents 10 000 V obtained on open-circuit with a field current of 50 A.

55 Using the data in Problem 47, draw V-curves for the motor for input currents having constant active components of (*a*) 200 A, (*b*) 100 A. Use the ampere-turn method.

56 A 1 000-h.p., 6 000-V, 3-phase, star-connected synchronous motor has a synchronous impedance of $1{\cdot}5 + j\,16{\cdot}0$ Ω per phase. It is excited to develop an open-circuit electromotive-force of 5 000 V. Draw the locus diagram of the current for loads up to 1 250 h.p., with constant excitation. Determine the maximum value of the power factor. *[0·89.]

Synchronous-induction Motors

57 The no-load and short-circuit tests of a 20 h.p., 400-V, 50-c/s, 3-phase induction motor gave the following line values :—

No-load test :—400 V, 9 A, $\cos\phi = 0{\cdot}2$.
Short-circuit test :—200 V, 50 A, $\cos\phi = 0{\cdot}4$.

The machine is to run as a synchronous motor to give a full load leading power factor of 0·9. Find the full load stator current, the equivalent rotor excitation, the synchronous pull-out torque and the power factor at one-half and at one-quarter full load.
*[26 5 A ; $30\sqrt{2}$ A ; 138% full load torque ; 0·56, 0 33.]

58 A 400-V, 3-phase, 50-c/s, induction motor has a magnetizing current of 15 A. When at rest, the voltage between slip-rings of the star-connected rotor is 150 V. As a synchronous-induction motor, direct current is supplied between one slip-ring and the other two joined together. Find the rotor current when the motor takes 60 A at a power factor of 0·8 leading. [264 A.]

59 A 240-V, 3-phase, 50-c/s, slip-ring induction motor takes 2·5 A and 80 W on no load, and 8·5 A at a power factor of 0·84 lagging on full load. Stator and rotor have the same number of turns per phase, and each is star-connected. If a direct current is supplied between one slip-ring and the other two

joined together when run as a synchronous-induction motor, find the value of rotor current necessary to raise the full load power factor to unity. [9·9 A.]

60 A 100-h.p., 50-c/s, 440-V, 3-phase, star-connected synchronous-induction motor is supplied by an exciter rated at 8 kW. What is the lowest leading power factor that can be obtained when both machines are delivering full load? The stator/rotor transformation ratio is 1·5, the rotor resistance is 0 05 Ω per phase and there is a drop of 1 V per brush at the slip rings. The no-load and short-circuit test results of the machine when run as an induction motor are as follows :—

No load : 440 V, 30 A, cos ϕ = 0 15 ;
Short-circuit : 220 V, 200 A, cos ϕ = 0 25.

Determine also the maximum kVA which the machine can draw on no load and the corresponding power factor. *[0·8 ; 88 5 kVA, 0 035.]

61 A 50-h.p., 440-V, 50-c/s, 3-phase, 6-pole. star-connected synchronous induction motor gave the following results when tested as an induction motor :—No-load : 440-V, 20 A, 1 400 W ; short-circuit : 220 V, 110 A, 12 000 W. The rotor resistance is 0 11 Ω per phase, and the stator/rotor transformation-ratio is 1·2. Draw the circle diagram and determine the full load power factor and pull-out torque when running as (*a*) an induction motor, (*b*) a synchronous motor with two of the rotor phases connected in series and supplied with 5 kW from the exciter. Allow a voltage drop of 1 V per brush at the slip rings. (*c*) Find also the exciting current required to give unity power factor at full load.

[(*a*) 0 85 lag, 487 lb-ft ; (*b*) 0·56 lead, 525 lb-ft ; (*c*) 72 A].

CHAPTER XXX

INDUCTION MACHINES

Induction Generator

1 An induction generator having a synchronous speed of 750 rev per min is driven at 800 rev per min, and, with the rotor short-circuited, the stator output is 9 kW. Determine (*a*) the rotor copper loss, (*b*) the rotor resistance per phase, given that the voltage across slip-rings at standstill equals 260 V. Neglect stator resistance and rotor reactance. (*c*) By the addition of external resistance in the rotor circuit the stator output is reduced to 4 kW. Determine the value of the external resistance inserted per phase. [(*a*) 0 6 kW ; (*b*) 0 5 Ω ; (*c*) 0·63 Ω]

2 A cage-rotor induction generator is driven successively at two speeds, n_1 and n_2. The stator outputs corresponding to these speeds are 9 kW and 4 kW respectively. Given that the ratio n_1/n_2 is 1·043 and that the synchronous speed of the generator is 750 rev per min, find (*a*) n_1 and n_2, (*b*) the rotor resistance per phase, and (*c*) the rotor copper loss corresponding to each output. Rotor voltage per phase at standstill = 150 V. Neglect rotor reactance and stator resistance.
[(*a*) 812 ; 778 rev per min ; (*b*) 0 62 Ω ; (*c*) 0·743 kW, 0·149 kW.]

3 A 3-phase induction generator, star-connected to 500-V mains, works at a negative slip of 2·0 per cent. The equivalent rotor standstill impedance is $0{\cdot}06 + j\ 0{\cdot}21\ \Omega$/phase, and the magnetizing-current component is $2 - j\ 36$ A. The stator impedance and the mechanical losses are negligible. Calculate per phase (*a*) the rotor input, (*b*) the rotor losses, (*c*) the stator power output, (*d*) the terminal power factor, and (*e*) the lagging kilovars taken by the stator and their distribution
[(*a*) 28·5 kW ; (*b*) 0·56 kW ; (*c*) 27 4 kW ; (*d*) 0·91 ; (*e*) 12 4 kVAr, 10·4 for magnetization, 2·0 for rotor.]

Induction Motors : Magnetic Circuit and Design

4 Estimate the effective gap-area per pole of a 12-pole induction motor with a bore of 90 cm ; core length 25 cm ; 108 stator slots with 3 mm opening ; 144 rotor slots with 2 mm opening ; gap length 1 mm ; two ducts each 1 cm wide in both stator and rotor. [516 cm^2.]

5 Calculate the requisite single air-gap excitation required for a sine-distributed mean gap flux of 1 8 megamaxwells (0 018 Wb) in an induction motor with the following particulars :—8 poles ; stator bore, 45 cm ; core length, 22 cm ; 2 radial ventilating

ducts, each 1·5 cm ; 9 stator slots and 7 rotor slots per pole ; slot openings, 1 8 mm ; single air-gap, 0·8 mm.
[550 ampere-turns.]

6 The magnetic circuit of a 440-V, 6-pole, 3-phase, star-connected, 50-c/s induction motor has the following particulars :— core-length, 15 cm ; stator teeth, length 3 cm ; width at one-third length from bore, 0·7 cm ; rotor teeth, length 1·5 cm, width at one-third length from root, 1·05 cm ; stator bore, 40 cm ; gap, effective length 0·9 mm ; stator and rotor core depths, 6 5 cm ; mean 60° lengths of magnetic circuit per pole-pair, stator 25 cm, rotor 16 cm. The stator has 72 slots with 8 conductors per slot. The rotor has 49 slots. Neglecting voltage drop, estimate the magnetizing current, using the curve for armature steel plates, Figs. 1 and 2. *[14 A.]

7 A 10-h p , 6-pole induction motor has an efficiency of 85%. Its maximum power factor of 0·82 occurs at full load. Estimate the effect on (i) the maximum power factor, (ii) the full-load output, (iii) the full-load efficiency, of (*a*) a reduction of 25% in the air-gap ; (*b*) rewinding for 4 poles with the original gap Neglect the reluctance of the iron path.
[(*a*) (i) 0 86 ; (ii) 10 5 h.p. ; (iii) 85% ; (*b*) (i) 0·87 ; (ii) 15 9 h.p. ; (iii) 90%.]

8 A 100-h.p. induction motor with a synchronous speed of 750 rev per min at 50 c/s has 96 stator slots with 4 conductors per slot, and 120 rotor slots with 2 conductors per slot. If the full-load efficiency be 0·92 and power factor 0 9, calculate the current per phase when the stator is star-connected (i) to a 3-phase, 500-V supply, and (ii) to a 440-V, 2-phase supply. Find the number of turns per phase on stator and rotor in each case. The rotor is wound 3-phase.
[(i) 104 A ; (ii) 102 5 A ; stator turns, 64, 96 ; rotor turns, 40.]

9 Find suitable dimensions, D and L, for a 10-h.p., 220-V, 50-c/s, 3-phase, cage-type induction motor to run at a speed near to 1 500 rev per min and to be started by means of a star-delta switch. [$D = 18$ cm, $L = 13{\cdot}5$ cm.]

Losses, Efficiency

10 A 6-pole, 50-c/s, 3-phase induction motor running on full load develops a useful torque of 120 lb-ft, and it is observed that the rotor electromotive force makes 90 complete cycles per min. Calculate the brake horse-power. If the mechanical torque lost in friction be 10 lb-ft, find the copper loss in the rotor windings, the input to the motor and the efficiency. Stator losses total 750 W. [22 1 h.p. ; 550 W ; 19·2 kW ; 86%.]

11 The power input to a 3-phase induction motor is 60 kW. The stator losses total 1 kW. Find the total mechanical power

developed and the rotor copper loss per phase, if the motor is running with a slip of 3%. [57·2 kW ; 590 W.]

12 A 500-V, 3-phase induction motor has a stator impedance of $0{\cdot}062 + j\ 0{\cdot}21\ \Omega$. The equivalent rotor impedance at standstill is the same. The magnetizing current is 36 A, the core loss is 1 500 W, and the mechanical loss is 750 W. Estimate the output, efficiency and power factor at a slip of 2·0%.

[100 h.p. ; 0·926 ; 0·89.]

13 A 25-h.p., 6-pole, 50-c/s, 3-phase, slip-ring induction motor runs at 960 rev per min on full load with a rotor current per phase of 35 A. Allowing 250 W for the copper loss in the short-circuiting gear and 1 000 W for mechanical losses, find the resistance per phase of the 3-phase rotor winding. [0·153 Ω.]

14 The power input to the rotor of a 440-V, 50-c/s, 6-pole, 3-phase induction motor is 80 kW. The rotor electromotive force is observed to make 100 complete alternations per min. Calculate (*a*) the slip ; (*b*) the rotor speed ; (*c*) the mechanical power developed ; (*d*) the rotor copper loss per phase ; (*e*) the rotor resistance per phase if the rotor current is 65 A.

[(*a*) 0·033 ; (*b*) 967 rev per min ; (*c*) 103·5 h.p.]
(*d*) 880 W ; (*e*) 0·208 Ω.;

15 An induction motor has an efficiency of 0·9 when the load is 50 h.p. At this load, the stator copper and rotor copper loss each equals the iron loss. The mechanical losses are one-third of the no load loss. Calculate the slip. [0·312.]

16 A 3-phase, 500-V, 50-c/s induction motor with 6 poles develops 20 b.h.p. at 950 rev per min with a power factor of 0·86. The mechanical losses total 1 h.p. Calculate for this load (*a*) the slip ; (*b*) the rotor copper loss : (*c*) the input if the stator losses total 1 500 W ; (*d*) the line current.

[(*a*) 0·05 ; (*b*) 820 W ; (*c*) 18 0 kW ; (*d*) 24·0 A.]

17 The power input to a 500-V, 50-cycle, 6-pole, 3-phase induction motor running at 975 rev per min is 40 kW. The stator losses are 1 kW and the friction and windage losses total 2 kW. Calculate (*a*) the slip ; (*b*) the rotor copper loss ; (*c*) the brake horse-power ; (*d*) the efficiency.

[(*a*) 0·025 ; (*b*) 975 W ; (*c*) 48·4 h.p. ; (*d*) 90%.]

18 A 6-pole, 3-phase induction motor develops 30 h.p., including mechanical losses total 2 h.p., at a speed of 950 rev per min on 550-V, 50-c/s mains. The power factor is 0·88. Calculate for this load (*a*) the slip ; (*b*) the rotor copper loss ; (*c*) the total input if the stator loss[illegible]e 2 000 W ; (*d*) the efficiency ;

(*e*) the line current ; (*f*) the number of complete cycles of the rotor electromotive force per minute.
[(*a*) 0·05 ; (*b*) 1 175 W ; (*c*) 25·6 kW ; (*d*) 82% ; (*e*) 30·4 A ; (*f*) 150.]

Slip, Torque (Maximum and Starting)

19 A 12-pole, 3-phase alternator is coupled to an engine running at 500 rev per min. It supplies an induction motor which has a full load speed of 1 440 rev per min. Find the percentage slip and the number of poles of the motor. [4% ; 4 poles.]

20 If the electromotive force in the stator of an 8-pole induction motor has a frequency of 50 c/s, and that in the rotor $1\frac{1}{2}$ c/s, at what speed is the motor running and what is the slip ?
[728 rev per min ; 0 03.]

21 A 3-phase induction motor has a 4-pole, star-connected stator winding. The motor runs on a 50-c/s supply with 200 V between lines. The rotor resistance and standstill reactance per phase are 0·1 Ω and 0 9 Ω respectively. The ratio of rotor to stator turns is 0 67. Calculate :—(*a*) total torque at 4% slip ; (*b*) total mechanical power at 4% slip ; (*c*) maximum torque ; (*d*) speed at maximum torque ; (*e*) maximum mechanical power. Prove the formulæ employed, neglecting stator impedance.
[(*a*) 4·08 kg-m ; (*b*) 8 05 h.p. ; (*c*) 6·5 kg-m ; (*d*) 1 335 rev per min ; (*e*) 12 0 h.p.]

22 A 200-h.p., 1 000-V, 25-c/s, star-connected induction motor has a star-connected, slip-ring rotor with a transformation ratio of 3 6. The rotor resistance per phase is 0 01 Ω and inductance 0 64 mH. The stator losses may be neglected. Find (*a*) the rotor starting current per phase on normal voltage with slip-rings short-circuited ; (*b*) the rotor power factor at starting ; (*c*) the rotor current at 3% slip ; (*d*) the rotor power factor at 3% slip ; (*e*) the necessary external resistance per phase to obtain a starting current of approximately 300 A in the stator.
[(*a*) 1 600 A ; (*b*) 0 1 ; (*c*) 460 A ; (*d*) 0 96 ; (*e*) 0·1 Ω.]

23 A 3-phase induction motor with star-connected rotor has an induced electromotive force of 60 V between slip-rings at standstill on open-circuit with normal voltage applied to the stator. The resistance and standstill reactance of each rotor phase are 0·6 Ω and 4 Ω respectively. Calculate the current per phase in the rotor (*a*) when at standstill and connected to a star-connected rheostat of resistance 5 Ω and reactance 2 Ω per phase ; (*b*) when running short-circuited with 4% slip.
[(*a*) 4·22 A ; (*b*) 2·22 A.]

24 A 3-phase induction motor has a 4-pole, star-connected, stator winding, and runs on a 220-V, 50-c/s supply. The rotor

resistance is 0·1 Ω and reactance 0·9 Ω. The ratio of stator to rotor turns is 1·75. The full load slip is 5%. Calculate for this load (*a*) the total torque ; (*b*) the horse power. Find also (*c*) the maximum torque ; (*d*) the speed at maximum torque.

[(*a*) 4·28 kg-m, 31 lb-ft ; (*b*) 8·4 h.p. ; (*c*) 5·78 kg-m, 41·5 lb-ft ; (*d*) 1 330 rev per min.]

25 A 3 000-V, 24-pole, 50-c/s, 3-phase, star-connected induction motor has a slip-ring rotor of resistance 0 016 Ω and standstill reactance 0·265 Ω, per phase. Full load torque is obtained at a speed of 247 rev per min. Calculate (*a*) the ratio of maximum to full-load torque ; (*b*) the speed at maximum torque. Neglect stator impedance. [(*a*) 2 61 ; (*b*) 235 rev per min.]

26 A slip-ring induction motor runs at 290 rev per min on full load when connected to a 50-c/s supply. Calculate (*a*) the number of poles ; (*b*) the slip ; (*c*) the slip for full-load torque if the total resistance of the rotor circuit be doubled.

[(*a*) 20 ; (*b*) 3·3% ; (*c*) 6·6%.]

27 The rotor resistance and standstill reactance of a 3-phase induction motor are respectively 0·015 Ω and 0 09 Ω per phase. At normal voltage the full-load slip is 3%. Estimate the percentage reduction in stator voltage to develop full-load torque at one-half of full-load speed. What is then the power factor ? [22·5% ; 0 31.]

28 On a short-circuit test, a 12-pole, 3-phase, 50-c/s induction motor, with an equivalent standstill rotor resistance equal to the stator resistance, took 250 A and 100 kW. Find the starting torque developed. [705 lb-ft.]

29 The following data refer to a 12-pole, 420-V, 50-c/s, 3-phase, mesh-connected induction motor :—$R_1 = 2\ 95$, $X_1 = 6{\cdot}82$, $R_2' = 2{\cdot}08$, $X_2' = 4{\cdot}11\ \Omega$ per phase. On no load the line value of magnetizing current is 6 7 A, and the total core loss is 269 W. Determine power factor, input current, equivalent rotor current, and torque of the motor at a slip of 3% from (*a*) the "approximate" equivalent circuit ; (*b*) the "rigid" equivalent circuit.

[(*a*) 0·78 ; 13 0 A ; 5·75 A per phase ; 97 lb-ft ; (*b*) 0·78 ; 11·9 A ; 5·35 A per phase ; 88 5 lb-ft.]

30 A 400-V, 3-phase, star-connected induction motor has an equivalent T-circuit consisting of a stator impedance of $1 + j\ 2\ \Omega$, an equivalent rotor impedance at standstill of $1{\cdot}2 + j\ 1{\cdot}5\ \Omega$, and a magnetizing branch impedance of $4 + j\ 40\ \Omega$ per phase. Determine the current, efficiency, power factor and output when the slip is 5%. Assume friction losses of 250 W.

[10·8 A ; 80·8% ; 0·815 ; 6·7 h.p.]

31 An 8-pole, 50-c/s, 3-phase induction motor has an equivalent rotor resistance of 0·07 Ω/phase. If its stalling speed is 630 rev per min, how much resistance must be included per phase to obtain maximum torque at starting? Ignore magnetizing current. [0·37 Ω.]

32 In a 3-phase induction motor the stator reactance is equal to the equivalent rotor standstill reactance, whilst each resistance is one-fourth of this value. A torque of 300 lb-ft is developed at a slip of 3·0%: find the starting and maximum torques. Ignore magnetizing current. [165, 620 lb-ft.]

33 An 8-pole, 500-V, 50-c/s, star-connected induction motor has a stator impedance of $0{\cdot}0615 + j\,0{\cdot}21$ Ω and an equivalent rotor impedance at standstill of $0{\cdot}065 + j\,0{\cdot}214$ Ω. Neglecting the magnetizing current, find (*a*) the maximum gross power output and the slip at which it occurs, and (*b*) the maximum gross torque and its corresponding slip. Determine the corresponding values when the stator impedance is ignored.
[(*a*) 295 h.p., 0·128; (*b*) 2 395 lb-ft, 0·151. (*a*) 582 h.p., 0·226; (*b*) 5 500 lb-ft, 0·304.]

34 The following test results were obtained from a 3-phase 20-h.p., 400-V, 50-c/s, 6-pole induction motor. No load: 400 V, 10 A, power factor 0 08; short-circuit: 200 V, 50 A, power factor 0 415. The motor drives a load having a constant torque of 130 lb-ft. Estimate the possible percentage reduction in the supply voltage before the motor stalls. Assume that the copper losses are equally divided between the stator and the rotor. [18%.]

35 A 50-h.p. induction motor with a star-delta starter is supplied through a feeder from a 440-V, 50-c/s main. Owing to line drop the starting torque is found to be the same with star as with mesh connexions. Determine the resistance of the feeder. From a short-circuit test on the motor when mesh-connected the following data were obtained: $V = 200$ V; $I = 125$ A, $\cos \phi = 0{\cdot}4$ (line values). If a second feeder of the same cross-section be run in parallel with the original one, find the percentage increase in starting torque obtained with each connexion. [1·61 Ω; 37%, 119%.]

36 Each phase of the rotor of a 3-phase induction motor has a resistance of 1·0 Ω and a standstill reactance of 4·0 Ω. When running from a symmetrical 3-phase, 400-V supply at 5% slip, the torque developed is 20 lb-ft. Calculate the torque when running with the same slip from a supply having the following voltages to neutral: $V_A = 240\underline{/0^\circ}$, $V_B = 188\underline{/229^\circ}$, $V_C = 188\underline{/131^\circ}$. Ignore stator impedance. [14·6 lb-ft.]

37 Calculate the slip of a 100-h.p., 500-V, 8-pole, 3-phase, star-connected, cage induction motor from the following data: Stator winding: single-layer, 4 slots per pole per phase, 4 conductors per slot. Rotor winding: 67 slots, bars 11 mm dia × 29 cm long, end-rings 15 mm × 15 mm cross-section, 48·5 cm mean diameter. Full load efficiency and power factor may be taken as 0 92 and 0·89 respectively. Mechanical losses are 750 W. Take resistivity of copper as 0·02 Ω/metre/mm^2. [2·05%.]

38 A 40-c/s, 3-phase induction motor designed for voltage V_1 is switched direct on to 50-c/s mains of voltage V_2. Ignoring stator impedance, find (*a*) the ratio of currents and torques at starting and of maximum torques when $V_2 = V_1$, (*b*) the ratio of V_2 to V_1 to give the same values of starting current and torque at 50 as at 40 c/s. [(*a*) 0·8, 0·64, 0·64, (*b*) 1·25.]

39 A 50-h.p. induction motor has a standstill impedance of $1{\cdot}1 + j\ 2{\cdot}5\ \Omega$ per phase. The motor is fed from 440-V, 50-c/s mains through a feeder 500 yd long. Determine the minimum allowable cross-section of each conductor of the feeder in order that the starting torque shall not be reduced by more than 30% due to voltage drop in the feeder. The machine is started through a star-delta starter. [0·012 in^2.]

40 The cages of a double-cage induction motor have standstill impedances of $3{\cdot}5 + j\ 1\ 5\ \Omega$ and $0{\cdot}6 + j\ 7{\cdot}0\ \Omega$ respectively. The full-load slip is 6%. Find the starting torque at normal voltage in terms of full-load torque. Neglect stator impedance and magnetizing current. [300%.]

41 An induction motor has a double-cage rotor with equivalent impedances at standstill of $1\ 0 + j\ 1\ 0$ and $0\ 2 + j\ 4{\cdot}0\ \Omega$. Find the relative values of torque given by each cage (*a*) at starting, and (*b*) at 5% slip. [(*a*) 40 : 1 ; (*b*) 0·4 : 1.]

Torque/speed Curves

42 Plot the torque/speed curve of a 6-pole, 50-c/s, 3-phase induction motor. The rotor resistance and reactance per phase are 0 02 Ω and 0·1 Ω respectively. At what speed is the torque a maximum? What must be the value of the external rotor resistance per phase to give two-thirds of maximum torque at starting? [(*a*) 800 rev per min ; (*b*) 0·242 Ω or 0·018 Ω.]

43 Plot the torque/speed curve of a 3-phase induction motor with rotor resistance 0·02 Ω and reactance 0 1 Ω, each per phase. The motor has 4 poles and runs on a 50-c/s supply. Find (*a*) the speed at which maximum torque is developed; (*b*) the value of the external rotor starter-resistance per phase in order to

obtain maximum torque at starting; and (*c*) by what percentage the resistance in (*b*) will reduce the starting current.

[(*a*) 1 200 rev per min; (*b*) 0·03 Ω; (*c*) about 27%.]

44 The output of a 3-phase, 50-c/s, 6-pole induction motor is 5 h.p., at 935 rev per min. (*a*) Calculate the input if the stator losses are 400 W. (*b*) What starting torque will the machine develop when switched directly on to the supply if maximum torque is developed at 800 rev per min? Allow 1% for windage and friction. [4·03 kW; 18·6 lb-ft.]

45 The rotor of a 3-phase induction motor can have a ratio of resistance to standstill reactance per phase of (*a*) 0·1, (*b*) 0·5, (*c*) 1 0. Draw the torque/speed curves for these values and from them derive curves of mechanical power/speed.

46 An 8-pole, 50-c/s induction motor has a full-load slip of 2½% and a maximum torque of twice full-load torque. Draw to scale the torque/speed curve, taking maximum torque as unity. At what value of the slip does the maximum torque occur? [0·093.]

47 Plot the torque/speed curve of a 4-pole, 50-c/s, 3-phase induction motor with rotor resistance and reactance 0·03 Ω and 0·12 Ω per phase respectively. What is the value of the speed at which maximum torque occurs? Find the amount of external rotor resistance per phase to be inserted to obtain 75% of maximum torque at starting. By what percentage will this reduce the current, and what will now be the power factor at standstill?

[1 125 rev per min; 0·285 Ω or 0·025 Ω; 57 or 6·2%; 0·915 or 0·42.]

Circle Diagram

48 Draw the no load and short circuit diagram for a 20 h.p., 400-V, 50-c/s, 3-phase star-connected induction motor from the following data (line values):—

No load test:—400 V, 9 A, cos $\phi = 0{\cdot}2$.
Short circuit test:—200 V, 50 A, cos $\phi = 0{\cdot}4$.

From the diagram find (*a*) the line current and power factor at full load, and (*b*) the maximum horse power.

*[(*a*) 32·0 A, 0·85; (*b*) 29 h.p.]

49 Draw the no load and short circuit diagram for a 5 h.p., 200-V, 50-c/s, 4-pole, 3-phase, star-connected induction motor from the following test data:—

No load:—Line voltage, 200 V; line current, 5 A; total input, 350 W.
Short circuit:—Line voltage, 100 V; line current, 26 A; total input, 1 700 W.

Estimate from the diagram for full load conditions the line current and power factor; also the maximum torque and starting torque in terms of the full load torque. The rotor copper loss at standstill is half the total copper loss.

*[15 A; 0 83; 180%; 83%.]

50 A 40-h.p., 500-V, 6-pole, 3-phase, 50-c/s, mesh-connected induction motor has a phase turns-ratio of 2. The resistance per phase of the stator winding is 0·6 Ω and of the rotor winding 0·15 Ω. Draw the no load and short circuit diagram and find (*a*) the full load line current, slip, and power factor, (*b*) the maximum output, (*c*) the starting torque with rotor short circuited, (*d*) the pull-out torque. The no load and short circuit test data (line values) are :—

No load :—500 V, 18 A, total input 1 200 W.
Short circuit :—250 V, 100 A, total input 11 000 W.

*[(*a*) 46 A, 2 6%, 0·88; (*b*) 81 h.p.; (*c*) 165 lb-ft; (*d*) 483 lb-ft.]

51 Tests on a 5-h.p., 500-V, 3-phase, 6-pole, cage-type induction motor gave the following figures :—No load current, 3 A; short circuit current on 250 V, 12 A at power factor 0·6; full load current, 7 A; full load slip, 3·5%. Estimate (*a*) the maximum output; (*b*) the starting torque on normal voltage.

*[(*a*) 7 6 h.p.; (*b*) 11·2 lb-ft.]

52 Draw the no load and short circuit circle diagram for a 3-phase mesh-connected, 30-h.p., 500-V, 4-pole, 50-c/s, cage-type induction motor. The figures below give the measurements of line current and voltage and readings of two wattmeters connected to measure the power input :—

No load :—500 V, 8·3 A, + 2·85 kW, − 1·35 kW.
Short circuit :—100 V, 32 A, − 0·75 kW, + 2·85 kW.

Find from the diagram for full load (*a*) the line current, (*b*) the power factor. Calculate (*c*) the efficiency, (*d*) the maximum output.

*[(*a*) 33 A; (*b*) 0·9; (*c*) 88%; (*d*) 68 h.p.]

53 The following data refer to the design of a 100-h.p., 50-c/s, 8-pole, 500-V, slip-ring induction motor with 3-phase, star-connected stator winding :—Turns per phase: stator, 64; rotor, 35. Resistance per phase: stator, 0·062 Ω; rotor, 0 019 Ω. Reactance per phase: stator, 0·21 Ω; rotor, 0·064 Ω. Magnetizing current, 36 A per phase. Iron loss, 1 500 W. Friction and windage loss, 750 W. Draw the no load and short circuit circle diagram and deduce therefrom the line current, efficiency, power factor and slip for (*a*) full load and (*b*) half load

conditions. Find also (*c*) the maximum output and the pull-out torque.

*[(*a*) 104 A, 92·7%, 0·89, 0·022 ; (*b*) 60 A, 92·0%, 0·77, 0·011 ; (*c*) 280 h.p., 2 300 lb-ft.]

54 The test figures for a 3-phase, mesh-connected, 30-h.p., 500-V, 4-pole, 50-c/s, cage-type induction motor are as follows :—

No load :—

Line Voltage, V	Line current, A	Wattmeters, kW	
		1	2
564	9·8	− 2 07	3·69
515	8 6	− 1·55	3·05
480	7·8	− 1·22	2·62
400	6·5	− 0·70	1·95
320	5·4	− 0·25	1·40
220	4·2	+ 0·15	0 90
160	4 3	0·28	0 69
75	7 9	0 36	0·60

Short circuit :—

Line Voltage, V	Line current, A	Wattmeters, kW	
		1	2
60	18 6	− 0·27	0 83
100	32·0	− 0·75	2 35
140	44·8	− 1·50	4·65

The stator resistance (hot) is 0 68 Ω per phase. Draw the circle diagram and compute the separate losses, efficiency, power factor and slip for 50%, 100%, and 150% of full load. Find also the values of maximum torque, maximum output, and maximum power factor. *[2 800 lb-ft ; 68 h.p. ; 0·91.]

Induction-motor Speed Control : Resistance

55 A 3-phase induction motor with a synchronous speed of 1 000 rev per min develops 5 mechanical h.p. at 935 rev per min. What is the stator input if the stator loss is 400 W ? If a rheostat be included in the rotor circuit so that the converted mechanical power is 6 h.p. at 750 rev per min, what is the stator input, assuming that the stator losses are doubled ?

[4·4 kW ; 6·76 kW.]

56 The rotor of a 4-pole, 50-c/s, slip-ring induction motor has a resistance of 0·25 Ω per phase and runs at 1 440 rev per min at full load. Calculate the external resistance per phase which must be added to lower the speed to 1 200 rev per min, the torque being the same as before. [1 Ω.]

57 The rotor of a 6-pole, 50-c/s, slip-ring induction motor has a resistance of 0·2 Ω per phase, and runs at 960 rev per min on full load. Calculate the approximate resistance per phase of a rotor rheostat such that the speed is reduced to 800 rev per min for full-load torque. [0·8 Ω.]

58 The normal full load slip and shaft torque of a 500-h.p., 50-c/s, 3-phase induction motor are respectively 1·9% and 7 000 lb-ft. The rotor winding has a resistance of 0·25 Ω and a standstill reactance of 1·5 Ω per phase. Estimate the slip and power output for full-load stator current when external resistances of 2 Ω per phase are inserted in the rotor circuit. Neglect magnetizing current. [17·1% ; 422 h.p.]

59 A 50-c/s, 8-pole induction motor, equipped with a flywheel, supplies a constant load torque of 100 lb-ft, and, at wide intervals, an additional load torque of 300 lb-ft for 6 sec. The motor has a slip of 6% when supplying a torque of 300 lb-ft. What is the minimum moment of inertia of the flywheel if the motor torque is not to exceed 250 lb-ft ? How much energy is supplied by the flywheel at each retardation ? What period elapses after the removal of the additional load before the motor torque is reduced to 150 lb-ft ?

[17 700 lb-ft^2 ; 97 000 lb-ft ; 9·7 sec.]

Cascade Connexions

60 What are the synchronous speeds possible with two electrically- and mechanically-coupled induction motors with 6 poles and 8 poles respectively, running on a 50-c/s supply ?

[3 000, 1 000, 750, 429 rev per min.]

61 A cascade induction motor set consists of two machines : the main motor has 6 poles and the auxiliary motor 4 poles Find the possible synchronous speeds obtainable on a 50-c/s supply. [3 000, 1 500, 1 000, 600 rev per min.]

62 Two 3-phase, 50-c/s, 220-V induction motors are mechanically coupled and connected in cascade. One machine is rated at 10 h.p. and has 4 poles. The other is rated at 15 h.p. and has 6 poles. Calculate (*a*) the synchronous speed ; (*b*) the greatest load which can be developed without either machine taking more than full-load current ; (*c*) the relative torques of each machine at this load.

[(*a*) 600 rev per min ; (*b*) 10 h.p. ; (*c*) 1·5 : 1.]

XXX. INDUCTION MACHINES

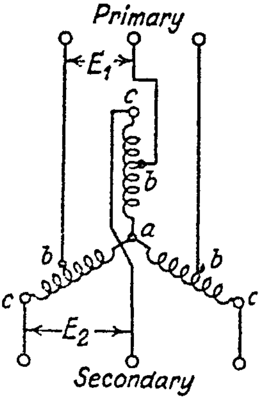
Multi-phase Transformers

63 A transformer is arranged to give a 13-phase voltage (phase-angle $= 2\pi/13$ radian), and a cage motor is provided with a stator winding suitable for tapping at 13 equi-distant points. Show that by suitable connexions it is possible to produce a multi-speed motor with 2, 4, 6 and 8 poles.

Induction Regulators : Rating

64 The voltage at the receiving end of a 3-phase feeder delivering 100 kVA varies between 475 V and 525 V. For what kilovolt-ampere capacity must an induction regulator be designed to maintain the voltage constant at 500 V ? [5 kVA.]

65 It is desired to regulate the voltage on a 3-phase feeder dealing with 1 200 kVA between the limits of 10 000 and 12 000 V by means of an induction regulator. Calculate the rating of the regulator. [109 kVA.]



66 A 3-phase induction regulator, connected as shown, is to vary the secondary voltage E_2 between the limits of $E_1 \pm e$. The power delivered is constant at W kW and power factor $\cos\phi$. Assuming an overall efficiency of η% determine the rating of the windings *ab* and *bc*. Evaluate for $E_1 = 440$ V, $e = 50$ V, $W = 10$ kW, $\cos\phi = 0{\cdot}8$, $\eta = 85\%$.
[*ab* = 3 48 kVA, *bc* = 1·6 kVA.]

Regulation

67 A 1-phase, 25-c/s induction regulator has a flux of 1·45 megamaxwells (0 0145 Wb). The stator phase has a spread of 120° and consists of 15 coils each of 2 turns. Find the root-mean-square value of the maximum electromotive force induced in the winding. If the regulator is used on an 880-V supply, find the maximum and minimum voltages available, and the rating of the regulator for a full load current of 50 A.
[40 V ; 920 V, 840 V ; 2 kVA.]

68 The resistance and inductive reactance of each winding of a 50-c/s split-phase induction motor are 80 Ω and 237·5 Ω respectively. In series with one winding is an additional resistance R and condenser C. Calculate C and R to give the same voltage across each winding with a phase difference of 90°
[10 μF ; 157·5 Ω.]

Commutator Motors

69 A "universal" series motor has a resistance of 80 Ω and an inductance of 0·5 H. When connected to a 250-V, direct-current supply and loaded to take 0·8 A it runs at 2 000 rev per min. Estimate its speed and power factor when connected to a 250-V, 50-c/s supply and loaded to take the same current.
[1 710 rev per min ; 0·87.]

70 A 1-phase series commutator motor has one turn per armature coil. The maximum permissible voltage in the coil short-circuited by the brushes is 4 V. Find the maximum value of the flux per pole when the supply frequency is $16\frac{2}{3}$ c/s. How does this restriction affect the design and the power factor of the motor? [3·83 megamaxwells (0·038 Wb).]

CHAPTER XXXI

CONVERTING MACHINES

Synchronous Convertors

Design

1 Obtain an expression for the limiting commutator voltage V_d in a synchronous convertor with frequency, f; circumferential pitch of commutator sectors, y cm; peripheral speed of commutator, v_c m per sec; and mean voltage between commutator sectors, $\bar{E}$ volts. If the maximum permissible peripheral speed of the commutator be 30 m per sec, the pitch y be 0·5 cm, and the mean voltage $\bar{E}$ be 25 V, estimate the commutator voltage for a 50-c/s machine. [$V_d = 100\ v_c\bar{E}/2fy$; 1 500 V.]

2 Prove that the direct voltage of a synchronous convertor is given by $\frac{50\ v\ \bar{E}}{fy}$ where v is the peripheral speed of the commutator in m per sec, $\bar{E}$ is the average voltage between commutator sectors, y is the pitch of the commutator sectors (copper plus mica) in cm and f is the frequency. Find the maximum voltage for which a 50-c/s convertor can be built with the following limitations, $\bar{E} = 15$, $v = 40$ and $y = 0{\cdot}4$. [1 500 V.]

3 Find the minimum number of poles that a 50-c/s, 1 000-kW synchronous convertor must have if the average voltage per commutator sector, the limiting peripheral speed of the armature in m per sec, and the ampere-conductors per cm of armature periphery are not to exceed 15, 35 and 400 respectively. [10.]

4 A 6-ring, 25-c/s, 1 100-kW synchronous convertor is to give 600 V at the commutator brushes when running unloaded, and 650 V on full load, voltage regulation being obtained by compounding in conjunction with an effective reactance (including transformer reactance) of 0 02 Ω in each slip-ring lead. The field system has 8 poles with 1 500 shunt turns per pole, and a shunt current of 5 A on no load produces 600 V on the commutator, the no load lagging current per slip-ring then being 500 A. If the estimated efficiency on full load be 95%, find the number of series turns per pole necessary, and the power factor at the slip rings, on full load. The armature has 168 slots, with 6 conductors per slot. Use the magnetization curve in Fig. 4, taking 100% voltage as 600 V.

[7 ; 0·89 leading.]

Voltage and Current Ratios, Connexions

5 Numbering the rings of a 6-ring synchronous convertor consecutively A, B, C, D, E, and F, tabulate the root-mean-square values of the available alternating voltages (*a*) in terms of the largest root-mean-square alternating voltage, and (*b*) in terms of the direct voltage.

tappings	(*a*)	(*b*)
180°	1·0	0·707
120°	0·866	0·612
60°	0·5	0·353

6 Evaluate the relations in an N-ring synchronous convertor between (*a*) the slip-ring and commutator voltages; (*b*) the alternating current in a slip-ring lead, and the direct current on the commutator side. Tabulate these values in terms of the voltage and current on the direct-current side for $N = 2, 3, 4,$ 6 and 12 rings, assuming unity power factor and efficiency.

N	(*a*)	(*b*)
2	0·71	1·41
3	0·61	0·94
4	0·5	0·71
6	0·35	0·47
12	0·19	0·235

7 Compare the root-mean-square phase voltage with the commutator voltage in (*a*) a 2-ring and (*b*) a 6-ring synchronous convertor under the following conditions of flux distribution in the gap:—(i) constant density over the whole pole-pitch; (ii) constant density over the middle two-thirds of pole-pitch and zero elsewhere; (iii) sinusoidally-distributed density over the pole-pitch. Assume a uniformly distributed winding.

[Ratio $\frac{V_a}{V_d}$:—(*a*) (i) 0 578 ; (ii) 0·746 ; (iii) 0·707.
(*b*) (i) 0·294 ; (ii) 0·373 ; (iii) 0·354.]

8 A 12-pole, 6-phase, lap-wound synchronous convertor delivers a current of 1 600 A on the direct-current side. It has 1 000 armature conductors, a flux of 6 megalines (60 mWb) per pole, and runs at 600 rev per min. Calculate (*a*) the voltage between slip-rings on the alternating-current side; (*b*) the current per slip-ring; (*c*) the current in the connexions between slip-rings and tapping points. Assume unity power factor on the alternating-current side and an armature efficiency of 95%.

[(*a*) 212 V ; (*b*) 795 A ; (*c*) 132·5 A.]

9 Find the voltage and current of one secondary phase of a 3-phase transformer feeding a 6-ring synchronous convertor loaded to 600 kW at 400 V on the commutator side. Diametral tappings are used. Take the efficiency as 95% and the power factor as 0·85. [283 V ; 875 A.]

10 Draw a diagram of the standard connexions for a 3-phase transformer and 6-ring synchronous convertor combination, the latter supplying a 3-wire direct-current system If the incoming high-voltage mains are at 10 000 V, and the high voltage side of the transformer is mesh-connected, find the number of turns per phase when there are 20 turns per phase on the low-voltage side. The commutator voltage is 550 V. [515.]

11 A transformer star-connected on the primary side to 6 600-V, 3-phase mains, supplies a 6-ring synchronous convertor having diametral tappings, and connected to a 3-wire, direct-current system with 500 V between the outers. Draw a diagram of the essential connexions. Calculate the ratio of turns on the primary and secondary sides of the transformer, and the voltage between opposite and successive slip-rings. [10·75 ; 354 V ; 177 V.]

12 A 3-phase, 250-kW synchronous convertor develops 250 V on the commutator and is supplied from a 2 200-V, 3-phase network through three 1-phase transformers mesh-connected on the high-voltage side and star-connected on the low-voltage side. The convertor operates on full load at power factor of 0 9 with an efficiency of 91%. Determine the voltage and current rating of both sides of the transformers. Neglect losses in the transformers.

[l.v., 88·5 V, 1 150 A ; h.v., 2 200 V, 46·2 A.]

13 On no load, a 3-ring synchronous convertor has a voltage ratio of 1·6, while its star-delta connected transformer has a voltage-ratio of 3 300/300 V. With the transformer secondary short-circuited and 400 V applied to the primary, the primary current was 25 A and the input was 2 100 W. Find the convertor voltage with an input current to rings of 250 A at (*a*) 0·9 power factor lagging ; (*b*) 0 9 power factor leading.

[(*a*) 450 V ; (*b*) 497 V.]

Voltage Control

14 A 50-c/s, 3-ring synchronous convertor has in each slip-ring lead a 2-Ω reactor of negligible resistance. The line voltage is 1 100 V on the alternating-current side. Calculate the commutator brush voltage when a load of 20 A (alternating) is taken with the field excited to give a power factor of (*a*) 0 866 lag ; (*b*) unity ; (*c*) 0·866 lead, at the transformer secondary terminals. [(*a*) 1 748 V, (*b*) 1 800 V, (*c*) 1 855 V.

15 A 6-ring synchronous convertor with diametral tappings is supplied from a transformer with a constant voltage of 353 V on each of its three low-voltage phases. A choking coil of 0 1 Ω reactance is included in each slip-ring lead. Find the change in voltage on the direct-current side when the power factor at the transformer terminals is changed from unity to 0·8 leading by

over-excitation of the convertor, which supplies a constant current of 750 A on the direct-current side. Neglect losses. [509 − 601 V = 92 V.]

16 A 3-phase synchronous convertor with reactance control has a lagging power factor of 0·85 when the current is 200 A per ring and the voltage at the slip-rings is 550 V. The reactance of each reactor is 0·25 Ω. Find the power factor when the current is 400 A and the slip-ring voltage is 570 V. The supply (transformer) voltage is constant. [1·0.]

17 A compound-wound, 3-phase, 3-ring synchronous convertor gave the following results on a load test :—

D C. load	0%	50%	100%
Reactive line current .	388 A lagging	30 A leading	388 A leading
Total line current . .	388 A	601 A	1 242 A

The transformer secondary voltage was constant at 876 V, and the reactance included in each line for voltage regulation was 0·06 Ω. Estimate the commutator voltage at no load, half load, and full load. [550 V, 626 V, 708 V.]

18 A 6-ring synchronous convertor with diametral tappings is supplied from a 33 000-V system through a transformer star-connected on its primary side and having a turns-ratio of 44·5. Each phase has an equivalent reactance of 0·2 Ω, considered all in the secondary. Find the change in commutator voltage due to a change in the power factor at the convertor slip-rings from unity to 0·8 leading. The direct current load is constant at 500 A. Assume a constant efficiency of 90%. [600 − 656 V.]

19 An 8-pole, 6-ring, compound-wound synchronous convertor, designed for reactance control, gives 600 V when supplying 1 500 A, the power factor at the slip-rings then being 0 9 leading. The field regulator is set so that the shunt winding produces 18 000 ampere-turns per pole at 600 V, while the excitation needed for this voltage on no load is 14 500 (= 100%, Fig. 4). The armature has 192 slots and 4 conductors per slot; the efficiency is 0·95; the transformer secondary voltage is 405 V. Find (*a*) the reactance in each of the 8 phases, (*b*) the number of series turns per pole. [(*a*) 0·061 Ω; (*b*) 8.]

Copper-loss Ratio

20 A 3-ring, 2-pole synchronous convertor supplies 100 A on the direct-current side. Calculate the root-mean-square value of the resultant current in a conductor (*a*) in the centre of a phase, and (*b*) at a phase tapping-point, under conditions of unity power factor and efficiency. State for each case the copper-loss ratio. [(*a*) 23·8 A, 0·225 ; (*b*) 55 A, 1·21.]

21 Find the copper-loss ratio of a synchronous convertor with N rings for $N = 2$, 3, 4, 6, 9, 12, and ∞, taking a power factor of 0·9 and an efficiency of 95%.

$N =$	2	3	4	6	9	12	∞
Ratio =	2·03	0·92	0·66	0·51	0·44	0·42	0·40

22 Calculate the copper-loss ratio for (*a*) a tapped coil, (*b*) a coil in the centre of a phase, and (*c*) the whole armature of a 6-ring convertor with diametral tappings, (i) at unity power factor ; (ii) at a power factor of 0 8.
[(*a*) (i) 0 42, (ii) 1·56 ; (*b*) (i) 0·19, (ii) 0·69 ; (*c*) (i) 0·27, (ii) 0·77.]

Inverted Convertor

23 A 4-pole, 1-phase, lap-wound synchronous convertor for 220 V on the commutator has 568 conductors and a polar flux of 2 63 megalines (26 3 mWb). At what speed (approximately) will it run alone as an inverted machine on no load, and what will be the slip-ring frequency ? Find also the approximate speed when armature reaction reduces the field flux to 1·75 megalines (17 5 mWb), due to a load at a low lagging power factor.
[885 rev per min ; 29·5 c/s ; 1 325 rev per min.]

Motor Convertor

24 A motor-convertor, consisting of an 8-pole induction machine driving a 6-pole direct-current machine, is fed from 11 000-V, 3-phase, 50-c/s mains, and supplies a direct-current network at 440 V. If the machine be loaded to 500 kW, calculate (*a*) the speed of the set ; (*b*) the input current ; (*c*) the output current ; (*d*) the power supplied electrically to the direct-current side from the alternating-current machine ; (*e*) the current per rotor phase in the alternating-current machine, which has 12 rotor phases. Assume unity power factor and neglect losses.
[(*a*) 430 rev per min ; (*b*) 26·2 A ; (*c*) 1 135 A ; (*d*) 214 kW ; (*e*) 114 A.]

Motor Generator

25 The 230-V generator of a constant-speed Ward-Leonard set has an armature resistance of 0·12 Ω. The armature resistance of the separately-excited main motor is 0·24 Ω. Assuming an armature current of 50 A and the O.C.C. shown in Fig. 4, find the range of generator field current required to vary the speed of the main motor from 100 to 1 200 rev per min.
[16 to 106% of normal.]

Frequency-Changers

26 An 8-pole synchronous machine connected to a 50-c/s system is coupled to a 6-pole induction machine connected to a 40-c/s system. How will power be transferred?

[Constant power from 40- to 50-c/s system, with 6·25% loss.]

27 (*a*) A 14-pole, 3-phase induction-type frequency-changer is driven at 3 000 rev per min and the stator is excited from a 50-c/s system. At standstill, the slip-ring voltage is 17·5 V. What frequencies and voltages are obtainable from the rotor? (*b*) If a 4-pole, 3-phase induction-type frequency changer with stator connected to a 50-c/s system gives 300 V at 100 c/s at the rotor slip-rings, at what speed must it be driven to give 125 c/s, and what will its slip-ring voltage then be?

[(*a*) 140 V at 400 c/s and 105 V at 300 c/s; (*b*) 2 250 rev per min, 375 V.]

28 A 1 000-kW frequency-changer set consists of a synchronous machine with p_s pole-pairs coupled to an induction machine with p_i pole-pairs. Find p_s and p_i for maximum speed of the set and show how the power is distributed, ignoring losses, (*a*) when the stator of the synchronous machine and the rotor of the induction machine are connected to a 40-c/s system, and the stator of the induction machine is connected to a 50-c/s system; (*b*) when both stators are joined to a 50-c/s system, and the induction-machine rotor to a 40-c/s system.

[(*a*) $p_i = 1$, $p_s = 4$; induction motor takes 1 000 kW at 50 c/s and supplies 800 kW at 40 c/s and drives synchronous generator which supplies 200 kW at 40 c/s. (*b*) $p_i = 1$, $p_s = 5$; induction motor takes 1 250 kW at 50 c/s of which 250 kW is supplied by synchronous generator.]

29 Two coupled synchronous machines are connected to 50- and 40-c/s systems respectively. As generators, the full-load displacement angles (electrical) from the neutral position, are 25° and 20° respectively. The power needed to run the set light is 10% of full load. Find the mechanical angle through which the stator of the 50-c/s machine must be rocked to obtain full load. [10·5°.]

CHAPTER XXXII

STARTERS

Shunt-motor Starters

1 A starter is required for a 250-V shunt motor. The maximum current limit is to be 67 A and the minimum limit about ¾ of this value. The armature resistance is 0·5 Ω. Find the number of sections of the starter and the resistance of each section.
*[7 sections ; 0·94, 0·70, 0·53, 0·39, 0·30, 0 21, 0·17 Ω.]

2 Calculate the resistance steps for the starter of a 500-V shunt motor, given : number of steps, 12 (i.e. 13 studs) ; armature resistance, 1 Ω ; maximum current limit, 20 A.
*[5·9, 4·5, 3·4, 2·6, 2·0, 1·5, 1·2, 0·9, 0·7, 0·53, 0·40, 0·31 Ω.]

3 Find the values of the five steps in a 6-stud starter for a 5-h.p., 200-V shunt motor. The current is not to exceed twice full load current. Full load efficiency, 88%. Armature copper loss constitutes half the total loss. Field current 0·6 A.
*[1·60, 1·06, 0·70, 0 46, 0·30 Ω.]

4 Find the 7 steps of an 8-contact starter for a 100-h.p., 500-V shunt motor. The lower current limit is to be full load current. The full load efficiency is 94%. The total copper losses are 3·7% of the input, and the field resistance is 250 Ω.
*[0·77, 0·47, 0 28, 0·16, 0·1, 0 06, 0·03 Ω.]

5 A 20-h.p., 500-V, shunt motor is to reach 1 000 rev per min from rest in 30 sec against full load torque. The motor efficiency is 87% at rated load and the armature and field resistances are 0·87 Ω and 250 Ω respectively. The moment of inertia of the moving parts is 10 ft-lb-sec^2. Design a starter which will limit the maximum line current to 52 A. It may be assumed that the average current during the starting period is equal to the mean of the minimum and maximum current.
[2·8 ; 2·02 ; 1·45 ; 1 04 ; 0·75 ; 0·54 ; 0·39 ; 0·28 ; 0·2 ; 0·15.]

6 A 4-section starter, for a 500-V shunt motor with an armature resistance of 0·4 Ω and full load current of 80 A, is designed for an upper limit of 100 A. Calculate the percentage speed at which the first changeover occurs (*a*) with the normal starter, (*b*) with the starter modified to make the initial current (i) 120 A ; (ii) 80 A. Show the speed changes graphically.
[(*a*) 50% ; (*b*) (i) 59·5%, (ii) 36%.]

7 A 500-V, 50-h.p., 1 000 rev per min, direct-current, shunt motor is to exert a maximum torque during the starting period of 150% full load torque. Design a suitable 5-section starter

and determine the current limits and the motor speeds at which notching takes place. If the initial current is to be (*a*) 170 A, (*b*) 100 A, what are the changes required in the starter, and what are the minimum current values at the first change-over if this is to occur at the same speed as in the original arrangement. Armature resistance 0·85 Ω. Assume motor efficiency constant at 83%.

[135, 84 A ; 1·391, 0 867, 0·542, 0·338, 0·212 Ω ; 401, 649, 804, 902, 968 rev per min (*a*) $r_1 = 0{\cdot}62\ \Omega$, $I_{min.} = 106$ A. (*b*) $r_1 = 2{\cdot}68\ \Omega$, $I_{min.} = 62{\cdot}4$ A.]

8 A 4-section automatic starter for a 460-V shunt motor operates on the current-limit principle. The armature resistance is 1 Ω. If the initial starting current is 27 A, and the peak on each of the other steps is 45 A, find the lower current limit and the resistance of each section. [22·2 A ; 8·62, 4·25, 2·10, 1·03 Ω.]

9 An automatic starter with sections of 18·4 Ω, 7·2 Ω and 2·9 Ω is used with a 460-V shunt motor having an armature resistance of 2 Ω. Find (*a*) initial starting current, (*b*) current peaks when contactors are set to close at a lower current limit of 12 A. [(*a*) 15·1 A ; (*b*) 30·2, 29·6, 29·5 A.]

Series-motor Starters

10 Assuming the magnetization curve to be a straight line passing through the origin, determine the resistance of each of the five elements of a 6-stud starter for a 500-V series motor having a total resistance of 0·25 Ω between terminals. The maximum current at starting is not to exceed 100 A.

*[0·95 Ω.]

11 Find the resistance of the various steps and the number of steps for starting a 20-h.p. series motor for 500 V, the current varying between 1·5 and 2 times full load current. Efficiency, 80% ; combined resistance of armature and field windings, 1 Ω. The flux increases by 10% as the current rises from 1·5 to 2 times full load current. *[4 steps ; 1·84, 1·52, 1·25, 1·03 Ω.]

12 Find the values of the 4 sections of an automatic starter for a 500-V series motor to operate at a lower current-limit of 56 A. The motor flux rises by 0·50% per A as the current rises from 56 to 80 A. The motor resistance is 0·5 Ω.

[1·98, 1·61, 1·31, 1·07 Ω.]

13 A 5-h.p., 200-V series motor has a magnetization curve as given in Fig. 4 : 100% flux is produced by 25 A. Find the resistance of each of 5 starter steps. The upper current limit is to be 25 A. The motor resistance is 1·86 Ω.

*[1·6, 1·4, 1·3, 1·2, 1·1 Ω.]

XXXII. STARTERS

Series-parallel Starters

14 Find the resistance sections for the series-parallel control of two 500-V, 250-h.p. railway motors. The lower current limit is to be 300 A per motor with both connexions, and there are to be 4 series and 3 parallel sections. The magnetization curve may be taken as rectilinear, increasing from 8·0 to 9·0 megamaxwells (80 to 90 mWb) per pole over the current range 300 to 400 A. Resistance per motor 0·1 Ω.

*[0·168, 0·148, 0·121, 0·103 Ω ; 0·221, 0·191, 0·167 Ω.]

15 Find the resistance sections for the series-parallel control of two 250-h.p., 550-V traction motors having a resistance of 0·11 Ω each. Maximum current per motor for both series and parallel connexion, 450 A ; number of steps in series connexion, 4 ; and in parallel connexion, 3. The magnetization curve for each machine is given in Fig. 4, where 100% flux is produced by a current of 300 A.

*[0·160, 0·134, 0·112, 0·094 Ω ; 0·211, 0·178, 0·150 Ω.]

Auto-transformer and Star-delta Starters

16 Compare the line currents at the instant of switching on a 3-phase cage-rotor induction motor by (*a*) direct switching (*b*) star-delta switch ; (*c*) auto-transformer with a p% tapping. Ignore the magnetizing current. [(*a*) 1 ; (*b*) 0·33 ; (*c*) $(p/100)^2$.]

17 Determine approximately the starting torque of an induction motor in terms of full load torque when started by means of (*a*) a star-delta switch ; (*b*) an auto-transformer with 50% tapping. Ignore magnetizing current. The short circuit current of the motor at normal voltage is 5 times the full load current, and the full load slip is 5%. [(*a*) 0·42 ; (*b*) 0·31.]

18 Determine a suitable auto-transformer ratio for starting an induction motor with a supply current not exceeding twice full load current. Use the following data :—short circuit current = 4 times full load current ; full load slip = 2·5%. Estimate the starting torque in terms of full load torque. Ignore the magnetizing current. [1·41 ; 0·2.]

19 Find the percentage tapping required on an auto-transformer starter for a cage-rotor induction motor to start the motor against ¼ full load torque. The short circuit current on normal voltage is 4 times the full load current, and the full load slip is 3%. Ignore the magnetizing current. [72·5%.]

20 A 3-h.p. induction motor with full load efficiency and power factor of 0·83 and 0·8 respectively, has a short circuit current of 3·5 times full load current. Estimate the line current at the instant of starting the motor from a 500-V supply by means of a star-delta switch. Ignore the magnetizing current. [4·54 A.]

21 The short circuit current of a 5-h.p. induction motor is 4 times its full load current, and the stator is arranged for star-delta starting. The supply voltage is 440 V, full load efficiency and power factor 0·85 and 0·8 respectively. Ignoring the magnetizing current, calculate the current taken from the mains at the instant of starting. [9·6 A.]

22 Find the ratio of starting to full load current in a 15-h.p., 400-V, 3-phase induction motor with star-delta starter, given : full load power factor, 0·85 ; full load efficiency, 0·88 ; short circuit current, 40 A at 200 V. Ignore the magnetizing current. [1·25.]

Rotor Starters

23 Calculate the steps in a 5-step rotor starter for a 3-phase induction motor. The slip at the maximum starting current is 2% with slip-rings short-circuited and the resistance per rotor phase is 0·015 Ω. *[0·40, 0·19, 0·09, 0·04, 0·02 Ω.]

24 Calculate the steps in a 4-step starter for a 3-phase slip-ring induction motor, from the following data :—maximum starting current = full load current ; full load slip = 2·5% ; rotor resistance per phase = 0·02 Ω. *[0·48, 0·19, 0 08, 0·03 Ω.]

Pony-motor Starters

25 A 12-pole, 50-c/s, 500-kW rotary convertor is to be started with uniform acceleration by means of a 10-pole induction motor, the operation to take 4 min. Assuming that the constant losses of the convertor amount to 3% of its output, determine the maximum power output required from the starting motor. The moment of inertia of the system is 80 000 lb-ft^2. [71·5 h.p.]

Single-phase Series-motor Starters

26 A 1-phase series motor has an impedance of $0{\cdot}1 + j\,0{\cdot}3\ \Omega$ when the current is 400 A. It is started by a multi-tap transformer between current limits of 300 and 400 A. Find the transformer tappings for the first two notches. Flux at 300 A = 85% of flux at 400 A. [127 V ; 152 V.]

CHAPTER XXXIII

HEATING AND COOLING OF ELECTRICAL PLANT

Natural Cooling

1 A coil of copper wire, having a resistance of 5 Ω at 0° C, maintains a steady mean temperature of 100° C above the surrounding air (20° C) when the continuous loss in it is 1 000 W. If a voltage of 100 V be applied, find the steady temperature attained. [144° C.]

2 A wire of radius 0·5 mm is made of material with a resistivity of 12·5 μΩ-cm, and a melting point of 250° C. If the wire loses heat at the rate of 0·00028 cal per sec per cm^2 per 1° C, find its fusing current in an ambient temperature of 15° C. [6·7 A.]

3 A rectangular copper bus-bar 0·25 in × 1·75 in carries 1 000 A. The emissivity is 0·01 W/in^2/°C. Find the rise of temperature above 20° C, given that the resistivity is 0·68 μΩ-in and resistance-temperature coefficient 0·004 at 20° C. [46° C.]

4 A block of wood, 50 cm × 50 cm × 15 cm thick is to be inserted between two parallel metal plates and heated by the application to the plates of an alternating voltage of frequency 15 megacycles per sec. Calculate the power, voltage, and current required to raise the temperature of the wood by 80° C in 80 min. Specific heat of wood = 0·35 cal/g/1° C, specific gravity = 0·41 g/cm^3, permittivity = 2·9, loss angle 2·2°. [1 000 W, 2 550 V, 10·2 A.]

5 A cylindrical coil 15 cm in diameter and 25 cm long has a resistance (hot) of 10 Ω and carries 5 A. (*a*) What will be its final outside temperature rise if only the outside cylindrical surface dissipates heat and radiates 0·003 W per cm^2 per 1° C? (*b*) If the allowable rise were 60° C, what current could be carried? [(*a*) 71° C; (*b*) 4·6 A.]

6 Find the full load temperature rise of the armature of a 10-pole, 550-kW, 550-V generator having an armature resistance of 0·018 Ω, an iron loss of 11 kW, and an armature radiating surface of 80 000 cm^2. Each square centimetre of armature surface will radiate 0·012 W per 1° C. [25° C.]

7 A 150-kW, 250-V generator has an armature capable of dissipating 0·006 W per cm^2 per 1° C. Calculate the temperature rise if the armature resistance is 0·009 Ω; the hysteresis loss is 620 W in the teeth and 1 140 W in the core; and the eddy

current loss is 75 W in the teeth and 140 W in the core. The radiating surface is 15 000 cm^2. [58° C.]

8 A 1 000-A shunt consists of 8 strips of a nickel-alloy connected in parallel, each having a cross-section of 25 mm × 2 mm. The normal volt-drop is 75 mV. The alloy used has the following characteristics : resistance = 0·4 Ω per m per mm^2, specific heat = 0·12 cal/g/°C, specific gravity = 8·0 g/cm^3, rate of heat dissipation = 0 01 W per cm^2 surface area per °C, resistance-temperature coefficient negligible. Determine the maximum temperature rise of the shunt and the time taken to reach 99% of the maximum value. [28·2° C, 2·85 min.]

9 Estimate the steady conductor temperature of a 1-conductor, lead-sheathed, buried cable with a conductor diameter of 1·8 cm, a paper-oil dielectric 1·9 cm thick, and a lead sheath 4 mm thick. The total loss per cm length is 100 mW presumed all in the conductor. The thermal resistivity of the dielectric is 600, and of the earth is 200° C per cm per W per cm^2. Assume an earth temperature of 13° C at a radius of 3 ft. [37° C.]

10 A 33 kV, single-core cable with 61 copper strands each 0·098 in in diameter has a dielectric thickness of 0·4 in, a metal sheath of thickness 0·15 in and a serving and armouring of thickness 0·35 in. It is laid 3 ft below ground, and the maximum temperature rise of the copper must not exceed 55° C above atmospheric temperature. The thermal resistivity of the dielectric is 600, of the outer cover 250, and of earth 150. Calculate the current which can be carried continuously, neglecting dielectric and sheath losses. The resistivity of the conductor is 0·8 μΩ-in. (Thermal resistivity is defined as the temperature difference in °C between opposite faces of a centimetre cube to cause the transfer of heat at the rate 1 J/sec.) [750 A.]

11 Estimate the maximum current that can be carried by a 3-phase circuit formed by three single-phase cables laid alongside and touching each other, having the following particulars : area of copper conductor, 0 5 in^2 ; thickness of dielectric 0·6 in, lead sheath 0·15 in, and covering 0·4 in. The thermal resistivities of the dielectric, the outer covering and earth are 550, 400 and 140 °C/cm^3/W respectively. The sheath loss and dielectric loss amount to 10% and 5% of the copper loss respectively. The cable is buried at a depth of 4 ft and its maximum temperature rise is to be limited to 50° C above the ambient temperature of 15° C. [614 A.]

12 A motor works on a 2-min load-cycle constituted as follows : 0 to 15 sec, load rising uniformly from 0 to 1 050 h.p. ; 15 to 85 sec, load constant at 600 h.p. ; 85 to 95 sec, regenerative braking, the h.p. returned falling uniformly from 200 to 0 h.p. ; 95 to 120 sec, motor stationary. What size of continuously-

rated motor would be suitable, assuming the rating to depend on (*a*) the average value, (*b*) the r.m.s. value, of the load?
[(*a*) 424, (*b*) 495 h.p.]

13 A 100-kVA transformer has a time-constant of 2 hours, a normal temperature rise of 75° C and a maximum efficiency of 98·5% at 80 kVA. Calculate the load which can be carried without exceeding the normal temperature rise under the following conditions: (*a*) 30-min run, starting cold; (*b*) continuous load cycle of 30 min on load, 30 min on no load.
[(*a*) 261 kVA; (*b*) 135 kVA.]

14 The outside of a 12 h.p. totally enclosed motor is equivalent to a cylinder of 30 in diameter and 36 in length. The machine may be considered to be of homogeneous material weighing 400 kg and of specific heat 700 J/kg/1° C. Assuming that the outer cylindrical surface dissipates heat at 0·008 W/in²/1°C, estimate the final temperature of the machine and its time constant when operating on full load with an efficiency of 90%. Prove all formulæ used. If this machine were semi-enclosed with a resultant improvement to 0·015 W/in²/1° C, find the load which could be delivered for the same temperature rise if the efficiency remains unchanged.
[36·7° C; 2·9 hr; 22·6 h.p.]

15 Assuming that a transformer tank in a substation approximates to the case of a small spherical body of emissivity 0·9, find the heat radiation from the tank when the temperature of the tank is 55° C and of the building is 20° C.
[214 W/m².]

Forced Cooling

16 Find the amount of cooling water in litres per minute required by a 5 000-kW transformer having an efficiency of 98·8%, when the water has to carry off 80% of the heat developed. Water inlet temperature, 15° C; outlet temperature, 35° C.
[35 l per min.]

17 The full load losses in a 16 000-kVA transformer are:—iron loss, 82 kW; copper loss, 134 kW. The tank is 3·6 m high × 2·9 m long × 1·35 m wide, and contains a cooling coil through which 200 l of water per min are passed. Assuming that the tank sides dissipate 7·5 W per m² of surface per 1° C, and that the average temperature-rise is 30° C, estimate the rise in temperature of the cooling water. [15° C.]

18 Calculate the volume of cooling air in cubic metres per second to carry off the losses of a 10 000-kW generator having an efficiency of 0·96, when the inlet temperature is 15° C, and the outlet temperature is 35° C. [17·2 m³/sec at inlet.]

19 Find the efficiency of a 3 000-kW turbo-alternator from the following full load test figures :—Air discharge = 6·5 m^3 per sec ; outlet temperature = 40° C ; inlet temperature = 15° C ; standard barometer. [94·3%.]

Thermal Time-constants

20 The temperature of an oil-cooled transformer, when disconnected, fell from 75° to 28° C in 2 hr : find its cooling time-constant. [2·03 hr.]

21 The temperature-rise of a loaded transformer is 20° C after 1 hr and 33·5° C after 2 hr on full load. Calculate the final steady full load temperature-rise and the time-constant. Assume an exponential law. If the loss in the iron equals the loss in the copper at rated continuous load, estimate the one-hour rating for the same temperature rise. [61·5° C; 2·55 hr ; 2·3 times.]

22 A field coil is supplied with a constant power input, and the temperature rises from 15° C to 60° C in 2 hr. Find the time taken for the coil to reach 75° C if its final steady temperature be 95° C. [3·38 hr.]

23 A transformer has a heating time-constant of 4 hr. Compare the permissible loadings of the transformer when rated continuously and on a 1-hr basis respectively, (*a*) if the iron losses are negligible in comparison with the copper loss, (*b*) if the iron and copper losses are equal at full load.
[(*a*) 1 : 2·13 ; (*b*) 1 : 2·83.]

24 A motor with a thermal time-constant of 45 min has a final temperature-rise of 75° C on continuous rating. (*a*) What is the temperature-rise after 1 hr at this load ? (*b*) If the temperature-rise on 1-hr rating is 75° C, find the maximum steady temperature at this rating. (*c*) When working at its 1-hr rating, how long does it take the temperature to increase from 60° to 75° C ? [(*a*) 55° C ; (*b*) 102° C ; (*c*) 20 min.]

25 At the beginning of a full load run the temperature of a transformer is 20° C. After 2 hr it is 50° C, and after 4 hr 65° C. Calculate the final full load temperature and temperature-rise. In what time after starting would the temperature reach 95% of its final steady value ? *[80° C ; 60° C ; 7·85 hr.]

26 The mean temperature-rise of a machine run on full load is given below :—

Time, hr . .	0 2	0·4	0·6	1·0	1·5	2·0	3·0
Temp.-rise, °C. .	7·0	13·5	19·3	29·1	39·1	46·7	57·5

Estimate graphically the final steady temperature-rise and the time-constant. *[74° C ; 2·0 hr.]

27 A machine on a short heat-run gave the following mean temperature-rise :—

Time, hr . .	0	0·25	0·5	0·75	1 0	1·25	1·5
Temp-rise, °C .	42·5	45·0	47·4	49·5	51·4	53·0	54·4

Estimate graphically the final temperature-rise and the time-constant. *[62° C ; 1·6 hr.]

28 Measurements made from a temperature/time curve of a transformer, in which the power dissipated is constant, show that the rate of temperature-rise is 0·0743° C per min and 0·055° C per min, when the temperature-rise is 19·8° C and 27° C respectively. Estimate the final steady temperature-rise and the heating time-constant of the transformer.
[49° C ; 400 min.]

29 A motor has a heating time-constant of 100 min, a cooling time-constant when stationary of 150 min, and a final steady temperature-rise on full-load of 50° C. Estimate the maximum temperature attained if the motor runs on full load for 20 min followed by a stationary period of 40 min, the cycle being repeated indefinitely. [24·5° C.]

30 An oil-cooled transformer equipped with radiators and fans attains, on full load, a temperature of 36 5° C after 120 min and a final steady temperature of 70° C. If the transformer operates on full load for 30 min at regular intervals, what time must be allowed between each load period if the maximum temperature rise is not to exceed 30° C ? Air temperature, 20° C. The transformer is disconnected from the supply and the fans are not in operation during the cooling period. Under these conditions the cooling time-constant is 400 min. [29 min.]

31 Calculate the maximum overload that can be carried by a 400-kVA transformer if its temperature is not to exceed 75° C after $\frac{1}{2}$ hr on overload. The temperature rise after 1 hr on full load is 40° C and after 2 hr 55° C. The ambient temperature is 15° C and the full load copper losses are twice the iron losses.
[710 kVA.]

CHAPTER XXXIV

MERCURY-ARC RECTIFIERS

Rating and Efficiency

1 Calculate the rating of the secondary of a rectifier transformer in terms of the direct-current output and the number of phases. Neglect arc-drop and assume the winding to be connected in a simple star.

$$\left[\frac{\pi V_d I_d}{\sqrt{2N}\,\sin\,(\pi/N)}.\right]$$

2 A mercury-arc rectifier has an arc drop of 25 V. What will be its internal efficiency when giving (*a*) 250 V, (*b*) 1 000 V, (*c*) 2 000 V on the direct-current side?

[(*a*) 90·90% ; (*b*) 97·56% ; (*c*) 98·77%.]

3 Draw a curve showing how the internal losses and the efficiency of mercury-arc rectifiers vary with the arc length l cm for values of l between 15 and 65 cm. Take the internal voltage drop as $14 + 0{\cdot}1\,l$ volts, the direct voltage as 250 V and the current as constant.

4 A 3-anode mercury-arc rectifier has a full load output of 20 A at 250 V. The arc drop is 24 V, and the loss in connexions, reactors, etc., is 50 W. A separate test on the transformer gave a core loss of 150 W on open circuit, and an I^2R loss of 40 W in each primary and each secondary phase when the (sinusoidal) output current was 11·55 A. Calculate the full load efficiency of the equipment for a highly-reactive load. The transformer has a mesh/star connexion. [85%.]

5 The following test results were obtained on a mercury-arc rectifier :

A.C. input, kW .	0·36	1·27	2·19	3·10	3·98	4 87	7 45	9·16
Direct voltage, V .	345				328			302
Direct current, A .	0	2·5	5·0	7·5	10 0	12·5	20	25

Calculate the arc drop, assuming that it is independent of the load. Draw a curve of efficiency to a base of output power. *[23 V.]

Regulation

6 A 3-anode, mercury-arc rectifier has an anode current overlap of 30° for full load current. Find the regulation expressed as a percentage of the full load voltage. Neglect arc drop. [7·2%.]

7 If the voltage drop in the arc of a mercury-arc rectifier is 25 V, the no load voltage 240 V, and the angle of overlap 30° at full load, find the voltage regulation. [20·6%.]

8 Find the no load and full load output voltages of a 3-anode, mercury-arc rectifier in which the overlap angle is 30° on full load. The secondary phase voltage is $1\,270/\sqrt{2}$ V, and the constant arc drop is 25 V. [1 025 V ; 953 V.]

9 A 6-anode mercury-arc rectifier gives 800 kW at 650 V on the direct current side, and is supplied from a transformer with secondaries in double 3-phase connexion with an interphase transformer. If the angle of the overlap at full load is 30°, find (*a*) the leakage reactance of each transformer phase, and (*b*) the percentage regulation between the transition point and full load. [(*a*) 0·159 Ω ; (*b*) 7·54%.]

Wave-form, Connexions

10 Assuming the direct voltage wave-form of a mercury-arc rectifier to consist of symmetrical parts of alternate half-cycles of each transformer phase voltage, determine (*a*) the root-mean-square value and (*b*) the arithmetic mean value of the direct voltage in terms of the maximum phase voltage for 2, 3, 4, 6, 12 and N anodes. Assume sinusoidal variation of the alternating voltage.

$$\left[\begin{array}{l}(a)\ 0{\cdot}71,\ 0{\cdot}84,\ 0{\cdot}90,\ 0{\cdot}95,\ 0{\cdot}99\ ;\ \left[\frac{1}{2}+\frac{N}{4\pi}\sin\frac{2\pi}{N}\right]^{\frac{1}{2}} \\ (b)\ 0\ 64,\ 0{\cdot}83,\ 0{\cdot}90,\ 0{\cdot}95,\ 0{\cdot}99\ ;\ \left[\frac{N}{\pi}\sin\frac{\pi}{N}\right].\end{array}\right]$$

11 Find the ratio between the mean value of the direct voltage and the secondary phase voltage in a 6-phase mercury-arc rectifier (*a*) with transformer secondaries connected in 6-phase star with neutrals forming the negative (diametral connexion), (*b*) with interphase transformer, the midpoint forming the negative. Ignore arc drop. [(*a*) 1·35 ; (*b*) 1·17.]

12 Assuming that the anode current wave-form consists of rectangular blocks of the constant rectified current of magnitude I_d, draw out the wave-forms of the transformer secondary currents for (*a*) a 3-anode, and (*b*) a 6-anode mercury-arc rectifier, and calculate in each case the root-mean-square value. (*c*) If an absorption choke is fitted to the 6-anode rectifier such as to double the duration and halve the magnitude of the anode current, draw out the corresponding wave-form and find its root-mean-square value. [(*a*) 0·58 I_d ; (*b*) 0·41 I_d ; (*c*) 0·29 I_d.]

13 A 3-anode mercury-arc rectifier has a direct full load current of 100 A. Estimate the equivalent inductance per secondary

transformer phase if the overlap angle on full load is to be limited to 45°. The load inductance may be considered large enough to maintain a sensibly constant output current. The transformer phase voltage is 400 V and the frequency 50 c/s. [4·6 mH.]

14 A 3-phase, grid-controlled mercury-arc rectifier has a sinusoidal phase voltage of 1 000 V r.m.s. Neglecting anode reactance, calculate the direct voltage output when ignition is delayed by 15°. [1 130 V.]

15 The ignition of a rectifier is retarded until the direct voltage is zero. By what additional angle must it be retarded to reduce the alternating current power to zero if the direct current is kept constant at 30 A ? The direct voltage with zero retardation is 200 V ; arc drop is 20 V and copper loss is 1 200 W. [17·5°.]

16 A 50-c/s, 3-anode rectifier has an effective inductance of 1 mH in each secondary phase, and the peak phase voltage is 1 000 V. Plot curves showing (*a*) the voltage during overlap, (*b*) the current in the anode, the conduction of which begins during overlap. Find from curve (*b*) the overlap angles for an output current of (i) 50 A, (ii) 100 A, (iii) 300 A ; check by calculation. Assume the load to be highly reactive, and neglect arc-drop. [11°, 15·5°, 27°.]

17 A 6-anode, grid-controlled, mercury-arc rectifier equipment is supplied from a 440-V, 3-phase, 50-c/s system. The transformer primary is mesh-connected, and the centre points of the three secondary phases are joined together. The phase turn-ratio is 55/53. Determine the mean output voltage with a highly inductive load when the grids are biassed to delay the ignition of the anodes by (*a*) 0, (*b*) $\frac{1}{4}\pi$, (*c*) $\frac{1}{2}\pi$. What is the r.m.s. value of the output voltage and the fluctuation expressed as a percentage of the mean voltage in (*b*)? Neglect arc-drop and leakage reactance.

[(*a*) 288 V, (*b*) 203 V, (*c*) 0, (*d*) 212 V, 104%.]

18 A mercury-arc rectifier is fed from a 440-V, 3-phase sinusoidal supply. Two wattmeters connected in the usual way to measure the input read 15 kW and 20 kW respectively, and a dynamometer ammeter connected in one line reads 57·5 A. Calculate the displacement, distortion and power factors of the equipment. [0·971 ; 0·824 ; 0·8.]

19 Draw a diagram of essential connexions for a 6-anode mercury-arc rectifier with interphase transformer, supplying a non-reactive load of 2 Ω. Neglecting all leakage reactances and assuming that the magnetizing current of the interphase transformer is zero, insert on the connexion diagram the instantaneous

currents and voltages for an instant 45° after the peak voltage of phase 1. Take the peak voltage per phase as 1 000 V and the arc-drop as 25 V. Draw the direct output-voltage wave.

20 Using the data in problem No. 19, plot a curve of output voltage and derive a curve showing the voltage in each half of the phase-equalizer winding. If a total variation of current in the rectifier phase groups is $\pm$ 2%, estimate the inductance of the phase-equalizer winding and its rating in volt-amperes. [0·21 H ; 100 kVA.]

21 An inverted 8-phase mercury-arc rectifier with a r.m.s. voltage of 125 V per phase when connected to alternating current mains on primary side, feeds into these mains from a 150 V battery. If the arc drop is 20 V, calculate the angle of ignition delay. Find also the maximum battery voltage for safe inversion, assuming there is no overlap. [153° ; 166 V.]

CHAPTER XXXV

PARALLEL OPERATION

Batteries and Generators in Parallel

1 The terminal voltage of a generator falls uniformly from 250 V on open circuit to 238 V when delivering 60 A. It is connected to a load in parallel with a battery having a constant electromotive force of 245 V, and a resistance of 0·1 Ω. What current will be supplied by the generator when the total output to the load is (*a*) 50 A, (*b*) 100 A, (*c*) zero?

[(*a*) 33·3 A; (*b*) 50 A; (*c*) 16·7 A.]

2 A shunt generator supplies a constant current of 10 A to a battery. During charge, the terminal voltage of the battery rises from 150 to 165 V. The generator field resistance is 160 Ω, the armature resistance 0·5 Ω, and the resistance of the connexions 0 6 Ω. Calculate (*a*) the percentage change in speed required if the field resistance is unaltered; (*b*) the percentage change in field resistance if the speed is kept constant. The change of flux can be taken as proportional to one-half the change in field current. [(*a*) 4·3%; (*b*) 7·5%.]

3 A battery having an electromotive force of 525 V and a resistance of 0·02 Ω is connected across the load at the end of a feeder cable, each conductor of which has a resistance of 0·05 Ω. The generator bus-bar voltage is 550 V. Calculate the potential difference at the terminals of the battery when the load is (*a*) zero, (*b*) 250 A, and (*c*) 750 A.

[(*a*) 529 2 V; (*b*) 525 V; (*c*) 516·7 V.]

4 A feeder supplying a traction load has a resistance of 0·2 Ω. At the station bus-bars a constant voltage of 600 V is maintained. A battery connected across the load-end of the feeder has a constant electromotive force of 550 V and internal resistance 0·1 Ω. Find the current supplied from the station bus-bars and the voltage at the feeding point when the load is (i) zero, (ii) 250A, (iii) 600 A with (*a*) the battery connected across the feeding end of the cable, and (*b*) the battery disconnected.

[(*a*) (i) 167 A, 566·7 V; (ii) 250 A, 550 V; (iii) 367 A, 526·7 V; (*b*) (i) 0 A, 600 V; (ii) 250 A, 550 V; (iii) 600 A, 480 V.]

5 Two shunt generators and a battery are all connected to common bus-bars. The open circuit voltage of the battery is 247 V. The open circuit voltage, armature and field resistances of the generators are 250 V, 0·24 Ω, 100 Ω and 248 V, 0·12 Ω, 100 Ω respectively. The two generators each supply the same

current when the load on the bus-bars is 40 A. Calculate the internal resistance of the battery. [0 0864 Ω.]

6 A 110-V battery is connected to the same bus-bars as a direct current generator. Battery resistance, 0 025 Ω; generator armature resistance, 0·1 Ω. The battery floats when load current is 100 A. Find the generator output (*a*) when the battery floats, (*b*) when the load current is 50 A, (*c*) when the load current is 250 A. Assume the open circuit voltages of generator and battery to be constant.

[(*a*) 11 kW ; (*b*) 10 kW ; (*c*) 14 kW.]

7 A load of 20 kW is taken from two batteries *A* and *B* in parallel. *A* has an open circuit voltage of 240 V and an internal resistance of 0 1 Ω. Battery *A* delivers a current 3 times as great as that of *B*. If *B* has an internal resistance of 0·2 Ω, find its open circuit voltage. [237·9 V.]

8 Four batteries of open circuit voltages 220, 230, 240 and 250 V, and internal resistances 0·1, 0·1, 0 25 and 0·2 Ω respectively, are joined in parallel across a 5-Ω resistor. Calculate the combined load. [10·5 kW.]

D.C. Generators in Parallel

9 Two shunt-wound generators running in parallel have each an armature resistance of 0·02 Ω and a field resistance of 50 Ω. The combined external load current is 5 000 A. The fields are excited so that the electromotive force induced in one machine is 600 V, and in the second machine 610 V. Calculate the bus-bar voltage and the output of each machine.

[554·6 V ; 1 252 and 1 520 kW.]

10 Two shunt generators in parallel supply together 2 000 A. The machines have armature resistances, 0·04 and 0 025 Ω; field resistances, 25 and 20 Ω; induced e.m.f.'s, 440 and 420 V respectively. Determine the output of each machine.

[430, 370 kW.]

11 Two separately-excited, direct-current generators running in parallel each supply a current of 200 A at 200 V. The armature resistance of each machine is 0·05 Ω. Calculate (*a*) the percentage change in the generated electromotive force of one machine necessary to reduce its load to zero; (*b*) the resulting percentage drop in terminal voltage, assuming that the total load current and the electromotive force of the other machine remain unchanged. [(*a*) 9·5% ; (*b*) 5%.]

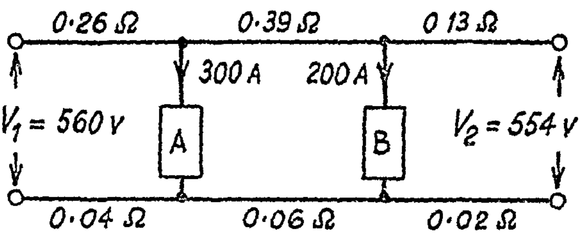
12 Two 220-V generators operate in parallel. One machine has a terminal voltage of 270 V on no load and 220 V at a load current of 35 A. The other has a voltage of 280 V at no load and 220 V at 50 A. The external characteristics are rectilinear. Calculate

the output current of each machine and the bus-bar voltage when the total load is (*a*) 60 A, (*b*) 20 A.
[(*a*) 23·6 A, 36·4 A, 236·2 V ; (*b*) 5·3 A, 14·7 A, 262·4 V.]

13 A station contains 6 generators operating in parallel, each having an armature current of 1 000 A, an induced electromotive force of 250 V, and a terminal voltage of 240 V. The external load has a constant resistance. Calculate the new terminal voltage and current of each generator if the electromotive force of one generator is raised by 5%, the others remaining unaltered. [242 V ; 2 050 A and 800 A.]

14 Two overcompounded, long-shunt, 480-kW generators, designed to operate in parallel, each develop a terminal voltage rising from 550 V on no load to 600 V on full load. The resistance of each armature is 0·01 Ω and of each series field 0 0015 Ω. The open circuit characteristic is given in Fig. 4, where 100% represents 550 V and 20 000 ampere-turns per pole. Neglecting armature reaction, determine the maximum allowable resistance of the equalizer bus-bar to ensure equal division of load at full load. [About 0 001 Ω.]

15 Stations V_1 and V_2 supply power at points *A* and *B*, as shown. Find the loads and load voltages.
[146, 103 kW ; 488, 515 V.]



Alternators in Parallel

16 Two identical 2 000-kVA alternators operate in parallel. The governor of the first machine is such that the frequency drops uniformly from 50 c/s on no load to 48 c/s on full load. The corresponding uniform speed drop of the second machine is 50 to 47·5 c/s. (*a*) How will the two machines share a load of 3 000 kW ? (*b*) What is the maximum load at unity power factor that can be delivered without overloading either machine?
[(*a*) 1 667 kW, 1 333 kW ; (*b*) 3 600 kW.]

17 Two identical, 3-phase, star-connected generators, operating in parallel, share equally a total load of 750 kW at 6 000 V and power factor 0·8. The synchronous reactance and resistance of each machine are respectively 50 Ω and 2·5 Ω per phase. The field of the first generator is excited so that the armature current is 40 A (lagging). Find (*a*) the armature current of the second alternator : (*b*) the power factor of each machine ; (*c*) the electromotive force of each machine.
[(*a*) 51·4 A ; (*b*) 0·9, 0·7 ; (*c*) 8 270 V, 9 780 V.]

18 An alternator of 1 500 kVA capacity runs in parallel with one of 1 000 kVA capacity. What load should each supply and at what power factor should each operate in order that the currents and power outputs shall be proportional to their ratings if the combined load is 2 000 kW at 0·8 power factor?

[1 200 kW, 800 kW, 0·8.]

19 An impedance $10 + j\ 5\ \Omega$ is supplied from two 1-phase alternators A and B connected in parallel. The induced electromotive force E of each machine is 220 V and E_A leads E_B by 20°. The equivalent synchronous impedances of the two machines are $Z_A = 0{\cdot}2 + j\ 3{\cdot}0\ \Omega$ and $Z_B = 0{\cdot}25 + j\ 4{\cdot}0\ \Omega$. Determine the current and power delivered by each machine; also the total load current and power.

[$I_A = 20{\cdot}0$ A, $P_A = 8\,960$ W; $I_B = 6{\cdot}47$ A, $P_B = -780$ W; $I = 17{\cdot}8$ A, $P = 8\,180$ W.]

20 Two 1-phase alternators work in parallel on a load of impedance $Z\ \Omega$. Their e.m.f.'s are E_1 and E_2 and synchronous impedances Z_1 and Z_2. Find the terminal voltage V in terms of these e.m.f.'s and admittances Y, Y_1 and Y_2. Find the terminal voltage and power output of each machine if $E_1 = 230$ V and is in phase with $E_2 = 250$ V. $Z = 7{\cdot}5 + j\ 10\ \Omega$. $Z_1 = Z_2 = 0{\cdot}5 + j\ 2{\cdot}5\ \Omega$.

$$\left[V = \frac{E_1Y_1 + E_2Y_2}{Y_1 + Y_2 + Y};\ 218\text{ V};\ 1{\cdot}32,\ 0{\cdot}91\text{ kW.}\right]$$

21 Two synchronous generators are connected to 50-c/s busbars having a constant voltage of $10\,000/\underline{0°}$ V. Generator A has an induced e.m.f. of $13\,000/\underline{22{\cdot}6°}$ V and a reactance of 2 Ω; generator B has an e.m.f. of $12\,500/\underline{36{\cdot}9°}$ V and a reactance of 3 Ω. Find the current, kW and kVAr supplied by each generator.

[(A) $2\,700/\underline{-21{\cdot}8°}$ A, 25 000 kW; 10 000 kVAr.
(B) $2\,500/\underline{0°}$ A; 25 000 kW; 0 kVAr.]

22 Two 3-phase alternators run in parallel and supply power to a transmission line symmetrically loaded. Two wattmeters connected to measure the power input to the line read 600 and 1 400 kW respectively. The reading on each instrument increases with increase of load. The currents in the line and in the two alternator circuits are in the ratio 5 : 4 : 3. Calculate the load taken by each alternator and the reading on each of 4 wattmeters connected in pairs to read the power output from each machine.

[First alternative: load, 1 388 and 612 kW; alternator A, 835 and 553 kW; alternator B, 836 and − 224 kW.

Second alternative: load, 1 950 and 50 kW; alternator A, 995 and 955 kW; alternator B, 415 and − 865 kW.]

23 Two 3-phase generators A and B supply a sub-station through lines 1 and 2. Sub-station load: $R = 50\ \Omega$ and $X = 40\ \Omega$ to neutral. If generator e.m.f.'s are: A, 10 000 V; B, 12 000 V; and A leads B by 10° elec., find currents delivered by A and B.
Generator A: rating 10 000 kVA, 11 000 V, 10% reactance.
Generator B: „ 20 000 kVA, 11 000 V, 15% „
Line 1: $X = 3{\cdot}8\ \Omega$; line 2: $X = 4{\cdot}1\ \Omega$.
Neglect resistance of lines and generators.

[166 A; 165 A.]

24 A farm is supplied with 30 kW at 0·8 power factor and 415 V from two small hydro-electric plants A and B. Station A has an alternator and is at the end of a 3-phase line of impedance $0{\cdot}6 + j\,0{\cdot}8\ \Omega$/phase. Station B, at the middle of this line, has an induction generator supplying 15 kW at power factor 0·9. Find the power supplied by the alternator.

[19·4 kW, power factor 0·47, and 500 V.]

Synchronous Machines on Constant-voltage and Constant-frequency Bus-bars: Maximum Output

25 Calculate the maximum load of a 5 000-kVA, 1-phase alternator having an equivalent reactance of 5 Ω when connected to 6 600-V bus-bars, if its excitation is such that the electromotive force on open circuit would be 6 000 V. Find the armature current and power factor at this load.

[7 900 kW; 1 780 A; 0·675 leading.]

26 A 6 600-V, 1-phase alternator has an armature reactance of 10 Ω and negligible resistance. After being connected in parallel with constant-voltage, constant-frequency bus-bars, the steam supply to the prime mover is gradually increased, the excitation remaining constant, until the machine falls out of step. Estimate the output, armature current and power factor at which this occurs. [4 350 kW; 935 A; 0·7 leading.]

27 A 3-phase, star-connected alternator with $R = 0{\cdot}4\ \Omega$ and $X = 6\ \Omega$ per phase delivers 300 A at power factor 0·8 to constant frequency, 10-kV bus-bars. If the steam supply is unchanged, find the percentage change in the induced e.m.f. necessary to raise the power factor to unity. Ignore change in losses. [15%.]

28 A 1-phase, 11 000-V turbo-alternator, having a reactance of 10 Ω, has an armature current of 220 A at unity power factor when running on constant-frequency, constant-voltage bus-bars. If the steam admission be unchanged and the e.m.f. raised by 25%, find the new values of current and power factor. At what power output would the alternator break from synchronism with this new value of excitation? Determine the corresponding current and power factor.

[360 A, 0·612 lagging; 15 400 kW; 1 780 A, 0·787 leading.]

XXXV. PARALLEL OPERATION

29 A 6 600-V, 3-phase, star-connected, 50-c/s, 1 500- rev per min synchronous generator is connected in parallel with the bus-bars, and the steam supply is increased till the output power equals 10 000 kW. When the field current is increased to 150%, the power factor of the machine becomes 0·8 lagging. Using Fig. 4, where 100% = 6,600 V, calculate the synchronous reactance of the machine. Neglect resistance.

[0·872 Ω/phase.]

30 An 11 000-V, 3-phase, star-connected turbo-alternator delivers 200 A at unity power factor when running on constant-voltage and constant-frequency bus-bars. If the induced electromotive force be raised by 25%, find the current and power factor at which the machine now works. Assume a constant steam supply and an unchanged efficiency. Armature resistance 0·5 Ω synchronous reactance 8 Ω. [297 A ; 0·67 lagging.]

31 A 10 000-kVA, 1-phase alternator has an impedance of 10 Ω and a resistance of 1 Ω, and operates in parallel with constant-voltage 10 000-V bus-bars. Calculate (*a*) the maximum external load that the machine can supply before dropping out of step when the machine is excited to give an electromotive force of 11 000 V ; (*b*) the armature current and power factor corresponding to this maximum load.

[(*a*) 10 000 kW ; (*b*) 1 410 A, 0·71 leading.]

32 A 20 000-kVA, 11 000-V, 3-phase, star-connected synchronous machine, running on constant-voltage, constant-frequency bus-bars, has a resistance of 1% and a synchronous reactance of 30%. If excited to give a terminal voltage of 13 000 V on open circuit, find (*a*) its maximum possible output power as a generator (in kW) ; (*b*) its output as a motor (in h.p.) when taking maximum input power. Core, windage, and friction losses at normal rating, 4%, presumed constant.

[(*a*) 76 500 kW ; (*b*) 100 000 h.p.]

Synchronizing

33 Synchronizing lamps are connected across two 440-V alternators, running at 50 c/s and 50 6 c/s respectively. How often will the lamps light up in 1 min and what is the periodicity of the high-frequency component ? Derive the formula used.

[36, 50·3 c/s.]

34 A 3-phase alternator is to be synchronized to constant-voltage, 50-c/s bus-bars. Three lamps, used to determine the speed of the alternator, are connected between the *R*, *Y* and *B* phases of the alternator and the *R*, *B* and *Y* phases of the bus-bars respectively. If the lamps light 20 times per minute in the sequence 1–2–3, determine the speed of the alternator.

[$49\frac{2}{3}$ c/s.]

Synchronizing Torque and Power

35 Deduce an expression for the synchronizing torque on no load of a 3-phase synchronous machine in terms of the line voltage V, the short circuit line current I_{sc}, the electrical angle of displacement θ, and the speed n in rev per sec.

[$\sqrt{3}\ VI_{sc}\theta/(2\pi n\ 9{\cdot}81)$ kg-m.]

36 A 5 000-kVA, 10 000-V, 1 500-rev per min, 50-c/s alternator runs in parallel with other machines. Its synchronous reactance is 20%. Find for (*a*) no load, (*b*) full load at power factor 0·8 lagging, the synchronizing power per unit mechanical angle of phase displacement, and calculate the synchronizing torque if the mechanical displacement is 0·5°.

[(*a*) 875 kW ; 2 050 lb-ft ; (*b*) 980 kW ; 2 300 lb-ft.]

37 A 1-phase, 10 000-V, 50-c/s alternator running at 1 500 rev per min has an equivalent armature reactance of 4 Ω. Calculate the synchronizing power for 1 mechanical degree of phase displacement when running in parallel with other machines.

[872 kW.]

38 A 10 000-kVA, 3-phase alternator has an equivalent short circuit reactance of 20%. Calculate the synchronizing power of the armature per mechanical degree of phase displacement when running in parallel on 10 000-V, 50-c/s bus-bars at 1 500 rev per min. [1 744 kW.]

39 A 2 000-kVA, 3-phase, 8-pole alternator runs at 750 rev per min in parallel with other machines on 6 000-V bus-bars. Find the synchronizing power on full load at power factor 0·8 lagging per mechanical degree of displacement and the corresponding synchronizing torque. The synchronous reactance of the machine is 6 Ω per phase. [505 kW ; 4 750 lb-ft.]

40 A 10 000-kVA, 6 600-V, 16-pole, 50-c/s, 3-phase alternator has a synchronous reactance of 15%. Calculate the synchronous power per mechanical degree of phase displacement from the full load position at power factor 0·8 lagging. [10 000 kW.]

41 Calculate the synchronizing power in kW per mechanical degree of displacement when supplying full load at 0·8 power factor lagging to 6 600-V bus-bars of a 3-phase, 2 000-kVA, 6 600-V, 50-c/s, 12-pole machine having a synchronous reactance of 25% and negligible resistance. [954 kW.]

42 A 250-kW synchronous motor runs at 100 rev per min on 50-c/s mains. On full load the angular retard of the rotor is 13 electrical degrees. Calculate the torque of the motor per mechanical radian of displacement, and the natural frequency of the machine on full load if the moments of inertia of the moving parts aggregate $1{\cdot}87 \times 10^6$ lb-ft².

[$2{\cdot}3 \times 10^6$ lb-ft ; 1 c/s.]

43 On no load the electromotive force of a 250-h.p., 2 200-V, 10-pole, 50-c/s, 3-phase, star-connected synchronous motor is numerically equal to and in phase with the terminal voltage. When a certain load torque is applied, the rotor is retarded by 1 mechanical degree. Calculate the armature current if the synchronous reactance per phase is 8 Ω. How far is the rotor retarded when the armature current is 50 A ?

[36·9 A ; 1·36° mech.]

44 A 1-phase, 440-V, synchronous motor has an effective armature resistance and synchronous impedance of 1 Ω and 8 Ω respectively. Estimate the minimum armature current, and corresponding electromotive force and angle of retard in electrical degrees for loads (including iron, friction, windage and excitation losses) of (*a*) 5 kW ; (*b*) 7·5 kW ; (*c*) 10 kW.

[(*a*) 11·7 A, 438 V, 12·2° ; (*b*) 17·75 A, 445 V, 18·4° ; (*c*) 24 A, 457 V, 23·6°.]

45 A 1-phase 500-V synchronous motor having an armature resistance of 2 Ω and a synchronous impedance of 10 Ω, is excited to give an electromotive force of 450 V. Plot to a base of γ, the angle of advance of the motor electromotive force, curves of (*a*) current, (*b*) input, (*c*) output, (*d*) efficiency, (*e*) external power factor. Assume that the excitation, iron and friction losses total 1 kW, and take values of γ between 0° and 120°.

Swinging

46 Calculate at full load and unity power factor the natural period of oscillation of a 2 000-kVA, 3-phase alternator having a synchronous reactance of 30%, running at 750 rev per min on 2 000-V, 25-c/s bus-bars. The moment of inertia of the complete rotating system is 200 000 lb-ft². Prove the formula used.

[1·4 sec.]

47 A 6 000-kVA, 5 000-V, 50-c/s, 3-phase alternator with 4 poles and a synchronous reactance of 25%, operates on constant-voltage and constant-frequency bus-bars. The moment of inertia of the whole rotating system is 400 000 lb-ft². Calculate the time of one complete oscillation for full load and unity power factor.

[1·48 sec.]

48 Calculate for full load and unity power factor the natural period of oscillation of a 50-c/s, 10 000-kVA, 6 600-V alternator driven at 1 500 rev per min and connected to constant-voltage, constant-frequency bus-bars. The steady short circuit current is five times the normal full load value, and the moment of inertia of the rotating mass is 400 000 lb-ft².

[1·025 sec.]

49 An alternator driven by a low-speed prime mover having a cyclic irregularity of frequency of 1·66 per sec, is working in

parallel with other machines. The natural period of oscillation of the alternator on constant-voltage, constant-frequency bus-bars is 1 sec. Calculate the ratio by which the amplitude of oscillation is increased by the synchronizing effects of parallel running. [1·57.]

50 An alternator driven by a reciprocating engine and connected to constant-voltage, constant-frequency bus-bars is found to resonate mechanically with the prime mover. Find the percentage increase in the moment of inertia of the rotating masses required to limit the amplitude of oscillations to three times that obtaining when the set is working independently. [50%.]

Transformers in Parallel

51 Two 100-kVA, 1-phase transformers are connected in parallel on both primary and secondary sides. One transformer has an ohmic drop of 0·5% and a reactive drop of 8% of the voltage on full load. The other has corresponding drops of 0·75% and 4% respectively. How will the following total loads be shared? (*a*) 180 kW at 0·9 lagging power factor; (*b*) 120 kW at 0·6 lagging power factor; (*c*) 200 kW at unity power factor.

[(*a*) 58 and 122 kW; (*b*) 36 and 84 kW; (*c*) 67 and 133 kW.]

52 A 500-kVA, 500-V, 1-phase transformer with a reactance drop of 4% and resistance drop of 1% is connected in parallel on the high-voltage side with a 250-kVA, 500-V, transformer having drops of 6% and 1 5% respectively. The secondary terminal voltage on open circuit is 510 V in the first and 500 V in the second transformer. With the secondary windings connected in parallel, calculate (*a*) the cross-current in the secondaries on no load, (*b*) the secondary current in each transformer when the total load is 700 kW at unity power factor, and (*c*) the terminal voltage at this load.

*[(*a*) 121·5 A; (*b*) 1 070 and 330 A approx.; (*c*) 501 V.]

53 The short circuit tests of two, 1-phase, 3 300/220-V transformers with equal voltage ratios are as follows:—

A. Primary voltage, 100 V; secondary current, 230 A; power, 600 W.

B. Primary voltage, 80 V; secondary current, 230 A; power, 1 100 W.

The transformers are run in parallel on the same primary and secondary bus-bars, and carry a total load (input) of 100 kW at 0·8 power factor (lagging). Find the approximate primary load currents, power factors, and power distribution between the two transformers.

*[*A*, 17·9 A, 0·51, 30 kW; *B*, 22·4 A, 0·95, 70 kW.]

54 Two 1-phase transformers, *A* and *B*, are connected in parallel to supply a load having a resistance of 5 Ω and an inductive

reactance of 2 Ω. The equivalent resistances referred to the secondary windings are 0 25 Ω and 0 3 Ω, and the equivalent reactances 1·5 Ω and 2 Ω respectively. The open circuit secondary voltages are in the ratio 100 : 98. Calculate the inductive reactance which, when connected in series with the secondary of transformer B, will cause the magnitudes of the currents delivered by A and B to be in the ratio of 1·8 : 1. [0·52 Ω.]

55 Two 6 600/440-V, 3-phase transformers, A of 250 kVA and B of 500 kVA, have the following particulars per phase referred to the secondary side :—A : $R = 0{\cdot}008$, $X = 0{\cdot}035\ \Omega$; B : $R = 0{\cdot}003$, $X = 0{\cdot}019\ \Omega$. How will they share a load of 600 kVA at a power factor of 0 8 lagging ?

[A : 210 kVA at p.f. = 0 825 lagging ;
B : 393 kVA at p.f. = 0·785 lagging.]

56 Two transformers A and B are connected in parallel to a load $2 + j\ 1{\cdot}5\ \Omega$. Their impedances in secondary terms are $Z_A = 0{\cdot}15 + j\ 0\ 5$ and $Z_B = 0{\cdot}1 + j\ 0\ 6\ \Omega$. Their no load terminal voltages are $E_A = 207\underline{/0^\circ}$ and $E_B = 205\underline{/0^\circ}$. Find the power output, and power factor of each transformer.
[A : 6 57 kW at power factor 0 834 ; B : 5·0 kW at power factor 0·795.]

57 Two star/star transformers are connected in parallel on primary and secondary sides. A balanced 3-phase mesh-connected load of 3 Ω per phase and power factor 0 8 lagging is connected to the secondary side. The open circuit secondary line voltage of one transformer is 400 V, and of the other 403 V. Resistance and reactance per phase of each transformer are 0·015 and 0·06 Ω. Find the current supplied by each transformer and the voltage at the load terminals. [102 5 A ; 125 A ; 394 V.]

58 Two equal-ratio, parallel-connected, three-phase transformers deliver 1 000 kVA at 11 kV, power factor 0 8 lagging. Their impedances are $Z_1 = 1 + j\ 5$, $Z_2 = 1 + j\ 8$, respectively. Calculate (*a*) the current, (*b*) the phase displacement between voltage and current, in each transformer secondary.
[20 2 A, lagging 41·2° ; 32·5 A, lagging 34·1°.]

CHAPTER XXXVI

TRACTION

Dynamics

1 Deduce the relations between (*a*) miles per hour per second and feet per second per second; (*b*) tractive resistance in pounds per ton and the corresponding energy consumption in watt-hours per ton-mile; (*c*) tractive resistance in pounds per ton and coasting retardation in miles per hour per second; (*d*) percentage gradient and the corresponding energy consumption in watt-hours per ton-mile.

[(*a*) 1·47; (*b*) 1 lb per ton ≡ 2·0 Wh per ton-mile; (*c*) 1 lb per ton ≡ 0 01 mile per hr per sec; (*d*) 1% gradient ≡ 44·8 Wh per ton-mile.]

2 The limiting safe peripheral speed for a railway motor is 40 m per sec. To what speed does this limit the train for a gear ratio of 15/69, an armature diameter of 35 cm and a wheel diameter of 85 cm? [76 km per hr.]

3 A tramcar, with its two series motors in parallel, takes 40 A and attains a steady speed of 30 m.p.h. on the level. Find the speed when, on an up-gradient, three times the torque is needed with the motors connected in series. The trolley voltage is 500 V and the resistance of each motor is 0·5 Ω. Consider the cases in which the magnetic circuit is (*a*) highly saturated, (*b*) unsaturated. [(*a*) 18·5; (*b*) 8·2 m.p.h.]

4 The two motors of a 15-ton tramcar each develop 20 h.p. when the car ascends an incline of 8%. Find the speed of the car if the tractive resistance is 20 lb per ton and the gearing efficiency is 90%. [10·3 m.p.h.]

5 A 200-ton train with 10% rotational inertia effect is started on a 2·5% up-gradient with uniform acceleration and reaches a speed of 30 m.p.h. in 25 sec. Find the torque exerted by each of the 8 motors if the wheels are 36-in diameter, the gearing efficiency is 90%, the gear ratio is 3·5, and the tractive resistance is 12 lb per ton. [2 420 lb-ft.]

6 A 100-ton, motor-coach train has 4 motors each developing a shaft torque of 2 000 lb-ft during acceleration from rest. Calculate the time taken for the train to attain a speed of 18 m.p.h., starting from rest on a 1·2% up-gradient. Gear ratio 3·5; gear efficiency 94%; wheel diameter 40 in; train resistance 12 lb per ton; addition for rotational inertia 8%. [16·6 sec.]

7 Find the approximate weight and number of axles of a locomotive capable of accelerating a 300-ton train up a 0·5% gradient with an acceleration of 0·8 m.p.h. per sec. Tractive resistance, 10 lb per ton; coefficient of adhesion, 0·23; axle weight, not more than 20 tons. Allow 5% for rotational inertia. [80 tons, 4 axles.]

8 Assuming a simplified rectilinear speed/time curve with constant-speed coasting, find the maximum speed of a train which maintains, between stations 2 miles apart, a schedule speed of 25 m.p.h. with 30-sec stops. The train accelerates at 1·5 m.p.h. per sec, and brakes at 2 m.p.h. per sec. Estimate the average energy output of the motors in watt-hours per ton-mile, if the tractive resistance averages 10 lb per ton, and the addition for rotational inertia is 8%. [30 m.p.h.; 33 Wh per ton-mile.]

9 An electric train has a mean speed between start and stop of 28 m.p.h. The acceleration is 1·5 m.p.h. per sec and the braking retardation 2·5 m.p h. per sec. Assuming a rectilinear speed-time curve, a mean tractive resistance of 12 lb per ton, and an average motor efficiency of 70%, estimate the specific energy consumption on a run of 0·5 mile between stations. Addition for rotational inertia, 8%. [102 Wh per ton-mile.]

Motor Characteristics

10 A 12-ton tramcar is ascending at 5·5 m.p.h. a gradient equivalent to 5% when tractive resistance is included. The motors are in series on 500-V mains and take 44 A. The motors have each a resistance of 0·6 Ω. Calculate (*a*) the overall efficiency of the car; (*b*) the steady speed of the car on the same gradient with the motors in parallel and a 1-Ω resistance in series with them. [(*a*) 67%; (*b*) 9·5 m.p.h.]

11 Plot to a base of current the curves of tractive effort and speed from the data given below. Derive curves of tractive effort and speed for a field weakening obtained by cutting out one-third of the field turns.

Current, A	50	75	100	125	150	200	250
Train speed, m.p.h.	45 5	37	31·5	28	26 5	23 5	22 3
Tractive effort, lb	460	1 100	1 760	2 430	3 100	4 500	5 900

12 A 250-h.p. railway motor, when geared to 33-in wheels, gives the following characteristics:—

Current, A	80	160	240	320	400
Speed, m.p.h.	45	27·7	22·4	19·6	18·0

Draw this curve, and from it derive a curve showing the effect of 1 in of total radial wear on the driving wheels. Compare the heating of two such motors on the same train when one motor drives new and the other old wheels at a train speed of 25 m.p.h. *[1·27.]

13 The two motors of a motor-coach have the following characteristics for a gear ratio of 20/70 and 40-in wheels :—

Current, A .	25	50	75	100	125
Speed, m.p.h. .	42	27·2	23·4	21 2	20
Tractive effort, lb .	400	1 310	2 320	3 340	4 380

Plot the characteristics of these motors on a coach with 42-in wheels and a gear ratio of 20/80.

14 The following table gives the speed/current relation for a direct-current series railway motor :—

Current, A . .	60	70	80	90
Speed, rev per min .	560	520	490	465

The torque/current curve is rectilinear, passing through 5 A for zero torque. If the motor takes 75 A when developing a certain draw-bar pull with 35-in wheels, estimate the speed and current of the motor when developing the same pull with 37-in wheels, using the same gear ratio. [495 rev per min ; 79A.]

15 An electric train accelerates from rest up a uniform 1% gradient. There are 12 motors, the dead weight of the train is 200 tons, and the train resistance is 12 lb per ton. Taking a 10% allowance for additional rotational inertia, calculate the time taken for the train to attain a speed of 30 m.p.h. The following data refer to the performance of one motor.

Current, A	174	145	116	87
Tractive effort at wheel rim, lb	2 400	1 830	1 320	820
Train speed, m.p.h. . .	23	25	28	34

The initial acceleration takes place with a constant current of 174 A until all resistance is cut out. If the normal line voltage be 650 V, estimate the energy consumption for this run, series-parallel control being employed. *[38·0 sec ; 10·7 kWh.]

16 An 8-ton trolley-bus is driven by a 600-V series motor. Its maximum speed on the level against a tractive resistance of 40 lb per ton is 26 m.p.h. Calculate the current taken and the

useful power developed by the motor, assuming that the efficiency of the drive is 84%. Find also the speed of the vehicle and the efficiency of the drive when manœuvring on the level, the motor being supplied from a 72-V battery. Assume that the gross torque remains constant. The motor resistance is 0·65 Ω.
[22·2 h.p.; 32·9 A; 2·28 m.p.h.; 61·4%.]

17 A 600-V, long-shunt, compound traction motor takes an armature current of 40 A when driving a trolley-bus at 37 km per hr. The combined armature and series field resistance is 0·6 Ω. Use the magnetization curve given by Fig. 4, where the normal excitation corresponds to a shunt excitation giving 50% of the total, and a series field current of 120 A. Calculate the speed of the bus and the current taken when manœuvring on a 72-V battery and exerting the same total tractive effort.
[3·4 km/hr; 67 A.]

18 An 8-ton trolley-bus, driven by a 600-V series motor, running at a steady speed of 30 m.p.h. on the level, develops a tractive effort of 40 lb per ton, with an overall efficiency at that load of 80%. The motor resistance is 0·65 Ω and the magnetization curve is given by Fig. 4, where 100% on the current scale corresponds to 150 A. Assuming that the mechanical and iron losses are proportional to the speed, plot curves of steady speed, current and efficiency to a base of gradient, for values from 0 to 10%. Determine the time required to cover a distance of 2 miles on a gradient of 8%. [6·4 min.]

19 A 150-ton train with 10% rotational inertia is equipped with four 250-h.p., 750-V series motors, the characteristics of each of which are :—

Current, A	80	160	240	320	400
Speed, m.p.h.	45	27·7	22·4	19·6	18·0
Tractive effort at wheel rim, lb.	400	1 440	2 800	4 100	5 500

The train starts from rest on level track with a mean starting current of 400 A per motor, and continues to run under power up to a speed of 32 m.p.h., during which time the tractive resistance can be assumed constant at 10 lb per ton. The train coasts after reaching 32 m.p.h., during which the tractive resistance is 14 lb per ton. Braking is effected at 2 m.p.h. per sec. Find by any suitable method (*a*) the distance travelled by the train in 120 sec, (*b*) the average speed over this distance, (*c*) the ton-mileage, (*d*) the approximate energy consumption in watt-hours, (*e*) the specific consumption in watt-hours per ton-mile.

*[(*a*) 0·8 mile; (*b*) 24 m.p.h.; (*c*) 120 ton-miles;
(*d*) 9 600 Wh; (*e*) 80 Wh per ton-mile.]

20 A train is hauled by two 3-phase, 1 000-h.p. electric locomotives equipped with induction motors having a slip at rated load of 3%. The wheels of one locomotive are 50 in diameter, while the wheels of the other are worn down to 49 in diameter. How will the locomotives share the following total loads? (*a*) 2 000 h.p.; (*b*) 1 000 h.p.; (*c*) 100 h.p.
[(*a*) 1 330 h.p., 670 h.p.; (*b*) 830 h.p., 170 h.p.; (*c*) 386 h.p., − 286 h.p.]

Battery Vehicles

21 An electric battery-vehicle weighs 4 tons fully loaded and has a tractive resistance on the level of 65 lb per ton. Calculate the output of the battery when the vehicle is running (*a*) at 16 m.p.h. on the level; (*b*) up a 1 in 30 incline at 8 m.p.h. Take the motor efficiency as 80% in each case. Estimate the necessary capacity in watt-hours of the battery to enable a run of 25 miles to be made on level roads with a single charge.
[(*a*) 10·3 kW; (*b*) 11·1 kW. About 16 000 Wh.]

22 An 8-ton trolley is to be manœuvred by means of a 60-V auxiliary battery over a distance of 4 miles along a level road at a speed of 10 m.p.h. If the tractive resistance is 40 lb per ton and the efficiency of the motor and gearing is 65%, determine the ampere-hours taken from the battery and the watt-hours required per ton-mile. [65·5 Ah, 123 Wh per ton-mile.]

23 If a 60-V battery uses 48 Ah to propel a bus weighing 7·875 tons a distance of 4·25 miles at 4·1 m.p.h., estimate (*a*) the tractive effort in lb/ton, (*b*) the total Wh/ton-mile, (*c*) current and horse-power. The overall efficiency of gearing and motor is 0·7. [(*a*) 30·1; (*b*) 86·4; (*c*) 46·3 A, 2·6 h.p.]

Braking

24 A tramcar has two magnetic track brakes, each with a contact area of 2·5 in^2 per pole-face. Find the braking effect in tons if the flux per track brake is 0 27 megaline (2·7 mWb) and the coefficient of friction is 0·22. What rate of retardation in feet per second per second would this give to a 12-ton car?
[0·159 ton; 0·424 ft per sec^2.]

25 A 800-ton train is travelling at 60 m.p.h. If regenerative braking is employed to reduce the speed of the train to 45 m.p.h., calculate the electrical energy generated. Allow 10% for rotational inertia and take the efficiency of the conversion as 80%. [11·7 kWh.]

26 A 400-ton train has its speed reduced by regenerative braking from 60 to 30 m.p.h. in 6 000 ft on a down gradient of 1 in 100. Allow 10% for rotary inertia and 12 lb/ton for tractive resistance. Find the energy in kWh returned to the line, ignoring motor losses. [42·6 kWh.]

XXXVI. TRACTION

Starting Characteristics

27 A 800-ton, multiple-unit train has 12 motors, each developing a net accelerating tractive effort of 8 000 lb and taking a mean current of 350 A during notching-up to a speed of 25 m.p.h. Series-parallel control of the motors is used, the line voltage being 600 V. The resistance of each motor is 0·171 Ω. Calculate for each motor during the period of initial acceleration (*a*) the total energy input from the line; (*b*) the energy lost in the rheostats; (*c*) the copper loss per motor; (*d*) the output (including friction). Estimate also (*e*) the time occupied in accelerating on rheostats, and (*f*) the speed at which transition from series to parallel connexion occurs. Allow 10% for rotational inertia.

[(*a*) 1·06 kWh; (*b*) 0·31 kWh; (*c*) 0·136 kWh; (*d*) 0·61 kWh; (*e*) 28·4 sec; (*f*) 11·1 m.p.h.]

28 Two traction motors for series-parallel operation take 850 A during the starting period of 18 sec at uniform acceleration. Line voltage = 600 V. Resistance per motor = 0·1 Ω. Find (*a*) the time during which motors are in series, (*b*) the energy loss in the rheostats during starting.

[(*a*) 8·44 sec; (*b*) 0·496 kWh.]

CHAPTER XXXVII

LIGHTING

Lamp Characteristics

1 Define the units of luminous intensity and of luminous flux. Find the total flux of light emitted from a source having a uniform intensity of 200 c.p. [2 510 lumens.]

2 (i) A lamp taking 3·5 A at 100 V emits 6 000 lumens. Calculate its efficiency in (*a*) mean spherical candle-power per watt, (*b*) lumens per watt. (ii) A lamp taking 0·5 A at 250 V is rated at 125 mean horizontal candle-power. Find its efficiency in (*a*) mean horizontal candle-power per watt, (*b*) lumens per watt (spherical reduction factor = 0·8).

[(i) (*a*) 1·36, (*b*) 17·1 ; (ii) (*a*) 1 0, (*b*) 10·0.]

3 A metal-filament lamp has the following distribution of intensity in any vertical plane :—

Angle from vertical line below lamp .	0	15	30	45	60	75	90	105	120	135	150	165
Candle-power . .	10	15	39	53	66	73	74	69	60	45	20	0

Estimate its mean spherical candle-power and the spherical reduction factor. How many lumens are emitted from the lamp below its horizontal plane ? If the total useful luminous flux is 200 lumens, calculate the utilization factor.

*[50 c.p. ; 0·68 ; 333 ; 0·32.]

4 Derive an expression to show how the dimensions (diameter d and length l) of an incandescent filament of given material and worked at given efficiency depend on candle-power c.p. and current I. Find the relative diameters and lengths of two filaments a and b of the same material and run at the same efficiency for 25 c.p. at 115 V and for 32 c.p. at 230 V.

[$d \propto I^{\frac{2}{3}}$; $dl \propto$ c.p. ; $d_b / d_a = 0{\cdot}74$; $l_b / l_a = 1{\cdot}73$.]

5 A 110-V lamp develops 16 c.p., and a lamp of the same material and worked at the same efficiency develops 25 c.p. on 220 V. Compare the diameter and length of the filaments.

[d, l ; 0·85 d, 1·84 l.]

6 A 60-c.p., 250-V metal-filament lamp has a measured candle-power of 71·5 c.p. at 260 V and 50 c.p. at 240 V. (*a*) Find the constants for the lamp in the expression $C = aV^b$, where C = candle-power, and V = voltage. (*b*) Calculate the change of

candle-power per volt at 250 V. (*c*) Determine the percentage variation of candle-power due to a voltage variation of ± 4% from the normal value.

[(*a*) $a = 1{\cdot}24 \times 10^{-9}$; $b = 4{\cdot}46$; (*b*) 1·1 c.p./V ; (*c*) 19%, − 16·6%.]

7 A 60-W, 110-V lamp is in series with a 75-W, 110-V lamp across a 220-V supply. Assuming constant resistance, and candle-power proportional to the fourth power of the current, calculate the candle-power of each lamp as a percentage of the candle-power at 110 V. [153% ; 62·5%.]

8 A lamp gives 1 000 c.p. in every direction below the horizontal and no illumination above the horizontal. (*a*) Find the total radiation sent vertically downwards. (*b*) If the lamp is suspended in a diffusing sphere 6 ft in diameter with mat-white walls, the reflecting coefficient of which is 0·9, find the illumination of the sphere at a point vertically below the lamp.

[(*a*) 3 140 lumens. (*b*) 611 f.c.]

9 A 9-in diameter globe of dense opal glass which encloses a lamp emitting 1 000 lumens has a uniform brightness of 428 millilamberts when viewed in any direction. What is the candle-power of the globe in any direction, and what percentage of the light emitted by the lamp is absorbed by the globe ?

[55·4 c.p. ; 30·5%.]

10 Calculate the brightness of the following : (*a*) a pearl-type lamp giving 150 c.p. over a projected area of 20 cm^2, (*b*) a clear-type lamp giving 150 c.p. with an effective filament surface of 40 mm^2, (*c*) an 80-W fluorescent-type lamp giving 35 lumens/W over a projected area of 6 000 cm^2.

[(*a*) 7·5 c.p./cm^2 ; (*b*) 375 c.p./cm^2, (*c*) 0·47 lumens/cm^2.]

Illumination

11 Define the "lux," and evaluate the relation between the lux and the foot-candle. How far must a 25-c.p. lamp be away from a normally-placed screen in order that the illumination shall be (*a*) 5 lux, (*b*) 10 lux, (*c*) 4 f.c. ?

[1 f.c. = 10 76 lux ; (*a*) 2·24 m ; (*b*) 1·58 m ; (*c*) 2·5 ft.]

12 (*a*) Calculate the solid angle subtended by an area of 1 500 cm^2 on the surface of a sphere of diameter 1 m. (*b*) Find the total flux of light emitted by a lamp having a mean spherical candle-power of 35 (*c*) A lamp has an intensity of 60 c.p. in a given direction. Find the illumination in foot-candles on a normal surface distant (i) 3 ft and (ii) 8 ft from the source.

[(*a*) 0·6 sph.radian ; (*b*) 440 lumens ; (*c*) (i) 6·67 f.c. ; (ii) 0·94 f.c.]

13 (*a*) A lamp emits a total flux of light of 1 000 lumens. What is its mean spherical candle-power? (*b*) The mean spherical candle-power of a lamp is 150. Find its total flux of light. (*c*) A plane surface is placed 10 ft from a 200-c.p. uniform source of light. Calculate the intensity of illumination on the surface when (i) normal, (ii) inclined at 60°, (iii) parallel to the rays.

[(*a*) 79·6 c.p.; (*b*) 1 885 lumens; (*c*) (i) 2 f.c. (ii) 1·73 f.c., (iii) 0.]

14 A conical reflector, with a semi-vertical angle of 30° and a reflection factor of 70%, illuminates a circular floor space by reflecting all the luminous flux below the horizontal of a lamp with an average candle power in a downward direction of 100. (*a*) Find the average illumination on the floor when the lighted area is 10 ft in diameter. (*b*) Over what area will the lamp give an average illumination of 5 f.c.? (*c*) How high must the lamp be above the floor in (*b*)?

[(*a*) 5·6 f.c.; (*b*) 10·6 ft dia.; (*c*) 9·2 ft.]

15 The illumination of a disc 100 ft in diameter increases uniformly from 1 f.c. at the edge to 2 f.c. at the centre. Calculate (*a*) the total light falling on the disc, (*b*) the average illumination of the disc, (*c*) the average candle-power of the source issuing from a 120°-angled cone that just illuminates the disc.

[(*a*) 10 500 lumens; (*b*) 1·33 f.c.; (*c*) 3 333 c.p.]

16 A lamp has a uniform candle-power of 300 in all directions, and is provided with a reflector which directs 50% of the total emitted light uniformly on to a flat circular disc of 20 ft diameter placed 20 ft vertically below the lamp. Calculate the illumination (*a*) at the centre, and (*b*) at the edge of the surface, (i) with and (ii) without the reflector. (*c*) What is the average illumination over the disc without the reflector?

[(i) (*a*) 6; (*b*) 6; (ii) (*a*) 0·75; (*b*) 0·535; (*c*) 0 63 f.c.]

17 The polar curve of a lamp about its vertical axis is as follows:—

Candle-power . .	500	560	600	520	400	300	150	50
Angle to the vertical .	0°	10°	20°	30°	40°	50°	60°	70°

Plot a curve showing the illumination on a horizontal line below the lamp, the vertical distance between the line and the lamp being 20 ft. What is the illumination at a point 20 ft from the vertical through the lamp? [0·3 f.c.]

18 A 1 000-c.p. lamp is placed 15 ft normally from a mat-white square of 1 ft side having a reflecting power of 0·9. Calculate the apparent candle-power of the square viewed from a direction making an angle of 60° with the normal to its surface. [0·64 c.p.]

19 Balance is obtained in a photometric bench measurement when a standard lamp of 30 c.p. in the horizontal direction is 100 cm and the test lamp is 150 cm from the photometer head. What is the candle-power of the test lamp ? If the light from the test lamp is reduced by 25%, (i) how much nearer to the photometer head must it be placed ? (ii) How much nearer must the photometer head be moved towards the test lamp if the two lamps remain 250 cm apart ?

[67·5 c.p. ; (i) 20 cm ; (ii) 9 cm.]

Layouts

20 Two similar lamps fitted with reflectors have each the following distribution curve :—

Angle below horizontal axis	15	30	45	60	75	90
Candle-power . . .	50	125	240	190	155	140

The lamps are 12 ft apart and 6 ft above a bench. Find the illumination on the bench beneath one of the lamps and at a point 4 ft from one lamp and 8 ft from the other. Find the corresponding values for lamps without reflectors assuming uniform candlepower of 150. [4·1, 4·8, 4·5, 3·3 f.c.]

21 A lamp having a uniform luminous intensity of 200 c.p. is suspended 20 ft above street level. What will be the illumination on the ground (*a*) vertically beneath the lamp, and (*b*) 20 ft away from (*a*). Plot a curve of illumination between the limits given. [(*a*) 0·5 f.c. ; (*b*) 0·177 f.c.]

22 A road 40 ft wide is lighted by units giving uniform candle-power in all directions below the horizontal. The lamps are placed 20 ft above the sides of the road at intervals of 160 ft. Find the minimum illumination in the middle of the roadway in the following two cases : (*a*) when the lamps are opposite each other and the road illumination midway between opposite lamps is 0·75 f.c. (*b*) When the same lighting units are uniformly staggered. [(*a*) 0·055 f.c. ; (*b*) 0·152 f.c.]

23 A minimum illumination of 12 f.c. is required over a circular area of diameter 25 ft, and is to be obtained by either (*a*) a large number of small lamps, each having an efficiency of 10 lumens per W, spaced uniformly over the area to be illuminated, or (*b*) a single lamp of efficiency 20 lumens per W placed over the centre of the area and fitted with a reflector which restricts the light output to and gives a uniform candle-power over an angle of 80°. The height of the lamps is 15 ft and the coefficient of utilization is 0·4 in each case. Estimate the power supplied for each scheme. [(*a*) 1 470 W ; (*b*) 1 075 W.]

24 Estimate the number and disposition of 1 000 W floodlight projectors to illuminate the upper 250 ft of one face of a 320-ft high tower, of width 43 ft, if the approximate illumination to be provided is 8 f.c. The projectors are to be mounted at ground level 170 ft from the base of the tower. The divergence of the projector beam is 10°, the coefficient of utilization 0·2, and the efficiency of the lamps 18 lumens per watt. [24.]

25 An illumination on the working plane of 3 f.c. is required in a room 270 ft × 50 ft in size. The lamps are required to be hung 15 ft above the work benches. Assuming a suitable space ratio, a utilization factor of 0·5, a lamp efficiency of 14 lumens per watt, and a candle-power depreciation of 20%, estimate the number, rating and disposition of the lamps.

[36 lamps each of 200 W.]

Photoelectric Cells

26 A photo-cell has a sensitivity of 12 μA/lumen and operates with a load of 1·5 MΩ. The projected area of the cathode is 2 × 1·5 in. Find the output voltage when the cell is illuminated by (*a*) a 60-c.p. lamp at a distance of 6 ft, (*b*) a 6-c.p. lamp at 20 in, and (*c*) a 100-W lamp having an efficiency of 20 lumens/watt at 2 m.

[(*a*) 0·625 V ; (*b*) 0·81 V ; (*c*) 1·39 V.]

27 The light output of a 4-V lamp varies as the third power of the applied voltage. At nominal voltage the output is 30 lumens. The lamp illuminates a photo-cell having a cathode area of 2 in^2 and a sensitivity of 15 μA/lumen. Find the voltage developed across a 330-kΩ load resistor if the lamp runs at 3·5 V and is placed 12 in from the cell. [110 mV.]

CHAPTER XXXVIII

AUTOMATIC CONTROL SYSTEMS

1 A direct current generator with an armature resistance of 0·1 Ω supplies a full load current of 50 A at a terminal voltage of 100 V when the field current is 0·5A, and is derived from a battery. When the field is supplied from an amplifier, the input to which is the difference between the battery voltage and one-tenth of the generator output voltage, the generator voltage-regulation is improved 100 times. Find the gain of the amplifier in output amperes per input volt. Assume constant speed.

[4 71 A/V.]

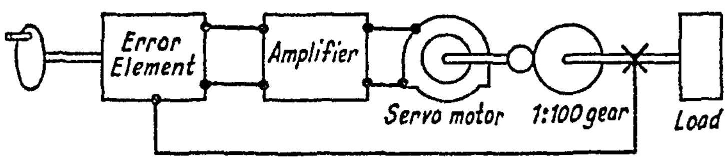
2 A critically-damped servomechanism has a maximum output speed of 120 rev per min. If the undamped natural frequency is 8 c/s, what is the largest possible error before the linear range of operation is exceeded ? [0·5 radn.]

3 A servomechanism has inertia $12{\cdot}5 \times 10^{-6}$ slug-ft^2, viscous friction 500×10^{-6} lb-ft per radn/s, and controller constant 0·005 lb-ft/radn error. Find an expression for the error angle as a function of time when the input begins to rotate at 3 rev per min. Find also the natural frequency of the system.

[$1{\cdot}8\,\{1 - (1 + 10t)e^{-20t}\}$ deg. ; 3·18 c/s]

4 A servo system used to position an aerial contains a motor of inertia 2×10^{-6} slug-ft^2, matched through a gear-box to the aerial which has an inertia of 0·02 slug-ft^2. The motor torque, proportional to the amplifier output voltage, is 0·01 lb-ft at 100 V, and the amplifier itself is supplied from an error element of sensitivity 1·5 V/deg error. The viscous friction measured at the motor shaft is 50×10^{-6} lb-ft per radn/s and the damping constant for the system is 0·25. If the wind exerts a static torque of 0·28 lb-ft on the aerial, what are (*a*) the static error in positioning the system, and (*b*) the motor voltage under these conditions ? [0·64° ; 28 V]

5 In the system shown, the friction torque is 68×10^{-6} Nw-m



per radn/s at the motor shaft ; the inertia is $1{\cdot}86 \times 10^{-6}$ kg-m^2 at the motor shaft ; the motor torque is 0·027 Nw-m per 100 V; and the error element produces 1 V/deg error. The damping is to be 0·25 of critical. Find the required voltage-gain of the amplifier, the natural damped frequency of the system, and the steady-state error for a constant input speed of 20 rev per min. [88; 97 radn/s; 0·6°.]

6 A servomechanism has a motor of inertia 4×10^{-6} slug-ft², and viscous friction 50×10^{-6} lb-ft per radn/s, excited from an amplifier and developing a torque of 0·025 lb-ft for 200 mA amplifier output current. The motor drives the output shaft (of negligible inertia) through a 100/1 gear-box, and the amplifier input is derived from a differential developing 0·5 V/deg error between input and output shafts. For a damping factor of 5/16, determine (*a*) the natural frequency of the system, (*b*) the amplifier gain in mA/V, and (*c*) the steady-state error for a constant input speed of 10 rev per min. If a load is connected to the output shaft which reduces the damping factor to 1/4, what will be (*d*) the inertia of this load, (*e*) the new natural frequency, and (*f*) the steady-state error of the system?
[(*a*) 3·18 c/s; (*b*) 44·8 mA/V; (*c*) 1·87°; (*d*) 0·0225 slug-ft²; (*e*) 2·54 c/s; (*f*) 1·87°.]

7 A viscous-damped servo system with damping 200×10^{-6} lb-ft per radn/s is driven by a 2-phase motor with a torque in lb-ft given by $(0{\cdot}5\ V - 0{\cdot}02\ N)10^{-3}$, where V is the applied voltage and N is the speed in rev per min. If the motor drives through a reduction gear of ratio 100/1 and the follow-up synchro produces 1 V/deg error between input and output shafts, find the amplifier gain required to give a steady-state error of 8° when the input rotates at 10 rev per min. [27·8.]

8 A servomechanism having an inertia of 20×10^{-6} slug-ft², an external viscous friction of 100×10^{-6} lb-ft per radn/s, and a damping ratio of 0·3, is required to have a steady-state error of 0·5° at an input speed of 10 rev per min. Find (*a*) the required controller gain, and (*b*) the proportion of error-rate damping to secure this result.
[(*a*) 0·012 lb-ft/radn; (*b*) 194×10^{-6} lb-ft per radn/s.

) A servomechanism is defined by the relation $p^2\theta_o + 10p\theta_o = 100\epsilon$, where $\epsilon = (\theta_i - \theta_o)$ is the error between the input and output positions θ_i and θ_o. Plot to a base of input frequency the ratio θ_o/θ_i in magnitude and phase. Calculate the maximum value of this ratio and the frequency at which it occurs.
[$1{\cdot}15\underline{/-55°}$; 1·13 c/s.]

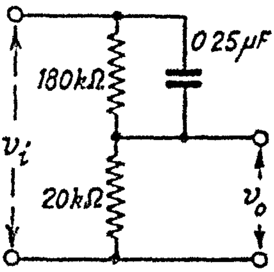
) A simple RC integral-control network is introduced into a servomechanism having a natural frequency of 5 c/s and damping factor 0·26, in order to reduce the steady-state error to 0·2°

for an input speed of 20 rev per min. If the series resistor of the network is 1 MΩ, what is the necessary shunt resistor, and the maximum value of the integrating capacitor, if the phase is not to be retarded by more than 5° at the resonance frequency of the system? [111 kΩ; 2·94 μF.]

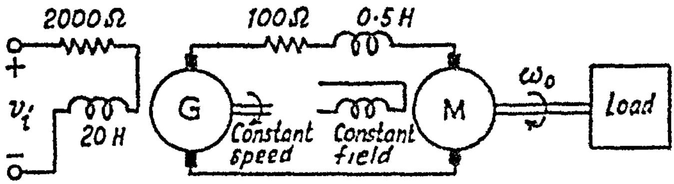
11 A servomechanism has a constant-amplitude variable-frequency sine input. The maximum output amplitude is 2·5 times the input amplitude and occurs at an input frequency of 8 c/s. Calculate (*a*) the damping ratio and (*b*) the natural frequency of the system. [(*a*) 0·205; (*b*) 8·4 c/s.]

12 The network shown is used to modify the error-signal voltage v_i of a servomechanism to a value v_o. Find the transfer function v_o/v_i neglecting any load on the output terminals. Evaluate the function (*a*) for a sine signal voltage of 1·0 V at angular frequency $\omega = 20$ radn/s, and (*b*) for a step-function input signal voltage of 1 0 V.

[(*a*) 0·134/37° V; (*b*) $0\ 1(1 + 9e^{-222t})$ V.]



13 For the small speed-control system shown, (*a*) find an expression relating the instantaneous angular velocity ω_o of the load, in radians per sec, to the input voltage v_i, neglecting friction.



(*b*) Show in a block diagram a possible method of modifying the system into a closed-loop mechanism. The required data are: generator e.m.f., 1 500 V per field A; motor e.m.f., 1·0 V per radn/s; motor torque, 0·32 Nw-m per armature A; total inertia of load and motor, $0{\cdot}44 \times 10^{-4}$ kg-m².

[$\omega_o = v_i\ 1{\cdot}09 \times 10^6/(p^3 + 300\,p^2 + 34\,000\,p + 1460\,000)$.]

14 A separately-excited direct current motor has an armature resistance of 20 Ω and negligible inductance; its inertia is 2×10^{-6} kg-m². A counter-e.m.f. of 100 V is generated for a speed of 100 rev per min, and the torque developed is 0·2 Nw-m per A. Find the equivalent electrical capacitance and time-constant of the motor. Neglect friction.

[1·05 μF; 21 μs.]